MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 2/1994

Algebraic Combinatorics
09.01.-15.01.1994

The conference was organized by E.Bannai (Fukuoka), A.A.Ivanov (Moscow), A.Kerber (Bayreuth) and U.Ott (Braunschweig). This was the first Oberwolfach meeting on Algebraic Combinatorics but in a sense it was a succesor of two earlier meetings on the subject held in Vladimir (Russia) in 1991 and in Fukuoka (Japan) in 1993. The conference brought together mathematicians from various countries working in different areas of Algebraic Combinatorics. There was always a common thread, namely an application of association schemes and their analogs. O.Tamaschke gave a series of two expository lectures on the theory of S-rings. This exposition of a subject fundamental to the theory of association schemes was very helpful for the participants. Numerous other talks were given on traditional areas of Algebraic Combinatorics such as the theory of (P and Q)-polynomial schemes, representation theory of symmetric groups and the theory of diagram geometries. Progress in these areas was reported, for instance the possibility of answering existence and uniqueness questions for the sporadic simple groups by studying their diagram geometries. Moreover, a number of new and rapidly growing areas of Algebraic Combinatorics were well represented at the conference. These new subjects include the theory of spin models, originally motivated by certain problems in theoretical physics, and the theory of Terwilliger algebras which appeared as a highlight of the theory of (P and Q)-polynomial association schemes. The high level of interactions between participants has amply fulfilled the hopes of the organizers in bringing together these diverse groups of researchers to focus on the theme of association scheme and representation theory.

The organizers wish to thank the "Mathematisches Forschungsinstitut Oberwolfach" for its very kind hospitality.

E. Bannai, A. A. Ivanov, A. Kerber, U. Ott

Vortragsauszüge

J. André:

Non-commutative Spaces and Algebraic Combinatorics

A (non-commutative) *space* is a structure (X, F) with $X \neq \emptyset, F \subseteq \mathcal{P}(X^2)$ and with:

(i) any $(x, y) \in X^2$ lies in exactly one $f \in F$,

(ii) $\{(x, x) \mid x \in X\} \in F$.

A *line* $x\Box y$ is defined by $x\Box y := \{x, \langle x, y\rangle\} \cup \{z \mid \langle x, z\rangle = \langle x, y\rangle\}$, where $\langle x, y\rangle = f \in F$. In general $x\Box y \neq y\Box x$. Examples are the homogeneous coherent configurations in the sense of D. G. Higman (*Geom.Ded.* 4(1975),1-32) which are also called *strongly skewaffine spaces*. Generalizations of them are the *skewaffine spaces*. A skewaffine space with $x\Box y = y\Box x$ is affine. Some modifications of the adjacency algebras belonging to those spaces are considered and some applications, especially on group-theory, are presented.

R. Bacher:

Generalized Hadamard Matrices

We are interested in constructing matrices $M \in \mathrm{GL}(n, \mathbb{C})$ which satisfy

$$M^{-1} = \frac{1}{n} M^{-!}$$

where $M^{-!}$ is defined by $(M^{-!})_{j,k} = (M_{k,j})^{-1}$. Our method is to look for a finite group G of order n and an element $\alpha = \sum \alpha_g g$ in the group algebra $\mathbb{C}G$ such that

$$\alpha^{-1} = \frac{1}{n} \sum \alpha_g^{-1} g^{-1}.$$

We shall give examples and we shall outline some relations with association schemes.

E. Bannai:

Spin Models and Association Schemes, II

This is a joint work with Etsuko Bannai and François Jaeger.
Using the results explained in Jaeger's talk, we get the following

Main Theorem. Let $\mathcal{X}(G)$ be the group association scheme of any finite abelian group G. (1) For each fixed dual map ψ of $\mathcal{X}(G)$ (i.e., for each character table P of $\mathcal{X}(G)$ satisfying $P = {}^tP$), we determined explicitly all the diagonal matrices Δ satisfying the modular invariance property: $(P\Delta)^3 = \lambda I$. (2) For each solution Δ in (1), we can associate a generalized spin model on the group G.

Remark. Generalized spin models on finite abelian groups constructed for any even $\mathbb{Q}$-lattice by V. Kac – M. Wakimoto all appear among those constructed in the main theorem. Also, generalized spin models on abelian groups which generate the Bose-Mesner algebra of the group association scheme are special cases of those constructed in the main theorem.

