MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 24/1993

Differentialgeometrie im Großen

23.5. bis 29.5.1993

Die Tagung fand unter der Leitung von W. Ballmann (Bonn), J.P. Bourguignon (Palaiseau) und W. Ziller (Philadelphia) statt.

Geometrie überzeugte als idealer kultureller und programmatischer Rahmen, in dem eine Vielfalt an Fragestellungen und Methoden in produktive Wechselwirkung treten können.
Neben genuin geometrischen Themen wird man auch algebraische, analytische, topologische und andere Orientierungen an den Vortragsauszügen erkennen.
Außerhalb des Vortragsprogramms gab es Gelegenheit zu vertieftem fachlichen und persönlichen Austausch.

Unter anderen Schwerpunkten seien hier genannt:

- Starrheit,
- negative Ricci- bzw. Schnittkrümmung,
- topologische Entropie.
- Holonomie,
- isospektrale Deformationen,
- elliptische Operatoren,
- positive Skalarkrümmung,
- isosystolische Ungleichungen.

Gefördert durch DFG Deutsche Forschungsgemeinschaft

Vortragsauszüge

K. GROVE

Curvature and Packing radii

It seems natural to divide metric invariants into two kinds: (1) Size invariants such as diameter, volume etc., and (2) Shape invariants like curvature, excess etc.

Typically the shape of a space will influence its size: "Big curvature implies small size". For example, if curv $M \geq 1$ then $diam M \leq \pi$.

The size of M can also be measured in terms of its sequence of packing radii, $pack_2 M \geq \ldots \geq pack_q M \geq \ldots \searrow 0$ where $2 \cdot pack_q M = max_{(x_i, \ldots x_q)} min_{i<j} dist(x_i, x_j)$. For each q standard comparison yields

$$pack_q M^n \leq pack_q S_1^n \quad , \quad if \quad curv M \geq 1.$$

If on the other hand $pack_q M$ is not too small we have
Theorem (Grove, Markvorsen). There is an $\varepsilon = \varepsilon(n) > 0$ such that any closed Riemannian n-manifold M with sectional curvature $curv M \geq 1$ and $pack_{n+2} M \geq \pi/4 - \varepsilon$ is homeomorphic to S^n or diffeomorphic to $\mathbb{R}P^n$.

The proof uses convergence techniques and the following

Theorem (G, M). Any n-dimensional Alexandrov space X with $curv X \geq 1$ and $pack_{n+1} X = \pi/4 = 1/2 diam X$ is isometric to S_1^n/H, where $H \subset (\mathbb{Z}_2)^{n+1}$ (the group of reflexions in coord. hyperplanes) acts without fixed points on S_1^n.

Another non-convergence application of singular spaces yields

Theorem (Grove, Wilhelm) Any n-manifold M with $curv M \geq 1$ and $pack_{n-1} M > \pi/4$ is diffeomorphic to S^n.

M. BERGER

Systoles & Applications According to Gromov

In 1949 Loewner proved that, for any Riemannian metric g on the 2-dimensional torus one has $\frac{Area(g)}{Sys^2(g)} \geq \frac{\sqrt{3}}{2}$, the equality holding exactly only for the equilateral flat torus.

Here $Area(g)$ is the area of g and $Sys(g) = systole$ of g denotes the length of the smallest curve which is not homotopic to zero.

This Loewner theorem, as Thom remarked in 1962, is the founding stone of a general program. Namely found universal inequalities relating the various lower bounds for homology, homotopy classes. The lower bound of a homology class, for example, for some Riemannian metric g, is the infinum of the volume for g of the various cycles representing that class (*systole*).

The talk will consist to give a survey of the various results (positive and counter examples) obtained since Loewner. All major theorems are due to Gromov in 1983 and 1992 - 1993.

Gromov also use the concept of systoles, in the case a general inequality is false, for two objectives.

The first is to characterize Riemannian metric on a Kähler manifold by their defect (measuring their defect) of Kählerianity.

The second is a purely geometric characterization of Jacobians among flat tori (abelian varieties).

W. GOLDMAN

Complex Hyperbolic Geometry

A bisector in complex hyperbolic n-space $H^n_{\mathbb{C}}$ is the set of points equidistant from a pair of points. Faces of Dirichlet fundamental polyhedra of discrete groups are regions in bisectors. A bisector is a real hypersurface which is not totally geodesic although it admits two foliations by totally geodesic submanifolds - complex hyperplanes and totally real submanifolds.

A construction is given of two bisectors in $H^2_{\mathbb{C}}$ whose intersection is the union of a complex geodesic $H^1_{\mathbb{C}}$ and a totally real geodesic submanifold $H^1_{\mathbb{R}}$. Such an intersection cannot be transverse; by perturbing it slightly, one obtains disconnected intersections of bisectors.

Another approach to the intersection of bisectors was the orthogonal projection onto the complex geodesics containing the spine. In particular compare the intersection of a geodesic with the projection of a bisector corresponding to the components of the intersection of two bisectors. It would be quite interesting to have exact formulas for the projection of a bisector in a complex geoedesic.

J. LOHKAMP

On the Geometry of Negative Ricci Curvature

Negative Ricci curvature (Ric) fulfills a large (and unexpected) variety of flexibility conditions.

Besides the general existence of $Ric < 0$-metrics on each manifold $M^n, n \geq 3$ we can get metrics fulfilling many additional properties. For instance: If M^n is open, we can find such a metric in each conformal class, or we can construct metrics with negative pinched Ricci curvature resp. with finite volume and $Ric < -const$.

Furthermore we can approximate any metric by $Ric < 0-$metrics and indeed in C^∞-topology for flat metrics. Moreover each two $Ric < 0$-metrics an be continously deformed into each other by a path consisting only of $Ric < 0$-metrics.

R.L. BRYANT

A Survey of the Holonomy Problem

The concept of the *holonomy* of a connection was introduced by É. Cartan as a measure of the global "non-flatness" of a connection. Precisely, given a principal bundle $B \to M$ and a connection ∇ on B, the holonomy of ∇ at x is the collection of all $P_\gamma^\nabla : B_x \to B_x$ where γ is a (piecewise C^1) loop in M based at x and P_γ^∇ is the ∇-parallel translation around γ. Fundamental results are that the holonomy at x, H_x^∇, is a subgroup of the structure group of B, in fact, a Lie subgroup, and that the identity component of H_x^∇ can be computed by parallel translation of the curvature of ∇ (this is the Ambrose-Singer Holonomy Theorem).

