MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 23/1993

Mathematical Problems in Viscoelastic Flows

May 17 - 21, 1993

Die Tagung fand unter der Leitung von J.A. Nohel (UW-Madison & ETH-Zürich) und M. Renardy (Virginia Tech. - Blacksburg) statt.

Viscoelastic materials with fading memory exhibit behavior that is intermediate between the nonlinear hyperbolic response of purely elastic solids and the strongly diffusive, parabolic response of viscous fluids. A deep understanding of these effects is fundamental to advanced materials engineering and process design involving high-strength polymers, suspensions, and emulsions in production of polymers, additives to lubricants, rubber and plastics, paints, printing inks, magnetic tape coatings, etc; process design includes spinning of synthetic fibers and injection molding.

The key to successful mathematical modeling of viscoelastic flows is the use and construction of constitutive relations (in some situations replaced or supplemented by appropriate molecular theory) that reflect the relevant physics and that are computationally tractable. The complex behavior of steady and unsteady motions of these materials continues to pose challenging problems in modeling, analysis and computation. Applied and computational mathematicians and analysts, engineers and scientists need to develop qualitative and numerical techniques for understanding solutions to the nonlinear equations governing particular motions, as well as effective computational algorithms and codes that are vital for the comparison of performance of mathematical models with results of careful experiments.

The purpose of the Tagung was to address these issues by assembling an interdisciplinary group of 41 researchers, including younger applied mathematicians, who are active in or interested in various aspects of this fascinating field. The resulting 38 presentations are abstracted below under the following headings: (i) experimental methods; (ii) numerical and computational methods; (iii) stability of flows; (iv) modeling and analysis of particular viscoelastic flows; (v) modeling and analysis of motions of viscoelastic solids; (vi) modeling and analysis of thermoviscoelastic materials; (vii) control problems in viscoelasticity; (viii) related mathematical problems. In additions to numerous public and private discussions, a much appreciated highlight was a special session on open problems, four of which are abstracted below.

Abstracts (arranged by topics)

Experimental Methods

JOACHIM MEISSNER:

Experimental Methods in Polymer Rheology

Conventional methods in polymer rheology have made it possible to study such phenomena as oscillations and extrusion through capillaries. New experimental methods giving more insight into the viscoelastic nature of polymer melts are being developed. But also the increase in precision of conventional methods reveals important phenomena, e. g., the additional elasticity due to interfacial tension in melts of polymer blends, and opens the way to measuring the two normal stress differences in simple shear flow even at higher temperatures. By changing the shear direction in a parallel plate rheometer, the "shear-induced rheological anisotropy" can be studied. Another, rather new area is polymer melt elongation. It is even possible to change the principal axes of the strain rate tensor during such a test and thereby gaining an interesting perspective for checking the applicability of constitutive equations.

JOSEF HONERKAMP:

Determination of Relaxation Spectra for Viscoelastic Fluids

There are several phenomenological equations in rheology relating experimentally accessible quantities to distributions $h(\tau)$ of relaxation times which are useful, e.g., for characterizing a material. The estimation of these distributions from experimental data is an inverse (or deconvolution) problem. The naive approach, treating the problem by least squares, is not consistent; the uncertainties of the esimated values $\{h(\tau_\alpha), \alpha = 1, ..., M\}$ increase to infinity by increasing the number M of esimated values.

To deal with this issue, we have adapted regularization methods, known as Tikhonov or maximum entropy methods, to rheological problems; we also developed more robust versions and generalized these to the case when the relation between experimental quantity and distribution is nonlinear. A nonlinear relation appears if, e.g., one tries to estimate the logarithm of a relaxation spectrum rather then the spectrum itself.

We demonstrate that nonlinear regularization works better than the linear version when determining relaxation spectra from dynamic moduli.

Numerical and Computational Methods

JACQUES BARANGER and DOMINIQUE SANDRI:

Numerical Analysis of Differential Models for Viscoelastic Flows

For the Oldroyd B model in steady flow (for example), it is known that if the continuous problem has a sufficiently smooth and small solution, then the approximate solution obtained by FEM and upwinding on a triangular mesh exists; moreover, error bounds are available. This has been extended to quadrangular meshes and to a scheme for unsteady flow. Some remarks are also made on a decoupled scheme for models with one and two (or more) relaxation times, and on the White-Metzner model; many problems remain open.

HANS CHRISTIAN ÖTTINGER:

"Smart" Polymers in Finite-Element Calculations

Being able to calculate the flow behaviour of complex fluids from molecular models has been a long-standing dream of many engineers. The CONNFFESSIT method described in this talk provides a tool for performing such calculations in a very simple and direct manner. Moreover, this new method allows us to solve problems and to determine material properties inaccessible by conventional methods for calculating the flow of complex fluids. The key idea behind CONNFFESSIT is to use a stochastic simulation of the molecular dynamics in order to obtain the stresses in the fluid required for solving the momentum balance equation - rather than using some rheological constitutive equation as in the conventional approach. As an illustration of the method, the time development of plane Couette flow is studied for a variety of models of polymer solutions and melts (Oldroyd-B, FENE, Doi-Edwards). Several mathematical implications of the CONNFFESSIT approach are discussed in detail.

F. DEBAE, V. LEGAT and M. J. CROCHET:

Numerical Simulation of Extrusion of Viscoelastic Fluids in Two and Three Dimensions

During recent years, we implemented different mixed finite element methods for computing viscoelastic flows. We found that the most efficient methods are: "MIX 1" which uses bilinear interpolation for the stresses and the velocities and piecewise constant approximation for the pressure; "4 by 4" with 4 by 4 sublinear interpolation for the stresses, biquadratic for the velocities and bilinear for the pressure; "EVSS P1-P1-P2-P1", where we apply the EVSS method with bilinear interpolation for the tensors S and

D and the pressure, as well as biquadratic interpolation for the velocities; and finally, "EVSS P2-P2-P2-P1" in which biquadratic interpolation is used for the tensors S and D. We compare the performance of these methods for some typical flows of a Maxwell fluid (i.e., around a sphere falling in a tube, in a corrugated tube, in a 4 to 1 circular contraction, and in a circular die swell) using the following formulations: Galerkin, SU (streamline upwinding), and SUPG (streamline-upwinding-Petrov-Galerkin). We also simulate non-isothermal viscoelastic flows with multiple relaxation files in experimental 2-D geometries, and we show most recent progress in calculation of extrusion in 3-D, and in calculation of the prediction of the shape of the die in order to produce a given shape of the extrudate.

