MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 20/1993

Low dimensional dynamics

25.4. – 1.5.1993

Die Tagung fand unter der Leitung von G. Keller (Erlangen) und Z. Nitecki (Medford) statt. Thema der Tagung waren analytische, metrische und kombinatorische Aspekte der Dynamik 1- und 2-dimensionaler dynamischer Systeme. Im Mittelpunkt standen die allgemeine Strukturtheorie stetiger Intervallabbildungen und verschiedene Ansätze, eine Sharkovskiische Theorie periodischer Trajektorien auf kompakten 2-dimensionalen Mannigfaltigkeiten zu begründen. Die 12 Hauptvorträge wurden durch 15 kürzere Beiträge und 2 abendliche "problem sessions" ergänzt.
Unter den 42 Teilnehmern aus Europa, Nord- und Südamerika und Japan waren viele, die die beiden oben skizzierten Forschungsgebiete in den letzten Jahren entscheidend prägten und nun zum ersten Mal persönlich Kontakt hatten. Darüberhinaus war es für viele der erste Aufenthalt in Oberwolfach.
Zum erfolgreichen Verlauf der Tagung haben die großzügigen Möglichkeiten, die das Mathematische Forschungsinstitut sowohl für Vorträge als auch für Diskussionen im kleinen Kreis zur Verfügung stellt, und die organisatorische Kompetenz und Hilfsbereitschaft der Institutsmitarbeiter entscheidend beigetragen. Dafür herzlichen Dank im Namen aller Teilnehmer!

Vortragsauszüge

V. Baladi:

The spectrum of randomly perturbed piecewise expanding maps
(Joint work with Lai-Sang Young)

Let $f : [0,1] \to [0,1]$ be a continuous, piecewise C^2, piecewise expanding (i.e. there exists $\lambda > 1$ with $|f'_{|(x_i,x_{i+1})}| \geq \lambda > 1$ for $0 = x_0 < x_1 < \ldots < x_{N-1} < x_N = 1$ a finite number of turning points), topologically weak mixing map. Let $\mu_0 = \rho_0 dx$ be its unique absolutely continuous invariant probability measure ($\rho_0 \in BV$) and $\tau_0 < 1$ be the expontential rate of decay of correlations for observables in BV. We consider a small random perturbation given by a positive L^1 function ψ_ϵ with support in $[-\epsilon, \epsilon]$, i.e. we study the Markov chain with transition probability $p^\epsilon(x, E) = \int_E \psi_\epsilon(f(x) - y)dy$. If $\mu_\epsilon = \rho_\epsilon dx$ is an a.c.i.p.m. for the random perturbation and $\tau_\epsilon < 1$ its rate of decay for BV observables, we show the following robustness results: If none of the turning points are periodic, then $\rho_\epsilon - \rho_0$ tends to zero in the L^1 norm as $\epsilon \to 0$, and τ_ϵ tends to τ_0 as $\epsilon \to 0$ whenever $\tau_0 > \sqrt{\theta}$, where $\theta = \lim_{n\to\infty} \sqrt[n]{\sup_x \frac{1}{|(f^n)'(x)|}}$ is the essential spectral radius of the Perron-Frobenius operator L_0 of f acting on BV; in fact we show convergence of the spectrum of L_ϵ, the "perturbed" transfer operator, to that of L_0 in the annulus $\sqrt{\theta} < |z| \leq 1$. If periodic turning points are present, stronger assumptions are needed, in accordance with counterexamples of Keller and Blank.

Ch. Bandt:

The Cantor set as Mandelbrot set

The Mandelbrot set seems so complicated since it combines most different structures. If we forget all conformal and metric properties, we obtain the abstract Mandelbrot set defined by Douady and Thurston as a quotient space of the disk. This approach can be described also by symbolic dynamics, as was done by C. Penrose, and by K. Keller and myself.

A still more abstract approach is explained here. We consider a Julia set as a compact space with a 2-to-1-map, except for a single critical point. This comprises tree-like quadratic Julia sets and other examples. These spaces can be classified by the branching degree of the periodic points under the shift map.

M. Benedicks:

Non uniformly hyperbolic dynamical systems in the plane

The talk gave a survey of the recent development in the theory of dissipative dynamical systems in the plane

1) The existence of strange attractors for a positive Lebesgue measure set of parameters (a, b) for the Hénon family

$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} 1 + y - ax^2 \\ bx \end{pmatrix}$$

(Carleson & Benedicks; Ann. Math. 131 (1991)),

2) The existence of strange attractors for one-parameter families of two-dimensional maps near a homoclinic bifurcation
(Mora & Viana; Acta Math. to appear),

3) The ergodic theory in the preceeding situations: Existence of unique Sinai-Bowen-Ruelle measures
(L.S. Young & Benedicks; Inv. Math. to appear),

Also some remarks concerning the "critical set" in these situations, i.e. tangencies between the unstable and stable foliation were given (G. Ryd, Benedicks). This critical set is locally located on a C^α curve $\alpha < 1$.

Furthermore some open questions were discussed

(1) Can this theory be extended to the area preserving case, i.e. the "standard map" family?

(2) Are almost all points in the basin of attraction of the strange attractor of the Hénon map for the "good parameters" generic for the SBR-measure of 3)?

(3) A kneading theory exists for the "good parameters", but can there also be symbolic dynamics in that case?

Ch. Bernhardt:

On the existence of fundamental representatives of cyclic permutations in maps of the interval

(Joint work with E. Coven)

The ideas behind the proof of the following theorem were outlined.

Theorem: Suppose f is a piecewise weakly monotone map of an interval. If it has a representative of a cycle (not a double) then it has a fundamental representative.