When the principal bundle is the coframe bundle of M, $J \to M$, a connection has a first order invariant, the *torsion*. The connections of greatest interest are the torsion-free connections, and the fundamental problem is to determine the possibilities for the holonomy of torsion-free connections.

This talk is a survey of what is known in the case that the holonomy acts irreducibly on the tangent bundle of M. For simplicity, I assume that M is 1-connected, so that H is connected.

First I explain how the torsion-free connection defines a holonomy bundle $B \subseteq J$ which is an H-structure on M. I then recall the notion of intrinsic torsion for an H-structure, which is a section of an associated bundle $B \times \rho_0 H^{0,2}(h)$ and remark that the holonomy bundle B of a torsion-free connection is torsion free (i.e., its intrinsic torsion vanishes). Conversely, starting with a torsion-free H-structure B, the space of compatible torsion-free connections is an affine space modeled on the sections of a bundle $B \times \rho_1, h^{(1)}$. I survey the list of irreducibly acting subgroups $H \subseteq GL(V)$ for which either $H^{0,2}(h) = 0$ (noting that the standard list omits $(Sp(2,\mathbb{R}) \subseteq GL(4,\mathbb{R}))$ or else $h^{(1)} \neq 0$ (combined work of Cartan, Kobayashi and Nagano) and determine which of these can occur as holonomy, and which of these torsion-free H-structure are necessarily flat.

In the remaining cases, where $h^{(1)} = 0$, torsion-free connections with holonomy H are essentially the same as torsion-free H-structures and the latter problem can be studied by techniques from exterior differential systems. I review the lists of Berger and point out serveral new "exotic" examples. Highlights are: $Sp(n,\mathbb{R}) \cdot SL(2,\mathbb{R}) \subseteq S0(2n,2n)$ can occur as holonomy, two noncompact forms of E_6 can occur as holonomy in $\mathbb{R}^{27}$, there are non-flat $SL(p,\mathbb{R}) \cdot SL(2,\mathbb{R})(\subseteq SL(2p,\mathbb{R}))$-structures which are related to twistor constructions, an example of an $H \subseteq GL(6,\mathbb{R})$ for which the torsion-free equations are not formally integrable.

* The spaces $h^{(1)}$ and $H^{0,2}(h)$ are the kernel and cokernel, respectively, of the Spencer map $\delta : h \otimes V^* \to V \otimes \Lambda^2 V^*$ where $h \subseteq gl(V)$ is the Lie algebra of $H \subseteq GL(V)$.

T. H. COLDING

Ricci diameter sphere theorem

If (M,g) is a Riemannian manifold with sectional curvature greater than or equal to one and $diam \geq \pi$, then M is isometric to the round sphere by Toponogov's maximal diameter theorem. If $sec \geq 1$ and $diam > \pi/2$ then M is homeomorphic to a sphere by the generalized sphere theorem of Grove-Shiohama. In the case of Ricci curvature one have the analog of Toponogov's maximal diameter theorem, namely Cheng's maximal diameter theorem. It is therefore natural to ask what minimal geometric condition, if any, togehter wioth a Ricci diameter condition give topological stability.

The purpose of this talk is to answer this, namely we show:

Theorem (C)

Given $\Lambda > 0$ and an integer $m \geq 2$ there exists an $\varepsilon(\Lambda, m) > 0$ (which can be explicitly estimated) such that any R.M., of dimension m, whith,

$$Ric \geq (m-1), sec \geq -\Lambda^2 \text{ and } diam \geq (\pi - \varepsilon)$$

is homeomorphic to a sphere.

Grove-Petersen proved this with the additional assumption of a lower volume bound. Their work generalized a theorem of Eschenburg where a lower bound on the injectivity radius was assumed. On the other hand, Anderson and Otsu showed independently that if the lower bound on the curvature is replaced with a lower bound on the volume then the conclusion of the theorem does not hold. Namely they showed that $S^m \times S^n$, $L(p,q) \times S^n$, $\#_{i=1}^k S^2 \times S^2$, $\mathbb{C}P^n$ and $\mathbb{C}P^n \# \mathbb{C}P^n$ admit sequences of metrics with diameter converging to π, volume uniformly bounded away from zero and Ricci curvature greater or equal to the one of the round sphere of the same dimension. G. Perelman has independently shown the above theorem.

E. CALABI

The Weighted Isosystolic Problem for closed surfaces

Let M be a closed surface of genus $\mathcal{G} \geq 2$, γ a non-trivial homotopy class of free closed curves, and g a Riemannian metric on M. The local systole $\text{sys}_\gamma(M,g)$ is the greatest lower bound of the length of closed paths representing γ, in terms of the metric.

If $\Gamma(M)$ denotes the set of all free homotopy classes of paths in M (including the trivial class $\{e\}$), and $A(M,g)$ the area of M with respect to g, then the **systolic ratio** $\rho(M,g)$ is defined by

$$\rho(M,g) = \sup_{\gamma \in \Gamma(M) \setminus \{e\}} (A(M,g)/\text{sys}_\gamma(M,g)).$$

It is known (Gromov) that, for each value of the genus g there is a positive constant $c(\mathcal{G}) > 0$ defined by

$$c(M) = \inf_{(g)} \rho(M, g)$$

called the **extremal systolic ratio**, achieved by at least one **extremal isosystolic** metric g_0, that may be scaled so that either $A(M,g)$ or $\inf_\gamma(\mathrm{sys}_\gamma(M,g))$ is a suitable level. A **weight** function $\mu : \Gamma \to \mathbb{R}^+$ is a function, $\mu(\{e\}) = 0, \mu(\gamma) > 0$ otherwise subject to other suitable axioms (symmetry, strong subadditivity, etc.), that helps define a weighted (μ-weighted) isosystolic ratio

$$\rho_\mu(M,g) = \sup_{\gamma \in \Gamma(M)\backslash\{e\}} \{A(M,g)\mu^2(\gamma)/(\mathrm{sys}_\gamma(M,g))^2\} > 0$$

with the property that the space of all metrics g in M such that $\sup_\gamma(M,g) \geq \mu(\gamma)$ for every γ and $A(M,g) \leq e$ is compact (Gromov-Hausdorff topology) and non-empty for any $c \geq c_0 > 0$; there exists therefore a compact family of **extremal or isosystolic** metrics. This generalization of the isosystolic problem is essential in order to analyse the structure of solutions of the original isosystolic problem (unweighted). Indeed, it is important to know the set of **critical** free homotopy classes $S_\mu \subset \Gamma(M)$, i.e. the set of $\gamma \in \Gamma(M)$ such that, in extremal μ- isosystolic metric with

$$\mathrm{sys}_\gamma(M,g) = \mu(\gamma)$$

be finite, and the paths representing the shortest representatives in each critical class γ be in generic position, in order to derive the variational stability equations. Time permitting, the Euler equations are derived.