DIETMAR KRÕNER

Discretization of Convection Dominated Flows

Convection terms play a dominant role in many flow problems in computational fluid dynamics, including, e.g., in computing viscoelastic flows governed by the upper convected Maxwell model. In these problems, convection terms are usually discretized using the characteristic and other finite element methods; finite volume schemes can also be used. For a scalar conservation law, where only convection terms are present, the finite volume scheme can be justified by a direct convergence proof, also for a higher order scheme. For systems, one can still establish the expected order of convergence if the exact solution is known. We have used this technique to simulate flow problems with complex geometries in 3-D; mesh refinement and mesh alignment can also be done easily in this context.

Stability of Flows

MICHEAL RENARDY:

Linear Stability of Viscoelastic Flows

From a rigorous mathematical point of view, the stability of viscoelastic flows, other than the rest state, is largely an open problem. One of the principal issues is the relationship between linear stability and spectrum. Some first results in this direction are presented. In particular, the linear stability of plane Couette flow of an Oldroyal B fluid is discussed. An abstract result of a more general nature is also given. Unfortunately, applications of this result are limited to problems in one space dimension.

JEAN-CLAUDE SAUT:

Stability Issues in Viscoelastic Flows

We consider differential models for viscoelastic fluids (mainly of Jeffreys type) with a

"Newtonian" contribution to the extra stress. In this setting, we review known results concerning three notions of stability: linear stability, nonlinear stability, and spectral stability. One of the main issues (largely open) is the link netween linear stability and spectral stability.

YURIKO RENARDY:

Stability of the Interface in Two-Layer Couette Flow of Upper Convected Maxwell Liquids

The flow of two superposed liquids between parallel plates, the top plate moving, is discussed. A basic flow is $(U(z), 0)$ in each liquid, where $U(z)$ is identical to the Newtonian case. A linear stability analysis of this flow with a flat interface was studied by Y. Renardy (88) and Kangping Chen (91). A critical situation may occur for a single wave number α_c, with other wave numbers linearly stable. One may then calculate the Stuart-Landau constant to determine whether this bifurcation is supercritical. If so, one can continue the study of weakly nonlinear interactions to see if the traveling wave solution is stable to sideband perturbations. The method of analysis for the sideband problem is analogous to that for the two-layer Newtonian flow. The amplitude equations describe the evolution of the traveling wave and also that of the long-wave node (M. and Y. Renardy, Phys Fluids, '93).

Modeling and Analysis of Particular Viscoelastic Fluid Flows

JOHN HINCH:

Uncoiling of a Polymer Molecule by a Strong Flow

Computer simulations have been performed of an isolated random-coiled polymer unravelling when it is placed in a strong straining motion. The simulations have several hundred freely hinged links subjected to a Brownian motion and viscous forces, with some recent simulations including full hydrodynamic interactions. The stress is found to be mainly viscous rather than elastic in nature, i.e., proportional to the instantaneous strain-rate rather than being dependent on it. A rapid build up of this viscous stress with the total strain is shown to come from the growth of segments of a fully stretched chain. The evolution of these segments, the growth in their size along with the reduction in their number, is examined with a simplified 'kinks dynamics' model. The small elastic component in the stress, seen by suddenly switching off the imposed extensional motion, is found to be bounded. The above rheological behaviour in transient strong extensional flows is not described by the standard constitutive relations for dilute polymer solutions, such as the Oldroyd-B fluid and the FENE dumbbell models. A suitable modification is suggested which gives large strain-dependent viscous stresses.

A. RUSSELL DAVIES:

Corner Singularities in Viscoelastic Flows

Exact perturbation series solutions are presented for steady, incompressible corner flows of viscoelastic model fluids in the plane. The differential constitutive models are of the Johnson-Segalman-Oldroyd type, and both reentrant and non-reentrant sectors are considered. Singular perturbation series are required for reentrant corners, whereas regular perturbation series suffice for non-reentrant corners.

For reentrant corners, it is shown that a Stokesian velocity field gives rise to non-integrable normal stresses on the walls forming the corner, in the case of an Oldroyd-B fluid, unless separation takes place at the corner. A non-Stokesian velocity field together with viscometric stress behavior at the walls is given which satisfies the governing equations and no-slip boundary conditions. The normal stresses on the walls are infinite but integrable. For non-reentrant corners, the flow is Newtonian-like away from the walls.

DAVID S. MALKUS:

Shear-flow Instabilites in Viscoelastic Fluids

Intriguing instabilities have been observed in experiments involving shear flows of highly elastic and very viscous fluids. Many researchers have interpreted these observations as "slip" or "apparent slip", i.e. loss of adhesion of the fluid to the wall. This research, joint with J. Nohel and B. Plohr, offers the alternative explanation that such flow instabilities have a common origin in bulk material properties rather than in adhesive properties. One-dimensional shear-flows, pressure and piston-driven flow in a slit die, as well as Couette Flow are modeled mathematically as viscoelastic fluids with fading memory. The goal is to model "spurt" (a sudden increase in the volumetric flow rate) and related phenomena in pressure-driven flow, oscillations in piston-driven flow at a fixed volumetric flow rate, and stress relaxation under step-strain in Couette flow. The same basic system of time-dependent, quasi linear partial differential equations is used to model all three flows. The characteristic feature of the fluid model employed, stemming from the differential Johnson-Segalman-Oldroyd constitutive equations, is a non-monotonic relation between steady shear stress and strain rate. Analysis and numerical simulations, aspects of which are discussed in the presentations by Plohr and Pego abstracted below, show that in all three flows, the polymer system changes state in a thin layer near the wall giving the appearance of a slip layer. In Couette flow with a step shear strain applied by moving one wall while the other is held fixed, one observes an anomolous stress relaxation, accompanied by an inhomogeneous flow which starts some time after the moving wall has stopped. Calculated solutions are in good agreement with experiments.