Fundamental representatives are defined from the fundamental loop in the Markov graph and have the following properties: (i) They are periodic orbits of the correct cycle type, (ii) associated to each point is a side, and (iii) associated to each point and side is a given orientation.

It was shown how the existence of fundamental representatives lies behind a polynomial-time algorithm for deciding whether one cycle forces another.

L. Block:

Zero entroy permutations
(Joint work with A.M. Blokh and E.M. Coven)

Given a permutation π on $\{1,\ldots,n\}$, there is a unique piecewise linear map f of the interval $[1,n]$ to itself such that $f(k) = \pi(k)$ for $k = 1,\ldots n$ and f is linear on each interval $[k,k+1]$ for $k = 1,\ldots n-1$. We define the entropy of a permutation π to be the topological entropy of this map f. We give several different characterizations of zero entropy permutations as well as a procedure for constructing all of them. We also give some information about the number of zero entropy permutations.

P. Boyland:

On dynamics of annulus homeomorphisms
(Joint work with T. Hall and J. Guaschi)

Theorem: If $\phi : A \to A$ is pseudo-Anosov relative to $O(x,\phi)$ and $\rho(x,\phi) = \frac{p}{q}$ then $\frac{p}{q} \in$ Int $(\rho(\phi))$.

This implies an analogous theorem for general homeomorphisms of the annulus with periodic orbits of pseudo-Anosov type. The main lemma asserts that under the same hypothesis $\frac{m}{n} \in$ Int $\rho((\phi)) \iff T^{-m}\tilde{\phi}^n$ has a dense orbit in $\tilde{A} \iff$ the leaf attached to the lift of a $\frac{m}{n}$-periodic orbit is dense in $\tilde{A}$. Here $T : \tilde{A} \to \tilde{A}$ is the deck transformation of the universal cover $\tilde{A}$ of the annulus A.

M. Denker:

Ergodic theory of parabolic rational maps
(Joint work with J. Aaronson and M. Urbánski)

There are two general notions permitting the study of measure theoretic properties of parabolic rational maps on the Riemannian sphere. First, the conformal structure can be described by a measure m on the Julia-set J. Its conformal dimension equals the

Hausdorff-dimension. The construction principle is originally due to S.J. Patterson, here we give a different approach through dynamical quantities. Secondly, we introduce Markov fibred systems (MFS) with the Schweiger property (SP) (relative to this measure) which allows to describe the combinatorial structure of the map as well as the analytic properties of m.

The fundamental theorem for MFS with SP states: If the map T is "irreducible" and conservative, then

(a) T is ergodic

(b) $\exists \mu \sim m$ σ-finite, $\mu \circ T^{-1} = \mu$ and $\log \frac{d\mu}{dm} \in L^\infty(B)$ for every "good" B.

(c) Every "good" B is a Darling-Kac set with continued fraction mixing.

(d) If T is "aperiodic", then T is exact.

Applied to a parabolic rational map we derive e.g. the following: $\frac{1}{2} < h < 2$; if $h \geq 1$ then m is a multiple of the Hausdorff measure; a criterion when μ is finite and various new ergodic theorems.

R.L. Devaney:

The complex standard map
(Joint work with N. Fagella)

We study the complex standard family

$$z \rightarrow z + \alpha + \beta \sin z$$

where $\alpha \in \mathbb{C}$, $\beta \in \mathbb{R}^+$. When α is real, the unit circle is invariant, but this is not so for α non-real. We study a special limiting case where $\beta \rightarrow 0$ and the standard family is scaled to yield the family $z \rightarrow \lambda z e^z$. We show that

- there are infinitely many curves of λ-values for which the Julia set is the entire plane,
- there are infinitely many Mandelbrot sets in the λ-plane,
- there is a special core Mandelbrot set whose "arms" extend to ∞.

A.L. Fel'shtyn:

Dynamical zeta function, Nielsen theory and Reidemeister torsion
(Joint work with R. Hill)

We continue to study the Reidemeister and Nielsen zeta functions. We prove rationality and functional equations of the Reidemeister zeta function of an endomorphism of any finite group and of a self-map of a polyhedron with finite fundamental group. The same results are obtained for eventually commutative endomorphisms of groups, and for eventually commutative self-maps of compact polyhedra. We connect the Reidemeister zeta function of a group endomorphism with the Lefschetz zeta function of the Pontryagin dual endomorphism, and as a consequence obtain a connection of the Reidemeister zeta function with the Reidemeister torsion. We also obtain arithmetical congruences for the Reidemeister and Nielsen numbers similar to those found by Dold for the Lefschetz numbers.

J. Franks:

The rotation set for toral homeomorphisms

If $f : T^2 \to T^2$ is a homeomorphism homotopic to the identity and $F : \mathbb{R}^2 \to \mathbb{R}^2$ is a lift, we define $\rho(x, F) = \lim \frac{1}{N}(F^N(x) - x)$. Also define $R(F)$ = limit points of $\left\{\frac{F^{n_i}(x_i)-x_i}{n_i}\right\}$ where $n_i \to \infty$ and $x_i \in \mathbb{R}^2$. It is known that $R(F)$ is compact and convex and that if $\nu \in \text{int } R(F)$ is rational there is a periodic point z_0 for f such that for $z \in \Pi^{-1}(z_o)$ $\rho(X, F) = \nu$. (Here interior is with respect to $\mathbb{R}^2$). We show the same is true when $R(F)$ is a line segment containing at least two rational points and $\nu \in R(F)$ is rational.