Y. C. DE VERDIERE

UNSTABLE CLOSED GEODESICS AND SEMI-CLASSICAL LIMITS

Le problème considéré est l'étude du comportement asymptotique des fonctiones propres du laplacien sur une variété riemannienne compacte lorsque $\lambda j \to +\infty$. On introduit les mesures de probabilité $v_j = |\varphi_j|^2 dx$ et leur relévement microlocal μ_j, mesures de probalités sur le fibré unitaire cotangent, definies par:

$$\mu_j(a) = < Op^+(\tilde{a})\varphi_j | \varphi_j >,$$

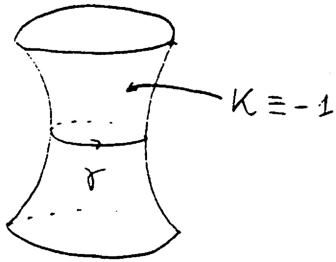
où $\tilde{a}$ est le prolongement homogène de degré 0 de a à $T^*X\backslash 0$ et $Op^+(\tilde{a})$ est donné par une quantification positive. Les μ_j sont canoniques à O $(\frac{1}{\sqrt{\lambda_j}})$ près. Leurs limites vogues sont invariantes par le flot géodésique.

On a le théorème *ergodique semi-classique*: si le flot géodésique est ergodique, il existe une sous-suite de densité 1 λ_{j_k} de λ_j telle que μ_{j_k} tend vers la mesure de Liouville (Schnirelman-Zelditch-CdV).

La question est de savoir s'il y a d'autres limites faibles, par exemple les mesures de Dirac sur les géodésiques fermeés. Rudnick et Sarnak ont prouvé qui c'est impossible dans le cas arithmetique si les φ_j sont aussi propres pour les opérateurs de Hecke. Nous avons récemment prouvé avec Bernard Parisse (Grenoble) que des fonctions propres peuvent se concentrer sur une géodésique instable d'une surface de révolution à courbure -1. De plus, nous avons un résultat qui minore par

$$k \equiv \frac{1}{log\lambda_j}$$

la vitesse de concentration.



figure

B. LEEB

Closed 3-manifolds with(out) metrics of nonpositive curvature

Thurston's Hyperbolization Conjecture combined with Mostow's Rigidity Theorem asserts that closed, orientable, irreducible, atoroidal 3-manifolds correspond bijectively to closed orientable hyperbolic (i.e. of constant sectional curvature -1) 3-manifolds. Hence, the class of hyperbolizable 3-manifolds can be described by simple topological conditions.

We raise the corresponding question of existence of a metric of nonpositive curvature. The right class of closed orientable 3-manifolds to look at, i.e. where the well-known topological obstructions are not present, are irreducible ones with infinite fundamental group. By the Jaco-Shalen-Johannson topological decomposition theorem, such a manifold M can be cut along disjoint, imcompressible 2-tori into pieces which are atoroidal or Seifert. A minimal decomposition is unique. We consider the case complementary to hyperbolization, where the topological decompositon is non-trivial, i.e. one cuts along more than zero tori. Then the pieces of the decomposition admit geometric structures of nonpositive curvature: the atoroidal pieces admit complete hyperbolic metrics of finite volume on their interior (by Thurston's Hyperbolization Theorem for Haken manifolds); the Seifert pieces admit geometric structures modelled on $\mathbb{H}^2 \times \mathbb{R}$ resp. $\mathbb{R}^3$ according to whether the base orbifold

Gefördert durch DFG Deutsche Forschungsgemeinschaft

is Euclidean or hyperbolic. We ask when M can be weakly geometrical in the sense that one can put an *npc* (nonpositively curved) metric on all of M. We present partial results, but do not see an easy-to-formulate criterion.

Existence: If in the topological decomposition of M occurs at least one hyperbolic piece, then M admits an *npc* metric. Hence the delicate case is the case of graph-manifolds (i.e. manifolds glued from Seifert pieces). The reason for this is that every *npc* metric arises up to isotopy by putting flat metrics on the Jaco-Shalen-Johannson tori and extending those into the geometric (hyperbolic or Seifert) pieces. Hyperbolic pieces are flexible in the sense that any prescribed flat metric on the boundary can be extended to an *npc* metric on the interior. For Seifert pieces instead, the flat metrics on the boundary tori are interrelated coming from the fact that an *npc* metric with totally geodesic flat boundary on a Seifert piece splits locally as a product. This rigidity can lead to obstructions to the existence of *npc* metrics on graph-manifolds.

Non-existence: We give examples of graph-manifolds with linear gluing graph built from arbitrarily many Seifert pieces which do not admit an *npc* metric.

P. TONDEUR (joint work with Y. MAEDA / K. U / S. ROSENBERG)

Minimal Gauge Orbits

Consider an isometric action of a compact Lie group on a Riemannian manifold (M, g). If an orbit O is of extremal volume among nearby diffeomorphic orbits, then O is a minimal submanifold.

Gauge theory provides an infinite dimensional analogue for this set-up. The Faddeev-Papov ghost determinant gives a regularization of the volume of a gauge orbit, so we can define what it means for a gauge orbit to have extremal volume among nearby orbits in terms of the infinitesimal variation of the ghost determinant. This allows to prove that an irreducible gauge orbit O with extremal volume among nearby orbits has vanishing trace for the second fundamental form of the orbit O inside all gauge fields. Using work of Fintushal-Stern, this yields the existence of minimal gauge orbits of flat $\mathbf{SU}(2)$-connections over certain Seifert homology 3-spheres.

