MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Mathematical Foundation and Numerical Methods for Transonic Flows

18.–24.4.1993

The conference was organized by Cathleen S. Morawetz (New York), Jindrich Nečas (Prag) and Wolfgang L. Wendland (Stuttgart).

The participants came from computational fluid mechanics and engineering, from numerical and applied analysis. The mathematical foundation of modelling and computational algorithms for transonic flows still remains one of the challenging frontiers with many open problems. The following topics have been discussed.

The simplest mathematical model for stationary transonic flows is the **full potential equation** for isentropic inviscid ideal gas flows. In fact, this is a limiting model which already admits change of type, discontinuous shock solutions and the necessity of an entropy selection criterion revealing already the main difficulties since existence and uniqueness are still open. Also the correct choice of function spaces is yet not clear. Nevertheless, several decisive special results have been achieved during the last years in mathematical theory as well as for computational algorithms concerning error analysis, reliability and efficiency. The improvement of reliable numerical methods was only possible as far as a new theoretical foundation became available. The following new theoretical results have been presented:

A global new existence result for the variational problem for subsonic flow enjoying the selection property by Glowinski and Nečas provides some new hope for the correctness of the model and, perhaps, the existence of transonic solutions (Gittel): The rigorous justification of the Glowinski–Nečas selection principle based on bifurcation analysis of an associated dynamical system (Keyfitz) and the clarification of shock reflexion (Morawetz).

Since now some compensated compactness results for the stationary variational transonic flow model are available, convergence results for the finite element approximation of the penalized variational formulation have been obtained including variational crimes (Berger and Feistauer) and for the coupling with a linearized far field model (Berger, Warnecke, Wendland). Moreover, the analysis and application of error and shock indicators and of error estimators has been developed (Warnecke).

Also, finite differences and improved approximation of shock conditions for the transonic full potential equation (Lifshitz) and subsonic potential methods with integral equations (Lifanov) belonged to this type of topics.

Because of the entropy production, instationary problems are more complicated. Here, many aspects of the **Euler equations** have been discussed. For the theoretical foundation, standard Riemann problems and their explicit solution in higher dimensions as the basis of efficient numerical methods have been considered (Klingenberg).

Finite volume methods with their various different aspects still pose many open problems. Since the functional analysis and appropriate topologies are rigorously justified only for scalar conservation laws, also the error analysis for corresponding methods is still in a preliminary state. For finite volume methods, a first rigorous error analysis was presented (Morton, Sũli) and for one scalar equation convergence has been shown (Krõner, Noelle, Rokyta). In close connection with rigorous analysis, there arose many activities in the creation of error and shock indicators and estimators for the rigorous justification of adaptive feedback algorithms (Felcman, Krõner, Morton). Model adaptive modifications of algorithms (Sonar) and a general theoretical foundation of feedback methods (Johnson) have also been presented. Since artificial and numerical viscosity, entropy production, dissipation and stability on one hand and the relation between computed results and the mathematical solution of the model equations on the other hand is still a largerly open question, the relation between numerical and physical viscosities in the framework of singular perturbation theory (Tobiska) and the validation of different numerical procedures in careful mutual comparison as well as in comparison with practical measurements was one of the central topics of interest (Kozel, Foȓt, Hãnel). Further problems at the frontier between mathematical theory and the design of computational schemes were non-reflecting artificial boundary conditions (Sofronov) and a modified boundary element method combining potential flow and vorticity transport (Iemma).

Also further developements of the **hodograph method** in higher dimensions concerning existence and uniqueness (Popivanov) and time continuous finite element Galerkin schemes for the Tricomi problem (Schneider) show the variety of mathematical analyses in this field.

The continuous discussions between practioners and theoreticians turned out to be particularly stimulating. Here, a comparison of the full potential model with Euler equations (Schippers), finite difference upwind schemes used for the wake–rotor interaction in helicopter flow computations (Wagner) and the various experiences with a multiple–field industrial computational fluid dynamics code (Rill) have been presented.