J.-M. Gambaudo:

Boundary of chaos in the Hénon map

A $(\frac{l_n}{q_n})$ infinitely renormalizable homeomorphism f of the two-disk is an orientation preserving map such that there exists an infinite sequence of nested disks $\ldots D_n \subset D_{n-1} \subset D^2$ satisfying D_n, $f(D_n), \ldots, f^{q_n-1}(D_n)$ are disjoint and $f^{q_n}(D_n) \subset D_n$. (Here l_n is the number of loops D_n is doing around D_{n-1} before coming back into itself).

In this talk, we first discuss some Denjoy theory associated to this type of maps. More precisely, let $K = \bigcap_{n \geq o} \bigcup_{i=0}^{q_n-1} f^i(D_n)$. We give some sufficient condition on the smoothness of the map and the geometry of K so that $f_{|K}$ does not possess a wandering domain. Assuming now that the diameters of the $f^i(D_n)$ go to zero uniformly with n, a necessary condition for C^1 smoothness is that the sequence $\frac{l_n}{q_n}$ converges.

Finally, we show how these infinitely renormalizable maps occur at the boundary of positive topological entropy in the Hénon model.

T. Hall:

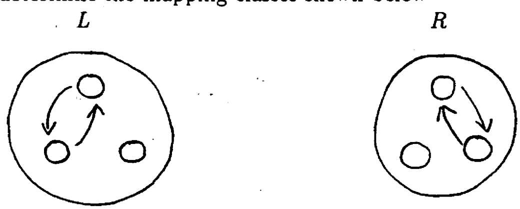
Period multiplying cascades for diffeomorphisms of the disc
(Joint work with J.-M. Gambaudo and J. Guaschi)

An orientation-preserving $C^{1+\epsilon}$ diffeomorphism of D^2 with positive topological entropy has zero-entropy cascades of periodic orbits of infinitely many distinct types. This generalizes a well-known result in one-dimensional dynamics.

M. Handel:

The pseudo-Anosov partial order as the mapping class group of the three times punctured disk.

Conjugacy classes for pseudo-Anosov elements of the mapping class of the three times punctured disk are in one-to-one correspondence with cyclic words in the letters L and R where L and R determine the mapping classes shown below



Theorem: A pseudo-Anosov conjugacy class determined by the word W_1 forces a pseudo-Anosov conjugacy class determined by the word W_2 if and only if W_2 is obtained, as a cyclic word, by removing letters from W_1.

S. Kolyada:

Topological dynamics of triangular maps of the square

Let I be a compact real interval and let $C_\Delta(I^2, I^2)$ be the set of all continuous maps of the square I^2 into itself, which are triangular, i.e. of the form:

$$F : (x,y) \longmapsto (f(x), g(x,y))$$

for any point $(x,y) \in I^2$.
We study the topological dynamics of triangular maps of the square, in particular the maps of the type 2^∞ and their topological entropy, and lower bounds for the topological entropy of transitive maps.

J. Llibre:

Periods for transversal maps via Lefschetz zeta function

We develop a modified Lefschetz number for analyzing if a given period belongs to the set of periods of a self-map. Essentially we work with the Lefschetz numbers for periodic points instead of the usual Lefschetz number for fixed points. If $L(f)$ denotes the usual Lefschetz number, the Lefschetz number of period m, $l(f^m)$, is defined as follows

$$l(f^m) = \sum_{d|m} \mu(d) L(f^{\frac{m}{d}})$$

where $\sum_{d|m}$ denotes the sum over all positive divisors d of m, and μ is the Möbius function. We present some theorems.

Theorem A: Let f be a transversal map. Suppose that $l(f^m) \neq 0$ for some $m \in \mathbb{N}$.

(a) If m is odd then m is a period of f.

(b) If m is even then either m, or $\frac{m}{2}$ is a period of f.

Theorem B: Let f be a transversal map of the n-sphere ($n \geq 1$) of degree D with $|D| > 1$. If m is a positive integer and $m \neq 2r$ with r odd, then f has a periodic point of period m.

Theorem C: Let M be a compact manifold such that $H_0(M;\mathbb{Q}) \approx \mathbb{Q}$, $H_1(M;\mathbb{Q}) \approx \mathbb{Q} \oplus Q$ and $H_q(M;\mathbb{Q}) \approx 0$ for $q \neq 0, 1$. Let $f : M \to M$ be a transversal map and let f_{*1} be the endomorphism induced on the first homological level. We denote by t and d the trace and the determinant of f_{*1}.

Assume that the eigenvalues of f_{*1} are real

(a) If $(t,d) \neq (1,0)$ then the set of periods of f, $\mathrm{Per}(f)$, is non-empty.

(b) If $t \neq 1$ then $1 \in \mathrm{Per}(f)$

(c) If $(t,d) \notin \{(\pm 1, 0), (\pm 2, 1), (0,d)$ with $d \leq 0\}$, then $\mathrm{Per}(f) \supset \{3,5,7,9,\ldots\}$.

(d) If $(t,d) \notin \{(\pm 1, 0), (\pm 2, 1), (0,0), (0,-1)\}$, then for any odd q at least one of each consecutive pairs of the sequence $q, 2q, 4q, 8q, \ldots$ is a period of f.

Theorem D: Let $f : T^n \to T^n$ be a continuous map of the n-torus, and suppose that f_{*1} has no roots of unity as eigenvalues. If $\ell(f^{pk}) \neq 0$ for every prime number p and every $k \in \mathbb{N}$, then $\mathrm{Per}(f) = \mathbb{N}$.

M. Lyubich:

Structure of quasi-quadratic maps

We study the dynamics of non-renormalizable S-unimodal maps with non-degenerate critical point. Our main result says that the scaling factors characterizing the geometry of the map exponentially decrease. It follows that the return maps near the critical point are becoming purely quadratic. This resolves the problem of non-linearity control at small scales. As the first applications of this result we prove that quasi-quadratic maps don't have "strange" attractors in the sense of Milnor, and that maps with the same combinatorics are quasi-symmetrically conjugate. The proofs strongly involve ideas from holomorphic dynamics and renormalization theory.

