MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 25/1993

Funktionalanalysis und nichtlineare partielle Differentialgleichungen

30.5.93 bis 5.6.93

The meeting was organized by H. Amann (Zürich). The central theme of the conference was the interplay between functional analysis and nonlinear partial differential equations. It presented the opportunity of bringing together specialists of both areas so that all could learn of and discuss the latest developments in the field. Thirtysix lectures were given on a wide variety of topics, including the following: systems of reaction-diffusion equations, dynamical systems, parabolic equations, free boundary problems, Navier-Stokes equations, abstract evolution equations, and models from biology and physics.

We very much regret that Professor Peter Hess, who originally was the co-organizer of this meeting, is no longer with us. He died on November 29, 1992. It is only fitting that this "Tagungsbericht" should be dedicated to his memory.

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VORTRAGSAUSZÜGE

Stable transition layers for the periodic bistable equation with small diffusion

N.D. Alikakos, Knoxville

In this talk we consider the equation

$$u_t = \varepsilon^2 u_{xx} + u(1-u)(u - \alpha(x,t)) \quad , \quad 0 < x < 1$$

with Neumann boundary conditions, $\gamma(x,t)$ T-periodic in t and for convenience $\gamma(x,t)$ decreasing in x for each t. We are interested in those γ's for which the equations

$$\bar{\gamma}(x) = \frac{1}{2} \quad , \quad \bar{\gamma}(x) := \frac{1}{T}\int_0^T \gamma(x,t)\,dt$$

has a (unique necessarily) solution $x_0 \in (0,1)$. Under these hypotheses we prove Theorem (Joint work with Xiufu Chen and Peter Hess).

Let $\{u_\varepsilon\}$ be a family of nontrivial T-periodic solutions. Then

$$(1) \qquad u_\varepsilon \longrightarrow \begin{cases} 0 & x < x_0 \\ 1 & x > x_0 \end{cases}$$

Let (2) $\eta = \frac{x-x_0}{\varepsilon}$, $U_\varepsilon(\eta,t) = u_\varepsilon(x_0+\varepsilon\eta,t)$, then $U_\varepsilon(\eta,t) \longrightarrow \phi(\eta+V(t))$ as $\varepsilon \to 0$ where $\phi(s) = \frac{1}{1+e^{-s/\sqrt{2}}}$, $\dot{V}(t) = \sqrt{2}(\frac{1}{2} - \gamma(x_0,t))$ where the convergence in (1) is uniform away from x_0 , and the convergence in (2) is unform over compacts. The main point in this work is the solution of the "stretched" equation about $\bar{x}$: $U_t = U_{\eta\eta} + U(1-U)(U-\gamma(\bar{x},t))$, $\eta \in \mathbb{R}$, $t \geq 0$. The existence-uniqueness or nonexistence of travelling waves is fullly settled. Regretably in our considerations the specific nonlinearity is of paramount importance. Existence of nontrivial solutions presently is established under the (presumably unnecessary) extra assumption that $\gamma(0,t) > \frac{1}{2}$, $\gamma(1,t) < \frac{1}{2}$ for all $t \in [0,t]$. A modification of γ away from x_0 to meet this requirement should not be essential. This work together with previous work of the author with P. Hess and work of Dancer and Hess on $U_t = \varepsilon^2 \Delta u + m(x,t)\,u(1-u)$ strongly suggests that the transition of locations of periodic solutions of $u_t = \varepsilon^2\Delta u + f(u,x,t)$, T-periodic in t , are determined by $\bar{f}(x,u) = \frac{1}{T}\int_0^T f(x,u,t)\,dt$.

Elliptic equations in L^1

Philippe Bénilan, Besançon

We consider the problem

$$(E) \qquad -div\, a(x, Du) = f \text{ on } \Omega\ ,\ u = 0 \text{ on } \partial\Omega$$

where $a(x,\xi)$ is measurable in $x \in \Omega \subset \mathbf{R}^N$, continuous strictly monotone in $\xi \in \mathbf{R}^N$ and satisfies the Leray-Lions condition for some $1 < p < \infty$:

$$a(x,\xi)\cdot\xi \geq \lambda|\xi|^2\ , \quad |a(x,\xi)| \leq a_0(x) + \Lambda|\xi|^{p-1}\ .\ a_0 \in L^{p'}(\Omega)$$

We want to solve (E) for $f \in L^1(\Omega)$. For simplicity we assume her Ω bounded. In the linear case $(p = 2, a(x,\xi)$ linear in $\xi)$ it is classical by duality of Di Giorgi-Nash theorem, that for any $f \in L^1(\Omega)$, then musts

(1)

$$u \in \cap_{1 \leq q < N(p-1)/N-1} W_0^{1,q}(\Omega) \quad , \quad -div\, a(x, Du) = f \in D'(\Omega)$$