B. BAUMEISTER:

Flag–transitive Dual Extended Grids

Let G be a group acting flag–transitively on a dual extended grid Γ ($\circ\!=\!\circ\overset{c}{-}\circ$: we have points, lines and grids, and two lines which intersect in a point define exactly one grid.)
There are the following examples:
A family of flag–transitive geometries, whose members have as collinearity graph the Hamming graph, the sporadic example $G \cong J_3$ and its triple cover $3J_3$ and finally the extended ternary Golay Code, whose universal 2–cover is a member of the family mentioned above.
We proved

> If G_{Π/K_Π} is an almost simple group, Π a grid, and $K_P = 1$, P a point, then $G \cong J_3$ or $3J_3$.

How can we generalize the assumption of almost simplicity of G_{Π/K_Π}? For the moment it seems reasonable to demand faithful and primitive action of G_{Π/K_Π} on the two parallel classes of lines of the grid Π. We already know, that G_{Π/K_Π} is not an affine group.

A. BLOKHUIS:

Extensions of Rédei's Theorem

Let $f : \mathbb{F}_q \to \mathbb{F}_q, q = p^n$, and $N := |\{\frac{f(x)-f(y)}{x-y} : x, y \in \mathbb{F}_q, x \neq y\}|$.
In 1970 Rédei showed that the value of N is restricted to a few intervals. A. E. Brouwer, T. Szönyi and the speaker further restricted the possibilities in 1992. Recently T. Szönyi observed that the same result applies if f is a partial function, assuming that $|Dom(f)| > q - \sqrt{q}/2$. A generalization in the other directions gives that if X is a set of $q+m$ points such that there are less than $(q+1)/2 - \sqrt{q}$ directions corresponding to a passing line, then $m > \sqrt{q}/2$ or all lines in other directions intersect X in 1 mod p points.

M. CLAUSEN:

Fast Fourier Transforms and Efficient Construction of Irreducible Representations

(Joint work with U. Baum). By Wedderburn Theorem, the complex group algebra of a finite group G is isomorphic to an algebra of block-diagonal matrices. Every such isomorphism $D : \oplus D_i : \mathbb{C}G \to \oplus_{i=1}^{n} \mathbb{C}^{d_i \times d_i}$ is called a Discrete Fourier Transform (DFT) for $\mathbb{C}G$. It involves a complete list $D_1, \ldots, D_n$ of pairwise inequivalent irreducible representations of $\mathbb{C}G$. The linear complexity $L(G)$ of G is the minimum number of arithmetic operations (addition, subtraction, scalar multiplication) to evaluate a suitable DFT for G at a generic (input) vector. (If only multiplication by scalars of absolute value ≤ 2 is allowed, we get the 2-linear complexity $L_2(G)$.) Trivially, $|G| - 1 \leq L(G) \leq 2|G|^2$.

Theorem

(1) $L(G) = O(|G|^{3/2})$, for any finite group G.

(2) $L_2(G) = o(|G| \log^3 |G|)$, for $G \in \{S_n, A_n\}$.

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(3) $L_2(g) = O(|G| \log |G|)$, if G is supersolvable.

(4) $L_2(G) > 1/4|G| \log |G|$, for any finite G.

(5) $L_2(G) > 1/2|G| \log |G|$ if G is abelian.

(6) If the supersolvable group G is given by a power-commutator presentation, then the irreducible representations can be constructed in time $O(|G| \log |G|)$.

More information and proofs can be found in our book: M. Clausen / U. Baum: Fast Fourier Transforms, BI, 1993.

A. E. Cohen:

On Chen Zhijie's Conjecture

Take $k = \overline{\mathbb{F}_3}, G = GL(4,k), V = Sym^3k^4/\langle\text{cubes}\rangle$. Then $dimG = dimV = 16$. This module came up in Liebeck's classification of group actions (G, V) with finitely many orbits of G in V, but wasn't actually settled as an example. It was studied by Chen Zhijie in "A new prehomogeneous vector space of char p", *Chinese Annals of Math.* **8B**(1987), 22-35, where he presented several orbits and conjectured that these are all. In joint work, David B. Wales and I have shown (computations need to be double checked) that his conjecture is true in spirit (there are finitely many orbits), but wrong in the sense that an additional orbit was found.
As a byproduct, it was found that, for any characteristic, $GL(4,k)$ has an invariant polynomial of degree 8 on Sym^3k^4.

C. Godsil:

p-Ranks of Projective Planes

Let B be the incidence matrix of a projective plane $\mathcal{P}$ of order n. Bruen and Ott proved that, if p divides n, the rank of B over GF(p) is at least $n^{3/2}+1$. (If p divides n then $\mathrm{rk}_p(B) \geq n^2+n$.) We improve Bruen and Ott's bound to

$$\begin{cases} n^{3/2}+1+\lfloor\frac{n+1}{2}\rfloor, & n\,\text{even}\,; \\ n^{3/2}+\frac{\sqrt{8n^3+1}+1}{2}, & n\,\text{odd}\,. \end{cases}$$

This is obtained by estimating the p-rank of $B \otimes B^T$, making use of the fact that there is a 5-class association scheme on the antiflags of $\mathcal{P}$.