After lectures and in the breaks many vigorous discussions developed. Also on Thursday evening the whole group gathered for a round table discussion on open problems. We missed J. Nečas very much who was not able to come. Although this was a small conference, the discussions have been rather vivid and there developed a particularly cordial and stimulating atmosphere. The wonderful atmosphere was also due to the staff of the institute whom we all are thanking for the continuous and friendly care. I want to express the participants' gratitude to everybody who helped to make possible this stimuating unforgettable conference.

ABSTRACTS:

M. FEISTAUER :

Finite element variational crimes in the solution of transonic flow
The paper presents the results obtained in the cooperation of the author with H. Berger concerning the theory of finite element approximations of two–dimensional transonic potential flow in a general bounded domain. In the discretization, the basic finite element variational crimes are committed: the curved boundary is approximated by piecewise linear curves and numerical integration is used. The second law of thermodynamics is represented by a new improved version of the entropy condition which leeds to good numerical results. The discrete problem is reformulated with

the aid of least squares and entropy penalization. The convergence of approximate solutions to an exact one is investigated in detail.

J. FELCMAN:

Adaptive remeshing for Euler equations

The paper presents a numerical method for the simulation of 2-dimensional inviscid compressible fluid flow in a channel or in a region around a profile. A finite volume flux-vector splitting scheme of Godunov type is used to deal with the hyperbolic problem. Some first and second order methods on triangular, quadrilateral and dual meshes are investigated. The comparison of the numerical results using an adaptive strategy is presented.

J. FOŘT:

Numerical modelling of steady and unsteady inviscid transonic flow

A few problems described by the system of Euler equations are numerically solved by two finite volume methods based on cell centered and/or cell vertex schemes of Lax–Wendroff type. The steady transonic flow through a plane cascade is solved as the basic problem; the results are compared with experimental data. The model of flow through a cascade lying on an arbitrary rotationally symmetric surface of variable thickness (S_1) and its numerical solution is presented. The unsteady flow through a cascade with fixed inlet conditions and periodical time dependence of the outlet pressure (low frequences) were simulated by both methods. We deal with the influence of some parameters, e. g. the frequency of motions and the distance of the outlet boundary.

H.-P. GITTEL:

Local entropy conditions in transonic flows

The transonic expansion procedure for irrotational steady two-dimensional flows of an inviscid gas past a slim profile leads to the basic transonic system:

$$\begin{aligned} u_y - v_x &= \mathrm{rot}(u,v) = 0\,, \\ uu_x + v_y &= \mathrm{div}(\frac{1}{2}u^2, v) = 0\,. \end{aligned}$$

The variational formulation of this problem in a rectangle $\Omega = [-a,a] \times [0,b]$ with appropriate boundary conditions for (u,v) on $\partial\Omega = \Gamma_1 \cup \Gamma_2$ (Γ_1-horizontal part,

Γ_2–vertical part) reads as follows: Let

$$V = \left\{(p,q) \in L^2(\Omega, \mathbb{R}^2) \middle| \ \mathrm{rot}(p,q) = 0 \ \text{in distributional sense}, \ q = 0 \ \text{in} \ H^{-\frac{1}{2}}(\Gamma_2)\right\}.$$

$$\text{Determine } (u,v) \in V : \iint_\Omega \left((\frac{1}{2}u^2, v) - (u_0, v_0) \right) (p,q) dx dy = 0 \ \forall (p,q) \in V ,$$

where $(u_0, v_0) \in V$ is given. The last relation ist the EULER–LAGRANGE–equation for the variational problem:

$$J(u,v) := \iint_\Omega \left(\frac{1}{6}u^3 + \frac{1}{2}v^2 - (u_0 u + v_0 v) \right) dx dy \rightarrow \min_{(u,v)\in V} .$$

We minimize $J(u,v)$ over a closed bounded set $K \subset V$ which is defined by bounds for $u^- = \min\{0, u\}$ and by the local entropy conditon: $(u^-)_x \leq M$ in distributional sense (with a constant $M \geq 0$). The functional J is not convex on K and K is not compact in V. Using compensated compactness methods we are able to prove the existence of a minimum of J on K.