But uniqueness of a solution u for (1) seems an open problem for $N \geq 3$, Ω non smooth or $a(x,\xi)$ non continuous. In the nonlinear case, for $p > 2 - \frac{1}{N}$ $(i.e. \frac{N(p-1)}{N-1} > 1)$, the same existence result has been proved by Boccardo-Gallout: uniqueness is a forteriori open for $N \geq p$. If $p \leq 2 - \frac{1}{N}$, one can easily see that there exists $f \in L^1(\Omega)$ such that the problem $u \in W_1^{1,1}(\Omega)$, $-diva(x, Du) = f \text{ in } D'(\Omega)$ has no solution. We give an uniqueness-existence result for (E) in the general case by introducing the class

$$T_0^{1,p}(\Omega) = \{u : \Omega \to \mathbf{R} \text{ measurable } T_k(u) \in W_0^{1,p}(\Omega) \text{ for any } k > 0\}$$

when $T_k(\Gamma) =$ (sign Γ) min $(|\Gamma|, k)$ in the classical turnover. For $u \in T_0^{1,p}(\Omega)$ there exists a unique $h : \Omega \to \mathbf{R}^N$ measurable such that $DT_k(u) = h\chi_{\{|u|<k\}}$ for any $k > 0$; furthermore $h \in L^1_{loc}(\Omega)$ iff $u \in W^{1,1}_{loc}(\Omega)$ and then $h = Du$. For general $u \in T_0^{1,p}(\Omega)$, we will call h the generalized gradient of u and denote it by Du. We prove in the general case

Theorem. For any $f \in L'(\Omega)$ there exists a unique solution $u \in T_0^{1,p}(\Omega)$ such that

$$\int_{|u-v|<k} a(\cdot, Du)\cdot(Du - Dv) \leq \int fT_h(u-v)$$

for any $v \in D(\Omega)$, $k > 0$.

Furthermore $a(\cdot, Du) \in L^1(\Omega)$ and $-diva(\cdot, Du) = f \in D'(\Omega)$. These results are joint wirk with Boccardo, Gallout, Gaziepy, Pierre and Vazquez; we actually handle unbounded open set Ω and f depending on u.

Multi-spike solutions to a singularity perturbated bistable elliptic equation

P.W. Bates, Provo

We consider

$$(1) \begin{cases} \Delta u - \lambda g(u) & = & 0 \in \Omega, \Omega \quad \text{bounded open in } \mathbf{R}^n, \lambda > 0 \\ \partial u/\partial n & = & 0 \text{ on } \partial\Omega \end{cases}$$

g is bistable, for example $g(u) = u^3 - u + c$ for some constant c. We are particularly interested in solutions which have interior spikes, i.e. $u(x) \to m$, a constant, except at points $\xi^1, \cdots, \xi^N \in \Omega$ where $u \to m_1 \neq m$ as $\lambda \to \infty$. We actually consider the more general problem of finding equilibria to the Cahn-Hilliard equation which have prescribed average value and N spikes:

$$(\text{CH}) \quad \begin{cases} u_t & = & -\Delta(\varepsilon^2 \Delta u - f(u)) & \text{in } \Omega \\ \partial u/\partial n & = & 0 = \partial \Delta u/\partial n & \text{on } \partial\Omega \end{cases}$$

where f is balanced bistable nonlinearity, e.g. $f(u) = u^3 - u$. Equilibria of (CH) satisfy (1) with $g = f +$ constant and $\lambda = 1/\varepsilon^2$. Since (CH) conserves the integral of u, we seek solutions with a fixed average value, m. We consider m, in the metastable region: $f'(m) > 0$, m between the extreme zeros of f. The main idea is to construct a manifold of N-spike functions, the manifold parameterized by the spike locations, $\xi^1, \cdots, \xi^N$ and then show the existence of an "almost invariant" manifold, M, as a graph over the first. The flow on M is such that equilibria of that finite dimensional flow correspond to equilibria for (CH). A careful estimate of the flow on M allows one to deduce the existence of equilibria and to give the asymptotic location of the spikes. These locations are related to a sphere-packing problem. (In joint work with Paul Fife and Giorgio Fusco.)

Well-posedness of the dynamic von Kármán equations

M. Böhm, Berlin

The time dependent evolution of the vertical displacement u of a clamped plate and the associated AIRY stress function Ψ are governed by the (nonlinear) von Kármán (vK) equations. By means of nonlinear interpolation methods, applied to appropriate regularizations of the initial-boundary value problem of the vK-equations, and a suitable limiting process we show that the map

$$\begin{array}{rcl} (\text{initial values } u(0), u_t(0)) & \mapsto & \text{solution } u \text{ of } vK \\ \overset{\circ}{H}{}^2 \cap H^{2+s} \times \overset{\circ}{H}{}^s & \mapsto & C_{loc}([0,\infty); H^{2+s}) \end{array}$$

exists and that it is Hölder-continuous for $s \in (0, 2]$.

