MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 1/1994

MODELLTHEORIE

02.01. bis 08.01.1994

Die Tagung fand unter der Leitung von U. Felgner (Thübingen), A. Prestel (Konstanz) und M. Ziegler (Freiburg) statt.

Im Vordergrund standen die Themen "Modelltheorie von Gruppen", speziell die 'small index property' für Strukturen, sowie Hrushovski's Beweis der Mordell-Lang Vermutung im Funktionenkörperfall. Zu beiden Schwerpunkten fanden mehrere Hauptvorträge statt.

Darüberhinaus wurden in Kurzvorträgen Ergebnisse aus der Stabilitätstheorie und aus der Modelltheorie spezieller Strukturen (Gruppen, Moduln, o-minimale Strukturen, Graphen) vorgestellt.

Vortragsauszüge

The field $\mathbb{Q}^{tr}$ of totally real algebraic numbers is decidable

Dan Haran, Tel Aviv

(A joint work with M. Fried and H. Völklein.)
The result in the title relies on two facts:

Theorem A (Pop): $\mathbb{Q}^{tr}$ is pseudo real closed (PRC).

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Theorem B: The absolut Galois group $G(\mathbb{Q}^{tr})$ of $\mathbb{Q}^{tr}$ is the free profinite product $\hat{D}$ of copies of $\mathbb{Z}/2\mathbb{Z}$ over the Cantor space. Equivalently (as $\mathbb{Q}^{tr}$ is countable) $G = G(\mathbb{Q}^{tr})$ has the following Embedding property (EP):

i) G is generated by $\mathrm{Inv}(G)$, the set of involutions in G;

ii) Given an epimorphism of finite groups $\alpha: B \to A$, a continuous epimorphism $\varphi: G \to A$, and a subset $I_B \subseteq \mathrm{Inv}(B)$ closed under the conjugation in B and such that $B = \langle I_B \rangle$ and $\alpha(I_B) = \varphi(\mathrm{Inv}(G))$, there is a continuous epimorphism $\psi: G \to B$ such that $\alpha \circ \psi = \varphi$ and $\psi(\mathrm{Inv}(G)) = I_B$.

By now standard model-theoretic considerations we give an axiomatization of $\mathrm{Th}(Q^{tr})$:

$M \models \mathrm{Th}(Q^{tr})$ if and only if

(1) M is PRC;

(2) $G(M)$ has EP;

(3) $\mathbb{Q}^{tr}$ is the absolute part of M; and

(4) $M/\mathbb{Q}^{tr}$ is totally real.

Finite covers with finite kernels

David M. Evans (Norwich)

Suppose C, W are countable $\aleph_0$-categorical structures and $\pi: C \to W$ is a surjection with finite fibres. If the map $\rho_\pi: \mathrm{Aut}(C) \to \mathrm{Sym}(W)$ given by $\rho_\pi(g)(w) = \pi(g(\pi^{-1}(w)))$ ($g \in \mathrm{Aut}(C), w \in W$) is well-defined and has image $\mathrm{Aut}(W)$, we say that π is a ***finite cover*** of W, with ***kernel*** $\ker(\rho_\pi)$. We are interested in describing, for fixed W, all the finite covers in which this is finite.

Apart from degenerate cases, we know of two types of constructions of these finite covers. In the first type, the covers arise as coverings of an invariant digraph on W. The second construction is modelled on the example of a vector space covering its projective space.

We give various conditions on W which guarantee that all finite covers of W with finite kernel arise in one of these ways.

Valuation theory for o-minimal expansions of real closed fields

Lou van den Dries (Urbana)

Let T be a complete o-minimal theory extending the theory RCF of real closed fields, let $(R, V) \models T_{\mathrm{convex}}$, let $\overline{V} = V/m(V)$ be the residue field and let $\Gamma = v(R^x)$ be the value group. In "T-convexity and tame extensions" (A. Lewenberg & L. van den Dries, to appear in JSL) we made $\overline{V}$ into a T-model and Γ into a model of a certain complete theory T_{vg} ("the theory of the value group") which depends only on T, not on the structure (R, V), and which extends the theory of ordered abelian groups. The following results answer some questions left open in the paper just cited. 'Definable' below means 'definable with parameters'.

Th. If $S \subseteq R^n$ is definable in (R, V), then $\overline{S} \subseteq \overline{V}^n$ is definable in the T-model $\overline{V}$, where $\overline{S}$ = image of $S \cap V^n$ under the canonical map $V^n \to \overline{V}^n$.
If moreover S is definable in R, then $\overline{S}$ is closed in $\overline{V}^n$ and $\dim_R(S) \geq \dim_{\overline{V}}(\overline{S})$.

Th. Suppose T is power bounded with field of exponents K (so that Γ is an ordered K-linear space). Then T_{vg} is an extension by definitions of the theory of nontrivial ordered K-linear spaces. If $S \subseteq (R^*)^n$ is definable in (R, V), then $v(S) \subseteq \Gamma^n$ is definable in the T_{vg}-model Γ.

Th. Suppose T is power bounded and $(R, V) \succ (S, W) \models T$convex with $rk(S/R) < \infty$. Then $rk(S/R) \geq rk(\overline{W}/\overline{V}) + \dim_K(v(S^*)/v(R^*))$.

Besides the results of the paper cited the main tools are the Marker-Steinhorn theorem, and in the power-bounded case, a result of Loveys-Peterzil, and the result that if $f: V \to R$ is definable in (R, V), then $v(fx)$ is ultimately constant as $x \to +\infty$ in V.

Zariski Geometries

David Marker (Chicago)

We survey the work of Hrushovski and Zilber. A Zariski geometry on a field D is a family of Noetherian topologies on $D, D^2, \ldots$ satisfying the following properties:

Z0) If $f(x) = (f_1(x), \ldots, f_n(x))$ $f: D^n \to D^n$ and each f_i is either constant or a coordinate projection, then f is continuous

Z1) If $C \subseteq D^n$ is closed and nonempty and $\pi: D^n \to D^m$ is a projection, then there is a proper closed $F \subset \overline{\pi(C)}$ such that $\pi(C) \supseteq \overline{\pi(C)} - F$

Z2) If $C \subseteq D^m \times D$ is closed an for $a \in D^m, C(a) = \{x: (a, x) \in C\}$ than there is an N such that for all $|C(a)| < N$ or $C(a) = D$

Z3) If $C \subseteq D^n$ is closed irreducible and $\dim C = k$ then every nonempty irreducible component of $C \cap \{x \in D^n: x_i = x_j\}$ has dimension at least $k - 1$.

If D is a Zariski geometry and there is a sufficiently rich family of curves in D^2 then D interprets a pure field K. Conditions are given to insure D is isomorphic to the Zariski geometry of a smooth curve.

Fields with Higher Derivations

Margit Messmer (South Bend)

joint work with C. Wood:

We define a first-order theory SHF_e of separably closed fields of finite degree of imperfection e which carry an infinite stack of Hasse derivations.
The language: $\{+, -, \cdot, ^{-1}, 0, 1\} \cup \{D_{p^i} : i < w\}$ where D_{p^i} are unary function symbols.