J. JOST

Equilibrium between metric spaces

The aim of this lecture ist to generalize the concept of harmonic maps between Riemannian manifolds to maps betwen metric spaces. Thus, let (M, d) and (N, d) be metric space, N a complete length space. Let μ be a positive measure on M, $\mu_x^\varepsilon, (\varepsilon > 0, x \in M)$ be measures on M. We define the ε-energy of $f : M \to N$ as

$$E_\varepsilon(f) := \frac{\int_M d^2(f(x), f(y)) d\mu_x^\varepsilon(y)}{\int_M d^2(x, y) d\mu_x^\varepsilon(y)} d\mu(x)$$

A typical case is $\mu_x^\varepsilon = \mu \mid B(x, \varepsilon)$. If M is a Riemannian manifold, then the usual energy is

$$E(f) = \Gamma - lim_{\varepsilon \to 0} E_\varepsilon(f)$$

where the $\Gamma - limit$ is taken in the sense of de Giorgi.

If the measures μ and μ_x^ε satisfy a certain symmetry condition (satisfied if e.g. $\mu_x^\varepsilon = \mu \mid B(x,\varepsilon)$), then f is a critical point of E_ε iff $f(x)$ is the mean value of the measure $f_* \mu^\varepsilon x$ for all $x \in M$. This suggests the following discrete evolution process: Given $f_0 : M \to N$, define

$$f : M \times \mathbb{N} \to N \quad by \quad f(x, n+1) = \text{mean value of } f_*(\cdot, n)\mu_x^\varepsilon \quad f(x, 0) := f_0(x).$$

This process converges to an ε-equilibrium-map, i.e. a critical point of E_ε if N has nonpositive curvature in the sense of Alexandrov, and certain obvious conditions on the measures μ_x^ε are satisfied. Taking Γ-limits then gives critical points of E, i.e. harmonic maps. For the special case of M a Riemannian manifold, such a result was also obtained by Korevaas-Schoen by a different method.

J. DODZUIK

Spectral degeneration of hyperbolic 3-manifolds

I consider the spectrum of the Laplace operator Δ on compact hyperbolic manifolds of three dimensions. Let $M_n \to M_0$ be a sequence of such manifolds converging to a complete manifold M_0 with cusps and of finite volume. Since $\text{Spec}(\Delta, M_0) \supset [1, \infty)$ one expects that the spectra of M_n's accumulate in $[1, \infty)$. I discuss the proof of a precise estimate of the accumulation rate. If

$$N_n(x) = \#\{\lambda \in \text{Spec}(\Delta, M_n) \mid |1 \leq \lambda \leq x^2\}$$

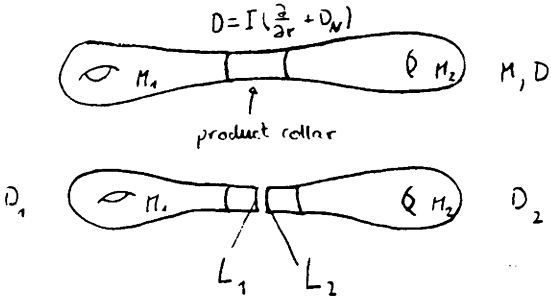
then

$$N_n(x) = \frac{x}{2\pi} \text{diam}(M_n) + 0_x(1).$$

U. BUNKKE

Verklebeformeln für η-Invariante und Spektralfluß

Wir betrachten verallgemeinerte Dirac-Operatoren in der folgenden geometrischen Situation:



Abbildung

$(V, I, < I, >), V = KerD_N$ symplektisch.

D_i wird mit einer APS-Randbedingung unter Verwendung des Raumes $L_i \subset V$ mit $L_i \oplus^{\perp} IL_i = V$ definiert. Seien $\eta(D), \eta(D_i)$ die η-Invarianten der Dirac-Operatoren.

Theorem: $\eta(D) - \eta(D_1) - \eta(D_2) = m(L_1, L_2) + dim ker D + 2d + dim L_1 \cap L_2 - dim Ker D_1 - dim Ker D_2$
mit
(1) $m(L_1, L_2) = \int_{Aut(V,I,<,>)} \tau(kL, L_1, L_2) dh$, τ – Maslovindex
(2) $d \in \mathbb{Z}$ – ein Index

Sei $L_{M_i} = \{pr_V \varphi|_N | \varphi \in ker D_i, \varphi \mid N \in E_{D_N}(-\infty, 0)\}$. L_{M_i} ist Lagrangsch. Sei $\overline{D}_i$ der Operator auf der Verlängerung von M_i zu einer vollständigen Mannigfaltigkeit durch Aufkleben eines unendlichen Zylinders. Unter den Regularitätsbedingung R : $L^2 ker \overline{D}_i = \{0\}$, $L_{M_1} \cap L_{M_2} = \{0\}$ gilt: Definiert man die selbstadjungierten Erweiterungen D_1 mit L_{M_2} und D_2 mit L_{M_1} und ist der zylindrische Teil von M lang genug, so gilt

Theorem: $\eta(D) - \eta(D_1) - \eta(D_2) = m(L_1, L_2)$.

H. PESCE

Representation theory and isospectral manifolds

Since Milnor's famous example of flat tori which are isospectral but non- isometric, one knows that the spectrum of a closed riemannian manifold doesn't determine the manifold. After this example, other examples arising from number theory were constructed (lower dimensional flat tori, hyperbolic manifolds ...).

The situation changed in a dramatic way in 1989 when Sunada gave a general method to construct isospectral manifolds. The situation he considers is the following: (M,m) is a closed Riemannian manifold, $H_1, H_2 \subset K \subset \mathrm{Isom}(M,m) = G$ and if the following condition "for each $k \in K, \#([k]_K \cap H_1) = \#([k]_K \cap H_2)$ where $[k]_K = \{\ell k \ell^{-1}; \ell \in K\}$" is satisfied then the manifolds $H_1 \backslash H$ and $H_2 \backslash M$ (with the induced metrics) are isospectral (in fact, they are isospectral for every natural operator: Hodge-De Rham, Lichnérowicz Laplacian ...).