Problems involving undefinite weight function

K.J. Brown, Edinburgh

The problem $-\Delta u = \lambda g(x) f(u)$ for $x \in \mathbf{R}^n$, $0 < u < 1$ where g changes sign on $\mathbf{R}^n$ with $g(x) < 0$ for large $|x|$ and $f(u) > 0$ for $0 < u < 1$ with $f(0) = 0 = f(1)$ arises in population genetics. The existence of solutions for various values of λ can be proved by the method of sub and supersolutions: this existence theory is very different in the two cases $n \geq 3$ and $n = 1, 2$. Supersolutions can be contructed for all $\lambda > 0$ and all n; if $n \geq 3$ these supersolutions $\rightarrow 0$ as $|x| \rightarrow \infty$. Subsolutions can be constructed provided

$$\lambda > \lambda^* = \inf \{ \frac{\int_{\mathbf{R}^n} |\nabla u|^2 dx}{\int_{\mathbf{R}^n} gu^2 ax} : u \in H_1^0(\mathbf{R}^n) \text{ and } \int_{\mathbf{R}^n} gu^2 dx > 0 \}.$$

If $n = 1, 2$ and $\int_{\mathbf{R}^n} g dx > 0$, $\lambda^* = 0$ and so there exist solutions for all $\lambda > 0$; these solutions $\rightarrow 0$ as $|x| \rightarrow \infty$ if and only if $g(x)$ does not approach 0 too fast as $|x| \rightarrow \infty$. If $\int_{\mathbf{R}^n} g dx < 0$ or $|g(x)| = 0(|x|^{-2-\epsilon})$ and $n \geq 3$ an inequality of the form

$$\int_{\mathbf{R}^n} |\nabla u|^2 dx \geq \alpha \int_{\mathbf{R}^n} gu^2 dx \; \forall u \in C_0^\infty(\mathbf{R}^n)$$

holds; this inegality implies that $\lambda^* > 0$ and may be used to establish nonexistence results when λ is small. Existence results can also be obtained by proving the bifurcation of solutions from the zero solution; in order to do so the existence of a principal eigenvalue for the corresponding linear problem is first investigated. An appropriate space in which to consider these problems is the closure of the $C_0^\infty(\mathbf{R}^n)$ functions in the norm $\int_{\mathbf{R}^n} |\nabla u|^2 dx - \frac{\alpha}{2} \int_{\mathbf{R}^n} gu^2 dx$.

Elastic deformation of a membrane by a rolling ball

M. Chipot, Metz

Let (φ, h) be the positive of the center of a ball of radius r sitting on an elastic membrane $\Omega \subset \mathbf{R}^2$ ($(\varphi, h) \in \mathbf{R}^2 \times \mathbf{R}$). When dist $(\varphi, \partial\Omega) > r$ the energy of the configuration is

$$E(\varphi, h) = \frac{1}{2} \int_\Omega |\nabla u|^2 - Gh$$

where G is the weight of the ball, u the solution to the obstacle problem

$$\begin{cases} u \in K = \{v \in H_2^1(\Omega) : v(x) \leq h - \sqrt{r^2 - |x - \varphi|^2} \text{ on } B(\varphi, r), x = (0, y)\} \\ u \text{ minimizes } \frac{1}{2} \int_\Omega |\nabla v|^2 \quad \text{on } K. \end{cases}$$

We show that there exists an equilibrium positive (φ, h) minimizing E among the (φ, h) such that dist $(\varphi, \partial\Omega) > r$, $h \in \mathbf{R}$. (Remark that this set is not compact). We show in particular that E decreases when φ moves towards the interior of Ω in certain directions. (Joint work with J. Bemelmans - RWTH Aachen.)

Population models with diffusion and the effect of domain shape on the number of positive solustions

E.N. Dancer, Armidale

In this talk, we discuss the existence uniqueness and stability of positive solutions of the competing species system

$$\begin{aligned} -\Delta u &= u(a-u-cv) \\ -\Delta v &= v(d-v-eu) && \text{in } \Omega \\ u &= v = 0 && \text{on } \partial\Omega \end{aligned}$$

where Ω is a smooth bounded domain in $\mathbf{R}^n$. Here by stability we mean stability for the corresponding parabolic system. We include discussion of the case of large interactions and we discuss briefly the corresponding problem for 3 or more equations.

Eigenvalue problems on $\mathbf{R}^N$

D. Daners, Zürich

We shall be concerned with the stability of the zero solution of the linear parabolic problem

$$(1) \begin{cases} \partial_t u - \Delta u = \lambda m u & \text{on } \mathbf{R}^N \times (0,\infty) \\ u(\cdot,0) = u_0 & \text{on } \mathbf{R}^N \\ \lim_{|x|\to\infty} u(x) = 0, \end{cases}$$

where $\lambda > 0$ is a parameter, m some weight function and u_0 an initial condition. The stability is understood as stability in the $\|\cdot\|_\infty$-norm. The question of stability of the zero solution is closely related to the existence of a principal eigenvalue for the elliptic eigenvalue problem

$$(2) \begin{cases} -\Delta\varphi = \lambda m\varphi & \text{on } \mathbf{R}^N \\ \lim_{|x|\to\infty} \varphi(x) = 0. \end{cases}$$

By a principal eigenvalue we mean a $\lambda > 0$ such that (2) has a positive solution φ. The function φ is then called principal eigenfunction. Theorem. Let m be Hölder continuous having compact support and being radially symmetric. Moreover, suppose that $N \geq 3$. Then, there exists a unique principal eigenvalue $\lambda_1 > 0$ for (2) and the zero solution of (1) is asymptotically stable with respect to initial values in

$$C_0(\mathbf{R}^N) := \{u \in C(\mathbf{R}^N) : \lim_{|x|\to\infty} u(x) = 0\}$$

if $0 \leq \lambda < \lambda_1$ and unstable if $\lambda > \lambda_1$. We give a simple proof using only the corresponding result in bounded domains and easy comparison arguments.
(Joint work with K.J. Brown, Edinburgh and J. López-Gómez, Madrid.)

