MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 3/1994

Gruppentheorie (Permutationsgruppen)

16.1. - 22.1.1994

Die Tagung fand unter der Leitung von Prof. Dr. Otto H. Kegel (Freiburg) und Prof. Dr. Peter M. Neumann (Oxford) statt.

An der Tagung, bei der Themen im Zusammenhang mit endlichen und unendlichen Permutationsgruppen im Mittelpunkt standen, nahmen 39 Mathematiker aus 10 Ländern teil.

Die Themen der 32 Vorträge waren weit gestreut; so reichten sie z. B. von Automorphismengruppen von (partiell) geordneten Mengen oder Mengen mit einer Relation bis zum Zusammenhang zwischen Permutationsgruppen und Untergruppenwachstum.
Neben den Vorträgen ergaben sich allabendlich intensive Diskussionen, ein reger Austausch von Ergebnissen und Perspektiven, sowie Gelegenheit zur konzentrierten Zusammenarbeit. Dazu trug die Atmosphäre des Mathematischen Forschungsinstitutes wie auch die großzügigen Möglichkeiten entscheidend bei.

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Vortragsauszüge

J. Bamblett

Groups of product type

We study primitive permutation groups which "look like" wreath products in their primitive action. We define systems of product imprimitivity according to Kovács, "Wreath decompositions of finite permutation groups", Bull Austral. Math. Soc. 40 and, having defined a partial order on the set of all such systems for a given group, we show that in many cases this poset is in fact a lattice. This helps us with the question of recognising computationally groups of product type as we do not have to search for systems of product imprimitivity but can then just compute; the minimal element of the lattice of systems can be found from the socle of the group.

B. Baumeister

Groups of product type with an application to C_2c geometries

I mentioned two propositions about permutation groups which I used then for the classification of flag transitive C_2c-geometries supposing an almost simple or affine stabilisator of a plane.

Proposition 1: Let H be an almost simple group and suppose $H = AA^{\alpha}$ for an subgroup A of H and $\alpha \in \mathrm{Aut}(H)$. Then $\mathrm{Soc}(H) \cong P\Omega_8^+(q)$, M_{12}, $Sp_4(q)'$, q even.

Proposition 2: Let $V \cong p^m$ be an irreducible and faithful module for a group K with $F(K) = 1$. If K has a subgroup U, such that $|K : U| = p^a$, $a \geq m$, then $p = 2$, $\mathrm{Soc}(K) \cong L_3(2) \times \ldots \times L_3(2)$ and V is a direct sum of natural modules for $L_3(2)$.

G. Behrendt

Automorphism groups of ordered sets

The talk gives a survey of recent results and open problems in some areas of the theory of automorphism groups of ordered sets. The subjects are the following:
Representations of groups as automorphism groups of "small" ordered sets; representations of groups as automorphism groups of ordered sets with forbidden induced suborders; homogenity conditions for finite ordered sets.

St. Bigelow

The truth about supplements of $B\lambda$

Let $\lambda \leq \kappa$ be infinite cardinals. A group G acting on a set Ω of size k is a supplement of $B\lambda$ if for every permutation s of Ω there is a $g \in G$ which agrees with s on all but fewer than λ points. Semmes has shown that if the Generalised Continuum Hypothesis holds then there is a simple and powerful description of all such groups. Macpherson and Neumann published a theorem giving a slightly weaker description which holds true in all models of set theory. As a corollary this yields the exact set theoretic assumptions required for Semmes' result to hold. Unfortunately I have discovered a mistake in the proof of their theorem and can construct a counterexample in certain models of set theory. However I have an alternative proof that the corollary still holds. I will discuss these results and some ingredients that went into their proof.