The axioms for $F \models \mathrm{SHF}_e$:

–Axioms for fields ofcharp

$-\forall x, y D_{p^i}(x + y) = D_{p^i}(x) + D_{p^i}(y)$

$-\forall x, y D_{p^i}(x \cdot y) = \sum_{\nu+\mu=p^i} D_\nu(x) \cdot D_\mu(y)$

where D_ν is some expression in the$D'_{p_i}s$

$-\forall x D_{p^i}(D_{p^i}(x)) = D_{p^i}(D_{p^i}(x))$

$-\forall x (D(x) = 0 \rightarrow \exists y\, y^p = x)$

$-\forall x D_{p^i}^{(p^e)}(x) = 0$

$-\exists x D_{p^i}^{(p^e-1)}(x) \neq 0$

$-F$ is separably closed

We show that SHF_e has quantifier elimination and eliminates imaginaries.

Definable Valuations

Jochen Koenigsmann (Konstanz)

The talk contributed to the following question:
Which valuations on a field K are intimately enough related to the arithmetic of the field K to be first-order definable in the language of fields (with parameters from K)?
On number fields, for example, all valuations are definable (even by an existential formula), whereas on $\mathbb{Q}_p$ only the canonical henselian valuation and on separably or real closed fields no valuation at all is definable. The following theorem has been reported on: A henselian field (K, v) which is not separably or real closed admits a definable valuation which is dependent with v (i.e. induces the same topolgy on K), provided (K, v) is not exceptional, where exceptional means: $\mathrm{Gal}(\tilde{K}/K)$ is pro-p, $\mathrm{char}K = 0$, the residue field K_v is algebraically closed, $\mathrm{char}K_v = p$ and the value group is divisible. We do not know whether or not the theorem extends to the exceptional case. For the proof, one has to work with t-henselian fields as introduced by Prestel-Ziegler (Crelle 1978) and study various notions of compatibility of an arbitrary multiplicative or additive subgroup G of a field K with a valuation. Here the idea that any such subgroup induces a topology on K plays a crucial role.

Mittag-Leffler modules

Philipp Rothmaler (Kiel)

Following Raynaud and Gruson a left R-module M is called *Mittag-Leffler* if the canonical map

$$\pi_i N_i \otimes M \longrightarrow \pi_i(N_i \otimes M)$$

is injective for every family of right R-modules N_i. Using Ivo Herzog's criterion of being zero in tensor products in terms of positive primitive (pp) formulas and duals of them, we show that the Mittag-Leffler modules are exactly the "positively atomic" modules (i.e. modules in which the pp type of every tuple is implied – modulo the theory of all left R-modules – by a single pp formula).
Using this we derive various known features of Mittag-Leffler modules and characterize – among others – the rings over which every reduced product of Mittag-Leffler modules is Mittag-Leffler. These turn out to be precisely the left pure-semisimple rings (= rings of left pure global dimension zero). (This answers a question of A. Facchini.)
We also point out that model-theoretically it is the Mittag-Leffler modules rather than the pure-projective ones that play a role dual to that layed by the pure-injectives (= "positively compact" modules).
Finally, certain realizations – of both, algebraic and model-theoretic nature – are mentioned.

Automorphisms of countable strongly minimal sets

Dugald Macpherson (QMW, London)

A paper by D. Lascar "Automorphismes d'un ensemble fortement minimale" (JSL 1992) was surveyed. The setting is a countable saturated structure M algebraic over a 0-definable strongly minimal set $D(M)$. If $A \subseteq M$ then an *A-strong automorphism* of M is an automorphism fixing setwise the classes of any finite A-definable equivalence relation, and $\mathrm{Aut}f_A(M)$ denotes the group of these automorphisms.
An automorphism g of M is *bounded* if there is $n \in \mathbb{N}$ such that for all $X \subseteq M$, $\mathrm{Dim}(X^g/X) \leq n$, and $B(M)$ denotes the normal subgroup of bounded automorphisms. The following theorems were proved in Lascar's paper.

Theorem 1 If $H \leq \mathrm{Aut}\, M$ and $|\mathrm{Aut}\, M : A| < 2^{\aleph_0}$ there is finite $A \subseteq D(M)$ such that $H \geq \mathrm{Aut}\, f_A(M)$

Theorem 2 The group $\mathrm{Aut}\, fM/B(M) \cap \mathrm{Aut}\, f(M)$ is simple.

A characterization of ω-stability via automorphisms

Michael C. Laskowski (Univ. of Maryland, College Park)

For any structure M and any $A \subseteq M$, let $G = \text{Aut}(M)$ and let $G_{\{A\}}$ denote the setwise stabilizer of A.

Theorem: Let M be any countably saturated structure.
The following are equivalent:

(1) For all $A \subseteq M$, $G_{\{A\}}$ induces a closed subgroup of $\text{Sym}(A)$
(2) For all $X \subseteq M$ and all $p \in S_1(X)$, $\{g(p): g$ an elem. permutation of $X\}$ is countable.
(3) $Th(M)$ is ω-stable.

This work is joint with E. Bouscaren.

Topological automorphism groups

Wilfrid Hodges, QMW, London

The talk was an introduction to the talks of Evans, Herwig and Macpherson on aspects of the small index property. It defined automorphism groups as topological groups and as complete metric spaces. The following theorem (from Hodges, Hodkinson, Lascar and Shelah, 'The small index property for ω-stable ω-categorical structures and for the random graph', J. London Math. Soc. 48 (1993) 204-218) was proved:
If the topological group G is a complete metric space and H is a measure subgroup of G, then H has index $\geq 2^{\omega}$ in G.

Permutation Groups of Finite Morley Rank (fMr)

Ali Nesin, UCI

We will survey the state of permutation groups of fMr.
The motivation is the Cherlin-Zil'ber conjecture: A simple group of fMr is an algebraic group over an algebraically closed field.

1) **Frobenius Groups**: A group B is a Frobenius group if it acts transitively but not regularly on a set X of cardinality ≥ 2 in such a way that only the identity element fixes 2 distinct points of X.

If B is a Frobenius group and T is a one point stabilizer, then $T < B$ and for all $b \in B, T^b \cap T \neq 1 \Rightarrow b \in T$. Conversely if B has such a (proper) subgroup, then B is a Frobenius group. We will say that $T < B$ is a Frobenius group if T has the properties above.

Ex: A sharply 2-tr. group is a Frobenius group.
Ex: Let K be a field, let $T < K^*$. Then $K^+ \rtimes T$ is a Frobenius group.
Ex: Let B be a bad group (simple group of fMr where any proper connected subgroup is nilpotent). Then B is a Frobenius group. (This is a theorem).
Ex: Let B be the free group on 2 generators & T a cyclic subgroup. Then $T < B$ is a Frobenius group.
Fact: If $T < B$ is a finite Frobenius group, then $B = U \rtimes T$ for some $U \triangleleft B$ which is nilpotent.

Conjecture: A Frobenius group of finite Morley rank splits as $U \rtimes T$ where U is nilpotent.