D. HÄNEL:

Viscous, transonic flows

A critical consideration is presented about numerical predictions of viscous, transonic flows around airfoils. The numerical simulaton is based on the Navier–Stokes equations, solved by finite–volume methods with different upwind or central discretizations. The first part of the paper deals with the difficulties arising by the numerical simulation of wind tunnel experiments. Several examples demonstrate the strong sensitivity of the flow with respect to small disturbances, the problems in describing the free or forced transition to turbulence, and the uncertaincies near maximum lift. In the second part of the lecture, numerical problems are discussed, which refer to the influence of the numerical approximations of the solution of the Navier–Stokes equations.

Several examples are presented, which demonstrate the significant influence of different elements of a method, as they are e. g. flux limiters, entropy corrections or far field boundary conditions.

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U. IEMMA:

Boundary integral equations for transonic flows
A boundary integral equation approach for the solution of transonic potential flows is presented. The formulation is valid for unsteady, three–dimensional flows. The nonlinear full–potential equation is derived from the governing equations (conservation of mass, momentum and energy) using both, the conservative and the non–conservative forms of the continuity equation. An artificial dissipitation term is introduced into the hyperbolic region of the domain of integration. Several forms for this artificial viscosity have been tested and validated by means of the comparison with existing numerical results obtained in *field methods* (i. e. finite–difference, finite–volume, finite–element methods).
The extension of the formulation to rotational inviscid flows (Euler equations), currently under investigation, will also be outlined. This consists of decomposing the velocity field into an irrotational field (described as usual by a scalar potential) and a rotational field confined with the *shadows* of the shock wave.

C. JOHNSON:

Numerics and hydrodynamic stability: Towards error control in CFD
We initially review the available error analysis in Computational Fluid Dynamics and come to the conclusion that the existing error estimates are meaningless in most cases of interest. We propose a new approach to error analysis in CFD aiming at reliable and efficient adaptive quantitative error control. This is based on a precise analysis of hydrodynamic stability coupled with Galerkin orthogonality. We prove a–priori and a–posteriori type error estimates in a model case for pipe flow, formulate corresponding adaptive algorithms, and discuss the potential of this approach for adaptive error control in CFD.

B. KEYFITZ:

Shocks near the sonic line in steady and unsteady flow
We analyze stability of shocks between a state where flow is hyperbolic (supersonic) and one where the flow is elliptic (subsonic) by using bifurcation theory and dynamical systems. The key observation is that the structure of the wave cone is opposite to the two cases. In steady transonic flow, the forward wave cone widens at the sonic line to become a half–space; in unsteady flow which changes type it degenerates to a singular line. One consequence of this is that the viscous profile

admissibility criterion for a steady flow can be described by a one–dimensional vectorfield for all states near the sonic line; the upstream state is always hyperbolic and the lower–speed (higher density) state is always downstream. On the other hand, the self–similar travelling wave solutions for viscous pertubations of unsteady flow require a two–dimensional vectorfield unfolding, equivalent to Takens–Bogdanov; the downstream state is always hyperbolic; the upstream state may be elliptic, but not all elliptic states lead to admissible shocks.

Ch. KLINGENBERG:

On two–dimensional selfsimilar gas dynamics
In modelling compressible, inviscid two–dimensional gases through the two–dimensional Euler equation, we address questions of 1.) uniqueness and 2.) existence.

1. We consider oblique shock reflection say of a planar shock impinging onto a wedge. Through shock polar analysis one can depict regimes of the parameters describing the incident shock which allows for two solutions, the so–called regular and Mach reflection. The author attempts to unify the many criteria in the literature selecting one of these solutions as follows. Find a relationship between the given data of the problem and an angle α, which decribes the first possibility when the Mach stem may turn the flow by this angle α.

2. Next, special solutions of two–dimensional Riemann problems are presented. Here, the initial datum is constant in wedges meeting at the origin. We give a complete list of elementary waves, the purported building blocks of the two–dimensional Riemann solutions. Finally, we show solutions of small perturbations of these solutions, by finding solutions to the selfsimilar, small disturbance equations.