B. Brewster

Groups 2-transitive on a set of their Sylow subgroups

By Sylow's Theorem each finite group G is transitive on $Syl_p(G)$, the set of Sylow p-subgroups of G, via conjugation. We present some results that indicate how strong a condition it is for this action to be 2-transitive. The work is joint with M. Ward (Bucknell U.).
Using the classification of 2-transitive groups cited by Cameron, Bull. London Math. Soc. 13 (1981), aided by methods presented by Hering, J. Algebra 93 (1985), we obtain a precise characterization of the groups G which are faithful and 2-transitive on $Syl_p(G)$ for some prime p. From this we deduce:

G is 2-transitive on $Syl_r(G)$ for each r and faithful on $Syl_p(G)$ for some p $\Leftrightarrow$

(i) $G \cong \{x \to ax + b \mid a,b \in GF(q^n), a \neq 0\}$ where q is prime, $q \neq p$, q is not Mersenne or $n \neq 2$

(ii) $G \cong H \leq \Gamma(q^n)$ with $|\Gamma(q^n):H| \mid 2$ if q is Mersenne and $n = 2$

(iii) $G \cong S_4$, $p = 3$

or (iv) $G \cong [C_3 \times C_3] \cdot SL(2,3)$, $p = 2$

Consequently if G is 2-transitive on $Syl_r(G)$ for each r, $G \in N^3$ and if either $|Syl_3(G)| = 1$ or $|Syl_2(G)| = 1$, $G \in N^2$.

A. R. Camina

Linear spaces with line-transitive, point-imprimitive automorphism groups

I wish to discuss the following result (proved jointly with S. Meschke).
Let S be a line-transitive, point-imprimitive linear space with k (number of points on a line) < 9. Then S is one of the following:

(a) A projective plane of order 4 or 7,
(b) one of 2 linear spaces with 91 points and $k = 6$ or
(c) one of 467 linear spaces with 729 points and $k = 8$.

A. R. Camina (for J. Siemons)

Modular Homology in the Boolean Algebra and Group Actions

Let Ω be a set and denote by $2^{[\Omega]}$ the finite sets of Ω. Let R be a ring and consider the R-module $R\,2^{[\Omega]}$. Let $\Delta \in 2^{[\Omega]}$ and let $\partial(\Delta) = \sum_{r_i \leq \Delta} r_i$, $|\Delta| = k$, $|r_i| = k - 1$.

Let $M_k = \{\Sigma\, r_\Delta \Delta \mid \Delta \leq \Omega, |\Delta| = k, r_\Delta \in R\}$. Consider the chain

$$o \leftarrow o \ldots \leftarrow M_0 \leftarrow M_1 \leftarrow M_2 \leftarrow \ldots$$

If R has char p the $\partial^P = o$.

<u>Theorem</u> All subsequence of the kind

$$\leftarrow M_{k-p} \leftarrow M_{k-p+i} \leftarrow M_k \leftarrow M_{k+i} \leftarrow M_{k+p} \leftarrow \ldots$$

are exact for arbitrary k, $0 < i < p$ as long as $2(k + p) \leq |\Omega|$.

M. Droste

McLain-groups over arbitrary rings and orderings
(joint work with R. Göbel, Essen)

We investigate McLain-groups $G(R,S)$ over arbitrary rings R and posets $(S,\leq)$. The group elements can be viewed as upper triangular matrices with only finitely many entries $\neq O$ from R, indexed by S. If R has no zero-divisors $\neq O$ and $(S,\leq)$ is locally linear (i.e. each interval is a chain), we can recover the structures of R and $(S,\leq)$, up to isomorphism or anti-isomorphism, from $G(R,S)$. Also, $Aut(G(R,S))$ can be determined, and we can characterize when $G(R,S)$ is characteristically simple. As a consequence, using non-linear posets (trees) $(S,\leq)$ we obtain for each prime p, continuously many countable characteristically simple locally finite p-groups $G(F_p,S)$.

T. Gardiner

Imprimitive graphs. Regular maps. Coverings of complete graphs

I shall present a 'geometrical' approach to the study of imprimitive graphs which makes it possible to decompose certain graphs as the 'product' of a quotient graph and a 1-design induced on each block. I shall analyse some of the simplest cases in detail.