Differentiability of semigroups generated by semilinear operators

J.R. Dorroh, Baton Rouge

Two notions of differentiability in abstract Banach spaces are given: α-differentiability and β-differentiability. Under suitable conditions an evolution system generated by an α-differentiable operator is β-differentiable. There is a "good calculus" for β-differentiability (chain rule, etc.). Sufficiently smooth partial differential operators are α-differentiable if the Banach spaces are chosen approximately. These notions are applied to semigroups generated by semilinear operators. The differentiability notions involve a pair of Banach spaces, one densely and continuously embedded in the other.
(Joint work with S. Oharu.)

On a free boundary problem in porous media

J. Escher, Besançon

We consider a standard model for the motion of a fluid in a fully saturated porous medium. The corresponding mathematical formulation leads to a free boundary problem for the surface separating the dry and the wet region. This free boundary problem is reduced to a quasilinear equation for the unknown defining the free boundary. We prove the existence of a unique maximal classical solution of this problem. This result improve considerably earlier results due to Kawavada and Koshigoe who proved the existence of a local weak solution.

Life after life for quasilinear parabolic equations

M. Fila, Ames

We consider two classes of parabolic equations with superquadratic growth in the gradient. For one class derivative blowup (without L^∞-blowup) occurs on the boundary and for the other one in the interior. We describe the profile of the solution at the blowup time and show that it is possible to extend the solution beyond the blowup time.
(Joint works with G. Liebermann and S. Angenent.)

Method of sub- and super-solutions for some elliptic systems involving the p-Laplacian

J. Fleckinger, Toulouse

Consider the system:

$$(S)\quad \begin{cases} -\Delta_p u_i &= \sum a_{ij}|u_j|^{p-2}u_j + f_i \quad \text{in } \Omega \\ u_i &= 0 \quad \text{on } \partial\Omega, \end{cases}$$

where Ω is a smooth and bounded domain in $\mathbf{R}^N$, Δ_p the p-Laplacian is defined by $\Delta_p u := div(|\nabla u|^{p-2}\nabla u)$, $p > 1$, and where the coefficients $a_{ij}(1 \le i,j \le n)$ are constant and $a_{ij} \ge 0$ for $i \ne j$ (cooperative systems). We prove necessary and sufficient condition for the Maximum Principle to some cooperative quasi-linear elliptic systems involving the p-Laplacian. We also show that under the same conditions, we have existence of solutions for any $f_i \in L^{p'}$. The same condition appears for a cooperative system defined on an unbounded domain Ω :

$$\begin{array}{llll} (T.a) & (-\Delta + q)u = a\rho u + b\rho v + f & x \in \Omega & \\ (T.b) & (-\Delta + q)v = c\rho u + d\rho v + g & x \in \Omega & \\ (T.c) & \lim_{|x|\to+\infty} u(x) = \lim_{|x|\to+\infty} v(x) = 0, & x \in \Omega; & u(x) = v(x) = 0 \quad x \in \partial\Omega. \end{array}$$

q is a non negative function, ρ is a positive function, and ρ/q tends to 0 at infinity. (Survey of joint works with L. Boccardo, J. Hernandez, R. Manasevich, F. de Thélin.) (Joint work with L. Cardoulis, A. Djellit, H. Serag.)

Motion of a graph by nonsmooth weighted curvature

Y. Giga, Sapporo

Geometric evolutions of curves represented by graphs are studied when the interface energy is not necessarily smooth. The resulting equation is of the form

$$u_t = g(u_x)_x$$

with nondecreasing g but not necessarily continuous. We adapt the theory fo nonlinear semigroups to formulate the problem and prove the existence of global solution for Lipschitz initial data. To avoid technical difficulty we impose periodic boundary condition. We also calculate the generator of the evolution when interface energy is crystalline. It turns out that usual adhoc evolution law for crystalline interface energy is justified. Our theory applies to

(i) non admissible initial data and
(ii) nonsmooth energy not necessarily crystalline.

(Joint work with my student T. Fulani.)

Spectral theory and positive semigroups generated by differential operators

M. Hieber, Zürich

Let $\mathcal{A}_p$ be a differential operator with constant coefficients and maximal domain on $L^p(\mathbf{R}^n)$ $(1 < p < \infty)$. Assume that $\mathcal{A}_p$ generates a positive semigroup $(T_p(t))_{t\geq 0}$ on $L^p(\mathbf{R}^n)$. We show that in this case the spectral mapping theorem holds, i.e. we have $e^{t\sigma(\mathcal{A}_p)} = \sigma(T_p(t))\setminus\{0\}$ for all $t \geq 0$. Moreover, the spectrum $\sigma(\mathcal{A}_p)$ can be calculated explicitly and we see that in particular $\sigma(\mathcal{A}_p)$ is independent of p. Applications to the theory of bounded imaginary powers are given.