BRADLEY J. PLOHR:

Persistent Oscillations in Piston-Driven Flows of Non-Newtonian Fluids

In their recent experiments on piston-driven flow at a fixed volumetric flow rate of a highly elastic and very viscous non-Newtonian fluid, F. Lim and W. Schowalter [J. Rheology 33 (1989), 1359] have observed pesistent, nearly periodic oscillations in the particle velocity at the channel wall. This periodicity has been characterized as a stick/slip phenomenon caused when the fluid fails to adhere to the wall.

The purpose of this work, in collaboration with D. Malkus and J. Nohel, is to offer an alternative explanation for these oscillations. The basis for our explanation is the differential Johnson-Segalman-Oldroyd constitutive law that exhibits a non-monotonic relation between steady shear stress and strain rate; the constitutive model being used to model piston-driven flow has also been used successfully to explain spurt [G. Vinogradov *et al*, J. Polymer Sci., Part A-2 10 (1972), 1061] and related phenomena in pressure-driven flow [D. Malkus, J. Nohel, and B. Plohr, SIAM J. Appl. Math. 51 (1991), 899 - 929].

Piston-driven flow is modeled as an instantaneous, globally well-posed feedback-control problem, the control being the volumetric flow rate and the feedback being the pressure gradient. In the inertialess approximation, the equations governing the flow can be viewed as a continuous family of quadratic ode's coupled by the non-local constraint that fixes the volumetric flow rate; all solutions of this system are bounded, even for large, discontinuous initial data. Numerical simulations demonstrate that beyond a critical flow rate, the time-asymptotic behavior is cyclic. Large shear strain rates are observed in a thin, but macroscopic, layer near the wall, just as in pressure-driven spurt flow.

Further analysis of the inertialess dynamical system links the observed oscillations to the apparent occurrence of a Hopf bifurcation that spawns a periodic orbit beyond a critical flow rate. To provide evidence for this, we observe that the dynamical system admits discontinuous steady states when the volumetric flow rate is fixed (corresponding to the wall stress between the steady local shear stress maximum and minimum), and we linearize the system around a discontinuous steady state with a single jump. Parametrizing such discontinuous steady states by wall stress and flow rate, there are regions in parameter space in which the eigenvalues of the discrete spectrum of the linearized operator change from having negative real parts to having positive real parts, and there is a separating curve along which the real part of the eigenvalue vanishes. Proving the existence of a stable periodic orbit beyond a critical flow rate is a challenging open problem.

JOHN A. NOHEL and ROBERT L. PEGO:

Nonlinear Stability and Asymptotic Behavior of Shearing Motions of a Non-Newtonian Fluid

The goal is to study the asymptotic behavior as $t \to \infty$ of solutions to an initial-boundary value problem in one space dimension governing pressure-driven shear flow of a highly elastic and very viscous non-Newtonian fluid in a channel, and to establish the nonlinear stability of discontinuous steady states. In this model, the total shear stress consists of a non-Newtonian part satisfying the Johnson-Segalman-Oldroyd differential constitutive law for the shear and normal stresses, a small Newtonian viscous contribution and a pressure term, and the flow is assumed to be symmetric about the centerline. The initial-boundary value problem is globally well-posed in time for initial data of arbitrary size, and permitting discontinuities in the initial velocity gradient and the stresses. The key feature of the model is that the total steady shear stress depends on the steady shear strain rate in a non-monotone fashion, resulting in steady states having discontinuous stress components and steady velocity profiles with discontinuities in the steady velocity gradient (strain rate). Such solutions are observed in shear flows described in the presentation by Malkus.

In a regime where the Reynolds number is small compared to Deborah number, we prove that every solution of the governing system tends to some, possibly discontinuous, steady state as $t \to \infty$; moreover, we show that discontinuous steady states that take their values on the increasing parts of the total steady shear stress *vs* strain rate curve are nonlinearly stable with respect to symmetric one-dimensional perturbations of initial data [J. Nohel and R. Pego, SIAM J. Math. Anal. - to appear]. These results provide a part of the analytic explanation of the spurt phenomenon described in the presentation by Malkus.

DANIEL ŠEVČOVIČ:

Explanation of Spurt via Geometric Singular Perturbation Theory

We study the model problem

$$\begin{aligned} \alpha v_t &= v_{xx} + \sigma_x + f \\ \sigma_t &= -\sigma + g(v_x) + \nu^2 \sigma_{xx} \quad (-1 < x < 0\,, t > 0)\,, \end{aligned}$$

with suitable boundary conditions at $x = -1, 0$ and prescribed initial data for v and σ. If $\nu = 0$, and for a particular choice of g, the model problem which neglects normal stresses, has the same behavior in steady shear as the more realistic model based on the the Johnson-Segalman-Oldroyd differential constitutive law discussed in the presentations by Malkus, Plohr and Pego, abstracted above. Here, the term $\nu^2 \sigma_{xx}$ represents a small diffusion term modeling stochastic effects of polymer molecules in Brownian motion and leads to a rich structure of steady state solutions. Using geometric theory of singular pertubations, we show that the model problem captures the spurt phenomenon on cyclic loading of the pressure gradient f, and it also exhibits hysteresis on subsequent unloading.

COLETTE GUILLOPÉ:

Differential Models with an Order Parameter

We introduce a class of models, the Kwon-Shen models, that are claimed to describe the behavior of polymers better than popularly used models. These models contain the Giesekus and Leonov constitutive models as particular cases. An internal parameter, the rigity parameter, is introduced; it satisfies a differential equation that relaxes with time. Existence governed by such a flow is proved for short time, as well as for long time provided the initial data are sufficiently small. Stable, slow steady flows are also shown to exist.