D. Mayer:

Dynamical zeta functions for Artin's billiard and the Venkov-Zograf factorization formula

In the thermodynamic formalism approach to Selberg's theory for Fuchsian groups the Selberg zeta function gets expressed in terms of Fredholm determinants of transfer operators for the geodesic flow on the corresponding surface. For the modular group $PSL(2,\mathbb{Z})$ the Selberg function can be written as $Z_S(s) = \det(1 - L_s)\det(1 + L_s)$ with L_s the generalized Frobenius-Perron operator for the continued fraction map $Tx = 1/x \bmod 1$. By using the reduction theory of indefinite binary quadratic forms I. Efrat proved recently, that the above factorization of the Selberg function corresponds exactly to the decomposition of the spectrum of the Laplacian on the modular surface into even and odd eigenfunctions with respect to the reflection $Jz = -z^*$ in the upper complex plane. By using the close connection between the geodesic flow on the modular surface and the billiard flow on Artin's billiard table we give a simple dynamical proof of Efrat's result. This approach shows at the same time, that the above factorization is a special case of the Artin-Venkov-Zograf formula relating the zeta function for a subgroup of a Fuchsian group to the ones with nontrivial representations for the latter one.

W. de Melo:

Sullivan's renormalization theory

We discuss Sullivan's proof of the contraction of the renormalization operator on the space of infinite renormalizable maps of bounded combinatorial type. We consider the space $\mathcal{U}$ of unimodal maps of the interval of the form $f = h \circ Q$ where Q is a quadratic polynomial and h a C^2 diffeomorphism. The domain of the operator is the open set $\mathcal{D} = \mathcal{D}_0 \cup \mathcal{D}_1 \subset \mathcal{U}$ characterized by the following inequalities of the first iterates of the interval point c_0 : $f \in \mathcal{D}_0 \Longleftrightarrow \overline{c_2\ c_0\ c_4}\ \overline{c_3\ c_5\ c_1}$ and $f \in \mathcal{D}_1 \Longleftrightarrow \overline{c_2\ c_5}\ \overline{c_3\ c_0\ c_6}\ \overline{c_4\ c_7\ c_1}$. The renormalization operator $\mathcal{R} : \mathcal{D} \to \mathcal{U}$ is defined by $f \in \mathcal{D}_0 \longmapsto$ rescaled $(f^2_{|[c_2,c_4]})$ and $f \in \mathcal{D}_1 \longmapsto$ rescaled $(f^3_{|[c_3,c_6]})$.
Theorem (Sullivan): There exists a compact invariant subset $\Lambda \subset \mathcal{D}_\infty = \{f \in \mathcal{D} : f^n \in \mathcal{D}\ \forall n\}$ such that

1. If $f \in \mathcal{D}_\infty$ then $\text{dist}(\mathcal{R}^n f, \Lambda) \to 0$ as $n \to \infty$.

2. The restriction of $\mathcal{R}$ to Λ is a homeomorphism conjugated to Smale's horse shoe.
3. The maps in Λ are real analytic and have holomorphic extensions which are quadratic-like in the sense of Douady-Hubbard.
4. For $f, g \in \mathcal{D}_\infty$, distance$(\mathcal{R}^n f, \mathcal{R}^n g) \to 0$ if and only if there exists $m \in \mathbb{N}$ such that $\mathcal{R}^m f$ and $\mathcal{R}^m g$ have the same combinatorial type.

M. Misiurewicz:

Minor cycles for interval maps

We consider minor cycles (i.e. periodic orbits) of continuous interval maps. "Minor" stands for "Minimal Non-Reducible". "Non-reducible" means that there is no non-trivial block structure. "Minimal" means that our cycle does not force any other non-reducible cycle of the same period.

In a similar theory for disk homeomorphisms, there is a problem of finding pseudo-Anosov cycles of a given period with minimal entropy. Here "pseudo-Anosov" means that the homeomorphism is isotopic to a pseudo-Anosov one relative to this cycle. The entropy of a cycle is the infimum of topological entropies of homeomorphisms exhibiting it. This motivates a similar question for cycles in interval maps. There the role of pseudo-Anosov cycles is played by non-reducible ones. Clearly, the non-reducible cycles with minimal entropy must be minor.

We classify all minor cycles of a given period and then find among them the ones with minimal entropy. It turns out that they are all unimodal. The forcing among them gives the ordering of periods: $4, 6, 3, 8, 10, 5, 12, 14, 7, \ldots$ If we look at the closest to $\frac{1}{2}$ fractions with a given denominator, we get the same ordering. This can be explained by A. Blokh's theory of rotation sets for interval maps.

H.E. Nusse:

The occurrence of Wada basins in simple dynamical systems and the fractal dimension and uncertainty dimension of basin boundaries

(Joint work with J.A. Yorke)

In dynamical systems examples are common in which two or more attractors (basins) coexist, and in such cases the basin boundary is nonempty. First, we consider diffeomorphisms of the plane to itself, for which there are at least 3 basins. A boundary point x of a basin in called a *Wada point* if every open neighborhood of x has a nonempty intersection with at least 3 basins. A basin B is called a *Wada basin* if every boundary point of B is a Wada point. Assuming each boundary point of basin B which is accessible from basin B is on the stable manifold of some hyperbolic periodic orbit, we have: The unstable manifold of each of the periodic points that is accessible from basin B intersects at least 3 basins if and only if B is a Wada basin. Examples of the Henon map and the forced damped pendulum illustrate the occurrence of Wada basins in simple systems (work in progress).