Invariant for fully nonlinear parabolic equations

H. Koch, Evanston/Heidelberg

Classical L^p estimates allow a direct approach to center manifolds for fully nonlinear parabolic equations, avoiding the problem of nonconstant domain of generators. It is crucial to cut off the equation and to work in large Banach spaces.

On the uniqueness of coexistence states for some two species reaction-diffusion systems

J. López-Gómez, Madrid

We consider the following reaction diffusion models

$$\left.\begin{array}{lcl} \frac{\partial u}{\partial t} - d_1\Delta u &=& (\lambda - au - bv)u \\ \frac{\partial v}{\partial t} - d_2\Delta v &=& (\mu \pm cu - dv)v \end{array}\right. \quad \Omega \times [0,\infty)$$

$$\begin{array}{lcl} u/\partial\Omega = v/\partial\Omega &=& 0, \quad t > 0, \\ u(x,0) = u_0(x) \geq 0, && v(x,0) = v_0(x) \geq 0, \end{array}$$

where $d_i > 0$, $i = 1,2, a > 0$, $b > 0$, $c > 0$, $d > 0$ and Ω is a bounded domain of $\mathbf{R}^N$, $N \geq 1$, with regular enough boundary. We obtain the following results: Predator-prey model (+): when $N = 1$, we show that the model has a unique coexistence state (which is stable if t is small enough). [Joint work with R. Pardo, Madrid.] When Ω is an arbitrary ball or annulus of $\mathbf{R}^N$ we show that the model has a unique radially symmetric coexistence state which is nondegenerate. [Joint work with E.N. Dancer, Armidale and R. Ortega, Granada.] Competition model (-): We characterize whether the model has a unique coexistence state. When this is the case the coexistence state is a global attractor, so describing the dynamics of the model.
(Joint work with R. Pardo, Madrid and J.C. Sabina, Tenerife.)

Regularity via general kernels and semilinear evolution equations with shocks

G. Lumer, Mons

We recall some recent results in modelling of periodic heat shocks (with specific applications to communication satellites); where ill-posed problems arise which can be handled using 2-times integrated solutions. In this and other situations one is in fact regularizing the Banach space equation $u' = Au + F(t)$, $u(0) = f$, $(in X)$, via the (convolution) kernels $K(t) = t^{n-1}/(n-1)!$ We develop a general regularization scheme which contains the regularization of the above kind and regularizations the C-semigroup type. Consider $t \mapsto K(t) \in \mathcal{B}(X)$ i.e. operator-valued kernels, strongly C^1, commuting and with $K(t)A = A\,K(t)$ on $D(A)$. Define $Z_k = \{f \in X : \exists v_k(t,f)$ classical solution of $v_k' = Av_k + K(t)f$, $v_k(0) = 0\}$ and $S_k(t)f = v_k'(t,f)$ for $f \in Z_k$. Then for f, $F(\cdot) \in Z_k$, the "variation of constants" formula holds under weak assumptions: $\exists$ a solution v_k of $v_k' = Av_k + K(t)f + F_k(t)$, $v_k(0) = 0$, $(F_k = K * F)$, and (though $S_k(\cdot)$, Z_k, need not be closed, Z_k need not be dence) $w(t) = v_k'$ is given by

$$w(t) = S_k(t)f + \int_0^t S_k(t-s)F(s)\,ds\,.$$

Many properties can be proved at this very general level; also with some appropriate additional assumptions, setting $K(0) = C$, $K(t) = C + K_0(t)$, one shows that:

$$S_k(t) : Z_k \cap Z_c \to Z_k \cap Z_c\,, \text{ and}$$

$$S_k(s)S_k(t) = \int_t^{t+s} S_k(r)K'(t+s-r)\,dr - \int_0^s S_k(r)\,K'(t+s-r)\,dr$$

$$+ \int_0^s S_0(r)CK'(t+s-r)\,dr \;+\; S_0(s)CS_{K-c}(t) \;+\; C\,S_C(t+s)\,,\ \text{on } Z_k \cap Z_c\,.$$

One of the main question now is classify the Kernels K_t, and comparing the difference K_t, concerning "regularizing strength".

Floquet theory and center manifolds for elliptic PDEs

A. Mielke, Hannover

We consider elliptic problems in a cylinder with periodic dependence on the axial variable. Studying the linearized problem leads to a Floquet theory which is more complicated than in parabolic problems. In fact, systems with nondiscrete Floquet spectrum exist. We summarize this theory and give examples where this behavior appears or can be excluded. The bifurcation of nonperiodic solutions from periodic ones is then reduced by the help of a center manifold which carries the a time-periodic flow. Thus, subharmonic branching as well as the existence of solutions homoclinic to a periodic solution can be established.