P. de la Harpe:

Jones' Basic Construction and Invariants of Combinatorial Objects

Let X be a square complex matrix such that ${}^tXX^{-!} = nI$ (where $({}^tX)_{j,k} = X_{k,j}$ and $(X^{-!})_{j,k} = (X_{j,k})^{-1}$). Mimicking part of V. Jones' theory for subfactors and operator algebras, we associate to X a "commuting square" of semi-simple algebras. Invariants of this square (such as the Weyl group and the higher relative commutants) provide interesting invariants of X. They can be computed for several examples related to Hadamard matrices and stronlgy regular graphs.

T. Ito:

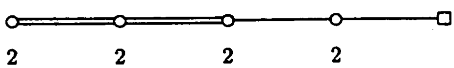
The Terwilliger Algebras of Cyclotomic Schemes

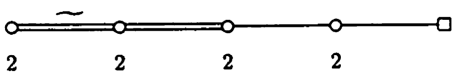
The Terwilliger algebra of an association scheme was introduced by Paul Terwilliger in his study of P- and Q-polynomial schemes and he himself called it a T-algebra or a subconstituent algebra. It is a semi-simple algebra containing the Bose-Mesner algebra, and it captures something about one point stabilizers when the scheme comes from a group action.
Terwilliger established the representation theory of T-algebras for thin P- and Q-polynomial schemes. For other classes of association schemes, not much is known about the Terwilliger algebras and the investigation of their representations has just recently started. Here we analyze the representation of the Terwilliger algebras for cyclotomic schemes by use of the Jacobi sums and determine the irreducible representations completely when the class is two.

A. A. Ivanov:

On Geometries of the Fischer Groups

The Fischer groups $\mathrm{Fi}_{22}, \mathrm{Fi}_{23}$ and Fi_{24} are associated with a number of diagram geometries some of which have been recently characterized. Exploiting a relationship between these geometries we obtain some further characterizations. In particular we prove the simple connectedness of the Ronan-Smith 2-local parabolic geometry of the largest Fischer group Fi_{24} and of its analogue related to the nonsplit triple cover $3 \cdot \mathrm{Fi}_{24}$. The diagrams of these geometries are the following:

Fi_{24}:


$3 \cdot \mathrm{Fi}_{24}$:


F. Jaeger:

Spin Models and Association Schemes, Part I

This talk presents some joint work with Eiichi and Etsuko Bannai. In 1989 Vaughan Jones introduced spin models as a way to construct invariants of links in 3-space, and we deal here with the nonsymmetric generalization due to Kanaga, Munemasa, and Watatani. Such spin models can be defined as pairs (w^+, w^-), where w^+, w^- are matrices in $M_n(\mathbb{C})$ which satisfy certain equations. With every spin model we associate two subspaces $\mathcal{M}$, $\mathcal{H}$ of $M_n(\mathbb{C})$ containing I, J, w^+, w^-, ${}^t w^+$, ${}^t w^-$. $\mathcal{M}$ is closed under ordinary product and $\mathcal{H}$ is closed under Hadamard product.

1. $\mathcal{M}$ and $\mathcal{H}$ are isomorphic as algebras.

2. When $\mathcal{M} = \mathcal{H}$, we have the Bose -Mesner algebra of a self-dual association scheme.

3. In this case, if P is the first eigenmatrix of the scheme, there exists a diagonal matrix Δ and $\lambda \in \mathbb{C} - \{0\}$ such that $(P\Delta)^3 = \lambda I$ (modular invariance).

4. for abelian group schemes, each such Δ gives a spin model.

G. D. James:
Immanants of M-Matrices

Given a partiton λ of n, one obtains a normalized immanant d_λ of an $n \times n$ matrix $M = (M_{ij})$ by defining

$$d_\lambda(M) = \sum_{\pi \in S_n} \frac{\chi_\lambda(M)}{\chi_\lambda(1)} m_{1,1\pi} m_{2,2\pi} \dots m_{n,n\pi} ,$$

where χ_λ is the irreducible character of S_n which is indexed by λ. For various classes of matrices, there appear to be many inequalities between normalized immanants. It is known, for example, that $\det M \leq d_\lambda(M)$ for all Hermitian positive semidefinite matrices, and the conjecture that $d_\lambda \leq \mathrm{perm} M$ for these matrices remains an intriguing open problem. In our talk, based on joint work with C. R. Johnson and S. Pierce, we discuss the problem of finding inequalities between immanants of M-matrices. The main result concerns the existence of a set of "test-matrices" for such inequalities. For certain, readily defined, matrices M_α (one for each partition α of n), we have $d_\lambda(M) \geq d_\mu(M)$ for all $n \times n$ M-matrices M if and only if $d_\lambda(M_\alpha) \geq d_\mu(M_\alpha)$ for all partitions α of n. Several examples are given which illustrate the power of this theorem.