Now we consider one dimensional maps and two dimensional diffeomorphisms which are hyperbolic on the basin boundary. We present a result that the box-counting dimension, the Hausdorff dimension and the uncertainty dimension of the basin boundary, in any given region that intersects the basin boundary, are all equal. (H.E. Nusse and J.A. Yorke, *The equality of fractal dimension and uncertainty dimension for certain dynamical systems*, Comm. Math. Phys. 150 (1992), 1-21)

M. Pollicott:

Automatic groups and symbolic dynamics
(Joint work with R. Sharp)

Following work of Cannon, Thurston, Epstein et al, we know that the fundamental group of a compact negatively curved manifold is an "automatic group", i.e. the elements of $\pi_1(M)$ are in bijective correspondence with the paths in a directed graph (the "automaton") starting at a distinguished vertex, the elements of $\pi_1(M)$ being the concatenation of labels on edges from a finite symmetric set of generators (cf. Lectures of Ghys-de la Harpe)

Using the directed graph (with some added edges and vertices) we define a subshift of finite type.

To the group theory we add some geometry by considering "weightings" $\omega : \pi_1(M) \to \mathbb{R}$ (subject to some "Hölder" assumption). Examples of such weightings are: word length g; and displacement $d(gx, x)$ on M.

We associated to the weight function a Hölder function f on the subshift such that $f + f\sigma + \ldots + f\sigma^{n-1}$, $n \geq 1$, evaluated on appropriate points gives the weightings.

By applying well-known results for subshifts of finite type we can show the following limit exists:

$$A = \lim_{n \to +\infty} \sum_{|g|=n} \frac{d(gx, x)}{|g|} / \sum_{|g|=n} 1.$$

A.N. Sharkovsky:

Spatial self-stochasticity

If we have a family of dynamical systems given by maps $f_\alpha : X \to X$, $\alpha \in A$, and a map $h : Y \to A$ where X, Y, A are some topological spaces, then we obtain a dynamical system on the space $K(Y, X)$ of functions $\varphi : Y \to X$ given by the map $\varphi(y) \longmapsto f_{h(y)}(\varphi(y))$. We use a special metric which allows to measure closeness between deterministic functions and random ones in order to complete the space $K(Y, X)$ by random functions. We give some conditions under which the attractor of the dynamical system (in the completed space) contains random functions. Thus, here random functions appear as result of deterministic evolution of deterministic functions.

K. Simon:

Computation of the Hausdorff dimension of the basic sets of non-invertible maps

To compute the Hausdorff dimension of the attractor of some near-hyperbolic, non-invertible maps of the surface e.g. "Yakobson's twisted horseshoe map": $F(x,y) = (\varphi(y) + \lambda(x - \frac{1}{2}), g(y))$ where $||g(y) - 4y(1-y)||_{C^2} < \epsilon$, and slanting baker transformation (examined previously by Falconer)

$$T(x,y) = \begin{cases} (\lambda_1 x + \mu_1 y + c_1,\ 2y - 1) & \text{if } \ y \geq 0 \\ (\lambda_2 x + \mu_2 y + c_2,\ 2y + 1) & \text{if } \ y < 0 \ , \end{cases}$$

we set up some conditions such that:

a) the maps above and their small perturbations in the first component satisfy these conditions,

b) under these conditions the Hausdorff-dimension of the attractor can be computed by the pressure formula.

J. Smital:

Sequence topological entropy of chaotic maps

Recently in a joint paper with Franzova' (Proc.Amer.Math.Soc. 112 (1991), 1083-1086) we proved that a continuous map f of the interval is chaotic in the sense of Li and Yorke iff it has positive sequence topological entropy $h_T(f)$ with respect to a suitable sequence T of positive integers.
In the two-dimensional case as expected the situation is more complex. We will give some results concerning triangular maps of the square.

S. van Strien:

Fibonacci maps and invariant measures
(Joint work with H. Bruin)

In this talk we consider S-unimodal maps f of the interval and show that two conjugate maps with different orders of non-flatness at the critical point can have different metric behaviour: one map may have an absolutely continuous invariant probability measure while the other does not. In the first part of the talk a necessary condition for the existence of such a measure was given. Let c_{-i} be the point in the set $f^{-i}(c)$ to the right of and nearest to c. Let $S_0 = 1$ and define inductively S_i to be the smallest integer larger than S_{i-1} such that $c_{-S_i} \in (c, c_{-S_{i-1}})$. Let $A_i = [c_{S_i}, c_{S_{i-1}}]$. If f has an absolutely continuous invariant probability measure then

(*) $$\sum S_{i-1}|A_i| < \infty.$$

Next this result is applied to maps with the Fibonacci dynamics. This means that $S_1, S_2, \ldots$ is the sequence of Fibonacci integers, $1, 2, 3, 5, \ldots$. If in this case we let the order of non-flatness l at the critical point be 2, then Lyubich and Milnor have shown that $Df^i(c_1)$ grows sufficiently fast so that the condition of Nowicki and van Strien for the existence of such a measure is satisfied. Moreover Nowicki and Keller have shown that if the order l of the critical point with the dynamics of Fibonacci type is not too much larger than 2 then – even though this condition fails – these measures still exist. We show that for large l the situation is quite different: the summability condition (*) is false.