K. KOZEL:

Numerical solution of inviscid and viscous transonic flow
The work deals with the numerical solution of several steady and unsteady problems of outer and inner aerodynamics. The mathematical models are the Euler and the Navier–Stokes equations. Numerical results were computed by finite volume methods using:

α) C-grid, H-grid or triangular unstructured grids
β) MacCormack or Ni or multistage Runge–Kutta schemes and some TVD type schemes (MacCormack predictor-corrector, Lax–Wendroff on step and Hwang-Liu scheme)
γ) cell vertex form for the Ni scheme and cell centered forms for other schemes.

This work presents for inviscid flows:

α) the comparison of results achieved by older type schemes (now TVS) and TVD type schemes for channel flows;
β) the comparison of steady cascade flows of turbine type (three methods) with experimental results of the Institute of Thermodynamics of the Czechoslovak Academy of Sciences;
γ) the simulation of unsteady flows through a compressor cascade of DFVLR Kōln with $M_\infty = M_\infty(t)$ and $p_2 = p_2(t)$.

The work also presents numerical solutions of viscous laminar transonic flows for

α) unsteady transonic flows around NACA 0012 with $M_\infty = 0.85$; $Re = 10^4$;
β) steady transonic flows through a turbine cascade of the VKI Brussels.

D. KRŌNER:

Finite volume methods with local mesh alignment in 2–D
For the simulation of complex flow phenomena in 3–D it is necessary to use self-adapting-grid techniques and numerical schemes of higher order. The first demand implies to use unstructured grids and finite volume or finite element methods. Since it is very easy to include the upwinding discretization to the finite volume scheme we decided to use this one although it is not obvious how to define higher order schemes in this context. For the Cauchy problem of the nonlinear scalar conservation law we get the convergence of first and higher order finite volume schemes. In the proof we use the theory of measure valued solutions very extensively.
For more complicated problems like the nonlinear system of the Euler equations of gas dynamics we have done some numerical experiments for finite volume schemes with local mesh adaption. In particular, we have used a local mesh refinement algorithm for the fluctuation distribution scheme of Roe and Deconinck in case of the shock reflection problem. Furthermore, near the shocks the triangles are aligned

with the discontinuity and this gives a very good resolution of the shocks. This is a method which was used previously by Kornhuber for linear scalar conservation laws.

I.I. LIFANOV and I.K. LIFANOV:

Numerical solution of boundary singular integral equations and some problems in aerodynamics and diffractions of waves

It is known that a lot of problems from aerodynamics of ideal inviscid fluids and the diffraction of waves can be reduced to the solution of the exterior Neumann problem for the Laplace or the Helmholtz equation. These Neumann problems are reduced to strongly singular integral equations, by using the double layer potential on a piecewise smooth open and also on a closed curve in the planar case and surfaces in the spatial case, correspondigly. For these integral equations, numerical solution methods are presented which are analogous to the method of discrete vertices .

Y.B. LIFSHITZ:

Numerical simulation of viscous transonic flow over an airfoil

The lecture presents a concise itemize of a new approach to the viscous–invisid method for the computation of transonic flow over an airfoil with large Reynolds numbers. This method has been included in the AEROFOIL code library used for the computation of a lot of airfoil flows, which were obtained in wind tunnels earlier and are known now as a division of the Experimental Data Base for Computer Program Assessment (AGARD–AR–138). A comparative analysis of the flows over all the airfoils of the data base allows to conclude that our code and a wind tunnel simulate the unbouded airfoil flow to about the same extent. The results also point to the new circumstances in the problem of errors arising when the wind tunnel data are transferred to the unbounded transonic flow. The effect of test section blockage is extremely important in this case, and an estimation of its impact on the aerodynamic characteristics becomes a challenging problem.