Proposition. Let $T < B$ be a Frob. group of fMr.
a) T is definable. (so that we may assume the action of B is interpretable).
b) If $B = U \rtimes T$ then U is definable.
c) If U is solvable then U is nilpotent.
d) If B is solvable then the conjecture holds. Furthermore T^0 is abelian and all the complements of U are conjugate.

Theorem (Epstein-N.) If $T < B$ is a Frob. group of fMr with T finite, then B splits: $B = U \rtimes T$.

Theorem (Epstein-N.) Let $T < B$ be a Frob. group of fMr. Assume every definable proper section of B which is a Frobenius group splits. Then either B splits or B is simple and a counterexample to the Cherlin-Zil'ber conjecture.

Theorem (Epstein-N.) Let $T < B$ be a Frob. grp of fMr. Assume every simple and definable proper section of B is an algebraic group. Then either 1) B splits as $U \rtimes T$ with U nilpotent and T is abelian-by-finite, or 2) B is simple and a counterexample to the Cherlin Zil'ber conjecture.

Proposition (Delahan-N.). Let $T < B$ be a Frob. grp. of fMr, then T has finitely many involutions.

2) Sharply 2-transitive groups

Conjecture An infinite sharply 2-transitive grp. of finite Morley rank is isomorphic (i.e. equivalent) to the following permutation grp: Let K be an alg. cl. field. Let $G = \{\begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix} : a, b \in K, a \neq 0\}$, $X = \{\binom{x}{1} : x \in K\}$.
Let G be a sh. 2-tr. group and let H be a one point stabilizer. Recall that $H < G$ is a Frobenius group, so that if G has fMr then H is definable.

Proposition Let G be a sh.2-tr. grp of fMr.
a) If G is ∞ then H is connected.
b) If G is solvable then the conjecture holds.

Theorem (Cherlin-Grundhöfer-N-Völklein) Let $G = A \rtimes H$ be an ∞ split sharply 2-tr. grp of fMr. (A is necessarily abelian). If $C_{\text{End } A}(H)$ is infinite, then G is standard (i.e. the conjecture holds).

Corollary: G as above. a) If A has an element of infinite order then G is standard.
b) If $Z(H)$ is ∞ then G is standard.
c) If H has an ∞ normal solvable subgroup, then G is standard.

Theorem (Delahan-Nesin) Let G be an ∞ sh.2-tr. grp of fMr. Assume it has an involution and is nilpotent, then G is standard.

3) **Zassenhaus Groups** A doubly transitive permutation grp is called a Zassenhaus group if the stabilizer of any three distinct points is trivial.

Ex: Sh. 2-tr. groups, sh. 3-tr. groups are Zassenhaus groups.
We let G denote a Zassenhaus group. We take $x, y \in X$ with $x \neq y$. We set $B = G_x$, $T = G_{x,y} = \{g \in G: gx = x \& gy = y\}$. We have $T \leq B \leq G$ and $T < B$ is a Frobenius group. If $B = U \rtimes T$ for some U, we say that G is a *split* Zassenhaus group.

Conjecture: An ∞ Zassenhaus group of fMr where $T \neq 1$ is $\mathrm{PSL}_2(K)$.

Theorem (Delahan-Nesin) Let G be an ∞ split Zassenhaus grp. If T has an involution then the conjecture holds.

Theorem A sharply 3-tr. group of fMr is isomorphic to $\mathrm{PSL}_2(K)$.

Theorem (De Bonis-N) If G is an ∞ split Zassenhaus group of fMr and if U has a central involution, then $G \cong \mathrm{PSL}_2(K)$.

4. BN-pairs

Theorem (Boronik-Corredor-Davis-De Bonis-N) Assume G is a simple BN-pair of fMr and of Tits rank ≥ 3, then G is an algebraic group over an alg. cl. field.

5) CIT and CN groups

Theorem (De Bonis-N.) Let G be an ∞ and nonsolv. group of fMr with $0_2(G) = 1$ (i.e. has no normal nontrivial 2-subgroups). Assume there exists $U \leq G$ nilpotent and with involutions such that for $u \in U^*, C_G(u) \leq U$, then one of the following holds:
i) U has ∞ many involutions and $G \cong SL_2(K)$, K alg. cl of char 2
ii) U has a unique involution and G is a Frob. compl. with U as a Frob. compl.

Corollary (De Bonis-N.) Let G be an ∞, nonsolv. grp with involutions. Assume that G is a CN-grp (i.e. the Centralizer of any nontrivial element is Nilpotent). Then one of the following hold
i) $G \cong SL_2(K)$ for some K alg. closed field of char 2.
ii) G is a Frob grp. with the centralizer of an involution forming the Frob. complement.

Theorem (Boronik-De Bonis-Nesin) Let G be an ∞ group of fMr. Assume G is a CIT group (i.e. G has involutions and the centralizer of any involution is a 2-grp.) Then one of the following hold:

i) $0_2 G \neq 1$ (i.e. G has a normal, nontrivial, 2-subgroup.)
ii) $G = H \rtimes S$ where H is definable, abelian, $2^{\perp}$-subgroup and S is a finite 2-Sylöw of G with a unique involution that inverts H.
iii) $G \cong SL_2(K)$ for some alg. cl. field K of characteristic 2.

Jordan Groups and Going Forth

Peter M. Neumann (Queen's College, Oxford)

When we speak of a 'back and forth' construction for proving isomorphism of two countable structures we call to mind the famous theorem of Cantor about countable dense linearly ordered sets. Cantor's proof, however, in fact requires only the 'going forth' part of the construction, and Adrian Mathias, when he noticed this, asked for a classifiction of those $\aleph_0$-categorical theories for which 'forth suffices'. Peter Cameron heard of the problem in 1986. His (not yet complete) answer involves properties of the automorphism groups of the countable models and leads to interesting questions about permutation groups.
Let Ω be a set, G a permutation group on Ω. A subset Σ of Ω is a *J-set* if the pointwise stabilizer $G_{(\Omega-\Sigma)}$ is transitive on Σ (it is a *Jordan set* if also $|\Sigma| > 1$). Cameron asks for the groups G with the property that for every finite set Φ, the pointwise stabilizer $G_{(\Phi)}$ has only finitely many orbits and each orbit is a J-set. It turns out to be easier to work with a wider class of groups: we say that G has *property* P if for any finite set Φ, the complement $\Omega - \Phi$ is a union of finitely many J-sets.

THEOREM Suppose that G is closed in the natural topology on $\mathrm{Sym}(\Omega)$, and that G is primitive on Ω. If G has property P then one of the following holds:
(1) Ω is finite;
(2) $G = \mathrm{Sym}(\Omega)$;
(3) $\Omega = PG(d,q)$ and $PGL(d+1,q) \leq G \leq P\Gamma L(d+1,q)$ or
$\Omega = AG(d,q)$ and $AGL(d,q) \leq G \leq A\Gamma L(d,q)$
for some cardinal number d and prime-power q;
(4) $G = \mathrm{Aut}(\Omega, R)$ where R is a dense homogeneous linear order relation,
linear betweeneness relation,
cyclic order relation,
cyclic separation relation,
C-relation [chains in a semilinear ordering],
D-relation directions in a general betweeness relation].