Generic asymptotic properties in strongly monotone discrete-time dynamical systems

J. Mierczyński, Wroclaw

We investigate the asymptotic behavior of a generic point in strongly monotone discrete-time dynamical systems. Such systems are generated, among others, by second order parabolic partial differential equations for which the strong maximum principle holds, and by some weakly coupled systems of such equations. The main tool used are the so-called p-arcs /p for positive/, that is, totally ordered invariant compact sets diffeomorphic to the real interval. The results improve on and follow up those recently obtained by P. Hess, P. Poláčik, P. Takáč, I. Tereščák, and others.

On stability of exterior stationary Navier-Stokes flows

T. Miyakawa, Fukuoka

Stability property is discussed for stationary solutions of the Navier-Stokes equations in three-dimensional exterior domains, under the assumption that the fluid velocity vanishes at $x = \infty$. First, it is shown that the derivatives of the stationary flow belong to the space $L^\infty \cap L^{3/2}_w$. Using this fact, as well as the well-known properties of the stationary flow, it is then shown that the perturbation tends to 0 as $t \to \infty$ in L^2 and L^∞, with a definite rate, provided that the stationary flow is small enough.

Singular perturbations of quasilinear parabolic equations

B. Najman, Bellingham/Zagreb

Let $A(t,u)$, $B(t,u)$ be uniformly elliptic operators of order $2m$ and $2m'$, $m > m'$. Let $A_\varepsilon(t,u) = \varepsilon A(t,u) + B(t,u)$. The Cauchy-Dirichlet problem

$$\begin{aligned} \frac{\partial u_\varepsilon}{\partial t} + A_\varepsilon(t,u_\varepsilon)u_\varepsilon &= f(t,u_\varepsilon) \\ u_\varepsilon(0) &= u_{0\varepsilon} \end{aligned} \qquad 1 > \varepsilon \geq 0$$

on a domain Ω is considered. Under appropriate conditions on the data it is shown that there exist a solution $u_\varepsilon \in C(\bar{J}, H^{q,p}) \cap C'(J, H^{q-2m'})$ on a common interval J with $q < m' + \frac{1}{p}$, p sufficiently large. Moreover the solutions u_ε converge to u_0, uniformly in $H^{q,p}$ norm: for every $\delta > 0$ there exists $C_\delta > 0$ such that

$$\|u_\varepsilon - u_0\|_{c(J,H^{q,p}} \leq C_\delta(\varepsilon^{h(q)-\delta} + \|u_{0\varepsilon} - u_{00}\|_{H^{q,p}})$$

$$\text{where } (2m-2m')h(q) = \begin{cases} 1 & q \leq m' + \frac{1}{p} - 1 \\ m' + \frac{1}{p} - q & m' + \frac{1}{p} - 1 < q < m' + \frac{1}{p} . \end{cases}$$

Partial differential equations, dynamical systems

P. Poláčik

Parabolic problems of the form

$$(1) \qquad \begin{aligned} u_t &= \Delta u + f(t,u,x), \quad x \in \Omega \subset \mathbf{R}^N, \\ u|_{\partial\Omega} &= 0 , \end{aligned}$$

with f τ-periodic in t will be considered. It is known that a typical solution of such a problem converges as $t \to \infty$ to a periodic solution (of period possibly bigger than τ). Other solutions, however, may exhibit very complicated behavior. Results on high dimensional ω-limites and chaos in (1) will be given. Then for a special class of problems, $f = f(t,u)$, $\Omega - a$ ball in $\mathbf{R}^N$ it will be shown that all nonnegative bounded solution converge to a radially symmetric τ-periodic solution.

Stationary solutions via dynamical methods

P. Quittner, Bratislava

Using dynamical methods we prove existence and multiplicity results for positive solutions of the boundary value problem $\Delta u = |\nabla u|^2 - \lambda u^p$ in Ω, $u = 0$ on $\partial\Omega$, where Ω is a bounded domain in $\mathbf{R}^N$, $2 \geq p > 1$ and $\lambda > 0$.

Elliptic equations in $\mathbf{R}^2$ with nonlinearities in the critical range

B. Ruf, Milano

In two dimensions the notion of critical growth of nonlinearities is connected with the inequality of Trudinger and Moser, which states for an $H_0^1(\Omega)$ function u the integral $\int_\Omega exp(u^2)dx$ is finite. Sufficient conditions for the solvability of equations with nonlinearities which have subcritical and critical growth are provided. The proofs rely on global variational methods. In the application of these methods one encounters the problem of "lack of compactness"; the mentioned sufficiency conditions ensure that the critical points of the functional are in the level of compactness.