M. Giraudet

Convenient languages for groups preserving or reversing chains or cyclic orderings

(1) It is well known (Holland 1963) that the groups of permutations of chains provided with the pointwise order are exactly the ordered subgroups of the lattice ordered groups.
(2) Groups of monotonic permutations of chains can be characterized as structures in (1) provided with a decreasing group automorphism of order two (Joint with F. Lucas).
(3) Groups of permutations of total cyclic orders have charaterisations among structures in (1) enriched with a parameter in their center (Joint with M. Droste and D. Macpherson), or enriched with a predicate for a normal subgroup which is a Z-group.
(4) Above (2) and (3) can be combined.
(2), (3) and (4) provide languages which behave much better than the natural ones (= with pointwise relations) towards usual class operations: direct sums, wreath product, ...A beginning of exploration of the lattice of varieties in (2) is undertaken (joint with I. Rachunek).

Ch. Hering

On large prime divisors of the order of a finite linear group

Starting point is a result proved by Feit and Thompson (1961) via modular representation theory:
Theorem 1. Let r be a prime. If the finite group G has a faithful representation of degree n over the complex numbers and if $r > 2n + 1$, then the Sylow r-subgroup of G is an abelian normal subgroup of G .
This generalizes a Theorem of Brauer (1942) who treated the special case $r^2 \nmid |G|$. Also Blichfeldt (1903) solved the case $r > (2n + 1)(n - 1)$. We prove the following generalisation:
Theorem 2. Let $G \leq GL(n,K)$, G finite, r a prime divisor of $|G|$ different from the characteristic of K and $r > 2n + 1$. Then one of the following conditions holds:

a) $O_r(G) \neq 1$.

b) Char $K = p < \infty$ and $O_p(G) \neq 1$.

c) Char $K = p < \infty$ and G contains a subnormal quasisimple subgroup H such that $r \mid |H|$ and $H/Z(H)$ is a simple Chevalley group of characteristic p (of ordinary or twisted type) or $H/Z(H) \cong J_1$, and $p = 11$. The proof uses the classification of finite simple groups. It is joint work with P. Munke.

W. Ch. Holland

The partial orders of the group of order permutations of the real line

We let G denote the group of the title and **R** the real line. Among the interesting properties of this group is that it admits a lattice order, the socalled *pointwise order*: If we define, for $f,h \in G$, that $f \leq h$ iff for all $\alpha \in \mathbf{R}$, $\alpha f \leq \alpha h$, then G becomes a partially ordered group (whose order is preserved by the group operation). In fact, G is a lattice. It has long been known that every countable lattice-ordered group can be embedded in G .

In this paper, we investigate other partial orders on G under which G is a partially ordered group. There is exactly one other lattice order of G (the reverse of the pointwise order). Among all non-trivial partial orders of G there are exactly 16 minimal ones and 40 maximal ones. It is probable that the partially ordered set of all partial orders of G is *well partially ordered* by containment, that is that there is no infinite descending chain and no infinite antichain.

W. Knapp

On Burnside's Method

W. Burnside gave in the second edition of his book [1911] a proof of the theorem stating that any primitive permutation group G containing a transitive cyclic subgroup of composite prime power order is doubly transitive.
P. Neumann pointed out in his recent article "Helmut Wielandt on permutation groups" that Burnside's proof uses in the last but crucial step an argument about sums of roots of unity which is false. (P. Neumann gave explicit counterexamples and stated that Burnside's error 'is not as easy to put right as one would like'.)
In my talk it is shown that Burnside's error can be repaired by the method used in his own proof of the celebrated theorem on permutation groups of prime degree p and by an additional result concerning sums of p^m-th roots of unity.

A. Kreuzer

Loops with an automorphism group related to the relativistic velocity addition

The relativistic velocity addition $\oplus$ is a binary operation which is neither commutative nor associative. For $\mathbb{R}_c^3 := \{v \in \mathbb{R}^3 : |v| < c\}$, $(\mathbb{R}_c^3, \oplus)$ is a loop with automorphisms $A := \{\delta_{a,b} : a,b \in \mathbb{R}_c^3\}$, at what for any $a,b \in \mathbb{R}_c^3$ the automorphisms $\delta_{a,b}$ are defined by the equation $a \oplus (b \oplus x) = (a \oplus b) \oplus \delta_{a,b}(x)$. Such a loop, called K-loop, appears also as the additive structure of a neardomain, which is a generalisation of a nearfield. The notion of a neardomain was introduced by H. Karzel to describe infinite sharply 2-transitive permutation groups. Further a construction method for K-loops is given.