In this talk, one shows that this condition is equivalent to the fact that the representations $\pi^K_{H_1}$ and $\pi^K_{H_2}$ are equivalent ($\pi^K_{H_4}$ is the representation induced by the trivial representation of H_i, i.e. $(\pi^K_{H_i}(g)\varphi)(s) = \varphi(sg)$ if $\varphi \in L^2_{\mathbb{C}}(H_i \backslash K)$) and one proves that Sunada's Theorem can be easily proved by the use of the classical Frobenius reciprocity theorem. Finally one looks at the converse of Sunada's theorem: assume that finite subgroups H_1 and H_2 of the isometry group G are such that $H_1 \backslash M$ and $H_2 \backslash M$ are isospectral, does it imply that the representations $\pi^G_{H_1}$ and $\pi^G_{H_2}$ are equivalent? The general answer to this question is no but one shows that it is "generically" true for small groups (i.e. $2 \dim \rho \leq \dim M$ for each irreducible representation ρ of G).

L. SCHWACHHÖFER

Connections with exotic Holonomy

R. Bryant gave the first examples of "exotic holonomies", i.e. holonomies of torsion free affine connections that do not occur on the classically known list compiled by Berger.

These holonomies are $\rho_3(S\ell(2,\mathbb{R})) \subseteq G\ell(\mathbb{R}^4)$ * and $\mathbb{R} \cdot \rho_3(S\ell(2,\mathbb{R})) \subseteq G\ell(\mathbb{R}^4)$. A subbundle $B \subseteq \mathcal{F}$ where $\mathcal{F}$ is the coframe bundle of a 4-manifold M^4 and where the fiber of B is one of the above representations will be called an H_3 and G_3-structure on M^4 resp.

The local existence of these structures was shown by Bryant using the method of exterior differential systems. We give some more global properties of these structures. For example, H_3-structures can be neither complete nor can they exist globally on closed manifolds. Moreover, we give descriptions of those H_3-structures which admit a 3- or 5-dimensional symmetry group:

The (unique) space with 5-dimensional symmetry group can be naturally identified with the space of parabolas in $\mathbb{R}^2$ acted on by $AS\ell(2,\mathbb{R})$, structures with 3-dimensional symmetry

* ρ_3 is the (unique) irreducible representation of $S\ell(2,\mathbb{R})$ in dim. 4.

groups are realized on certain $\mathbb{R}^2$-vector bundles over surfaces Σ^2, where Σ^2 carries some homogeneous bilinear form; e.g. we can choose

- $M^4 = T\mathbb{H}^2$
- $M^4 = T\Sigma^2$ with $\Sigma^2 = \{x^2 + y^2 - z^2 = 1\} \subset \mathbb{R}^3_{(2,1)}$ which carries a metric of sign. $(1,1)$
- $M^4 = T\Sigma^2$ where $\Sigma^2 = \mathbb{R}^2$ with the (degenerate) bilinear form $< x, y >= x_1 y_1$
- $M^4 = \mathrm{Taut}(\mathbb{C}^1\mathbb{P}^1)$ where $(\mathbb{C}\mathbb{P}^1)$ carries the metric with $K \equiv 1$.

The symmetry group of the connection is always the isometry group of the bilinear forms. Also, note that in the first and last example ($M = T\mathbb{H}^2$ or $M = \mathrm{Taut}(\mathbb{C}\mathbb{P}^1)$) the H_3-structure degenerates on the 0-sections of either bundle.

For G^3-structure, we note that the symmetry group can only have dimension four or less. Thus, any homogeneous G_3-structure must be a left-invariant structure on a 4-dimensional Lie group. These structures can be classified. The underlying Lie groups are $G\ell(2,\mathbb{R})$ and a certain 2-step nilpotent group. In particular, since both groups admit compact quotients there exist (locally homogeneous) G_3-structures on **compact** manifold, in contrast to the result for H_3-structures.

G. PATERNAIN

Geometric characterization of the topological entropy of the geodesic flow

Let M be a compact Riemannian manifold and for $(p,q) \in M \times M$ and $\lambda > 0$, let $n(p,q,\lambda)$ denote the number of geodesic segments between p and q with length $< \lambda$. If $(p,q) \in M \times M$ is generic, $n(p,q,\lambda)$ is finite and we consider here "geodesic entropy" defined as

$$h_{\mathrm{geod}} \stackrel{\mathrm{def}}{=} \limsup_{\lambda \to +\infty} \frac{1}{\lambda} \log \int_{M \times M} n(p,q,\lambda) dp dq.$$

Now let h_{top} denote the topological entropy of the geodesic flow on the unit sphere bundle. Then as a result of combined efforts of the author and Mañé we have:

Theorem $h_{\mathrm{geod}} = h_{\mathrm{top}}$ provided the metric is C^∞.

As a result for a generic metric close to the standard metric on S^2, the geodesic entropy is positive. As a concrete example take the following metric g on the ellipsoid

$$E, \frac{x^2}{a_1} + \frac{y^2}{a_2} + \frac{z^2}{a_3} = 1 : g = \frac{(1 - \epsilon x)}{a_1 a_2 a_3 (\frac{x^2}{a_1^2} + \frac{y^2}{a_1^2} + \frac{z^2}{a_3^2})} g_E$$

where g_E is the metric on E induced by the embedding $E \subset \mathbb{R}^3$, and ϵ sufficiently small $(a_3 > a_2 > a_1 > 0)$.

Gefördert durch DFG Deutsche Forschungsgemeinschaft

GANG TIAN

Cone structure of complete Ricci-flat manifolds at infinity

This is a joint work with J. Cheeger. Let M be a complete Ricci-flat manifold satisfying: 1) the volume growth of $B_r(p)$ is like er^n, where e is a constant, $n = \dim_R M$; 2) the curvature decays at quadratic order. Bando-Kasue-Nakajima proved that any sequence of pointed rescaled manifolds $(M, p, r_j^{-2} g)$ has a subsequence which converges to a metric cone $(M_\infty, p_\infty, g_\infty)$, where p is a fixed point of M, $\lim \Gamma_j = \infty$, g is a fixed Ricci-flat metric on M. The problem we concern is whether or not the cone $(M_\infty, p_\infty, g_\infty)$ is independent of choices of $\{\Gamma_j\}$, i.e., the uniqueness problem of the cone structure of M at infinity. We proved that if M is one of the manifolds given as above and some of tangent cones is integrable in the sense: the restriction of g_∞ to one cross-section is integrable, then the cone $(M_\infty, p_\infty, g_\infty)$ is unique, moreover, in this case, one has the following asymptotic estimate:

$$g = dr^2 + r^2 g_N + 0(r^{-a}) \quad \text{for some} \quad a > 0$$

where g_N is an Einstein metric on the cross-section which has distance one from p_∞. In case M is a Kähler Ricci-flat manifold, the cone M_∞ is also Kähler. We proved that if the dimension of holomorphic Killing fields on M_∞ is one, then M_∞ is integrable, consequently, M can be complex analytically compactified.