JAMES M. GREENBERG:

A Simple Model for Melt Fracture – Relaxation Oscillations in Polymer Melts

We develop a system of equations describing the mean flow of a polymer melt using singular pertubation methods. Slipping of the melt at the wall of the pipe is permitted, and an evolution equation for the boundary slip parameter is postulated. The combined system governing cross-sectional flow and slip parameter is shown to exhibit relaxation oscillations if the entrance boundary conditions are properly tuned.

PIOTR RYBKA:

Modeling Phase Transitions by Means of Viscoelasticity in Many Dimensions

We study the initial-boundary value problem for the quasilinear pde

$$u_{tt} = \nabla \cdot (\sigma(\nabla u) + \nabla u_t) \ ,$$

where σ is the primitive of a smooth function W, with either no-traction or no-displacement boundary conditions imposed on the smooth boundary of a bounded region Ω in R^n, and with suitable intial data $u(x,0)$ and $u_t(x,0)$ for $x \in \Omega$. We assume that the function W is not convex. We show global in time existence of solutions when σ is Lipschitz continous. We also study the stability of equilibria and the asymptotic behavior of solutions as $t \to \infty$. Specifically, we show that discontinuities in ∇u do not move.

ATHANASIOS E. TZAVARAS:

Viscoelasticity of Rate Type

The nonlinear system of partial differential equations

$$\begin{aligned} v_t &= \left(\mu(\theta,u)v_x^n\right)_x \\ u_t &= v_x \\ \theta_t &= \mu(\theta,u)v_x^{n+1} \end{aligned}$$

is a very simple model that incorporates the basic mechanism proposed for the explanation of shear band formation in high speed plastic deformation of metals. In this lecture, I discuss various results obtained using techniques of partial differential equations with the goal of understanding the features of the instability meechanism. These include conditions on $\mu(\theta,u)$ and on the strain-rate sensitivity n that lead to a stable response, as well as conditions that lead to the development of non-uniform response of the field variables.

RICHARD C. MAC CAMY:

Differential Approximations for Linear Viscoelasticity

Linear, isotropic viscoelasticity is considered. The models can be described by two scalar elastic moduli. The idea is to appropriate these moduli in a way what produces differential models instead of those having memory terms in the form of integrals. The approximations are designed to produce the same qualitative properties of the exact solutions, including dissipation, as well as short and long time behavior.

GIANPIETRO DEL PIERO:

Characterization of the Relaxation Functin in Linear Viscoelasticity in Terms of Work

This contribution is related to a series of papers by Breuer & Donat, Gurtin & Herrera, and Day, dating to the sixties, in which various a priori restrictions on the relaxation function of linear viscoelasticity, induced by general laws of thermodynamics are determined. Three classes of relaxation functions are considered: monotonic, completely monotonic, and exponential. Each of them is characterized by a corresponding property of the work done in particular deformation processes. More precisely, it is shown that the relaxation function is: monotonic if and only if the work done in any rectilinear monotonic process is decreased by retardation; it is completely monotonic if and only if the work done in retraced paths is increased by delay; and it is exponential if and only if the work done in all processes which are closed in stress-strain space is non-negative. The first and third of these results are original, while the second generalizes a result of

Day. The above results hold under the following assumptions: the restriction of deformation process to any time interval to any interval $(-\infty, t)$ is a function of bounded variation, and the relaxation function is Lebesgue integrable on $(0, \infty)$.

OLOF J. STAFFANS:

Well-Posedness of a Linear Viscoelastic Equation in Energy Space

We prove well-posedness in a new setting for the the linear viscoelastic equation

$$\begin{aligned} v'(t) &= -D^*\sigma(t) + f(t) \\ \sigma(t) &= \nu Dv(t) + \textstyle\int_{-\infty}^{t} a(t-s)Dv(s)ds\,, \quad t \geq 0\,. \end{aligned}$$

In this setting, the state of the system consists of two components: the velocity $v(t)$ and the Laplace transform of the initial function $Dv(s)\,, s < 0$. The transform is defined in parts of the left half plane. Different norms of the second component are considered, such as the absorbed energy norm and the internal energy norm. The latter is defined for a new class of general relaxation moduli a that includes the class of completely monotone kernels as a special case, and that can be analysed in detail. In particular, we characterize the spectrum of the semigroup in terms of a frequency domain condition, and we show that the growth of the semigroup is determined by the spectrum of its generator.

EDUARD FEIREISL:

Forced Vibrations in Nonlinear 1-D Viscoelasticity

We prove the existence of weak t-periodic solutions to the initial - boundary value problem

$$\begin{aligned} U_{tt} &= \left((\sigma(U_x) - \int_0^\infty k(s)\sigma(U_x(t-s,x))\right)_x ds \;+\; f(x,t) \\ U(0,t) &= U(1,t) = 0 \end{aligned}$$

with zero initial data, where f is t-periodic and possibly large. The main ingredients of the proof are: finding invariant regions for obtaining suitable a priori estimates for the application of the compensated compactness method.

DEBORAH BRANDON:

Global Existence for a Quasilinear Hyperbolic Volterra Equation with Semidefinite Equilibrium Modulus

We consider a nonlinear hyperbolic Volterra equation in one space dimension with a semidefinite equilibrium stress modulus (i.e., $\sigma'(\cdot) - k(\infty)\psi'(\cdot)$ in the notation of the presentation by Feireisel, except that here the material functions inside and outside the integral need not be the same!). In joint work with W. J. Hrusa, global existence of solutions for smooth data sufficiently close to equilibrium is obtained via energy methods. The main difficulty lies in obtaining suitable estimates on terms that do not involve time derivatives. We make essential use of properties of strongly positive kernels that satisfy additional assumptions; the latter are needed to obtain bounds on the spatial derivative u_x using an appropriate resolvent kernel. Large-time asymptotic behavior of solutions, including that of a term involving u_x, is also obtained.