M. H. Klin:
On Classical and Directed Versions of Strongly Regular Graphs

According to A. Duval a directed regular graph Γ of valency k with v vertices is called a directed strongly regular graph (d.s.r.g.) or a (v, k, μ, λ, t)-graph if

$$A^2 = tI + \lambda A + \mu(J - I - A) ,$$

where $A = A(\Gamma)$ is the adjacency matrix of Γ. The case $t = k$ is equivalent to the classical definition of s.r.g.

1. For each $n \geq 3$ there exists $(n(n-1), 2n-3, 2, n-2, n-1)$-graph which is invariant with respect to the natural action of the symmetric group S_n on $n(n-1)$ points. The case $n = 5$ gives a positive answer to a question of Duval.

2. To each projective plane of order q there corresponds a $((q^2+q+1)(q+1), q^2+q, q, q-1, q)$-graph which is obtained via merging of classes of the association scheme on flags.

3. There exists a primitive rank 3 s.r.g. (in a classical sense) with parameters $v = 81$, $k = 40$, $\lambda = 19$, $\mu = 20$, which admits a regular action of $\mathbb{Z}_9 \times \mathbb{Z}_9$. This gives a positive answer to a question of K. T. Arasu, D. Jungnickel, S. L. Ma, A. Pott.

R. LAUE:

Double-Cosets

For constructive purposes it is not sufficient to determine the number of double cosets. Instead a set of representatives is needed. Of course, a general solution is difficult and would for example solve the graph isomorphism problem. Therefore we discuss special situations using the homomorphism principle via a chain of subgroups or by prescribed stabilizers. A combination of both yields a tool which has been successfully applied to the construction of new 6-designs by Schmalz and will now be developed for double cosets more generally. As an example the construction of cycle permutation graphs is shown.

V. I. LEVENSHTEIN:

A New Lower Bound on the Size of Classical t-Designs

A system $S = S_\lambda(t, k, v)$ of k-subsets of a v-set possessing the property that any t-subset of the v-set belongs to λ k-subsets of S is referred to as a (classical) t-design. A problem of obtaining lower bounds on the size of a t-design is considered. The following bounds for t-designs are well known:

$$|S| \geq \binom{v}{t} \Big/ \binom{k}{t} \quad \text{(Steiner, 1853)},$$

$$|SS| \geq \binom{\frac{v}{k}}{}^{\gamma} \binom{v-\gamma}{l} \quad \text{(Wilson, Ray-Chaudhuri, 1971)},$$

where $t = 2l + \gamma$, $\gamma \in \{0, 1\}$, $2k \leq v$.
A new lower bound on the size of t-designs is obtained which for some classes is attained and better than the above mentioned bounds. The proof of the bounds is based on *(i)* Delsarte's inequalities for codes and designs in P- and Q-polynomial association schemes, *(ii)* the solution of the code problem for the system Q of orthogonal polynomials and *(iii)* a duality of bounding the sizes of codes and designs for finite spaces.

S. LÖWE:

Generalized Quadrangles with a Regular Point and their Association Scheme

If Γ is a generalized quadrangle with regular point we can associate with it an association scheme $\mathcal{A}(\Gamma)$ of class 4. Given an association scheme $\mathcal{A}$ with the same parameters as $\mathcal{A}(\Gamma)$, is it possible to reconstruct Γ from $\mathcal{A}$? A partial answer to this question uses the characterization of Γ as a cover of a net. From $\mathcal{A}$ we construct two geometric objects $\mathcal{X}$ and $\mathcal{N}$ and a projection map $p : \mathcal{X} \to \mathcal{N}$. Then the following are equivalent:

1. Γ can be reconstructed from $\mathcal{A}$.

2. p is a triangle free cover of a net $\mathcal{N}$ with suitable parameters.

3. Maximal cliques of $\mathcal{A}$ with respect to one of the relations are of constant size depending on the parameters of $\mathcal{A}$, and $\mathcal{N}$ is a net with suitable parameters.

In this context $\mathcal{N}$ is a net if and only if any two of its lines meet in a unique point and any two of its points are joined by a unique line.