Le HONG VAN

Symplectic fixed joints, the Calabi invariant and Novikov homology

This is a joint work with K. Ono. We prove the following generalized Arnold conjecture.

Theorem: Let (M^{2n}, ω) be a closed symplectic manifold. Suppose that its first Chern class satisfies the following monotonicity condition:

$$c_1|_{\pi_2(M)} = \lambda[\omega]|_{\pi_2(M)}, \lambda \neq 0$$

moreover if $\lambda < 0$ then the minimal Chern number N is at least $n-3$. Then the number of fixed point of a symplectomorphism φ isotopic to the identity through symplectomorphism can be estimated from lower by the sum of the Betti number of the Novikov homology over $\mathbb{Z}_2$, corresponding to the Calabi invariant of φ.

It is well-known that symplectomorphism φ is exact if and only if its Calabi invariant $= 0$. In this case the Theorem (also Arnld conjecture) was partially proved by Conley-Zehnder (1983), Floer (1986 - 1988) Ono (1993). To prove the Theorem we construct the Floer-Novikov for φ and prove that

$$F-NH(\varphi)_k = \oplus_{j=k(mod 2)} \text{Nov} H([\varphi])$$

and obtain the Theorem. We also construct a non-trivial application of our Theorem (this means that our estimate is better than Leftschetz fixed point Theorem).

D. SCHÜTH

Isospectral Deformations

In 1991, the following result about isospectral deformations of 2-step nilmanifolds was proved independently by Hubert Pesce at He Ouyang:

If G is a 2-step nilpotent, simply connected Lie group, g^t a continuous family of left invariant metrics on it, and Γ a cocompact discrete subgroup of G, then isospectrality of the compact Riemannian manifolds $(\Gamma\backslash G, g^t)$ implies the existence of a continuous famil

$$\phi_t \in AIA(G,\Gamma) := \{\phi \in \mathrm{Aut}(G) | \forall\gamma \in \Gamma \exists a \in G : \phi(\gamma) = a\gamma a^{-1}\}$$

(almost inner automorphisms of G w.r.t. Γ) with $\phi_0 = \mathrm{id}$ and $g^t = \phi_t^* g_0 \forall t$. This result implies that compact quotients $(\Gamma\backslash H_m, g)$ of the classical Heisenberg groups H_m (where Γ is a discrete cocompact subgroup and g a H_m-left invariant metric) are infinitesimally spectrally rigid within the family of H_m-left invariant metrics, since there are no non-trivial almost inner automorphisms, i.e. $AIA(H_m;\Gamma) = \mathrm{Inn}(G)$. In contrast to this fact, we show that arbitrarily "near" to certain $\Gamma\backslash H_m$, there are nontrivially isospectrally deformable manifolds, namely:

Theorem: For every $m \geq 2$ there is a cocompact discrete subgroup Γ of H_m and a 2-parameter family g_α^t of metrics on $\Gamma\backslash H_m$ with the following properties: All the g_α^t are left invariant with respect to a certain solvable group structure on the manifold H_m, but only g_1^t is H_m-left invariant. For every fixed $\alpha \in (0,1), g_\alpha^t$ is a non-trivial continuous isospectral deformation (t ranges over $\mathbb{R}$); g_1^t is a trivial deformation. For every fixed $t \in \mathbb{R}, g_\alpha^t$ (where α ranges over $(0,1]$) is a continuous deformation, which is however not isospectral. It is an open question whether the Theorem of Pesce and Ouyang holds also in the n-step nilpotent or solvable case. We can prove the following weaker result:

Theorem: (a) If G is simply connected and solvable with only real roots, and if Γ_t is a continuous family of cocompact discrete subgroups of G s.t. all the quasi-regular repre sentations are unitarily equivalent, then there exists a continuous family $\phi_t \in AIA(G;\Gamma_0$ with $\phi_0 = \mathrm{Id}$ and $\Gamma_t = \phi_t(\Gamma_0)\forall t$.
(b) The same result holds if G is a 1-dimensional exponentially solvable extension of $\mathbb{R}^n$, if we assume that Γ_t is a C^1-family. Here, the resulting ϕ_t are even inner.

K. HABERMANN

Twistor spinors on Riemannian manifolds

We study n-dimensional Riemannian spin manifolds $(M^n, g), n \geq 3$, admitting non-trivial twistor spinors. A twistor spinor is a spinor field $\varphi \in \Gamma(S)$ satisfying the differential equation

$$\nabla_X \varphi + \frac{1}{n} X \cdot \mathcal{D}\varphi = 0$$

for all vector fields X, where $\mathcal{D}$ denotes the Dirac operator. A. Lichnerowicz introduced the twistor spinors as zeroes of the conformally invariant twistor operator $\mathcal{D}$ and started their systematical investigation.

Twistor spinors φ (together with $\mathcal{D}\varphi$) correspond to parallel sections in a certain bundle over M^n. Hence the space of all twistor spinors is finite-dimensional.

A special class of twistor spinors are Killing spinors φ defined by the equation $\nabla_X \varphi = \lambda X \cdot \varphi$ for all vector fields X and $\lambda \in \mathbb{C}$. If (M^n, g) admits a non-trivial Killing spinor, then (M^n, g) is an Einstein manifold. On connected manifolds Killing spinors have no zeroes, since they are parallel with respect to $\nabla_X - \lambda X$. On compact manifolds the space of all twistor spinors coincides - up to a conformal change of the Riemannian metric - with the space of all Killing spinors. Now, twistor spinors with zeroes are possible.

Th. Friedrich proved that the set of all zeroes of a twistor spinor is a discrete subset of M^n. Any twistor spinor defines a conformal deformation to an Einstein manifold (on the complement of the zeroset). If (M^n, g) is a complete connected Einstein manifold admitting a twistor spinor vanishing at some point then (M^n, g) is isometric either to the standard sphere, to the Euclidean space or to the hyperbolic space.