This theorem can be proved by inspection if one uses the very deep classification theorem for primitive Jordan groups proved by Samson Adeleke + Dugald Macpherson. It does, however, also have a quite short and direct proof.
An arbitrary group with property P is "essentially" a direct product of transitive groups with property P. And a transitive group with property P is "essentially" a wreath product

of finitely many finite covers of the groups in the theorem. Thus Cameron's question will be completely answered when we know all the finite covers of those groups (and David Evan's techniques should give these very easily) and when we know what "essentially" means precisely.

On atomic sets

Ludomir Newelski

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Assume T is *stable, small,* $\Phi(x)$ *is a formula.* We study the relationship between $a \subseteq \mathbb{C}$ and $\Phi(\mathbb{C})$, and between $T|\Phi$ and $T(a)|\Phi$. We consider mainly the cases of ω-stable $T|\Phi$, which is 1-based or bounded.
For example we prove that if $T|\Phi$ is ω-stable ω-categorical then $T(a)|\Phi$ is such.
We generalize a Steinhorn result about existence of prime models over indiscernible sets.
Now let $Q = \Phi(N)$ for some countable N. Pseudo-type on $S(Q)$ are the orbits of the action of Aut($\mathbb{Q}$) on $S(Q)$. We discerned countable many "τ-stable", "good" pseudo-types of $S(Q)$. The τ-stability conjecture says that if $I(T, X_0) < 2^{X_0}$, then every good pseudo-type on $S(Q)$ is τ-stable.
Using the above results we prove the τ-stability conjecture in case when:
$(*)$ Q is atomic and $T|\Phi$ is (superstable of finite rank or ω-stable) and (1-based or bounded).
Conjecture 1. Assume $Q = \Phi(M)$ is countable, atomic and $I(T, X_0) < 2^{X_0}$. Then $\forall c \subseteq M \exists a \subseteq Q$ Q is atomic over ac
2. Assume $I(T, X_0) < 2^{\aleph_0}$ is $\aleph_0$-categorical. Then for every $c \subseteq \mathbb{C}$, $T(c)|\Phi$ is $\aleph_0$-categorical.

I proved conjecture 1 in case $(*)$, and conjecture 2 in case when $T|\Phi$ is ω-stable.

Highman's Embedding Theorem in general setting and its application to existentially closed algebras

Oleg V. Belegradek, Kemerovo University, Russia

Let $\mathbb{K}$ be a recursively axiomatizable quasivariety of L-algebras. We say that the *(Generalized) Highman Theorem* holds for $\mathbb{K}$ if (for every f.g. algebra B in $\mathbb{K}$) every algebra which is recursively presented in $\mathbb{K}$ (over B) can be effectively embedded into an algebra which is finitely presented in $\mathbb{K}$ (over B). We say that $\mathbb{K}$ has the *Internal Mapping Property* if

there is an L-term $t(x, \bar{y})$ such that, for every A in $\mathbb{K}$ and every $\alpha: A \to A$, there are $B \supseteq A$ in $\mathbb{K}$ and $\bar{b} \in B$ such that $\alpha(a) = t^B(a, \bar{b})$, for $a \in A$. We say that $\mathbb{K}$ has the *Internal Homomorphism Property* if, for every $n > 0$, there are an L-term $h_n(x, \bar{z})$ and a finite set of atomic L-formulas $S_n(\bar{x}, \bar{y}, \bar{z})$, where $\bar{x}$ and $\bar{y}$ are n-tuples, such that, for every A in $\mathbb{K}$ and n-tuples $\bar{a}, \bar{b} \in A$ the following are equivalent; (i) there are $C \supseteq A$ in $\mathbb{K}$ and $\bar{c} \in C$ such that $S_n(\bar{a}, \bar{b}, \bar{c})$ holds in C and $h_n(a_i, \bar{c}) = b_i$, for all i;
(ii) there is a homomorphism from $\langle \bar{a} \rangle$ into A sending $\bar{a}$ to $\bar{b}$.

Theorem. Suppose that $\mathbb{K}$ has the Joint Embedding Property, the Internal Homomorphism Property, the Internal Mapping Property, and the Highman Theorem holds for $\mathbb{K}$. Then

(1) the Generalized Highman Theorem holds for $\mathbb{K}$;

(2) for finitely generated A_1, A_2 in $\mathbb{K}$, the word problem for A_1 is Q-reducible to the word problem for A_2 iff A_1 is embeddable into any existentially closed algebra in $\mathbb{K}$ containing A_2.

Remark. The quasivarieties of groups, torsion-free groups, and semigroups satisfy the conditions of the Theorem.
For groups (1) and (2) are known.

The Ziegler, and other spectra

Mike Prest, University of Manchester

The right Ziegler spectrum, Z_{g_R}, over a ring R is a topological space whose points are the isomorphism classes of indecomposable pure-injective right R-modules and whose basic open sets are given by pairs $\phi \geq \psi$ of pp formulas. By results of Ziegler and Herzog, the quotient of this space by the relation of topological indistinguishable (points of Z_{g_R} belong to the same open sets iff they are elementary equivalent) is nearly spectral in the sense of Hochster.
The condition which fails is that the intersection of compact open sets need to be compact. Nevertheless, one may define the dual topology which has a sub-L basis open open sets the complements of compact Ziegler-open sets. It turns out that this dual topology is the Zariski topology on the set of (iso.classes) of indecompensable pure-injectives, where those are regarded, via the map $N(\in Z_{g_R}) \mapsto N \otimes - \in (R-\text{ mod }, Ab)$, as injective objects of the functor category $(R-\text{ mod }, Ab)$. The Zariski topology on this functor category is obtained by defining the Zariski spectrum over a coherent commutative ring in terms of the module category over that ring and applying this category-theoretic definition to $(R-\text{ mod }, Ab)$.

Randomness, genericity and near model completeness

John Baldwin, University of Illinois, Chicago

Shelah and Spencer proved the 0-1 law for first order sentences about random graphs with edge probability $n^{-\alpha}$ (α irrational). Call the theory of sentences with limit probability 1, T^α. Slightly modifying a construction of Hrushovski, Baldwin and Shi showed that the class K_α of all graphs such that $|H| - \alpha$ (number of edges of H) > 0 for all subgroups H of G is an amalgamation class such that the theory T_α of the generic model M_α is strictly (α-irrational)-stable. A theory T is nearly model complete if every formula is equivalent in T to a Boolean combination of existential formulas.
Theorem. $T_\alpha = T^\alpha$. This theory is nearly model complete, stable and not finitely axiomatizable.