W. Szlenk:

A simplified model of the growth of Baleen Whale population
(Joint work with F. Bofill)

The international Whaling Commision used the following formula for studying evolution of the Baleen Whale Population (see J.D. Murray: Mathematical Biology, ch. 2, §25):

(1) $$x_{t+1} = (1-\mu)x_t + R(x_{t-\tau}), \quad t = 0, 1, 2, \ldots$$

where μ is the probability that an individual does not survive to the next moment, τ is the time in which a new born individual becomes sexually mature ($\tau = 7$ years), and the function $R(x)$ is of the form

$$R(x) = (1-\mu)^\tau \cdot \{p + a[1 - (\frac{x}{k})^r]\}$$

where p, a, r, k are some constants. To study the dynamics of the process (1) is a difficult problem. So we propose the following simplification: $\tau = 1$, and we replace $R(x)$ by a tent function: $\varphi_\lambda(x) = \lambda x$ for $0 \le x \le \frac{1}{2}$ and $\varphi_\lambda(x) = \lambda(1-x)$ for $\frac{1}{2} \le x \le 1$, and $\lambda \in [0, 2]$, $\mu = \frac{\lambda}{2}$. Then the corresponding process is equivalent to the following dynamical system: the phase space is $Q = [0,1] \times [0,1]$ and the map $T_\lambda : Q \to Q$ is given by the formula: $T_\lambda(x, y) = (x_1, y_1)$ where

$$\begin{aligned} x_1 &= y \\ y_1 &= \varphi_\lambda(x) + (1 - \tfrac{\lambda}{2})y. \end{aligned}$$

The dynamics of the system (Q, T_λ) varies from regular to random behaviour while λ runs over its domain $[0, 2]$.

The system has two fixed points, namely (0,0) (a saddle point) and $\bar{p} = (\frac{2}{3}, \frac{2}{3})$ (a focus; stable for $\lambda < 1$ and unstable for $\lambda > 1$).
Let $\Delta_\lambda = \bigcap_{n=0}^{\infty} T_\lambda^n(Q)/W^u(0,0)$.
Theorem 1: If $0 < \lambda < 1$ then for each point $q \neq (0,0)$ holds

$$\lim_{n\to\infty} T_\lambda^n q = \bar{p},$$

i.e. $\Delta_\lambda = \{\bar{p}\}$.
Theorem 2: Let $\lambda_0 \in [1,2]$ be a root of the polynome $5\lambda^3+6\lambda^2-20\lambda+8$ ($\lambda_0 \approx 1.10738\ldots$). Then

(i) if $\lambda_0 < \lambda < 5-\sqrt{13}$ then Δ_λ is a pentagon,

(ii) $\lambda = 5-\sqrt{13}$ then Δ_λ is a quadrilateral,

(iii) if $5-\sqrt{13} < \lambda < 2$ then Δ_λ is a hexagon.

Theorem 3: For $\sqrt[3]{4} < \lambda < 2$ the system $(\Delta_\lambda, T_\lambda)$ admits an absolutely continuous invariant measure μ_λ and the system $(\Delta_\lambda, T_\lambda, \mu_\lambda)$ is exact (which corresponds to the random behavior).

M. Tsujii:

A remark on Milnor-Thurston monotonicity theorem

We give a proof of Milnor-Thurston monotonicity theorem, which slightly improves the proof in Milnor and Thurston's paper. We first introduce a formula which relates the movement of an iterated image of the critical point and the derivative of the so-called Thurston map. Then, applying the (generalized) Schwarz-Pick lemma w.r.t. Kobayashi metric to the (complexified) Thurston map, we obtain the theorem. What we want to point out is that, since the Schwarz-Pick lemma holds for (hyperbolic) complex manifolds, we have large choice of the domain of Thurston map. In fact, we use this in the final part of the proof when we consider a perturbation of Thurston map, and avoid the usage of Teichmüller's uniqueness theorem, which may be a most difficult part of the proof of Milnor and Thurston. A better choice of the domain, if exist, would give a more simple proof and a generalization of the proof.

Berichterstatter: G. Keller

Problem list:
(Compiled by Z. Nitecki and G. Keller)

P. Boyland: Suppose $f : A \to A$ is an area- and orientation-preserving homeomorphism of the annulus. Define the *mean rotation number* of f as

$$\rho_m(f) = \int [\pi_1 \circ f(x) - \pi_1(x)]dx,$$

and denote by $\rho(f)$ the usual rotation set (= set of rotation numbers for all orbits of f).

Question: Can $\rho(f) = [0, r]$ with $r > 0$? The answer is probably "no" if f is pseudo-Anosov relative to a finite set.

M. Misiurewicz: Suppose $\mathcal{C}$ is a smooth, convex oriented curve in the plane, and consider the following diffeomorphism of $\mathcal{C}$ to itself (formulated by A. Blokh): pick a basepoint a interior to $\mathcal{C}$, and for each $x \in \mathcal{C}$ take L a line through a parallel to the tangent at x to $\mathcal{C}$; then $f_a(x)$ is the endpoint of the segment of $\mathcal{C}$ starting from x and ending at the first intersection with L. Now, as a varies in the interior of $\mathcal{C}$, the rotation numbers for f_a vary over an interval $(0, r_+(\mathcal{C})]$. Similarly, define $r_-(\mathcal{C})$ using the same curve with opposite orientation.

Question: Are $r_+(\mathcal{C})$ and $r_-(\mathcal{C})$ always equal?

A somewhat weaker question is whether $r_+(\mathcal{C}) \geq \frac{1}{3}$ iff $r_-(\mathcal{C}) \geq \frac{1}{3}$. This has the following geometric content: it is known that $r_+(\mathcal{C}) \geq \frac{1}{3}$ if and only if there exists a circumscribed triangle ABC, with points of tangency to $\mathcal{C}$ at $A' \in AB, B' \in BC, C' \in AC$ for which

$$\frac{AA'}{AB} + \frac{BB'}{BC} + \frac{CC'}{CA} = 1.$$

M. Misiurewicz: Suppose $f : I \to I$ is a continuous map of the interval satisfying

1. Every point of I (possibly with a finite number of exceptions) has at least two preimages under f;
2. f is transitive.