C.S. MORAWETZ:

Reflection of a shock at a wedge: Potential theory

The reflection of a weak shock from a thin wedge is studied by solving for the lowest order term in the framework of a potential flow. The position of the shock and other properties are derived. It is shown that weak not strong reflection takes place in a certain regime of parameters but the Mach reflection is always possible. The main idea is to make use of the fact that the standard foliated shock polar shrinks to a

simple curve with discontinuous slope as the underlying flow becomes sonic; and the standard limit for weak shocks is not necessarily to be characteristic. These ideas come from Frankl's shock polar and the fact that the underlying flow is partly super and partly subsonic.
My other work is with the application of compensated compactness to transonic flow.

K.W. MORTON:

Finite volume approximation of steady transonic flow
The cell vertex finite volume method is a generalization of the classical box difference scheme. For the Euler equations it gives a very compact discretization of the conservation laws, which maintains its accuracy on distorted meshes and is well suited to the use of shock fitting or recovery techniques. In this lecture I will show how it has been developed for the modelling of transonic flows by both the Euler equations and the Navier–Stokes equations in two and three dimensions: I will show the need to introduce distribution matrices to combine cell residuals to four node–based residuals and the rôle that is played by artificial viscosity terms: The aim is to minimize the latter and I will describe various techniques that may be used to detect and model shocks and so eliminate the need for second order artificial viscosity. An outline will be given of the stages that will be necessary in the error analysis of the schemes, as well as an indication of the present state of progress.

N. POPIVANOV:

Overdeterminated problems for mixed type equations
Some three–dimensional BV–problem for mixed type equation is considered, which is similar to a plane problem, investigated previously by C. Morawetz. The three–dimensional problem was posed by M. Protter (1954), but still now there is no existence theory. As proved, the cokernel of the corresponding BVP in the hyperbolic part of the domain has an infinite dimension. The reason for that is the strong singularity, which appears in the solution of the original problem for an infinite number of smooth functions in the right–hand side. This singularity appears in the vertex of the characteristic cone and is not going along the cone.
To investigate this three–dimensional BVP, we consider a perturbation, in which we change the equation only in the hyperbolic part of the domain, and we add some nonlocal operator which connects each point in the domain with some corresponding point on the characteristic cone. The existence, uniqueness and infinite differentia-

bility with respect to one variable are proved. The connection between solutions of the local and nonlocal problems is established.

S. RILL:

Numerical flow simulation for complete transport aircraft — its impact on aerodynamic design

Due to considerable advances in CFG algorithms as well as supercomputing power and efficiency, it has become possible to solve the Euler and Navier–Stokes equations for complex geometries. At Deutsche Aeropace Airbus the algorithm MELINA, together with the grid generation system INGRID form the tool package for 3D inviscid and viscous compressible flow analysis. Airbus has gained considerable experience with optimization of aircraft components on the basis of 3D computational fluid dynamics in combinaton with dedicated wind tunnel experiments for validation of the predicted improvements. The validation experiments for the INGRID/MELINA package with wing / body / pylon / engine / flap track fairing geometries have shown that the reliability of CFD results is very high even for such complex geometries. This offers great chances for future developements of efficient, safe and environmentally clean airplanes.
The scope of the talk will be a concise itemize of the mesh generation system INGRID with its newest features and the flow solver MELINA, that has only recently been enhanced to treat viscous flows. The numerical results that will be presented are obtained with the solver running in the inviscid and viscous mode. We will discuss how the 3D simulation data are used in the aerodynamic design process.

M. ROKYTA:

Convergence of higher order finite volume schemes on unstructured grids for scalar conservation laws in 2D

The lecture presents results obtained by D. Kröner (Freiburg), S. Noelle (Bonn) and M. Rokyta (Prague). We consider higher order finite volume discretizations for hyperbolic conservation laws in several space dimensions; and for the specified class of schemes we prove the convergence to the uniquely determined admissible weak solution.
The main tool for proving the convergence is the concept of measure valued solutions, which was established by DiPerna. Using the properties of numerical flux we prove an L^∞–estimate of the approximate sequence. This estimate implies the weak–star convergence of the sequence to a Young measure. Then we prove that the Young

measure satisfies a set of entropy conditions. Finally, DiPerna's theory gives us that the Young measure reduces to the admissible weak solution in the sense of Kružkov. As an application, we consider the discontinuous Galerkin method. It turns out that the method can be viewed as an higher order finite volume scheme of the discussed class and, therefore, the convergence proof applies to the method.