F. Leinen

Irreducible representations of periodic finitary linear groups

Let V be a vector space over the field K. A *finitary representation* of the group G is a homomorphism $\sigma: G \to \mathrm{Aut}_K(V)$ such that, for every $g \in G$, the endomorphism $g\sigma - 1$ has finite rank. A group G is said to be *finitary linear*, if it has a faithful finitary representation. During the last few years the theory of finitary linear groups has been an area of intense and fruitful research. Here, we report about the following generalization of a well-known theorem of A. E. Zalesskii and D. J. Winter for linear groups.

Theorem. *Every irreducible finitary representation of a periodic group* G *over the algebraically closed field* K *is equivalent to a finitary representation of* G *over the smallest subfield of* K *containing the* m*-th roots of unity for every natural number* m *which occurs as the order of some element in* G.
The proof combines an ultraproduct argument with an induction on the cardinality of G. In this setup, the theorem of Zalesskii and Winter is used locally. The Theorem has some consequences concerning the size of periodic finitary linear groups.

Corollary. (a) *Every infinite periodic irreducible group of finitary transformations of a* κ*-dimensional vector space has cardinality at most* 2^κ.
(b) *Every irreducible periodic locally solvable finitary linear group is countable.*

M. Liebeck

Subgroups of exceptional groups

In joint work with A. Shalev, we complete the proof of a conjecture of J. D. Dixon made in 1969:
Theorem Let G_0 be a finite simple group, and let G be a group with $G_0 \leq G \leq \mathrm{Aut}\, G_0$. If P(G) denotes the probability that two randomly chosen elements of G generate a subgroup containing G_0, then $P(G) \to 1$ as $|G| \to \infty$.
This was proved by Dixon for G_0 alternating (using elegant combinatorial arguments) and by Kantor and Lubotzky for G_0 a classical group or a small exceptional group (using CFSG).
The proof is based on a new result concerning the orders of maximal subgroups of simple groups of exceptional type:
Theorem Let G_0 be of type $F_4, {}^2F_4, E_6, {}^2E_6, E_7$ or E_8 over F_q, and let $G_0 \leq G \leq \mathrm{Aut}\, G_0$. If M is a maximal subgroup of G such that either $|M| \geq |G|^{2/5}$ or soc(M) is non-simple, then M is known. In particular, there are at most $c + \log q$ conjugacy classes of such subgroups M.

A. Lubotzky

Subgroup growth and permutation groups

Let Γ be a f.g. group, $a_n(\Gamma)$ = number of subgroups of Γ of index n. Groups of polynomial subgroup growth (PSG) (i.e. with $a_n(\Gamma) = O(n^c)$ for some c) were characterized:
Theorem 1 (A. Lubotzky, A. Mann, D. Segal) Let Γ be a f.g. residually finite group. Then Γ is PSG iff Γ is virtually soluble of finite rank.
Other type of growth are provided by arithmetic groups satisfying the congruence subgroup property (CSP).
Let Γ be an arithmetic group, e.g. $\Gamma = SL_r(\mathbb{Z})$, $\gamma_n(\Gamma) = \#\{H \leq \Gamma \mid [\Gamma : H] \leq n$ and H is a congruence subgroup$\}$.

Theorem 2 If Γ is arithmetic in char = O, then $\exists\, c_1, c_2$ s.t.

$$n^{c_2 \log n/\log\log n} \le \gamma_n(\Gamma) \le n^{c_1 \log n/\log\log n}$$

Theorem 3 If Γ is arithmetic in char $p > O$ then $\exists\, c_3, c_4$ s.t.

$$n^{c_3 \log n} \le \gamma_n(\Gamma) \le n^{c_4 \log^2 n}$$

Theorem 4 If Γ is any f.g. linear group then $a_n(\Gamma) \ge n^{c \log n/\log\log n}$.

In the lecture we also present an example of a f.g. residually finite group which seems to have $a_n(\Gamma) \sim n^{c \log n/(\log\log n)^2}$. This beats the lower bound of linear groups. To finish the proof one needs to prove: $\forall r$: $a_n(S_r) \le n^{c \log n/(\log\log n)^2}$.