A. MUNEMASA:

Graphs of Alternating, Symplectic, and Quadratic Forms

Let q be a power of 2. We consider the graph $\mathrm{Alt}(n+1,q)$ (resp. $\mathrm{Sym}(n,q)$, $\mathrm{Quad}(n,q)$) whose set of vertices is the set of all alternating bilinear forms on an $n+1$-dimensional vectorspace over $\mathrm{GF}(q)$ (resp. all symmetric bilinear forms on an n-dimensional vectorspace over $\mathrm{GF}(q)$, all quadratic forms on an n-dimensional vectorspace over $\mathrm{GF}(q)$). Two vertices are adjacent iff $\mathrm{rank}(x-y)=2$ (resp. $\mathrm{rank}(x-y)=1$ or 2, $\mathrm{rank}(x-y)=1$ or 2). $\mathrm{Alt}(n+1,q)$ is isomorphic to $\mathrm{Sym}(n,q)$ as graphs but not isomorphic as Schur rings. Indeed, $\mathrm{Alt}(n+1,q)$ is self-dual as Schur ring, while $\mathrm{Sym}(n,q)$ has dual $\mathrm{Quad}(n,q)$, which is not isomorphic to $\mathrm{Alt}(n+1,q)$.

M. MUZYCHUK:

The Structure of Schur Rings over Cyclic Groups of Square-Free Order

Let $\mathbb{Z}_n$ be a cyclic group, n is a square-free number. We give an explicit description of the structure of S-ring over $\mathbb{Z}_n$. Every S-ring is uniquely determined by a pair $(\mathcal{F}, \{\mathcal{G}_F\}_{F\in\mathcal{F}})$, where $\mathcal{F}$ is a topology defined on the set of prime divisors of n and $\{\mathcal{G}_F\}_{F\in\mathcal{F}}$ is a family of subgroups of $\mathbb{Z}_n^*$ satisfying some additional conditions.

S. NORTON:

Transposition Groups

The main topic was 3-generated 6-transposition groups, with emphasis on the Monster. It was shown how the braiding operations could, by use of a standard homomorphism from the 3-string braid group to the modular group, lead to a way of associating a polyhedron, called a football with every set of 3-transpositions. Such a polyhedron would have trivalent vertices and its faces would have at most 6 sides. One possibility is for a genus 1 surface (called doughnut) where all faces were hexagons and there is no degeneracy; otherwise Euler's formula means the genus would have to be 0. By homological arguments the size of a football can be correlated with the order of the product of the 3 transpositions, and it is also possible in principle to calculate the total number of footballs - estimated to be about 10000 for the Monster. A possible association with Moonshine was discussed, but this is not believed to be fruitful.

D. V. PASECHNIK:

Combinatorial Characterizations of Geometries for Sporadic Groups

We survey recent results concerning characterizations of sporadic group geometries without assumptions on group actions, which may be viewed as an attempt to create a theory unifying buildings and affine polar spaces with sporadic geometries.

1. D. V. Pasechnik: *Geometric characterization of graphs from the Suzuki chain.* Eur.J.Comb. 14(1993), 491-499.

2. D. V. Pasechnik: *Geometric characterization of the sporadic groups* Fi_{22}, Fi_{23} *and* Fi_{24}. J.Comb.Th.(A), to appear.

3. D. V. Pasechnik: *Extended generalized octagons and the group He.* preprint 1993.

CH. E. PRAEGER:

On Hexagonal Graphs

This is joint work with Manley Perkel.

A hexagonal graph is a graph Γ with a set C of hexagons (cycles of length 6) such that each 2-arc of Γ lies in a unique hexagon in C. We studied hexagonal graphs such that $Aut(\Gamma)$ is transitive on 2-arcs and $Aut(\Gamma)_\alpha^{\Gamma(\alpha)}$ is 3-transitive. Two new infinite families of hexagonal graphs with valencies $2^d, d \geq 2$, were constructed. It was shown that the two smallest graphs, one from each family, are the only two hexagonal graphs such that $Aut(\Gamma)_\alpha^{\Gamma(\alpha)}$ is 3-transitive, and such that the set of vertices $\Delta(\alpha)$ which are antipodal vertices of the cycles in C containing α has minimal size $|\Gamma(\alpha)| - 1$. The two examples are:

1.) Valency 4, 32 vertices, $Aut(\Gamma) = S_5 \times S_3$, and

2.) Valency 8, 270 vertices, $Aut(\Gamma) = S_9$.

A. SALI:

On the Rigidity of Spherical t-Designs

Spherical t-designs were introduced by P. Delsarte, J.M. Goethals and J.J. Seidel.

Definition *A finite* $X \subset \mathbf{S}^d$ *is called a* spherical t-design in $\mathbf{S}^d$ *iff*

$$\frac{1}{|\mathbf{S}^d|}\int_{\mathbf{S}^d} f(x)d\omega(x) = \frac{1}{|X|}\sum_{x\in X} f(x)$$

holds for all polynomials $f(x)$ of degree at most t.