Furthermore, we give a sufficient condition for completeness of the metric (which is mentioned above) on the complement of the set of all zeroes. In addition to the conformal invariance of the twistor equation the existence of non-trivial twistor spinors forces properties of the conformal structure of the manifold. A twistor spinor defines a conformal vector field. Now Kühnel and Rademacher proved that a manifold with twistor spinor having a zero, such that the associated conformal vector field is non-vanishing, is conformally flat.

S. STOLZ

Manifolds of positive scalar curvature

Question: given a compact, smooth, closed manifold M^n, does it admit a Riemannian metric of positive scalar curvature?

For spin manifolds, the Weitzenböck formula implies that the kernel of the Dirac operator is trivial if the scalar curvature is positive. In particular, the index of the Dirac operator vanishes. For a spin manifold M^n with fundamental group π, Rosenberg constructed a Dirac operator whose index $\alpha(M)$ lives in $KO_n(C^*\pi)$, the real K-theory of the group C^*-algebra.

Conjecture (Gromov-Lawson, Rosenberg): A spin manifold M^n of dimension ≥ 5 has a positive scalar curvature metric iff $\alpha(M)$ vanishes.

Theorem (Stolz): Conjecture is true for simply connected manifolds.

This result follows from the Gromov/Lawson, Schoen/Yau surgery Theorem for positive scalar curvature metrics, and the following result which is proved using stable homotopy theory:

Theorem (Stolz): Let M^n be a spin manifold. Then $\alpha(M) = 0 \in KO_n(\mathbb{R})$ iff M is spin bordant to the total space of an $\mathbb{H}\mathbb{P}^2$-bundle.

Theorem (Rosenberg-Stolz): Conjecture true for $\pi_1 M \sim \mathbb{Z}_2$.

Theorem (Rosenberg-Stolz): The conjecture is stably true for finite $\pi_1 M$, i.e. if M^n is spin with $\pi = \pi_1 M$ finite, and $\alpha(M) \in KO_n(C^*\pi)$ vanishes, then $M \times \underbrace{b \times \ldots \times B}_{k}$ has a positive scalar curvature metric for some k, where the "Bott manifold" B^8 is a spin manifold with $\tilde{A}(M) = 1$.

C. PLAUT

Beginnings of Metric Geometry in Infinite Dimensions

An inner metric space X has curvature bounded below if every point lies in a region of curvature $\geq k$ for some k depending (possibly) on X. Our main example is topological groups:

Theorem: Every first countable locally compact group admits an invariant metric of curvature $\geq k$ for some k.

The proof uses the theory of locally compact groups of Yamabe. The basic theorem is an extension of the Hopf-Rinow theorem:

Theorem: For each $p \in X \exists$ dense G_δ subset J_p s.t. $\forall q \in J_p$, there is a unique, almost extendable minimal curve from p to q.

Almost extendable means there is a complementary direction in the space of directions at q. The first application is the global comparison theorem (Toponogov's Theorem) (proved independently by Burago, Perelman).

Theorem: If X has curve $\geq k$ then all of X is a region of curvature $\geq k$. The proof is by constructive induction when the comparison radius is bounded uniformly. From the uniform case the general case follows by

Theorem: If X has curv. ≥ 1, contains a spherical set of $2(n+1)$ elements, X contains an n-sphere. This is a generalization of Toponogov's maximal diameter theorem. We use it to find spheres in the space of directions. If the size of such spheres is bounded, the Hausdorff dimension is finite; i.e. the space is an Alexandrov space. If not, one can find separable infinite spheres at a dense G_δ set of points.

M. WEBER

Fundamental Domains for Picard-Mostow Complex Hyperbolic Surfaces

Consider Thurston's Description of these surfaces: Take 5 pts. $P_1, \ldots, P_5$ on an oriented S^2 and "curvatures" $\kappa_i \in (0, 2\pi), i = 1 \ldots 5$ with $\sum \kappa_i = 4\pi$. Set $\mathcal{S}(\kappa_1, \ldots, \kappa_s) = \{\text{metrics on } S^2\text{: flat on } S^2 - \{P_1 \ldots P_s\}\text{; cone-singularity around each } P_i \text{ with cone angle } = 2\pi - \kappa_i\text{; area} = 1\}/$ or. preserving diffeomorphisms of S^2 fixing each P_i.

For instance, Thurston proves: $\mathcal{S}(\frac{3\pi}{5}, \frac{3\pi}{5}, \frac{3\pi}{5}, \frac{3\pi}{5}, \frac{8\pi}{5}) =: \mathcal{S}$ is a complex compact hyperbolic orbifold of dimension 2. A fundamental domain is constructed as follows: Partition $\mathcal{S}$ into $\mathcal{S} = P_1 \cup \ldots \cup P_6$ and show that each P_i is isometric to a certain complex hyperbolic polyhedron, bounded by bisectors. The combinatorial structure of P_i can be described explicitly in terms of the κ_i. For instances, P_i has in the above example 8 faces which are solid tetrahedra. All these polyhedra can be described quite explicitly. This can be used to study submanifolds of the $\mathcal{S}(\kappa_i)$. One is the fixed point set of the mapping $S \mapsto \bar{S} \in \mathcal{S}$ which inverts the orientation of S. This turns out to be a $2-\dim_{\mathbb{R}}$ manifold, which is totally geodesic and non-orientable.

Berichterstatter: V. Cortés (Bonn)

Tagungsteilnehmer

Prof.Dr. Uwe Abresch
Mathematisches Institut
Universität Münster
Einsteinstr. 62

D-48149 Münster

Prof.Dr. Pierre Berard
Laboratoire de Mathématiques
Université de Grenoble I
Institut Fourier
Boîte Postale 74

F-38402 Saint Martin d'Heres Cedex

Prof.Dr. Ivan K. Babenko
Chair of Higher Geometry & Topology
Dept. of Mathematics and Mechanics
Moscow State University

Moscow 11 98 99
RUSSIA

Prof.Dr. Marcel Berger
IHES
Institut des Hautes Etudes
Scientifiques
35, Route de Chartres

F-91440 Bures-sur-Yvette

Prof.Dr. Christian Baer
Mathematisches Institut
Universität Bonn
Beringstr. 6

D-53115 Bonn

Prof.Dr. Jean Pierre Bourguignon
Centre de Mathématiques
Ecole Polytechnique
Plateau de Palaiseau