GUSTAF GRIPENBERG:

On Some Nonlinear Hyperbolic Volterra Integrodifferential Equations

The goal is to establish the existence of a global weak (distribution) solution to the initial-boundary value problem for the equation

$$\begin{aligned} v_t(t,x) &= \int_0^t k(t-s)(\sigma(v_x))_x(s,x)ds + f(t,x)\,, \quad t \geq 0, x \in (0,1)\,, \\ v(t,0) &= v(t,1) = 0\,, \end{aligned}$$

together with the initial condition $v(0,x) = v_0(x)$. The equation can be thought of as an interpolation between a wave equation and a diffusion equation. The kernel k is assumed to be locally integrable and log-convex on $(0,\infty)$, and σ' has only one local minimum which is positive. Because of the singularity of k at zero, one cannot differentiate the equation and apply the technique used in the presentation of Feireisl (above). The crucial part of the proof (and of the result) is to establish certain apriori L^∞-bounds on the solutions. To do this, the equation is rewritten in the form

$$\begin{aligned} \tfrac{\partial}{\partial t}\beta \star (w - w_0) &= v_x \\ \tfrac{\partial}{\partial t}\beta \star (v - v_0) &= \sigma(w)_x + g\,, \end{aligned}$$

where $\beta = \gamma\delta + b$, where b is nonincreasing and $\hat{\beta}(z) = (2\hat{k}(z))^{-1/2}$.

HANS-DIETER ALBER:

Existence of Global Solutions to Mathematical Models for Hardening of Metals

We consider a system of differential equations modeling viscoelastic behavior of metals for small strains and large stresses. The derivation of the governing system is based on the assumption that the state of the material can be characterized by a set of internal variables; the resulting constitutive equations take the form of time-dependent o.d.e's. If the right side of these o.d.e's is the gradient of a convex function, global existence can be proved by applying the theory of monotone operators. However, such an assumption is not satisfied in practice by most models, and global existence must be proved by other methods. In one space dimension, one can sometimes derive an a priori

L^∞ bound which implies global existence, provided one of the constitutive functions satisfies a suitable growth condition.

FRANK KLAUS:

An Existence Theorem for Miller's Equations

Miller's constitutive equations aim to model the inelastic behaviour in solid materials like steel, aluminium, alloys, etc. The strain E is additively decomposed into an elastic part $E - E^n$ and a inelastic part E^n. With the elasticity tensor Γ, the stress is equal to $\Gamma(E - E^n)$. To describe the inelastic strain E^n, Miller introduces differential constitutive laws for the evolution of E^n in terms of the internal variables R (kinematic hardening) and D (isotropic hardening). The goal is to establish the well-posedness of the governing system of the three coupled differential equations subject to Dirichlet and Neumann boundary conditions. The proof depends on obtaining suitable H^1-energy estimates. In one space dimension, a global existence result with bounded energy is given.

Modeling and Analysis of Thermoviscoelastic Materials

WILLIAM J. HRUSA:

On Thermoviscoelastic Materials of Single-Integral Type

We consider a class of nonlinear materials in which the stress and internal energy depend on the present values and on the past histories of strain and temperature. We study two situations regarding the heat conduction mechanism: (i) Fourier's Law and (ii) Nonconductors. We show that for constitutive equations of the form

$$\sigma(t) = \hat{\sigma}(\epsilon(t), \theta(t)) + \int_0^\infty \tilde{\sigma}(\epsilon(t), \theta(t), \epsilon(t-\tau), \theta(t-\tau))\, d\tau$$

(and similarly for the internal energy e), it is possible to give simple and direct conditions for compatibility with the Second Law of thermodynamics. For motions of such materials in one space dimension, we also discuss the existence of globally defined smooth solutions for smooth data that are close to equilibrium. Most of the results described here were obtained jointly with Morton Gurtin.

Control Problems in Viscoelasticity

JONG UHN KIM:

Control of Some Linear Viscoelastic Models

The question of exact controllability is discussed for two special models: the plate equation with memory and the wave equation with memory in a domain Ω in $\mathcal{R}^n$ with smooth boundary. The basic tool is the Hilbert Uniqueness Method. The main feature of the proof is to establish a certain "unique continuation property" associated with each of the two model equations.

ROBERT L. WHEELER (with K. B. Hannsgen & O. J. Staffans):

Rational Approximation of Compensators in Some Control Problems for Viscoelastic Systems

We study rational approximations of the transfer function $P(s)$ for a linear viscoelastic rod undergoing torsional vibrations, where input and output are at the same end of the rod. An approximation is carried out that is consistent with the theory of robust (H^∞) control for the construction of rational, sub-optimal compensators that approximate the ideal, optimal compensator. The transfer function can be constructed in the form $P(s) = s^{-2}\mathcal{B}(s)g(\mathcal{B}^2(s))$, where $\mathcal{B}(s)$ is in general a transcendental function determined by the viscoelastic stress relaxation modulus, and $g(\mathcal{B}^2)$ is an infinite product of fractional linear transformations; in the case of uniform densities, $g(\mathcal{B}^2) = coth\mathcal{B}$. The approximation takes three steps: First, $g(\mathcal{B}^2)$ is approximated by partial products $g_N(\mathcal{B}^2)$. For relevant values of $\mathcal{B}$, convergence rates are analysed in detail. For the control problem at hand, suitable convergence requires introducing an irrational correction factor the effects of which must be studied separately. In addition, the fractional linear factors in $\mathcal{B}^2$ appearing in $g_N(\mathcal{B}^2)$ must be replaced by something rational. When the damping is weak, it is possible to do this by separating the oscillatory modes from creep modes and ignoring the latter. Numerical examples illustrating the procedure are given.

Related Mathematical Problems

HANS ENGLER

Similarity Solutions to a Class of Hyperbolic Volterra Intregrodifferential Equations.