Definition *A spherical t-design X is called rigid iff any sufficiently close t-design X' to X is an orthogonal transform of X.*

The concept of rigid spherical t-designs was introduced by Bannai. He conjectured that there is a function $f(t,d)$ such that if X is a spherical t design in the d-dimensional Euclidean space so that $|X| > f(t,d)$, then X is non-rigid. Furthermore, he asked to find examples of rigid but not tight spherical designs. In the present talk we shall investigate the case of t=2 and find a series of non-tight but rigid designs. Also an important duality property of spherical 2-designs is discussed. In the second half we take X as an orbit of a finite reflection group and prove that X is rigid iff tight for the groups $A_n, B_n, C_n, D_n, E_6, E_7, F_4, I_3$.

J. SAXL:

Exceptional Polynomials

A polynomial in $\mathbb{F}_q[X]$ inducing a permutation on $\mathbb{F}_q$ is a *permutation* polynomial. It is an *exceptional* polynomial, if it is a permutation polynomial over $\mathbb{F}_{q^t}$ for infinitely many q^t. Carlitz (1965), based on work of Dickson (1896), conjectured that for n fixed, even, there is a constant c_n such that there are no permutation polynomials for q odd with $q > c_n$. This translates into: There are no exceptional polynomials of even degree in odd characteristic. This has been proved in a joined work with M. Fried and R. Guralnick (*Israel J. Math.* 82(1993), 157-225). Information was also obtained in the general case, which has been used since by S. Cohen and R. Matthews to construct a new family of exceptional polynomials in even characteristic of degree $n = 2^{s-1}(2^s - 1)$, s odd.

R. SCHARLAU:

Glueing Integral Lattices

We study even integral lattices L with a prescribed decomposable sublattice $L_1 \perp L_2 \subseteq L$. Assuming $L_2 = \mathbb{Q}L_2 \cap L$, such a lattice is of the form

$$L = L_1 \times_\phi L_2 := \{x + y \in L_1^\# \oplus L_2^\# \mid \overline{x} \in S, \phi\overline{x} = \overline{y}\}$$

for a unique subgroup $S \subseteq T_1$ and an isometry (of finite quadratic forms) $\phi : (S, q_1) \to (T_2, -q_2)$. Here, $T_i = T(L_i) := L_i^\#/L_i$ is the discriminant group of L_i, where $L_i^\# := \{y \in L_i \mid (L_i, y) \subseteq \mathbb{Z}\}$ denotes the dual lattice, and $q_i : T_i \to \mathbb{Q}/\mathbb{Z}$, $q_i(\overline{x}) = (x,x)/2 + \mathbb{Z}$, is the discriminant quadratic form. Assume moreover that L is selfdual and L_1, L_2 are characteristic sublattices. Then the possible L are described by double cosets $\overline{O}(L_2)\backslash Aut(T_2)/^{\phi}\overline{O}(L_1)$. We consider as a complicated example the case $n = \mathrm{rank}L = 32$, $r = \mathrm{rank}L_1 = 20$, $L_1 \supseteq 20A_1$(root system), $L_2 = L_1^\perp \cap L$, $\mathrm{min}L_2 = 4$ (in fact, $L_2 \cong {}^2\tilde{D}_{12}$).

J. J. SEIDEL:

Spherical Designs and Tensors

1. A spherical design of even index q is a finite set U such that

$$\int_\Omega \otimes^q u\, d\sigma(u) = \frac{1}{n}\sum_{u \in U} \otimes^q u$$

where $|U| = n$, $U \subset \Omega \subset \mathbb{R}^d$, unit sphere Ω, q^{th} tensor power $\otimes^q a = a \otimes \ldots \otimes a$ having coordinates the monomials of degree q in the coordinates $(a_1, \ldots a_d) = a$. Equivalently:

(1.1) U is extremal in the Sidelnikov's inequality:

$$0 \leq \left\| \int_\Omega \otimes^q u \, d\sigma(u) - \frac{1}{n} \sum_{u \in U} \otimes^q u \right\| = -\delta + \frac{1}{n^2} \sum_{u,v \in U} (u,v)^q .$$

(1.2) $\frac{1}{n^2} \sum_{u,v \in U} (u,v)^q = \delta := \frac{1 \cdot 3 \cdot 5 \cdots (q-1)}{d(d+2) \cdots (d+q-2)}$.

(1.3) $\frac{1}{n} \sum_{u \in U} h(u) = \int_\Omega h(u) \, d\sigma(u)$.