H. SCHIPPERS:

Numerical modelling of transonic flow around the NLR7301 airfoil

In this paper numerical modelling aspects will be discussed associated with the calculation of inviscid transonic flow around the NLR7301 airfoil.

At the supercritical design conditons $M_\infty = 0.721$ and $\alpha = -0.194^o$, the flow is shock free, i. e. the mainly subsonic flow has a supersonic region without a terminating shock wave on its downstream side. This flow can be described both by full–potential theory and by the Euler equations. This testcase is being used extensively by the developers of computational methods to validate their codes, since the exact solution is known from hodograph theory. It appears to be a difficult testcase, because many codes are unable to produce shock free pressure distributions. The effect of numerical modelling errors as well as of the shock free solution are discussed.

The flow conditions $M_\infty = 0.7$ and $\alpha = 2.5^o$ lead to transonic flow with a strong shock. Here, the full–potential theory leads to results where the shock is located too far aft on the airfoil in comparison with results obtained from Euler equation methods. This is mainly due to the neglect of shock–generated entropy and vorticity. It will be shown that these phenomena can be exactly modeled in full–potential theory by introducing a Clebsch variable.

M. SCHNEIDER:

A finite element method and stabilization method for a nonlinear Tricomi problem

Using the Faedo–Galerkin method, we prove the existence of a generalized solution of an initial–boundary value problem for the nonlinear evolution equation

$$(*) \qquad L_E[u] := \mathcal{T}u + R_E(x,y,t)u|u|^\varrho + \frac{\partial}{\partial t}\ell(u) = F_E(x,y,t,u)\,,$$

$0 \le \varrho \le 2$, in a cylinder $Q_T = \Omega \times (0,T)$, where $\mathcal{T}u = yu_{xx} + u_{yy}$ is the Tricomi operator and $\ell(u)$ a special differential operator of first order. We then show that the approximate generalized solution of problem (*) converges to the approximate generalized solution of the corresponding stationary boundary value problem for

$$L_T[u]\mathcal{T}u + R_T(x,y)u|u|^{\varrho} = F_T(x,y,u)$$

as $t \to \infty$.

I.L. SOFRONOV:

Transparent boundary conditions in windtunnel for unsteady transonic flow problem

The unsteady flow problem in a windtunnel with circular cross–section is considered. The calculation domain with a streamlined body is bounded by front and back cross–sections. In order to obtain the transparent conditions on these boundaries, the unsteady Euler equations system considered outside the computational domain is linearized around the uniform flow. For this linear system, we investigated the intial boundary value problems in infinite hemicylinders on the left of the front cross–section (L–problem) and on the right of the back cross–section (R–problem). These initial–boundary value problems are solved in explicit form and these solutions are used for the exact transmission condition of uniform flow from the left infinity to the inlet boundary (solution of L–problem) and the condition of uniform flow perturbed generally speaking by a non–zero vorticity from the right infinity to the outlet boundary (solution of R–problem). The questions of incorporating such conditions into a difference scheme used inside the computational domain are discussed.

[1] I.L. Sofronov, Full Transparancy Conditions on a Sphere for the 3–D Wave Equation. Soviet Math. Dokl.**326**, No 6 (1992), pp. 453–457.

T. SONAR:

Recovery techniques and adaptivity in the computation of compressible fluid flow

A box method for the computation of inviscid, compressible flow fields is presented. To increase the order of accuracy of the scheme, polynomial recovery algorithms are used which allow the representation of the numerical solution as being piecewise polynomial but discontinuous at the box boundaries.

Two different strategies of recovery are discussed, leading to total variation diminishing and essentially non–oscillatory methods, respectively. Adaptive concepts are represented and the use of residual-based refinement indicators is outlined. Numerical experiments are shown for steady transonic as well as for unsteady supersonic flow fields.