Question: Is the topological entropy of f at least equal to $\log 2$?

A. Blokh proved that for a transitive map of an interval, the entropy is at least $\frac{1}{2} \log 2$. This was generalized to n-ods and triangular maps of the square by L. Alseda, S. Kolyada, J. Llibre, and L. Snoha, who showed that for a transitive map of an n-od, entropy is at least $\frac{1}{n} \log 2$, and for a transitive triangular map of the square, it is at least $\frac{1}{2} \log 2$. They also showed that for a continuous transitive map of the square the entropy (which must be positive) can be arbitrarily small.

E. Coven (in absentia) Drop the second assumption, and strengthen the first to allow no exceptions.

Question: Same as above.

The second question was posed by Coven some time ago; with Nitecki, he showed that the lower bound $\frac{1}{2}\log 2$ always holds; also, there is a counterexample on the interval where all but one point have at least two preimages but the entropy is zero; this can be used (by M. Barge) to create a counterexample to the second question posed on the circle.

M. Misiurewicz: Let f_a denote the tent map with slope a:

$$f_a(x) = a|1 - x|.$$

Assume $\sqrt{2} \leq a \leq 2$; in particular, we know f_a is transitive on $[-1, 1]$. Misiurewicz and K. Brucks have shown that for (Lebesgue) almost all parameter values a, the trajectory of the turning point 0 is dense in $[-1, 1]$.

Conjecture: For almost every a, the turning point is *generic* for the absolutely continuous measure.

The following lemma arises in the proof of the result quoted above: let $\varphi_n(a) = (f_a)^n(0)$. Then there exists $\epsilon > 0$ such that for *almost all* a, and for all n sufficiently large, the lap (maximum interval of monotonicity) of φ_n containing a has image of length at least ϵ

Question: Does this hold for *all* a?

J. Franks: Consider a homeopmorphism f of the torus, isotopic to the identity, and F a lift of f to the plane. Then the *rotation vectors* of F are defined as for circle maps by

$$\rho(x, F) = \lim_{n\to\infty} \frac{F^n(x) - x}{n}.$$

Questions: Can the set of all rotation vectors for F be a line segment disjoint from the points with both coordinates rational? Equivalently, is there a homeomorphism without periodic points for which its lift has more than one rotation vector? A *stronger question* is: if f has no periodic points, does the rotation vector defined above exist for *every* point?

J.-M. Gambaudo: Let f be a C^∞ diffeomorphism of the plane, and K a compact invariant set. It is known that for every ergodic measure for $f|K$, almost every point has Lyapunov exponents.

Question: If $f|K$ is *uniquely* ergodic, does *every* point have a Lyapunov exponent?

M. Denker: For f a rational map of the Riemann sphere, expanding (the derivative is bounded below away from 1 on the Julia set) and m the maximal measure, it is known (J. Grigull) that the *Erdös-Renyi ergodic theorem* holds for Lyapunov exponents: by the usual (Birkhoff) ergodic theorem,

$$\frac{1}{n}\sum_{j=1}^{n} \log|f'| \circ f^j \longrightarrow \chi \text{ almost everywhere.}$$

However, it is useful to work somewhat independently of the point: the Erdős-Renyi theorem says that for all α sufficiently small there exist constants d_* and d^* such that (almost everywhere)

$$\lim_{n\to\infty} \max_{0\le i\le n-d^*\log n} \frac{1}{d^*\log n} \sum_{j=i}^{i+d^*\log n} \log(|f'|\circ f^j) = \chi + \alpha;$$

and similarly

$$\lim_{n\to\infty} \min_{0\le i\le n-d_*\log n} \frac{1}{d_*\log n} \sum_{j=i}^{i+d_*\log n} \log(|f'|\circ f^j) = \chi - \alpha.$$

Question: For which other dynamical systems does such a result hold?

In order to prove such a result, two problems need to be solved:

1. When does the free energy exist? For Gibbs measures m with potential ψ, one knows that the free energy

$$c(\phi) = \lim_{n\to\infty} \frac{1}{n} \log \int \exp[\sum_{k=0}^{n-1} \phi\circ f^k]dm = P(f,\psi+\phi) - P(f,\psi)$$

exists for every continuous function ϕ.

2. The system has good mixing – loosely speaking, the spectrum of the Perron-Frobenius operator is discrete and finite near the unit circle.

A.N. Sharkovsky: Consider the map f of the plane defined by

$$x' = (y-2)^2, \quad y' = xy .$$

This map arises in the study of the Schrődinger equation (Y.Avishai, D.Berend, MSRI-preprint #06008-91). It leaves the triangle Δ with corners $(0,0)$, $(0,4)$ and $(4,0)$ invariant. Denoting its sides by L_1 (on the y-axis), L_2 (on the x-axis) and Γ (joining $(4,0)$ and $(0,4)$) one sees that $f(L_1) = L_2$, $f(L_2) = \{(0,4)\}$ and $f(\Gamma) = \Gamma$. Also $f_{|\Gamma} : x \mapsto (x-2)^2$.

Questions: Has anyone investigated this map? Are the periodic orbits dense in Δ? Is $f_{|\Delta}$ transitive? Is Γ the attractor of $f_{|\Delta}$ in the sense of Milnor? Does there exist a point x such that $\omega(x)$ is unbounded but $\Gamma\cap\omega(x)\neq\emptyset$?