2. Linear map $F : \mathbb{R}^d \to \mathbb{R}^N$, $x \mapsto y = Fx$, $F = \{\text{rows of standard matrix}\}$. For F the following are equivalent:

(2.1) Condition (D): $\int_\Omega \otimes^q u \, d\sigma(u) = \delta \sum_{f \in F} \otimes^q f$.

(2.2) Cubature: $\int_\Omega h(u) \, d\sigma(u) = \delta \sum_{f \in F} h(f)$, $h \in \hom_q(\mathbb{R}^d)$.

(2.3) Waring: $\|x\|^q = \sum_{f \in F} (f,x)^q$, $x \in \mathbb{R}^d$.

(2.4) Isometry: $\|x\|_2 = \|y\|_q := \sqrt[q]{\sum_{\nu=1}^{N} y_\nu^q}$, $x \in \mathbb{R}^d$.

3. Hilbert's Lemma (1909) for the solution of Waring's problem implies:

Theorem: $\forall d \in \mathbb{N}$, $\forall q \in 2\mathbb{N}$, $\exists N \in \mathbb{N}$ and linear $F : \mathbb{R}^d \to \mathbb{R}^N$ satisfying (D), Cubature, Waring, Isometry and (multi)spherical design of index q.

L. H. SOICHER:

Low Rank Representations and Graphs for Sporadic Groups

(Joint work with Cheryl Praeger) We classify all transitive permutation representations ϕ of rank at most 5 of the sporadic almost simple groups (the groups T with $S \leq T \leq \mathrm{Aut}(S)$, S a sporadic simple group). One application is the completion of the classification of the primitive permutation representations of rank at most 5 of the finite almost simple groups. For each representation ϕ above, we determine the intersection matrices for the resulting orbital digraphs. We then classify the distance-regular graphs having a vertex-transitive action of rank at most 5 by a sporadic almost simple group, and discover some new distance-regular graphs of diameter 2.

S. Y. SONG:

Character Tables of Association Schemes and their Fusions and Fissions

For given commutative association schemes, we can construct many new commutative association schemes as their fusion schemes or as their fission schemes by using the character table of given schemes and their fusion and fission. In this talk, many known examples of fusion schemes and fission schemes of orbital (group-case) association schemes and some open problems related to the classification of commutative association schemes will be discussed.

O. TAMASCHKE:

SCHUR-Ring Theory as an Extension of Finite Group Theory.

Every subring T of the group ring $\mathbb{Z}G$ of any finite group G with the following properties is called a SCHUR-*ring* (*S-ring*) on G:
There exists a decomposition $G = T_1 \cup \ldots \cup T_t$ of G into non-empty trivially intersecting subsets such that

$$(S1) \qquad \forall i \in \{1,\ldots,t\}\ \exists j \in \{1,\ldots,t\}: \qquad T_i^{-1} := \{g^{-1} \mid g \in T_i\} = T_j$$

$$(S2) \qquad \forall i,j,k \in \{1,\ldots,t\}\ \exists z_{ijk} \in \mathbb{Z}: \qquad \mathcal{I}_i \mathcal{I}_j = \sum_{k=1}^{t} z_{ijk}\mathcal{I}_k$$

$$(S3) \qquad T = \sum_{i=1}^{t} \mathbb{Z} \cdot \mathcal{I}_i,$$

where $\mathcal{I}_i := \sum_{g \in T_i} g \in \mathbb{Z}G$.
After historical remarks on BURNSIDE-groups and motivations via the double coset S-rings as the centralizer rings of permutation representations the category of SCHUR-algebras over $\mathbb{C}$ is introduced, followed by a report on T-subgroups. T-normal subgroups and the homomorphism theorem for S-algebras over $\mathbb{C}$.

This is a brief account of a theory presented in the book

Olaf Tamaschke: *Schur-Ringe.*

B.I. Hochschulskripten 735/735a*; Bibliographisches Institut Mannheim, 1970.

O. TAMASCHKE:

A Generalized Character Theory for Finite Groups.

A brief account is given on the paper "On Schur-rings which Define a Proper Character Theory on Finite Groups" Math. Z. 117 (314-360) 1970. For any SCHUR-ring T on any finite group G, decomposed into its G-classes $G = T_1 \cup \ldots \cup T_t$ and any representation $F : \mathbb{C} \to (\mathbb{C})_n$ of the SCHUR-algebra $\mathbb{C}T$ by $n \times n$-matrices over $\mathbb{C}$ we set

$$\forall g \in G \qquad \phi(g) := trace\, F\left(\frac{1}{T_i} \cdot T_i\right) \quad \text{for } g \in T_i$$

and call $\phi : G \to \mathbb{C}$ the *T-character* of G belonging to F. Any g, $h \in G$ are called *T-conjugate* iff $\phi(g) = \phi(h)$ for every T-character ϕ of G. If r is the number of irreducible

T-characters and s the number of T-conjugacy classes, then $r \leq s$. The T-character table Φ_T is a $r \times s$-matrix of rank r and orthogonality relations hold for the rows of Φ_T. If $r = s$, then T is called a *CS-ring* in which case orthogonality relations also hold for the columns of Φ_T and T-character theory on G has all relevant properties of ordinary character theory on G in that case.