Stabilität von Raum-periodischen Gleichgewichtslösungen von Reaktions-Diffusionsgleichungen gegen L^2-Störungen

B. Scarpellini, Basel

Gegeben ist ein Reaktions-Diffusionssystem $(*)u_t = D\Delta u + F(u)$, $D = (\delta_{jk}r_k)$, $r_k > c; j,k \leq n$ und $F(u) = (f_1(u), \cdots f_n(u))$, eine Nichtlinearität die polynomial ist in $u = (u_1; \cdots, u_n); \Delta$ ist der Laplace auf $\mathbf{R}^m, m \leq 3$. Es wird angenommen, eine hinreichend glatte Gleichgewichtslösung $v = (v_1, :, v_n)$ von $(*)$ die bezüglich der Raumvariablen $x_1, \cdots, x_m$ L-periodisch sei; $D\Delta v + F(v) = c$. Es wird die Stabilität von v gegenüber glatten L^2-Störungen untersucht, d.h. Störungen aus $(H^2(\mathbf{R}^m))^n$. Es wird gezeigt: Theorem: Ist v periodisch instabil, so instabil gegen glatten L^2-Störungen. Wichtigstes Hilfsmittel: direkte Integrale.

The periodic parabolic logistic equation on $\mathbf{R}^N$

G. Schätti, Zürich

We consider the time-periodic version of the diffusive logistic equation of population dynamics on $\mathbf{R}^N (N \geq 1)$:

$$\begin{cases} \partial_t u - \Delta u &= u(a(x,t) - b(x,t)u) \quad \text{on } \mathbf{R}^N \times (0,\infty) \\ u(\cdot,0) &= u_0 \quad \text{on } \mathbf{R}^N . \end{cases}$$

The functions a and b are assumed to belong to the space $BUC^{\mu,\frac{\mu}{2}}(\mathbf{R}^n x\mathbf{R})$, $\mu \in (0,1)$, and to be periodic in time t with a given period $T > 0$. Moreover, b is everywhere positive and a is for large $|x|$ negative and bounded away from zero. Depending on the stability of the zero solution we give a complete description of the asymptotic behavior for nonnegative initial values in $BUC(\mathbf{R}^N)$. In particular, the existence of a unique nontrivial positive T-periodic solution is proved, provided the zero solution is linearly unstable.

Sobolev spaces of fractional order and superposition operators

W. Sickel, Jena

We consider the superposition operator $T_g : f \to G(f)$ with respect to Sobolev spaces of fractional order.

$$\text{Let } W_p^s = \begin{cases} W_p^m & s = m; m \text{ integer} \quad \text{(Sobolev spaces)} \\ B_{p,p}^s & s \neq \text{integer} \quad \text{(Besov-Slovodeckij spaces)} \end{cases}$$

We ask for optimal conditions on G to guarantee an embedding $T_G(W_p^s) \subset W_p^s$. A partly positive answer is given. We prove (in case $0 < s < \mu + \frac{1}{p}$, $\mu > 1$)

$$\| \, |f|^\mu \, |W_p^s \| \leq c \|f|W_p^s\| \, \|f|L_\infty\|^{\mu-1}$$

for all $f \in W_p^s \cap L_\infty$, where c does not depend on f.

Bounded imaginary powers of elliptic operators

G. Simonett, Los Angeles

We consider systems of elliptic differential operators on $\mathbf{R}^n$. Using results on Fourier multipliers and pseudodifferential operators, we prove the existence of bounded imaginary powers, provided the coefficients satisfy some (weak) regularity assumptions. (Joint work with H. Amann and M. Hieber.)

Elliptic equations in infinite cylinders

J. Solà-Morales, Barcelona

Let Ω' be a bounded domain in $\mathbf{R}^n$ and $\Omega = (s_o, \infty) \times \Omega'$. Let us consider the following problem

$$\begin{aligned} &\sum_{i,j=0}^{n} a_{ij}(x)\, U_{x_i x_j} + f(\nabla u, u, x) = 0 \quad \text{in} \quad \Omega \\ &u(x) = 0 \quad \text{for} \quad (x_1, x_2, \cdots, x_n) \in \partial\Omega' \\ &u(x) = \varphi(x_1, x_2, \cdots, x_n) \quad \text{for} \quad x_0 = s_0 \,, \ \varphi \in C_0^0(\bar{\Omega}') \\ &|u(x)| \ \text{uniformly bounded.} \end{aligned}$$

We claim that under appropriate smoothness assumptions on $\partial\Omega'$ and on the functions $a_{ij}(x)$ and $f(p, z, x)$, the previous problem is a well posed problem, as an initial value problem in $C_0^0(\bar{\Omega}')$ provided that

(i) $zf(0,z,x) < 0$ for $|z| \geq M_0$ and some $M_0 > 0$

(ii) $|f(p,z,x)| = 0(|p|^2)$ uniformly on $x \in \bar{\Omega}$ and bounded z

(iii) there exists an $\varepsilon > 0$ such that

$$\varepsilon^2 a_{00}(x) + \frac{\partial f}{\partial p_0}(p,z,x) + \frac{\partial f}{\partial z}(p,z,x) \leq 0 \;\forall\, (p,z,x).$$

(Joint work with A. Calsina and M. València.)