M. Lustig

How to analyse a free group automorphism via graphs and trees

We show that for every $\varphi \in Out(F_n)$ (the outer automorphism group of a f.g. free group) some power φ^t $(t \ge 1)$ can be decomposed canonically into "prime" factors which are either "Dehn-twists" or "partial pseudo-Anosovs" (explained in the talk). This is based on fundamental work of Bestwina-Handel and on older work of myself.
If two automorphisms φ_1 and φ_2 are conjugate in $Out(F_n)$ then there is a correspondence between the prime factors in the factorization of φ_1 and φ_2, such that corresponding factors are conjugate. For both types of prime factors there have been developed algorithmic solutions of the conjugacy problem, in both cases coming with nice geometric interpretations on graphs or trees (for Dehn-twists this is joint work with M. D. Cohen). Combining these algorithms properly gives a solution to the conjugacy problem in $Out(F_n)$.

H. D. Macpherson

Jordan Groups

If G is a permutation group on Ω, then $\Sigma \subseteq \Omega$ is a Jordan set if $|\Sigma| > 1$ and $G_{(\Omega\setminus\Sigma)} := \{g \in G:\ g|_{\Omega\setminus\Sigma} = id\}$ is transitive on Σ; Σ is a proper Jordan set if in addition, if $|\Omega\setminus\Sigma| = k \in \mathbb{N}$, then G is not $(k+1)$-transitive. We say that Σ is a primitive Jordan set if $G_{(\Omega\setminus\Sigma)}$ is primitive on Σ. A Jordan group is a group with a proper Jordan set. Around 1983, P. M. Neumann and Kantor independently classified finite primitive Jordan groups, using the classification of the finite simple groups (and the result of Jordan that every finite primitive Jordan group is 2-transitive). Around 1985 S. Adeleke and P. M. Neumann classified infinite primitive Jordan groups with proper primitive Jordan sets; that is, they showed that such a group is highly transitive, or preserves a linear order, circular order, linear betweeness relation, separation relation, semilinear order, or B,C or D relation (there are relational structures associated with semilinear orders). Recently, all infinite primitive Jordan groups were 'classified' in the above sense, in joint work with Adeleke. There are some additional families of examples which arise; namely, groups of automorphisms of Steiner systems and certain limits of B-relations, D-relations, and Steiner systems.

A. Mann

Counting primitive permutation groups and generating profinite groups

A profinite group G is positively finitely generated (PFG) if for some k, the set of k-tuples generating G has a positive Haar measure in G^k. E.g. a finitely generated pro-p group is PFG (easy) while a finitely generated free profinite group is not (Kantor - Lubotzky). I will discuss the proofs of the following two theorems:
Theorem 1. A finitely generated pro-soluble group is PFG.
Theorem 2. A profinite group is PFG if and only if the number of maximal subgroups of G of index n grows polynomially (at most) with n (Mann - Shalev).
In both proofs counting primitive permutation groups is essential. In the proof of Theorem 2 we also employ some probabilistic arguments.

P. Neumann

A problem about permutation groups

A J-set for a permutation group (G,Ω) is a subset Σ of Ω such that the pointwise stabiliser of $\Omega-\Sigma$ is transitive on Σ. Peter Cameron [Oligomorphic Permutation Groups, p. 129] has asked for those (G,Ω) with the property that for any finite Φ the pointwise stabiliser G_Φ has only finitely many orbits, and each of them is a J-set. We answer this question by classifying a slightly more general class of groups.

Ch. E. Praeger

Finite quasiprimitive permutation groups

A group $G \leq \mathrm{Sym}(\Omega)$ is said to be quasiprimitive on Ω if each nontrivial normal subgroup of G is transitive on Ω. Quasiprimitive groups have arisen in an essential way in the study of finite 2-arc transitive graphs: each finite, non-bipartite, 2-arc transitive graph is a cover of a finite non-bipartite graph admitting a (sub)group of automorphisms quasiprimitive on vertices and transitive on 2-arcs. A structure theorem, similar to the O'Nan - Scott theorem for finite primitive permutation groups has been proved for finite quasiprimitive permutation groups, and has been applied to describe finite quasiprimitive 2-arc transitive graphs. Several fundamental questions remain unanswered:

1. When is an imprimitive quasiprimitive group G contained in a primitive subgroup H of $\mathrm{Sym}(\Omega)$, where $H \not\trianglerighteq \mathrm{Alt}(\Omega)$?