Homogeneous 3-graphs

Alistair Lachlan (Simon Fraser University)

A *3-graph* is a pair (V, E) such that $E \subseteq [V]^3$. Let $\mathbb{F}$ denote the class of all finite 3-graphs. Subclasses of $\mathbb{F}$ are considered which are amalgamation classes in the sense of Fraissé. For $A, B \in \mathbb{F}$ such that $E_A \cap [V_A \cap V_B]^3 = E_B \cap [V_A \cap V_B]^3$ the *no-edges amalgam of* A and B is $(V_A \cup V_B, E_A \cup E_B)$. For $\mathbb{X} \subseteq \mathbb{F}$, the *no-edges closure* of $\mathbb{X}$, denoted $NE(\mathbb{X})$, is the least subclass of $\mathbb{F}$ closed under isomorphism, substructure, and no-edges amalgamation. Complementary notions: *all-edges amalgam* and *all-edges closure* are defined in the obvious way.
For $\mathbb{X} \subseteq \mathbb{F}$, $Exc(\mathbb{F})$ ("exclude" $\mathbb{X}$) denotes the class of all $G \in \mathbb{F}$ such that no number of $\mathbb{X}$ embeds in G.
et $D = (\{0,1,2\}, \{\{0,1,2\}\}) \in \mathbb{F}$ and $\mathbb{C}$ denote $NE(\{D\})$.

Conjecture 1. (The strong conjecture) Let $\mathbf{A} \subseteq \mathbb{F}$ be an infinite amalgamation class, i.e., infinitely many isomorphism types are represented. Then $\mathbf{A}$ is either no-edges closed or all-edges closed.

Conjecture 2. (The W-conjecture) $\mathbb{C}$ is minimal amongst the amalgamation classes $\subseteq \mathbb{F}$ which contain both D and its complement.

Conjecture 3. (The weak conjecture) Let $\mathbb{X} \subseteq \mathbb{F}$ be finite and $Exc(\mathbb{X})$ be infinite. Then $Exc(\mathbb{X})$ is either no-edges closed or all-edges closed if it is an amalgamation class.

Theorem. Conjecture 3 holds when $|\mathbb{X}| = 1$.

Abstract theory of quadratic forms and Boolean algebras

M.A. Dickmann - CNRS - Univ. of Paris VII, France

We began by introducing a new abstract theory of quadratic forms, called the theory of *special groups*. A *special group* (SG) is a group of exponent 2, G, with a distinguished element, - 1, and a quaternary relation on G (better, a binary relation on G^2) satisfying seven first-order axioms. The quaternary ("special") relation is intended to represent the "isometry" relation for binary (quadratic) forms with coefficients in G. The category of these objects (with natural homomorphisms) is equivalent to that of *abstract Witt rings* (Marshall). Below we consider SG's satisfying an extra axiom, called *reduced* SG's; The category (and the 1st order theory) of them is denoted SG_{red}.

Results:
(1) There is a *duality* – i.e. a contravariant functor – between the category SG_{red} and that of abstract spaces of orderings (Marshall) with natural morphisms. Indeed, both categories are (contravariantly) equivalent, even isomorphic.
(2) Every Boolean Algebra (BA) is a SG_{red} under symmetric difference, complement and a "special" relation naturally induced by the order.
(3) Every SG_{red}, G, is naturally embeddable in a "smallest" BA (namely, the BA of clopens of its space of order, see (1)), its *Boolean hull*, denoted by B_G. The correspondence $G \to B_G$ can be extended to morphisms so as to give a functor of the category SG_{red} to the category BA of BA's with homomorphisms. I.e., for every SG homomorphism $f: G \to H$ (where $G, H \models SG_{red}$) there is a (unique) BA homomorphism $B(f): B_G \to B_H$ such that the diagram

$$\begin{array}{ccc} G & \xrightarrow{f} & H \\ \epsilon_G \downarrow & & \downarrow \epsilon_H \\ B_G & \xrightarrow[B(f)]{} & B_H \end{array}$$

commutes (ϵ_G denotes the natural embedding of G into B_G).
(4) Thus, the duality of (1) gets further decomposed into a pair of adjoint functors (namely B and the functor $S_g: BA \to SG_{red}$ which assigns to every BA itself seen as a SG_{red}), followed by the good, old, Stone duality for BA's.
(5) The above allows to easily determine the injective, projective and free objects in the category SG_{red} (namely, complete BA's, fans and fans, respectively), and to characterize the monos and epis in this category (namely, injective homos. and homos $f: G \to H$ s.t. Im $(\epsilon_H \circ f)$ generates B_H as a BA, respectively).
(6) More generally, these tools allow a reinterpretation of the "funny" combinatorics used in the algebraic theory of quadratic forms in terms of a combinatorics of filters and ultrafilters in BA's. As an illustration we give a short, structural proof of the *"small" representation theorem.*

At the end we discussed some new examples of abstract spaces of orderins (joint work with F. Miraglia).

Relative regularly closed fields

Y. Ershov, Novosibirks University

Let W be a Boolean family of the valuation rings of a field F. F is *regularly closed relative to the family* W ($\langle F, W\rangle \in RC$ or $F \in RC(W)$) iff for every regular extension $F_0 \geq F$ the following equivalence holds:

$$F \leq_1 F_0 \Longleftrightarrow \forall R \in W(H_R(F) \leq_1 F_0 H_R(F))$$

(where $H_R(F)$ - the henselization F relative R: $H_R(F) = qn(R^h)$).
For a Boolean family W let $R_W \rightleftharpoons \cap\{R|R \in W\}$. Introduce the following conditions:
(BAP) For every partition $[\alpha_0, \ldots, \alpha_n]$ of W, every $a_0, \ldots, a_n \in F$, $\epsilon \in J(R_W) \setminus \{0\}$ ($J(R_W)$ - the Jackobson radical of R_W) there is $a \in F$ such that $v_R(a - a_i) > v_R(\epsilon)$ for every $i \leq n$ and $R \in W_{\alpha_i}$.
(THR) For every absolutely irreducible $f \in R_W[x, y]$ monic in x, for any $a, \bar{b} \in R_W$ if $f'_x(a, \bar{b}) \neq 0$ and $f(a, \bar{b}) f'_\infty(a, \bar{b})^{-1} \in J(R_W)$, then there is $c, \bar{d} \in R_W$ such that $v_R(\bar{b} - \bar{d}) > v_R(\epsilon)$ for all $R \in W (\Longleftrightarrow (\bar{c} - \bar{d})\epsilon^{-1} \in J(R_W))$ and $f(c, \bar{d}) = 0$, $(a - c) f(a, \bar{c}) f'_\infty(c, \bar{b})^{-1}$ is a unit in R_W.

Thm. 1 If $\langle F, W\rangle \models$ (BAP) $\wedge$ (THR) thus $\langle F, W\rangle \in RC$.

Thm. 2 If W is a Boolean family of the valuation rings of F, then there is a regular extension F_0 of F, a Boolean family W_0 of the valuation rings of F_0 such that the map $R_0 \mapsto R_0 \cap F$, $R_0 \in W_0$ is a homomorphism W_0 and W; $H_{R_0 \cap F}(F) \leq_1 H_{R_0}(F_0)$ for all $R_0 \in W_0$ and $\langle F_0, W_0\rangle \models$ (BAP) $\wedge$ (THR).