E. SŪLI:

The analysis of finite volume methods for hyperbolic poblems

Over the last two decades, finite volume methods have enjoyed great popularity in the computational aerodynamics community and have been widely used for the numerical simulation of transonic flow problems. In spite of this, the stability and accuracy properties of these methods are not well understood. For first order hyperbolic equations in two dimensions we restate the cell vertex finite volume method as a finite element method. On structured meshes consisting of distorted quadrilaterals, the global error is shown to be of second order in various mesh-dependent norms, provided the quadrilaterals are close parallelograms in the sense that the distance between the midpoints of the diagonals is of the same order as the measure of the quadrilateral. On tensor product non-uniform meshes the cell vertex scheme coincides with the familiar box scheme. In this case, second-order accuracy is shown without any additional assumption on the regularity of the mesh, which explains the insensitivity of the cell vertex scheme to mesh stretching in the coordinate directions, observed in practice.

L. TOBISKA:

A note on the artificial viscosity of numerical schemes

In order to give a realistic evaluation of the quality of a numerical solution of a parameter-dependent boundary value problem, not only the influence of the meshsize but also that of parameters (which are unfortunately often considered as fixed) has to be taken into account. It can happen — in particular for singularly perturbed problems — that the numerical solution for a given parameter ε and a given meshsize h is much closer to a continuous solution for a different parameter $\varepsilon + \mu$, measured in some norm. The new approach for defining the artificial viscosity of discretization schemes given in this note is based on this observation. For the one-dimensional singularly perturbed two-point boundary value problems, the amount of the so defined artificial viscosity for several stabilized discretization methods is discussed. Numerical tests show that the artificial viscosity can be reduced by using special nonuniform meshes.

S. WAGNER:

Numerical procedures to accurately solve the Euler equations for rotary–wing–flow applications

Due to the ability of Euler methods to treat rotational, nonisentropic flows and also to correctly transport on the rotation embedded in the flow field, it is possible to correctly represent the inflow conditions on the blade in hovering flight of a helicopter, which are significantly influenced by the tip vortices.
A robust Euler method was developed to compute the transonic flow around a rotor blade using the Wake–Capturing method. One of the central points of this work was to prove that the radial change of the transport equation at the rotor blade had to be taken into account in contrast to the method used in the calculation of fixed wings, where the tansport velocity q over the examined control surface of a discrete control volume is assumed to be constant. This resulted in far–reaching consequences and completely new algorithms. Furthermore, it is shown that also the very complex starting procedure of a helicopter rotor can be described by the Euler method presented.
The algorithm based on the procedure is an upwind scheme, in which the difference formulation orientates to the actual, local flow state that is to say to the typical disturbance expansion direction. Hence, the artificial dissipation required for the numerical stability is included in a natural way adapted to the real flow state over the break–up error of the difference equation and has not to be included from outside. This makes the procedure robust. An implicit solution is used, where the inversion of the coefficient matrix is carried out by means of point–Gauß–Seidel relaxation.
After a short presentation of the basic equations and the numerical solution method the principal results of the method are shown.

G. WARNECKE:

Adaptive methods for transonic potential flows

Adaptive methods for finite element discretizations of transonic potential flow were discussed. The adaption was achieved by mesh refinement and a moving nodes procedure. The mesh refinement was driven by error estimators. These were given as a residual or, alternatively, by suitable seminorms of the discrete second order differences of the flow potential. The talk highlighted the relation of the difference estimator to distributional derivatives, the asymptotic equivalence of the estimator and their relation to the error in terms of estimates. For solutions with shocks, the difference estimator measures the gradients of the velocity in the H^{-1}-norm. A moving nodes procedure near shocks was driven by a shock indicator. The method was demonstrated in numerical examples. The results are joint work with U. Göhner.