F-91128 Palaiseau Cedex

Prof.Dr. Werner Ballmann
Mathematisches Institut
Universität Bonn
Beringstr. 6

D-53115 Bonn

Prof.Dr. Jochen Brüning
Institut für Mathematik
Universität Augsburg

D-86135 Augsburg

Prof.Dr. Victor Bangert
Mathematisches Institut
Universität Freiburg
Hebelstr. 29

D-79104 Freiburg

Prof.Dr. Robert Bryant
Dept. of Mathematics
Duke University

Durham , NC 27706
USA

Ulrich Bunke
Institut für Reine Mathematik
Fachbereich Mathematik
Humboldt-Universität Berlin

D-10099 Berlin

Prof.Dr. Vicente Cortés
Mathematisches Institut
Universität Bonn
Beringstr. 6

D-53115 Bonn

Prof.Dr. Sergei V. Buyalo
St. Petersburg Branch of Steklov
Mathematical Institute - POMI
Russian Academy of Science
Fontanka 27

191011 St. Petersburg
RUSSIA

Prof.Dr. Jozef Dodziuk
Ph.D. program in Mathematics
Graduate Center (CUNY)
33 West 42nd Street

New York , NY 10036
USA

Prof.Dr. Eugenio Calabi
Department of Mathematics
University of Pennsylvania
209 South 33rd Street

Philadelphia , PA 19104-6395
USA

Prof.Dr. Patrick B. Eberlein
Dept. of Mathematics
University of North Carolina
Phillips Hall CB 3250

Chapel Hill , NC 27599-3250
USA

Prof.Dr. Toby Colding
Courant Institute
New York University
251 Mercer Street

New York , NY 10012
USA

Prof.Dr. Jost-Hinrich Eschenburg
Institut für Mathematik
Universität Augsburg

D-86135 Augsburg

Prof.Dr. Yves M. Colin de Verdière
Laboratoire de Mathematiques
Universite de Grenoble I
Institut Fourier
Boîte Postale 74

F-38402 Saint Martin d'Heres Cedex

Patrick Ghanaat
Mathematisches Institut
Universität Basel
Rheinsprung 21

CH-4051 Basel

Prof.Dr. William Mark Goldman
Department of Mathematics
University of Maryland

College Park , MD 20742
USA

Prof.Dr. Ursula Hamenstädt
Mathematisches Institut
Universität Bonn
Beringstr. 1

D-53115 Bonn

Prof.Dr. Detlef Gromoll
Department of Mathematics
State University of New York

Stony Brook , NY 11794-3651
USA

Prof.Dr. Ernst Heintze
Institut für Mathematik
Universität Augsburg

D-86135 Augsburg

Prof.Dr. Karsten Grove
Department of Mathematics
University of Maryland

College Park , MD 20742
USA

Prof.Dr. Hesheng Hu
Dept. of Mathematics
Fudan University

Shanghai 200433
CHINA

Prof.Dr. Chaohao Gu
University of Science and
Technology of China
Hefei

Anhui 230026
CHINA

Prof.Dr. Jürgen Jost
Institut f. Mathematik
Ruhr-Universität Bochum
Gebäude NA

D-44780 Bochum

Prof.Dr. Katharina Habermann
Institut f. Mathematik
Ruhr-Universität Bochum
Gebäude NA

D-44780 Bochum

Prof.Dr. Bruce Kleiner
School of Mathematics
Institute for Advanced Study

Princeton , NJ 08540
USA

Prof.Dr. Le Hong Van
Max-Planck-Institut für Mathematik
Gottfried-Claren-Str. 26

D-53225 Bonn

Dr. Xiao-Wei Peng
Institut f. Mathematik
Ruhr-Universität Bochum
Gebäude NA

D-44780 Bochum

Dr. Bernhard Leeb
Mathematisches Institut
Universität Bonn
Wegelerstr. 10

D-53115 Bonn

Prof.Dr. Hubert Pesce
Laboratoire de Mathematiques
Université de Grenoble I
Institut Fourier
Boite Postale 74

F-38402 Saint Martin d'Heres Cedex

Joachim Lohkamp
Max-Planck-Institut für Mathematik
Gottfried Claren Str. 26

D-53225 Bonn

Prof.Dr. Ulrich Pinkall
Fachbereich Mathematik
Technische Universität Berlin
Straße des 17. Juni 136

D-10623 Berlin

Prof.Dr. Maung Min-Oo
Department of Mathematics and
Statistics
Mc Master University
1280 Main Street West

Hamilton Ontario L8S 4K1
CANADA

Prof.Dr. Conrad Plaut
Dept. of Mathematics
University of Tennessee at
Knoxville
121 Ayres Hall

Knoxville , TN 37996-1300
USA

Dr. Gabriel Paternain
Max-Planck-Institut für Mathematik
Gottfried Claren Str. 26

D-53225 Bonn

Dr. Hans-Bert Rademacher
Institut für Mathematik
Universität Augsburg

D-86135 Augsburg

Prof.Dr. Ernst A. Ruh
Département de Mathématiques
Chemin du musée 23

CH-1700 Fribourg

Prof.Dr. Marcos Salvai
Mathematisches Institut
Universität Bonn
Beringstr. 6

D-53115 Bonn

Dorothee Schüth
Mathematisches Institut
Universität Bonn
Beringstr. 1

D-53115 Bonn

Lorenz Schwachhöfer
Dept. of Mathematics
Washington University
Campus Box 1146
One Brookings Drive

St. Louis , MO 63130-4899
USA

Dr. Stephan Stolz
Dept. of Mathematics
University of Notre Dame
Mail Distribution Center
P.O. Box 398

Notre Dame , IN 46556-5683
USA

Prof.Dr. Gudlaugur Thorbergsson
Dept. of Mathematics
University of Notre Dame
Mail Distribution Center
P.O. Box 398

Notre Dame , IN 46556-5683
USA

Prof.Dr. Gang Tian
Department of Mathematics
Princeton University
Fine Hall
Washington Road

Princeton , NJ 08544-1000
USA

Prof.Dr. Philippe Tondeur
Department of Mathematics
University of Illinois
273 Altgeld Hall MC-382
1409, West Green Street

Urbana , IL 61801-2975
USA

Matthias Weber
Mathematisches Institut
Universität Bonn
Beringstr. 6

D-53115 Bonn

Prof.Dr. Wolfgang Ziller
Department of Mathematics
University of Pennsylvania
209 South 33rd Street

Philadelphia , PA 19104-6395
USA