We construct similarity solutions to the Cauchy problem

$$u_{tt} - u_{xx} + \int_0^t a(t-s)u_{xx}(x,s)ds = 0 \quad (t>0, x \in \mathcal{R}),$$
$$u(x,0) = sgn(x), u_t(x,0) = 0,$$

in the explicit form

$$u(x,t) = \Psi\left(\tfrac{t-x}{|x|^{1/\alpha}}\right) - \Psi\left(\tfrac{t+x}{|x|^{1/\alpha}}\right),$$

under the assumption that a is a certain completely monotone kernel in $L^1(0,\infty)$ that behaves like $t^{-\alpha}$ near $t=0$, and like $t^{\alpha-2}$ as $t \to \infty$; here, $0 < \alpha < 1$ is arbitrary. The function Ψ turns out to be the cumulative distribution of a (one-sided) stable distribution (with parameter α) in the sense of P. Lévy. In particular, for $\alpha = 1/2$, $\Psi'(r) = \pi^{-1/2} r^{-3/2} exp(-1/4r)$. The asymptotic behavior (as $t \to \infty$) of these solutions is also studied.

STIG-OLOF LONDEN

On Some Nonlinear Parabolic Volterra Intregrodifferential Equations

Consider the initial value problem

$$\begin{aligned} u_t(t,x) &= \textstyle\int_0^t a(t-s)(h(u_x))_x(s,x)ds + f(t,x), \quad t \geq 0, x \in \Omega \subset \mathcal{R},, \\ u(0,x) &= u_0(x) \quad x \in \Omega , \end{aligned}$$

where h is a given smooth function on $\mathcal{R}$ satisfying $h'(0) > 0$, and the functions f and u_0 are given. If $a(t) \equiv 1$, the equation reduces to the quasilinear wave equation; if $a(t)dt$ is a point mass at the origin, the equation reduces to the quasilinear heat equation. If $a \in L^1_{loc}(\mathcal{R}^+)$, a is positive and in some sense singular at the origin, then the equation represents an intermediate case between these two extremes. For the purpose of a rough classification, we distinguish three types of kernels: (i) $0 < a(0+) < \infty, -\infty < a'(0+) < 0$; (ii) $0 < a(0+) < \infty, a'(0+) = -\infty$; (iii) $a(0+) = +\infty, a \in L^1_{loc}(\mathcal{R}^+)$.

We analyze the almost parabolic case (iii), taking a convex, and assuming $Re\,\hat{a}(m) \geq q|Im\,\hat{a}(m)|, m \in \mathcal{R}$. First, existence results for strong solutions with small data are established. These are based on lengthy estimates of parameter-dependent resolvents. The equation can then be taken in the more general div-grad form. Second, we prove the existence of global weak solutions for data of arbitrary size. Finally we discuss regularity of solutions, showing that $u_x(t,x), u_t(t,x)$ are continuous in t for a.e. x, and also continuous in x for a.e. t.

KENNETH B. HANNSGEN (with Sergiu Aizicovici, Athens, OH, USA):

Mild Solutions of Abstract Semilinear Volterra Intregrodifferential Equations

Local existence of mild solutions is proved for the initial value problem

$$\begin{aligned} x'(t) &= \int_0^t a(t-\tau)Lx(\tau)d\tau + (Fx)(t) \quad (0 \leq t \leq T), \\ x(0) &= x_0 . \end{aligned}$$

Here L is a densely defined linear operator that (i) generates a cosine family in a Banach space or that (ii) is self-adjoint and negative in a Hilbert space. F is a continuous (nonlinear) hereditary mapping on $\mathcal{C}([0,T])$. The scalar kernel $a(t)$ satisfies conditions that make the resolvent kernel for the linear case $(F = 0)$ compact.

PAVEL SOBOLEVSKII:

A Model Problem for Stabilizing Viscoelastic Fluid Motion

We consider the initial-boundary value problem

$$\begin{aligned} v_t + \sum_{k=1}^{n} v_k v_{x_k} &= \epsilon \Delta v + \int_0^t \rho exp[-\delta(t-s)] \Delta v(s,x) ds + \nabla p + f(t,x), \\ \nabla \cdot v &= 0 \quad (t \geq 0, x \in \Omega), \\ v(t,x) &= 0 \quad (t \geq 0, x \in \partial\Omega), \\ v(0,x) &= v^0(x) \quad (x \in \bar{\Omega}); \quad (p,1) = 0. \end{aligned}$$

Here, the unknown functions are $v = (v_1, ..., v_n), p$, and the funtions f, v^0 are given; Ω is an open bounded domain in $\mathcal{R}^n$, where $n = 2$ or $n = 3$, with smooth boundary $\partial\Omega$, and ϵ, ρ, δ are positive constants.

Under suitable assumptions, it is shown that the above problem is locally well-posed in time when $n = 3$ and globally well-posed in time when $n = 2$. Moreover, as $t \to \infty$, the solution tends to the unique solution to an appropriate limiting boundary value problem.

MICHAEL BÖHM:

Diffusion in a Finely Structured Medium

Let G be a region occupied by a medium with a doubly porous structure, i.e., G contains subregions G_1, G_2 with different types of pores and permeabilities. Assuming that the subregions G_1, G_2 are "well intermingled", one arrives at a set of two continuity equations for appropriate densities. Specification of the coefficients leads to various types of coupled p.d.e's (and o.d.e's) which are briefly discussed.

ALEXANDER BELYAEV:

Homogenization of a Dirichlet Problem for the p-Laplacian

Consider the following nonlinear Dirichlet problem in an ϵ-perforated domain:

$$-\Delta_p u_\epsilon^\mu \equiv -div(|\nabla u_\epsilon^\mu|^{p-2} \nabla u_\epsilon^\mu) = f, \quad u_\epsilon^\mu \in \Omega_\epsilon^\mu.$$

Here, $\Omega_\epsilon^\mu \equiv \Omega \backslash \bigcup \{\epsilon T_\mu + k\epsilon : k \in \mathcal{Z}^n\}, u_\epsilon^\mu \in W_0^{1,p}(\Omega_\epsilon^\mu), \Omega \subset \mathcal{R}^n$,
$\partial\Omega \in C^\infty, \quad 1 < p \leq n, \quad f \in L^\infty, \quad T_\mu \subset D = \{|\xi| \leq \frac{1}{2}\}, \quad \xi = \frac{x}{\epsilon}$.