2. If $G \leq \mathrm{Aut}\,\Gamma$ and G is quasiprimitive and imprimitive on vertice, and transitive on 2-arcs of Γ, when can $\mathrm{Aut}\,\Gamma$ be primitive; and when can $\mathrm{Aut}\,\Gamma$ not be quasiprimitive?

O. Puglisi

Outer automorphisms of hypercentral p-groups

In 1971 Zalesskii proved that an infinite nilpotent p-group always has a non-inner automorphism. This result can be viewed as a generalization of Gaschütz's theorem about outer automorphisms of finite p-groups. On the other hand Zalesskii himself produced an example of a torsion-free nilpotent group of class 2 such that $\mathrm{Aut}\,G = \mathrm{Inn}\,G$. It seems, therefore, that p-groups behave better than other groups in questions related to the existence of outer automorphisms. For this reason M. Dixon was led to ask, during the meeting "Groups 1993" in Galway, if it is true that a hypercentral p-group always has non-inner automorphism. In this talk we examine the case of groups of hypercentral type w.

L. Pyber

On random generation of the symmetric group

We prove that the probability $i(u,k)$ that a random permutation of an n element set has an invariant subset of precisely k elements decreases as a power of k, for $k \leq n/2$. Using this fact we prove that the fraction of elements of S_n which belong to transitive subgroups other than S_n or A_n tends to C when $n \to \infty$, as conjectured by P. J. Cameron. Finally, we show that for every $\varepsilon > 0$ there exists a constant C such that C elements of the symmetric group S_n, chosen randomly and independently generate invariably S_n with probability at least $1 - \varepsilon$. This confirms a conjecture of J. McKay. (Joint work with T. Luczak)

A. Seress

On the diameter of permutation groups

For $G = \langle S \rangle$, let $\Gamma(G,S)$ denote the undirected Cayley graph of G defined by S. The "worst-case" diameter of G is defined as

$$\mathrm{diam}(G) := \max_S \mathrm{diam}\,\Gamma(G,S).$$

Theorem 1: Let $G \leq S_n$. Then $\mathrm{diam}(G) \leq e^{\sqrt{n \log n}(1 + o(1))}$, and this bound is best possible.

Theorem 2: If $G \leq S_n$ is transitive then $\mathrm{diam}(G) \leq e^{c \log^3 n} \cdot \mathrm{diam}(A_m)$, where A_m is the largest alternating composition factor of G.

Theorems 1, 2 are joint work with L. Babai.

We also discuss reductions and partial results toward the conjecture that for transitive $G \leq S_n$, $\mathrm{diam}(G)$ is polynomial in n. In particular this holds if G is solvable.

A. Shalev

The fixity of permutation groups

We say that a permutation group G has fixity f if f = maximal number of fixed points of a nontrivial element of G. We study permutation groups of given fixity, thus generalizing the theory of Frobenius groups, Zassenhaus groups, etc. Our main result shows that if G is a (fin) primitive permutation group of fixity f, then either

(i) G has a soluble subgroup of f-bounded index and derived length ≤ 4, or

(ii) G is almost simple, and $F^*(G) = PSL_2(q)$ or $Sz(q)$ in their natural permutation representations.

(Joint work with J. Saxl)

A. Shalev

Subgroup growth and primitive permutation groups

We show that there are $\leq c^{\log^2 n}$ conjugacy classes of primitive subgroups of S_n (improving a result of Babai). This result has a number of applications in the study of the subgroup growth of infinite groups. (Joint work with L. Pyber)

S. Thomas

The cofinality of the infinite symmetric group

Definition. Let G be a group which is not finitely generated. Then the cofinality of G, written cf(G), is the least cardinal λ such that $G = \bigcup_{\alpha<\lambda} G_\alpha$ is the union of a chain of λ proper subgroups.