Thm. 3 If W consists on the π-valuation rings of f (for some fixed $\pi \in F$), then

$$\langle F, W\rangle \in RC \Longrightarrow \langle F, W\rangle \models (\text{BAP}) \wedge (\text{THR}).$$

Hrushovski's proof of the Mordell-Lang conjecture for function fields in all characteristics

E. Bouscaren, CNRS, Paris

We present a brief survey of recent work by E. Hrushovski:

Theorem: (in any characteristic):
Let K be a field, $k_0 \subset K$, k_0 algebraically closed. Let S be a semi-abelian variety defined over K, X a subvariety of S defined over K, and $\Gamma \subseteq S(K)$ a subgroup of finite type. Suppose $X \cap \Gamma$ is dense in X.

Then there is a rational homomorphism h, from a group subvariety of S into a semi-abelian variety S_0, defined over k_0, and a subvariety X_0 of S_0, defined over k_0, such that X is a translate of $h^{-1}(X_0)$.

In particular, if $tr\ K/k_0(S) = 0$, then X is a coset of a group subvariety of S.

We present the main steps of this model-theoretic proof, which is "uniform" in all characteristics, and the main model-theory results that are being used.
The starting point is to expand the field, to a differentially closed field in characteristic 0, and to a separably closed field in characteristic p. In these expansions, Γ can be replaced by a definable subgroup of S of finite dimension. At this stage many results and techniques developped these last years in "Geometric Stability Theory" are used: Lanski geometries, groups of finite dimension, non-orthogonality relation between dimension 1 types in particular.

Groups with identities

F. Point, Mons University, Belgium

Let us first define certain identities which extend monoidal identities, Engel identities, m-identities (an example of an m-identity is an identity of the form $[\chi, y^m, \dots y^m] = 1$) (see [Boffa M., Point F.], m-identities, C.R. Acad. Sci. Paris, t. 313, Série I, p. 909-911, 1991).

Let $\alpha(x,y)$ be a reduced nontrivial word in $\{x, y, x^{-1}, y^{-1}\}$. Let $H = \langle y^i x y^{-i} | i \in \mathbb{Z}\rangle$. Then $\alpha(x,y) = y^m \cdot \prod_{i=i_0}^{i_1} (y^i x y^{-i})^{a_i} \cdot \gamma$, where $\gamma \in H$ $a_i \in \mathbb{Z}, m, i_0, i_1 \in \mathbb{Z}$ and the i's increasing. With α, we associate $R_v = \sum_{i=i_0}^{i_1} a_i t^i \in \mathbb{Z}[t, t^{-1}]$. We will call $\omega(x,y) = 1$ an M_ℓ-identity if there exists $v(x,y)$ with $d_x v = d_y v = 0$ ($\omega = 1 \rightarrow v = 1$) and $R_v = t^{-n}(a_0 + a_1 t + \cdots + a_\ell t^\ell) = t^{-n} P_v$ is such that $\ell cm(a_i) = 1$. Moreover $M_\ell = \ell cm\{|G| : G$ is a group fo roots of P_v modulo p in $\tilde{\mathbb{F}}_p^*$, for each prime $p\}$.
We will say that G is of exponent m if G satisfies $\forall x (x^m = 1)$.

Transfer theorem (see [Boffa M., Point F.] & [Point F.]: Conditions of quasi-nilpotency in certain varieties of groups, to appear in Communications in algebra.)
Let $\mathcal{C}$ be a class of groups closed under taking subgroups, quotients and ultrapowers. Let p be a prime number, m a natural number and C_p the cyclic group with p elements.
TFAE:
(i) every finite group in $\mathcal{C}$ of the form $(C_p \times \cdots \times C_p)(h), (o(h), p) = 1$ is nilpotent-by-exponent m.
(ii) every finite solvable group in $\mathcal{C}$ is nilpotent-by-exponent m.
(iii) every nilpotent-by-finitely generated solvable group in $\mathcal{C}$ is nilpotent-by- a finite group of exponent m.

(iv) every solvable linear group in $\mathcal{C}$ is nilpotent-by- a finite group of exponent m.
(v) every ω_1-saturated solvable M_c-group in $\mathcal{C}$ (in particular every stable group in $\mathcal{C}$) is nilpotent-by-exponent m.

We apply this transfer theorem to the class $\mathcal{C}$ of groups which satisfy a M_ℓ-identity (or a finite disjunction of M_ℓ-identities). Call such a class an M_ℓ-class.

Proposition: Any M_ℓ-class satisfies (v) of the transfer theorem above with $m = M_\ell$. Using a criterion of Shalev for a finitely generated pro-p-group to be analytic, a result of Wilson on finitely generated residually finite-nilpotent groups and the fact that fo p a prime number the groups of the form C_p or C_{p^n} do not belong to an M_ℓ-class $\mathcal{C}$ if $p^n > \ell$, we obtain bounds on the nilpotency class of the finitely generated group of $\mathcal{C}$.

Proposition: Let $\mathcal{C}$ be an M_ℓ-class. Let G be a nilpotent group in $\mathcal{C}$ generated by r elements. The the nilpotency class of G is bounded in terms of r and ℓ.
Using the positive solution of the restricted Burnside problem, we show the following

Proposition: Let $\mathcal{C}$ be an M_ℓ-class. Let G a solvable group in $\mathcal{C}$ generated by r elements. Then G is (nilpotent of clas $\leq g(r,\ell)$)-by-(a finite group of exponent M_ℓ).

We also show that we can bound the exponent of the finite groups in an M_ℓ-class in terms of ℓ and M_ℓ.

Value Groups of Nonarchimedean Exponential Fields

F.-v. Kuhlmann, Heidelberg

Every expansion of the theory of the ordered field $\mathbb{R}$ has models on which the order is nonarchimedean. In that case, they have a natural valuation associating to every element its archimedean class. We study the valuation-theoretical structure of nonarchimedean models of the theory of the reals with exponentiation. In particular, the exponential induces a partial map on the value group which can be prolongated to a total map χ called a *contraction* and satisfying:
(1) $\chi x = 0 \Longleftrightarrow x = 0$ 2) χ preserves $\leq$ 3) $\chi(-x) = -\chi x$
(4) If x and y are archimedean and have the same sign, then $\chi x = \chi y$ 5) (nevertheless) χ is surjective.
The growth axiom scheme

$$\exp x > x^n \quad \text{for sufficiently large} \quad x \qquad (n \in \mathbb{N})$$

translates to the axiom (CP) $\forall x |\chi x| < |x|$; a contradiction satisfying (CP) is called *centripetal.*

Thm. 1: The theory of divisible ordered abelian groups with centripetal contraction is model complete, complete, decidable and admits QE.