Denote $< \cdot > = \int_D \cdot \, d\xi$; let

$$\lambda_\mu = \inf\{< |\nabla v|^p > / < |v|^p >: v \in C^\infty_{\text{per}}(D), v = 0 \text{ on } T_\mu\},$$

and let $w_\mu(\xi)$ be the minimizer of this spectral problem, where $< w_\mu >= 1$. Under suitable assumptions, we study the limiting behavior of the solution u_ϵ^μ to the above nonlinear Dirichlet problem as $\epsilon \to 0$, and we describe the sense in which the resulting limiting equation is satisfied by the limit function u_0.

Open Problems

MICHAEL RENARDY:

The Stresses of an Upper Convected Maxwell Fluid in a Newtonian Velocity Field near a Reentrant Corner

We investigate the stresses of an upper convected Maxwell fluid in the neighborhood of a reentrant 270 degree corner. It is assumed (incorrectly, of course) that the velocity field is Newtonian. Both asymptotic analysis and numerical solutions are presented. It is found that, for a fixed angle, the stresses behave approximately like $r^{-0.74}$, which contrasts with a behavior like $r^{-0.91}$ at the walls (the latter is simply the square of the Newtonian shear rate at the wall, where the flow is viscometric). The analysis shows that there are boundary layers near the walls, in which there is a transition from the viscometric behavior at the wall to a core region where the behavior is dominated by the convected derivative in the constitutive equation. Moreover, our computations show large spurious stresses downstream resulting from numerical errors.

JOHN HINCH:

The flow of an Oldroyd Fluid around a Sharp Corner

A similarity solution $\psi = r^{1+\alpha} f(\theta)$ is constructed for the flow of an Oldroyd fluid around a 270° corner. An upstream viscometric region near the wall away from the corner determines the distribution of the stress across the streamlines in the vicinity of the corner as $\underline{\underline{\sigma}} = -p\underline{\underline{I}} + G\psi^{-\beta}\underline{u}\underline{u}$, where $\beta = 2(n-\alpha)/(2n-1-\alpha)$ with $f \sim -\theta^n$ as $\theta \to 0$ (the wall). Satisfying the momentum equation yields a nonlinear eigenvalue problem for $f(\theta)$ and α, with the exact solution $\alpha = 5/9$ and $f(\theta) = -sin^{7/3}\frac{2}{3}\theta$. Thus the velocity is found to vanish like $r^{5/9}$ and the stress to be singular like $r^{-2/3}$.

KENNETH WALTERS:

Settling in Elastic Liquids

The conceptually simple non-Newtonian flow problem involving the settling under gravity of a solid sphere moving along the axis of a cylindrical container provides a number of interesting challenges. Experimentally, there are some unusual features that have yet to be predicted theoretically. For example, for a Newtonian fluid, the sphere often reaches its terminal velocity after travelling no more than a diameter. In contrast, thirty or fourty diameters are sometimes required for elastic liquids, with substantial overshoot. The dependences of the drag on the sphere on the Weissenberg number and on the ratio of the sphere and cylinder radii have not yet been predicted with anything representing quantitative agreement between experiment and theory.

CONSTANTINE DAFERMOS:

Redistribution of Damping

In viscoelasticity, effects of the memory induce damping. However, the presence of dissipation is not explicit in every conservation law. For example, consider the simple one-dimensional model system

$$\begin{aligned} \partial_t u - \partial_x v &= 0 \\ \partial_t v - \partial_x \sigma(u) &= \int_0^t k(t-\tau)\partial_x \sigma(u)(x,\tau)d\tau\,, \end{aligned}$$

with a given smooth kernel k. This system can be written in the equivalent form

$$\begin{aligned} \partial_t u - \partial_x v &= 0 \\ \partial_t v - \partial_x \sigma(u) &= -k(0)v + \int_0^t r'(t-\tau)v(x,\tau)d\tau - r(t)v_0(x)\,, \end{aligned}$$

where r is the resolvent kernel of k and v_0 is the initial datum of v. Under appropriate assumptions on k, damping is clearly present in the second equation but not the first. This causes problems when for instance, we try to construct solutions with shocks to the above system by the random choice method of Glimm.

The question is whether we may redistribute the damping between the two equations by replaving v by a new variable of the form

$$w(x,t) = v(x,t) + \int_0^t p(t-\tau)v(x,\tau)d\tau$$

for an appropriate kernel p. For example, note that if

$$k(t) = \tfrac{a^2}{a-b}e^{-at} - \tfrac{b^2}{a-b}e^{-bt}\,, \quad b \neq a\,, \quad a\,,b > 0\,,$$

the change of variable

$$w(x,t) = v(x,t) + a\int_0^t v(x,\tau)d\tau$$

reduces the original system to

$$\begin{aligned} \partial_t u - \partial_x w + au &= au_0 \\ \partial_t w - \partial_x \sigma(u) + bw &= 0 \end{aligned}$$

in which damping effects are well balanced between the two equations.