The equation of a vibrating plate

A. Stahel, Biel

In a smooth bounded domain $\Omega \in \mathbf{R}$ we consider the scalar function u, which describes the vertical deflection of a vibrating plate. A mathematical model of this physical system is given by the dynamic von Kàrmàn equations

$$u_{tt} + \Delta^2 u = -[u, \Delta^{-2}[u,u]]$$

with appropriate boundary and initial conditions, and

$$[u,v] = u_{xx}\, v_{yy} + u_{yy}\, v_{xx} - 2u_{xy}\, v_{xy}$$

Conservation of energy implies that $\|u(t)\|_{W^{2,2}}$ is globally (with respect to t) bounded. We show that $\|v\|_{W^{4,2}}$ grows at most exponentially. Using a Galerkin approximation we show that the problem admitts unique, global classical solutions for appropriate initial values.
(Joint work with H. Koch.)

Large-time behavior of monotone discrete-time dynamical systems

P. Takáĉ, Nashville

Typical examples of strongly monotone dynamical systems are those generated by (1) a single parabolic PDE; (2) an irreducible cooperative system of ODE's; and (3) an irreducible cooperative system of weakly coupled parabolic PDE's. If such an evolution equation is periodic in time, the corresponding period map T generates a discret-time dynamical system $\{T^n : n \geq 0\, integer\}$ in a subset X of a strongly ordered Banach space V. The mapping T is strongly monotone, i.e.

$$0 \neq y - x \geq 0 \implies Ty - Tx \in Int(V_+)\; \forall\, x,\, y \in X,$$

where $Int(V_+)$ denotes the interior of $V_+ = \{v \in V : v \geq 0\}$ in V. Using only the monotonicity and differentiability of T and the compactness of all trajectories we will show that almost all trajectories are stable and approach a cycle. We give a full description of the set of all stable (unstable, resp.) points. The set of all unstable point is the union of at most countably many Lipschitz hypersurfaces of codimension one in V and hence, it has zero Gaussian measure. Under additional hypotheses on T we obtain that every trajectory converges to a single point. However, if these hypotheses are dropped, asymptotically stable cycles can occur. We give a few examples of such cycles.

Blow up for some degenerate parabolic equations

M. Wiegner, Bayreuth

We consider for $p > 2$ degenerate parabolic equations of the (model-) type

$$u_t = u^p(\Delta u + u) \quad \text{on} \quad \Omega \times (0, T)$$

$u = 0$ on $\partial\Omega \times (0, T)$ and $u(x, 0) = \varphi(x) > 0$. If the domain is large (precisely if the first eigenvalue $\lambda_1(\Omega) < 1$) then we have blow up after some finite time and the life span can be estimated. If $\lambda_1(\Omega) > 1$, then the solution exists for all times and

$$\lim_{t\to\infty} u(x,t)(pt)^{\frac{1}{p}} = W(x) \in C_{2/p}(\bar{\Omega}) \text{ with } \Delta W + W = -W^{1-p}.$$

Abstract evolution equations and its applications

A. Yagi, Himeij

We shall adopt an approach of using the semigroup theory towards the strongly coupled parabolic systems in mathematical biology. In this talk two models will be discussed.

Spatial segregation of interacting species.

$$\begin{cases} \frac{\partial u}{\partial t} = div[\nabla(a_1 u + a_{11}u^2 + a_{12}uv) + b_1(\nabla\Phi(x))u] + c_1 u - \gamma_{11}u^2 - \gamma_{12}uv & \text{in } \Omega \times (0,\infty), \\ \frac{\partial v}{\partial t} = div[\nabla(a_2 v + a_{21}uv + a_{22}v^2) + b_2(\nabla\Phi(x))v] + c_2 v - \gamma_{21}uv - \gamma_{22}v^2 & \text{in } \Omega \times (0,\infty), \\ \frac{\partial}{\partial n}(a_1 u + a_{11}u^2 + a_{12}uv) + b_1 \frac{\partial\Phi(x)}{\partial n} u = 0 & \text{on } \partial\Omega \times (0,\infty), \\ \frac{\partial}{\partial n}(a_2 v + a_{21}uv + a_{22}v^2) + b_2 \frac{\partial\Phi(x)}{\partial n} v = 0 & \text{on } \partial\Omega \times (0,\infty), \\ u(0,x) = u_0(x) \quad \text{and } v(0,x) = v_0(x) & \text{in } \Omega. \end{cases}$$

Aggregation of cellular slime mold by chemotaxis.

$$\begin{cases} \frac{\partial a}{\partial t} = d_1\Delta a - \gamma\nabla\cdot[a\nabla p] & \text{in } \Omega \times (0,\infty), \\ \frac{\partial p}{\partial t} = d_2\Delta p + c_1 a - c_2 p & \text{in } \Omega \times (0,\infty), \\ \frac{\partial a}{\partial n} = \frac{\partial p}{\partial n} = 0 & \text{on } \partial\Omega \times (0,\infty), \\ a(0,x) = a_0(x) \quad \text{and } p(0,x) = p_0(x) & \text{in } \Omega. \end{cases}$$

On free boundary problems for equations of viscous compressible fluids

W.M. Zaiaczkowski, Warszawa

The motion of viscous compressible barotropic fluid is considered in a domain bounded by a free surface. There are considered two cases: with the surface tension and without it. We prove existence of global solution which is close to an equilibrium state for all time. By the equilibrium state we mean a solution of the considered problem such that the velocity vanishes, the density is equal constant and in the case of the surface tension the domain is a ball.

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