Theorem 1. (Joint work with J. Sharp)

If $\lambda \leq \kappa$ are uncountable regular cardinals, then it is consistant with ZFC that

$$cf(Sym(\mathbb{N})) = \lambda \leq \kappa = 2^\omega .$$

Theoem 2.

If M is a countable ω-categorical structure, then

$$cf(Aut(M)) \leq cf(Sym(\mathbb{N})) .$$

Open problem

Is it consistant that there exists a countable structure M such that cf(Sym(IN)) < cf(Aut(M)) ?

Theorem 3

$$cf(GL(\omega,q)) = cf(Sym(\mathbb{N})) .$$

Conjecture

It is consistant with ZFC that

$$cf(Aut\langle\mathbb{Q}, <\rangle) < cf(Sym(\mathbb{N})) .$$

J. K. Truss

Conjugate homeomorphisms of the rationals

I study the group Hom $\mathbb{Q}$ of homeomorphisms of $\mathbb{Q}$ to itself under the usual topology. The ultimate goal would be to characterize the conjugacy classes of Hom $\mathbb{Q}$ in terms of cycle structure, but this is too ambitious.
For $g \in$ Hom $\mathbb{Q}$ and $n \in \{1,2,\ldots\} \cup \{\infty\}$ let $Y_g^n = \{x \in \mathbb{Q}$: x lies in a cycle of g of length $n\}$.

Theorem 1: If $f,g \in$ Hom $\mathbb{Q}$ have cycles only of length 1 and n,n finite, then they are conjugate if and only if there is $h \in$ Hom $\mathbb{Q}$ such that $Y_f^i h = Y_g^i$ for $i = 1,n$.

Conjecture: If $f,g \in$ Hom $\mathbb{Q}$ have finite order then they are conjugate if and only if there is $h \in$ Hom $\mathbb{Q}$ such that $Y_f^i h = Y_g^i$, all i.

Theorem 2: There is $g \in$ Hom $\mathbb{Q}$ of type 1.3.5.7.9... not conjugate to g^2.

I also make some remarks about members of Hom $\mathbb{Q}$ having just one (infinite) cycle, and "locally generic" elements.

R. M. Weiss

Graphs which are locally Grassmann
(joint work with Vladimir I. Trofimov)

We report on progress toward proving the following conjecture: Let X be the set of m-dimensional subspace of an n-dimensional vector space over F_q. Let H denote the projective special linear group $L_n(q)$ in its action on X. Let Γ be a connected graph and let $x \in V(\Gamma)$. Let $G \le aut(\Gamma)$ be a group acting transitively on $V(\Gamma)$ with $|G_x| < \infty$. Suppose $G_x{}^{\Gamma_x}$ contains a normal subgroup isomorphic as a permutation group to H^X. Then the pointwise stabilizer in G of the ball of radius six around x is trivial.

J. Zhang

Finite groups with many conjugate elements

As everybody knows, the symmetric group S_3 of degree three has three conjugacy classes of length 1, 2 and 3 respectively. Thus distinct conjugacy classes of S_3 have different lengths. A finite group with the property is called a dc-group. It has long been conjectured that solvable dc-groups ($\neq 1$) are isomorphic to S_3. Intensive study has been made on the conjecture. We confirm the conjecture. This is in fact a by-product of our study with Prof. L. Puig on source algebras.
We will also talk about the finite full p-defective groups, which are related to a problem posed by Prof. C. E. Praeger.

P.-H. Zieschang

On buildings and generalized groups

Generalized groups are algebraic objects the relationship of which to ordinary groups reflects the relationship of generalized polygons to ordinary polygons (or, more generally, the relationship of buildings to Coxeter complexes). We define coset systems of generalized groups to abstract the way in which classical geometries arise from groups, e.g. in which certain buildings arise from groups with a (B,N)-pair. Together with the ordinary representation theory of finite generalized groups the theory of coset systems leads to structure theorems on generalized groups, e.g., it is possible to determine the "dihedral" generalized groups.

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