Berichterstatter: J.A. Nohel

Tagungsteilnehmer

Prof.Dr. Hans-Dieter Alber
Fachbereich Mathematik
TH Darmstadt
Schloßgartenstr. 7

D-64289 Darmstadt

Prof.Dr. Pavol Brunovsky
UAMVT
Univerzita Komenskeho
Mlynska dolina

842 15 Bratislava
SLOVAKIA

Prof.Dr. Jaques Baranger
Département de Mathématiques
Université Claude Bernard de Lyon I
LAN Bat. 101
43, Bd. du 11 Novembre 1918

F-69622 Villeurbanne Cedex

Prof.Dr. Constantine M. Dafermos
Division of Applied Mathematics
Brown University
Box F

Providence , RI 02912
USA

Prof.Dr. Alexander G. Belyaev
Department of Mathematics
Moscow State University

Moscow 119899
RUSSIA

Dr. Arthur Russell Davies
Dept. of Mathematics
University of Wales
Aberystwyth

GB-Dyfed SY23 3BZ

Dr. Michael Böhm
Institut für Angewandte Mathematik
Fachbereich Mathematik
Humboldt-Universität Berlin

D-10099 Berlin

Prof.Dr. Frederick Debae
Unité de Méchanique Appliquée
Université Catholique de Louvain
2, Place de Levant

B-1348 Louvain-La-Neuve

Prof.Dr. Deborah Brandon
Dept. of Mathematics
Carnegie-Mellon University

Pittsburgh , PA 15213-3890
USA

Prof.Dr. Gianpietro Del Piero
Facoltà di Ingegneria
Via Scandiana 21

I-44100 Ferrara

Dr. Hans Engler
Department of Mathematics
Georgetown University

Washington , DC 20057
USA

Prof.Dr. Kenneth B. Hannsgen
Dept. of Mathematics
Virginia Polytechnic Institute and
State University
460 McBryde Hall

Blacksburg , VA 24061-0123
USA

Dr. Eduard Feireisl
Institute of Mathematics of the
CSAV
Zitna 25

115 67 Praha 1
CZECH REPUBLIC

Prof.Dr. John Hinch
Department of Applied Mathematics
and Theoretical Physics
University of Cambridge
Silver Street

GB-Cambridge CB3 9EW

Prof.Dr. James M. Greenberg
Dept. of Mathematics/Statistics
University of Maryland
Baltimore County Campus

Baltimore , MD 21228
USA

Prof.Dr. Josef Honerkamp
An der Rothalde 21

D-79312 Emmendingen

Prof.Dr. Gustaf Gripenberg
Dept. of Mathematics
University of Helsinki
P.O. Box 4
Hallituskatu 15

SF-00014 Helsinki 10

Prof.Dr. William J. Hrusa
Dept. of Mathematics
Carnegie-Mellon University

Pittsburgh , PA 15213-3890
USA

Dr. Colette Guillope
Mathématiques
Universite de Paris Sud (Paris XI)
Centre d'Orsay, Bàtiment 425

F-91405 Orsay Cedex

Prof.Dr. Jong-Uhn Kim
Dept. of Mathematics
Virginia Polytechnic Institute and
State University
460 McBryde Hall

Blacksburg , VA 24061-0123
USA

Frank Klaus
Fachbereich Mathematik
TH Darmstadt
Schloßgartenstr. 7

D-64289 Darmstadt

Prof.Dr. Joachim Meissner
Institut für Polymere
ETH-Zentrum
ML-J19
Sonneggstr. 3

CH-8092 Zürich

Dr. Dietmar Kröner
Institut für Angewandte Mathematik
Universität Freiburg
Hermann-Herder-Str. 10

D-79104 Freiburg

Prof.Dr. John A. Nohel
Forschungsinstitut für Mathematik
ETH Zürich

CH-8092 Zürich

Prof.Dr. Stig-Olof Londen
Institute of Mathematics
Helsinki University of Technology

SF-02150 Espoo 15

Dr. Hans Chr. Oettinger
Institut für Polymere
ETH-Zentrum
ML-J19
Sonneggstr. 3

CH-8092 Zürich

Prof.Dr. Richard C. MacCamy
Department of Mathematics
Carnegie Mellon University

Pittsburgh , PA 15213
USA

Dr. Robert Pego
Dept of Mathematics
University of Maryland

College Park MD 20742
USA

Prof.Dr. David Malkus
Center for the Mathematical Science
University of Wisconsin-Madison
1308 W. Dayton Street

Madison , WI 53715
USA

Prof.Dr. Bradley J. Plohr
Dept. of Applied Mathematics
State University of New York
at Stony Brook

Stony Brook , NY 11794-3600
USA

Dr. Jan Prüss
FB 17: Mathematik/Informatik
Universität Paderborn
Warburger Str. 100

D-33098 Paderborn

Prof.Dr. Jean-Claude Saut
Mathématiques
Université de Paris Sud (Paris XI)
Centre d'Orsay, Bâtiment 425

F-91405 Orsay Cedex

Prof.Dr. Michael Renardy
Dept. of Mathematics
Virginia Polytechnic Institute and
State University
460 McBryde Hall

Blacksburg , VA 24061-0123
USA

Daniel Sevcovic
Institute of Applied Mathematics
Comenius University
Mlynska dolina

84215 Bratislava
SLOVAKIA

Prof.Dr. Yuriko Renardy
Dept. of Mathematics
Virginia Polytechnic Institute and
State University
460 McBryde Hall

Blacksburg , VA 24061-0123
USA

Prof.Dr. Pavel E. Sobolevskii
Institute of Mathematics
The Hebrew University
Givat Ram

91904 Jerusalem
RUSSIA

Prof.Dr. Piotr Rybka
Instytut Matematyki
Uniwersytet Warszawski
ul. Banacha 2

00-913 Warszawa 59
POLAND

Prof.Dr. Olof Staffans
Institute of Mathematics
Helsinki University of Technology
ESPOO 15

SF-021250 Helsinki

Dr. Dominique Sandri
Departement de Mathematiques
Universite Claude Bernard de Lyon I
LAN Bat. 101
43, Bd. du 11 Novembre 1918

F-69622 Villeurbanne Cedex

Prof.Dr. Athanasios E. Tzavaras
Center for the Mathematical Scienc.
University of Wisconsin-Madison
610 Walnut Street

Madison , WI 53705
USA

Prof.Dr. Kenneth Walters
Dept. of Mathematics
University of Wales
Aberystwyth

GB-Dyfed SY23 3BZ

Dr. Gerald Warnecke
Mathematisches Institut A
Universität Stuttgart
Postfach 801140

D-70511 Stuttgart

Prof.Dr. Robert L. Wheeler
Dept. of Mathematics
Virginia Polytechnic Institute and
State University
460 McBryde Hall

Blacksburg , VA 24061-0123
USA