W.L. WENDLAND:

Analysis of a FEM/BEM coupling method for transonic flow computations

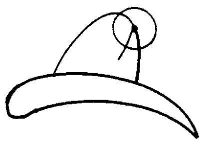
This is a joint paper by H. Berger, G. Warnecke and W.L. Wendland.
A sensitive issue in numerical calculations for exterior flow problems, e.g. around airfoils, is the treatment of the far field boundary conditions on a computational domain which is bounded. In this paper we investigate this problem for two-dimensional transonic potential flows with subsonic far field flow around airfoil profiles. We take the artificial far field boundary in the subsonic flow region. In the far field we approximate the subsonic potential flow by the Prandtl–Glauert linearization. The latter leads via the Green representation theorem to a boundary integral equation on the far field boundary. This defines a nonlocal boundary condition for the interior ring domain. Our approach leads naturally to a coupled finite element/boundary element method for numerical calculations. It is compared with local boundary conditions. The error analysis for the method is given and we prove convergence provided the solution to the analytic transonic flow problem around the profile exists.

PROBLEMS RAISED AT THE ROUND TABLE DISCUSSIONS:

1. The perturbation analysis of an almost sonic flow at the upper part of a profile by using a two-dimensional model and hodograph transformation (see also van Dyke: Circle Flows) is yet open (C. Morawetz).

2. The interaction of the shock and the sonic line in a transonic flow around a profile is yet not well understood. There is a problem with the bijectivity of the hodograph transformation (C. Morawetz).

   Where the interior characteristic meets the sonic point of shock generation in the hyperbolic subregion of a transonic flow, the first derivatives of the velocity admit a logarithmic singularity whereas the presentation in Guderley's book does not involve such a singularity (Lifshitz).



3. The following perturbation problem needs rigorous mathematical analysis including discussion of the corresponding hodograph transformations:
   Linearize the two-dimensional transonic potential flow about a smooth transonic Garabedian-Bauer shock-free profile by varying:

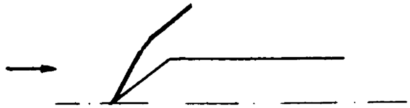
   (a) the angle of attack,

   (b) the travelling velocity M_∞,

   (c) the profile.

   This problem creates a singularly perturbed modified Tricomi equation in the hodograph plane where the bifurcation curve and corresponding estimates of a small shock need to be analized (Morawetz).

4. Consider the 2D or rotationally symmetric 3D inviscid flow modelled by the Euler equations for M_∞ near to M_∞^*.



Does the drag c_d admit an asymptotic behaviour

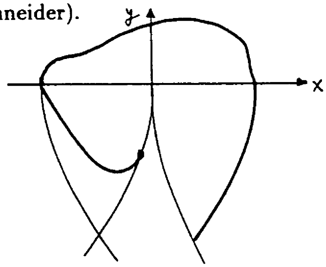
$$c_d = c_d(M_\infty^*) + A(M_\infty - M_\infty^*)^\alpha$$

This problem can been seen as a perturbation of self similar solutions (Lifshitz, Morawetz).

5. The uniqueness of the solution to the generalized Tricomi equation

$$k(y)u_{xx} + u_{yy} = f \quad \text{with} \quad yk(y) \geq 0$$

and mixed non-characteristic– characteristic boundary conditions is still open (Morawetz, Schneider).



6. For the perturbation of the transonic solution to the simplified transonic equation

$$\psi_x\psi_{xx} + \psi_{yy} = 0$$

one needs careful analysis of the corresponding bifurcation diagram. (Keyfitz)

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7. The existence of a variational solution u in the Sobolev space W_2^1 to the Neumann problem of the transonic full potential equation in a bounded domain satisfying weakly the selection condition $\Delta u \leq K$ and pointwise boundedness of speed $|\nabla u| \leq v_t < v^*$ is still open. Correspondingly, the question of existence of a finite element solution to the corresponding finite–dimensional nonlinear variational problem, which seems to be easier, is also still open (Feistauer, Wendland)

8. For two–dimensional supersonic flows around a profile, the Kutta–Joukovski condition at the trailing edge is still an unclear problem. If the flow is locally subsonic then for the full potential model local boundedness of the velocity is equivalent to the Kutta–Joukovski condition. For the Navier–Stokes and the Euler equations, viscosity including numerical viscosity produced by the truncation errror discards an additional condition at the trailing edge. Rigorous asymptotic analysis comparing the different models and including the supersonic case is needed (Feistauer, Wagner, Warnecke, Wendland).

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