Simulations suggest that Γ is indeed the attractor of $f_{|\Delta}$ in the sense of Milnor. But the transversal Lyapunov exponent of f on Γ is

$$\frac{1}{2\pi}\int_0^4 \frac{\log(4-x)}{\sqrt{x(4-x)}}\,dx = 0 .$$

General question: Under what conditions does the average Lyapunov exponent define the type of a set?

J. Franks (comment on computer simulations): The map T on $[0,1]$ defined by

$$T(x) = \begin{cases} \frac{x}{1-x} & 0 \leq x \leq \frac{1}{2} \\ \frac{1-x}{x} & \frac{1}{2} \leq x \leq 1 \end{cases}$$

has the property that $T^n(x) = x$ for sufficiently large n if and only if x is rational.

J. Llibre: Let M be a compact C^1-manifold and $f : M \to M$ a C^1-map such that $f(M) \subseteq M$ and f is transversal, i.e. for all $n > 0$ the graph of f^n is transversal to $\{(y,y) : y \in M\}$ at all points (x,x) with $f^n(x) = x$.

Theorem: If $f : S^n \to S^n$ is transversal $(n \geq 1)$ with degree$(f) \neq -1, 0, 1$, then Per$(f) \supseteq \mathbb{N} \setminus \{2r : r \text{ odd}\}$.

In fact, the theorem is true if M is a manifold such that $H_k(M;\mathbb{Q}) \approx H_k(S^n;\mathbb{Q})$.

Question: Is there for some n a transversal $f : S^n \to S^n$ such that Per$(f) = \mathbb{N} \setminus \{2r : r \text{ odd}\}$?
Conjecture: No. (For $n = 1$ either Per$(f) = \mathbb{N}$ or Per$(f) = \mathbb{N} \setminus \{2\}$.)

Question: Are there a manifold M with the homology of the sphere and a transversal $f : M \to M$ such that Per$(f) = \mathbb{N} \setminus \{2r : r \text{ odd}\}$?
Conjecture: Yes.

G. Keller: For the map T on $[0,1]$ discussed before by J. Franks (or for other maps T with $|T'| \geq 1$ and with indifferent fixed points) consider the transfer operator

$$\mathcal{L}_\beta f(x) = \sum_{y \in T^{-1}x} \frac{f(y)}{|T'(y)|^\beta} \quad (\beta \geq 0)$$

acting on the space of functions of bounded variation on $[0,1]$. We know that the spectral radius $r(\mathcal{L}_\beta) \geq 1$ for all $\beta \geq 0$ with equality if $\beta \geq 1$. Furthermore, $\log r(\mathcal{L}_\beta)$ equals the pressure $P(T, -\beta \log |T'|)$. For maps with two surjective C^2 branches T. Prellberg showed in his Ph.D. thesis that

$$(1) \qquad \log r(\mathcal{L}_\beta) = \text{const} \cdot \frac{1-\beta}{-\log(1-\beta)} \cdot (1 + o(1))$$

for $\beta \nearrow 1$.

Questions: Is the result of Prellberg true for more general maps? How is the rate in (1) related to other dynamical properties of T, e.g. to the wandering rate for the σ-finite absolutely continuous invariant measure as discussed in related work of Aaronson and Thaler. *Note added in proof:* O.A. Lopes seems to have related results.

S. Kolyada: Let f be a unimodal map with $Sf = 0$ (i.e. piecewise linear fractional). There are three possibilities for the asymptotic behaviour of typical trajectories: (1) An attractive cycle, (2) a minimal Cantor set, (3) a cycle of transitive intervals. A Feigenbaum-like attractor cannot occur.

Question: Is it possible to have case (2)?

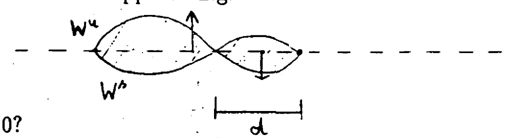
R. MacKay: Bifurcation of Denjoy minimal sets:
1. Let f be a continuous mapping of S^1 with degree 1 and nontrivial rotation interval $\rho(f) = [\rho_1, \rho_2]$, and let $\omega \in \operatorname{int}(\rho(f))$ be irrational. We know f posseses a rotationally-ordered Denjoy minimal subsystem Λ_ω of roation number ω (by cutting off at an apropriate level ).

Let $f_a = T_a \circ f$, $T_a(x) = x + a$. Let $a_c = \inf\{a : \omega \in \rho(f_a)\}$.

Question: How does Λ_ω approach the critical point as $a \to a_c$?

2. Let f be an area-preserving twist map of $\mathbb{T} \times \mathbb{R}$ with zero net flux, possessing a uniformly hyperbolic rotationally-ordered Denjoy minimal subsystem of rotation number ω. Let $f_a = T_a \circ f$, $T_a(x, y) = (x, y + a)$. Let $a_c = \inf\{a : \exists$ rotationally-ordered Denjoy minimal subsystem $\Lambda\}$.

Question: How does Λ disappear? E.g.



how does $d \to 0$?

Ch. Bandt: Denote by T the map $e^{ix} \mapsto e^{i2x}$ on the unit circle and let E_x be the diameter through the points e^{ix} and $e^{i(\pi+x)}$. E_x divides the unit circle into two halves, say L and R, R being the one which contains $T(e^{ix})$. A kneading sequence (with respect to E_x) is associated with the trajectory of x in the obvious way.

Question: Which kneading sequences occur?

N.B.: Not all R, L-sequences occur as kneading sequences. On the other hand the kneading sequences of unimodal maps are realized by those x whose trajectories avoid the arc from e^{ix} to $e^{i(\pi-x)}$ and its complex conjugate.

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