P. TERWILLIGER:

The Q-Polynomial Property

We investigate four conditions on a regular finite undirected graph $\Gamma = (X, E)$. They are, in order of increasing regularity:

(1) Γ is distance-regular.

(2) Γ is thin.

(3) Γ is Q-polynomial and thin.

(4) Γ is triply regular.

In fact (4) → (3) → (2) → (1). A graph Γ is said to be *thin* whenever for all distances a, b, i, j, k, and all vertices $x, y, z \in X$ such that $\partial(x,y) = a, \partial(x,z) = a, \partial(y,z) = b$, then

$$|\{w \mid w \in X, \partial(w,x) = i, \partial(w,y) = j, \partial(w,z) = k\}| \\ = |\{w \mid w \in X, \partial(w,x) = i, \partial(w,y) = k, \partial(w,z) = j\}|.$$

Theorem. Assume Γ is Q-polynomial and thin, with diameter $o \geq S$. Then for all distances a, i, j, k, and all vertices x, y, z such that $\partial(x,y) = 1, \partial(y,z) = a, \partial(x,z) = a+1$, then

$$|\{w \mid w \in X, \partial(w,x) = i, \partial(w,y) = j, \partial(w,z) = k\}|$$

depends only on a, i, j, k and not on x, y, z.

Z.-X. WAN:

Representations of Forms by Forms in a Finite Field

Let q be a prime power and $2 \not| q$. Let A and B be, respectively, $m \times m$ and $t \times t$ symmetric matrices over $\mathbb{F}_q$. Denote by $n^{(q)}_{m \times t}(A, B)$ the number of $m \times t$ matrices X over $\mathbb{F}_q$ satisfying ${}^tXAX = B$. Formulas for $n^{(q)}_{m \times t}(A, B)$ were obtained by L. Carlitz in 1954. The case when A and B are both skew-symmetric and the case when they are both hermitian (when q is a perfect square) were also studied by L. Carlitz in 1954 and by L. Carlitz and J. Hodges in 1955, respectively. Now the problem is studied in its full generality. On the one hand, the assumption $2 \not| q$ is removed. On the other hand, besides the above three cases studied by L. Carlitz, the cases of alternate forms, of quadratic forms when $2 \mid q$, of symmetric bilinear forms when $2 \mid q$, and bilinear forms are also studied and complete results are obtained.

V. WELKER:

Topological Proofs of Some Decompositions of Some S_n-Characters

In this talk we present two methods for obtaining filtrations of representations which arise in the theory of Coxeter groups (joint work with S.Sundaram and M.Wachs). The methods were developed in research on the theory of arrangements of linear subspaces and its relations to configuration spaces and singularity theory. The first method can be used to obtain a decomposition of the regular representation of a Coxeter group. We show that for S_n the resulting decomposition is the decomposition also derived from the Poincaré-Birkhoff-Witt theorem by considering the S_n-action on the variables. This method uses Coxeter-Arrangements and a G-module theoretic version of the Goresky-MacPherson formula. The second method, which can be used to obtain the filtration of the character of S_n on the multigraded part of the free Lie-Algebra, first calculated by Reutenauer, is based on a spectral sequence introduced by Hanlon. We apply this spectral sequence in the order complex of the partition lattice to retrieve Reutenauer's result.

P.-H. ZIESCHANG:

Geometric and Representation-Theoretic Methods in the Structure Theory of Association Schemes

Viewing finite groups as a distinguished class of association schemes we investigate the question which parts of finite group theory can be generalized to a useful contribution to the structure theory of association schemes. We introduce the concept of coset systems of association schemes to abstract the way in which classical geometries arise from groups, e.g., in which certain buildings arise from groups with a (B, N)-pair. Together with the theory of coset systems the ordinary representation theory of association schemes has strong implications in the structure theory of association schemes, e.g., it is possible to determine the association schemes generated by two involutions.

K.-H. ZIMMERMANN:

On a Basis for a Class of Weight Spaces

In my talk I present a basis for a class of weight spaces. These spaces are weight spaces of polynomial representations of the general linear group (in the usual sense) introduced by G. D. James (in his LNM-book, section 26).

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