Thm. 2: It is moreover weakly o-minimal.
The second theorem is obtained by an analysis of the terms buildt up in the language $\{0, +, -, <, \chi\}$ of contraction groups. As maps on the group, the terms built up without constants are equal to χ-polynomials $z_n\chi^n x + \cdots + z_1\chi x + z_0 x$ $(z_i \in \mathbb{Z})$; all these are monotone.
For every term built up *with* constants, there is a finite partition of the group into convex subsets s.t. on each of these subsets, the term is equal (as a map) to a "generalized χ-polynomial". Again, all generalized χ-polynomials are monotone. Example $(a \in G)t(x) = \chi(x + a) - \chi(x - a)$ has graph
$v=$ natural valuation of the ordered group G

The Frattini Subgroup

(Frank O. Wagner, Oxford)

In a finite group G, the intersection of all maximal subgroups form a characteristic subgroup $\Phi(G)$, the Frattini subgroup, H is also characterized as the set of all non-generating elements. If H is normal in G, then H is nilpotent iff $H\Phi(G)/\Phi(G)$ is nilpotent. This may be (need to deduce the existence of nilpotent supplements (i.e. if $N \triangleleft G$ and G/N nilpotent, there is $H < G$, $G = NH$ and H nilpotent) and ultimatively of Carter subgroups (i.e. self-normalizing nilpotent subgroups) in solvable groups. In a stable group, we define $\Phi(G)$, to be the union of all definable normal subgroups which do not have a supplement. This is a normal subgroup, and a union of definable ones (which, however, may vary with the model). We prove:
Theorem 1: If M is a family of uniformly definable normal subgroups without supplement, then modulo some definable normal subgroups N without supplement M generates a nilpotent group.

Theorem 2: If M is a type-definable normal subgroup of a saturated $\mathcal{R}$-group G such that $H^\Phi\Phi(G)/\Phi(G)$ is nilpotent, then there is N as above such that H^Φ/N is nilpotent.
The proofs use the existence of nilpotent supplements and Carter subgroups in the Frattini free component H^Φ of an $\mathcal{R}$-group. So the order of things is reversed in comparison to finite group theory.

Finally, $\Phi(G)$ arises in a different context for an abelian group as family of possible coherents of quasi-endomorphisms.

Extending partial isomorphisms

(Bernhard Herwig, Paris)

The aim of the talk is to present a theorem of Hrushovski, which is used in the proof of the small index property for the countable random graph as presented by Wilfrid Hodges and David Evans. We also present the following generalization of Hrushovski's Theorem:

Theorem: Let A be a finite relational structure. Let $p_1, \ldots, p_n$ be partial isomorphisms on A. There exists a finite structure B and partial isomorphisms $f_1, \ldots, f_n$ on B extending $p_1, \ldots, p_n$.

The theorem of Hrushovski is the same statement where A and B are graphs. In the same way Hrushovski's theorem can be used to prove the small index property for the countable random graph. The generalization can be used to prove the small index property for the countable random structure in every finite relational structure.

Universality of the automorphism group of the real line

Frieder Haug, Tübingen

In this talk we discuss the following question about the automorphism-group $\mathrm{Aut}(\langle \mathbb{R}, \leq \rangle)$ of the real line $\langle \mathbb{R}, \leq \rangle$:

Question (U. Felgner, 1991); Is $\mathrm{Aut}(\langle \mathbb{R}, \leq \rangle)$ universal?

Here we call the automorphism-group of a linear order $\langle \Omega, \leq \rangle$ *universal*, if for each linear order $\langle L, \leq \rangle$ with $|\mathrm{Aut}(\langle L, \leq \rangle)| \leq |\mathrm{Aut}(\langle \Omega, \leq \rangle)|$, $\mathrm{Aut}(\langle L, \leq \rangle)$ is embeddable into $\mathrm{Aut}(\langle \Omega, \leq \rangle)$.

We show that the above question cannot be answered just with the axioms of ZFC or $ZFC + CH_1$ by proving the following three theorems:

Theorem A ($ZFC + 2^{\aleph_0} < 2^{\aleph_1} + SH$): $\mathrm{Aut}(\langle \mathbb{R}, \leq \rangle)$ is unviversal.

Theorem B ($ZFC + 2^{\aleph_0} = 2^{\aleph_1}$): $\mathrm{Aut}(\langle \mathbb{R}, \leq \rangle)$ is not universal.

Theorem C ($V = L$): $\mathrm{Aut}(\langle \mathbb{R}, \leq \rangle)$ is not universal.

Here SH denotes the Saishi Hypothesis: if $\langle L, \leq \rangle$ is a non-separable linear order, then $\langle L, \leq \rangle$ contains an uncountable family of disjoint open intervalls.

If we view $\mathrm{Aut}(\langle \mathbb{R}, \leq \rangle)$ not just as a pure group, but also as a lattice-ordered group (ℓ-group), then we can define analogously as above the notion of a *ℓ-universal* automorphism group of a linear order. We can prove analogous theorems for this notion.

Definable subgroups of algebraic groups over finite fields

Anand Pillay, Notre Dame (USA)

I discuss joint work with Hrushovski.
Unless otherwise said F denotes a bounded perfect PAC field (bounded means small, i.e. there are only finitely many continuous homomorphisms from Gal$(\tilde{F}/F)$ into any given finite group.)
Let G be a connected algebraic group defined over F. We use model-theoretic and stability theoretic methods to study (definable) subgroups of $G(F)$, and we have various applications.

Theorem 1. Let $\{X_i; i \in I\}$ be a family of Zariski-irreducible definable (in F) subsets of $G(F)$, each containing identity.
Let G_1 be the subgroup of $G(F)$ generated by all the X_i. *Then* G_1 is *definable* (in F) (and is thus a finite-index subgroup of $H(F)$, for H some connected algebraic subgroup of G, defined over F).

Theorem 2. Let G_1 be a definable (in F) subgroup of finite index of $G(F)$. *Then* there is a connected algebraic group H defined over F, and a surjective F-rational homomorphism $f: H \to G$, with finite kernel, such that $f(H(F)) < G$, (and moreover $f(H(F))$ has finite index in G_1).

A special case is where F is a pseudofinite field. We have the following consequences

a) For F, G as above (i.e. F add perfect PAC); $G(F)$ is *definably* simple iff *abstractly* simple.

   Moreover if G is simple as an algebraic group, then there is a *definable* subgroup G_1 of $G(F)$ of finite index, such that G is *simple* on an abstract group.

b) (Nori) Let G be a connected simple algebraic group defined over $\mathbb{Z}$. For p a prime, let G_p be reduction of G mod p.
   Let $\Gamma < G(\mathbb{Z})$ be finitely generated and Zariski dense (in G). Then for all but finitely many p, $\Gamma/p = G_p(\mathbb{F}_p)$.

c) Let G be as in (h). Then the set of maximal subgroups of $G_p(\mathbb{F}_p)$ (p a prime) is *uniformly* definable (where p varies).

d) Let G be a *simply connected* almost simple algebraic group defined over F. Then $G(F)$ is simple (modulo its finite centre) as an abstract group.

The Buium-Manin homomorphism

Anand Pillay, Notre Dame

Let K be a differentially closed field of characteristic 0. Let A be an ***abelian variety***, defined over K. Identify A with $A(K)$.

Proposition 1. There is a DCF-definable homomorphism f from A into K^n (some n) such that Ker(f) has finite Morley rank (in the differentially closed field K).

Proposition 2. Let Γ be a subgroup of A which is the divisible hull of a finitely generated group. Then there is a ***finite Morley rank definable*** subgroup of A, say Γ_1, such that $\Gamma < \Gamma_1$.

We give an outline of the proofs of the above results.

Exponentiation and Formal Power Series

David Marker (UIC)

We report on joint work with Lou van den Dries and Angus Macintyre. Using formal power series we construct natural nonstandard models of $\mathbb{R}_{an}(\exp)$, the real numbers with exponentiation and analytic functions on bounded sets.

With these methods we prove:

1) $f(x) = \int_0^x e^{t^2} dt$ is not $\mathbb{R}_{an}(\exp)$ definable
2) Let $f(x) = \log x(\log\log x)$ and let g be its compositional inverse. Then g is not asymptotic to a composition of exp, log and algebraic functions.

O-minimal groups and rings

Charley Steinhorn (Vassar College)

We discuss joint work with Kobi Peterzil. Let $M = \langle M, <, \dots \rangle$ be an o-minimal structure such that $(M, <)$ is a dense linear order. A group $(G, *)$ is definable in M if G is a definable subset of M^n for some n and $*$ is a definable function. It was shown by A. Pillay that every definable group in an o-minimal structure admits a definable topology via an "n-manifold", with finitely many charts. Definable rings are given similarly.

We say that a manifold X is definably compact if for every path $p: (a, b) \to X$ definable in M, both $\lim_{t \to a^+} p(t)$ and $\lim_{t \to b^-} p(t)$ exist and are elements of X. About definable compactness, we prove

Theorem $X \subseteq M^n$ is definably compact - here X is a subset of M with the usual topology - if and only if X is closed and bounded.
It follows that the continuous image of a closed and bounded set under a definable mapping is closed and bounded. Turning now to groups and map, we have the following

Theorem Let $(G, *)$ be a definable group in an o-minimal structure M which is not definably compact. Then there is a definable $H \leq G$, $\dim H = 1$, and H is divisable and abelian.

Theorem Let $R = (R, +, *)$ be a definable ring in M which does not have zero divisors. Then R is either a real closed field of dimension 1, $R(\sqrt{-1})$ for a definable real closed field R (of dimension 1), or R is the ring of quaternions over a definable real closed field R (again of of dimension 1).

An unclassifiable unidimensional theory without the OTOP

Bradd Hart, McMaster University

Using Shelah's Main Gap for countable theories and related results, one can prove the following theorem, specialized in this case to a countable unidimensional theory:

Theorem (Shelah). If T is a countable unidimensional theory then equivalently

1) For every $M \models T$ there is $M_0 < M$, $|M_0| \leq 2^{|T|}$ and $N > M_0$ with $q = tp(N/M_0)$ dominated by a regular type such that M is primary over M_0 and a basis for q.
2) T is not unclassifiable
3) T does not have the OTOP

where T is unclassifiable if for every reluar $\lambda > |T|$ there are $M_1, M_2 \models T$ of cardinality λ such that $M_1 \not\cong M_2$ but may be forced isomorphic by a forcing which preserves all cardinals and add the new subsets of λ of cardinality $< \lambda$

and

T has the omitting types order property (OTOP) if there is an $\mathcal{L}$-type $p(x, y, z)$ such that for all λ there is $N \models T$ and $\langle a_\alpha : \alpha < \lambda \rangle \subseteq N$ such that N realizes $p(x, a_\alpha, a_\beta)$ iff $\alpha < \beta$.

In general, for an arbitrary language 1) $\Rightarrow$ 2) $\Rightarrow$ 3). It is hoped that in general 1) and 2) are equivalent. Ambar Chowdhury and I have constructed a unidimensional theory which is unclassifiable but does not have the OTOP so in general 3) $\not\Rightarrow$ 2).

The small index property for ω-categorical, ω-stable structures and for the random graph

David M. Evans (Norwich)

Continuing the discussion of the paper of Hodges, Hodkinson, Lascar and Shelah, introduced by Wilfrid Hodges, we presented a proof of
Thm (HHLS) Suppose M is either an ω-categorical, ω-stable countable structure, or the random graph. Then M has the small index property: If H is a subgroup of Aut(M) of index $< 2^{\aleph_0}$, then H contains the pointwise stabiliser in Aut(M) of a finite subset of M. The proof uses the existence of comeagre sets of generic automorphisms in $(\mathrm{Aut}(M))^n$, which in turn relies on known facts about M, due to Cherlin, Harrington and Lachlan and Hrushovski (for the ω-categorical, ω-stable M) and Hrushovski (for M the random graph).

On simple groups of finite Morley rank

A. Borovik (VMIST, Manchester)

Let G be a group of finite Morley rank (FMR). We say that G is *bad*, if every proper definable connected subgroup is nilpotent.
A *bad field* is a structure of FMR of the form

$$\langle K; +, \cdot, M \rangle,$$

where $\langle K; +, \cdot \rangle$ is an a.c. field and M is (a predicate for) proper infinite subgroup of K^*.
G. Cherlin conjectured, specifying the well-known Cherlin-Zilber conjecture on groups of FMR, that simple groups of finite Morley rank which do not interpret bad field or bad groups are simple algebraic groups over a.c. fields.
We call a simple group of FMR *tame*, if it does not interpret a bad group or a bad field and if, in addition, every proper definable connected simple section of G is a simple algebraic group over an a.c. field. Obviously a minimal counterexample to Cherlin's conjecture is a tame group.
The talk was devoted to discussion 2-Sylow theory and signalizer functors in tame groups. First results in the theory of tame groups are very instiring and promising and give hopes of eventual classification of this group.

Elementary theories closed to modules and coverings by classes of permutable equivalences

E.A. Palyutin (Russia, Novosibirsk)

Equivalences α, β are permutable if the composition $\alpha \circ \beta$ is an equivalence. Let P be a sublattice of the lattice of all equivalence relations on set A and all $\alpha, \beta \in P$ are permutable. Equivalence α has finite index in set Y, if $\alpha \restriction Y$ has finitely many classes.

Def. A set X has *finite multiindex* in a set Y relative to the lattice P iff there exists a chaine $\alpha_0 \subseteq \cdots \subseteq \alpha_n$ of P-equivalence relations, s.t. X is an α_0-class, Y is an α_n-class, $X \subseteq Y$ and for any $i < n$ equivalences α_i have finite index in $\alpha_{i+1}X$.
Next theorem is a generalization of the famous Neuman's lemma.

Theorem Let $Y, X_1, \ldots, X_n$ be classes of equivalence relations from P, $Y \subseteq \bigcup_1^n X_i$ and $Y \not\subseteq \bigcup_2^n X_i$. Then X_1 has finite multiindex in Y. This theorem is used for the proof of quantifier elimination for Hom stable theories with NDOP via positive primitive formulas.

Topological automorphism groups

Wilfrid Hodges, QMW, London

The talk was an introduction to the talks of Evans, Herwig and Macpherson on aspects of the small index property. It defined automorphism groups as topological groups and as complete metric spaces. The following theorem (from Hodges, Hodkinson, Lascar and Shelah, "The small index property for ω-stable, ω-categorical structures and for the random graph", J. London Math. Soc. 48 (1993), 204-218) was proved: If the topological group G is a complete metric space and H is a neagre subgroup of G, then H has index $\geq 2^{\omega}$ in G.

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