MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 5/1994

Nichtstandardanalysis und Anwendungen

30.01. bis 05.02.94

Die Tagung fand unter der Leitung von S. Albeverio (Bochum), J. L. Callot (Mulhouse), D. Laugwitz (Darmstadt) und W.A.J. Luxemburg (Pasadena) statt.

Die Teilnehmer von 14 Ländern haben in 36 Vorträgen gezeigt, wie fruchtbar die Methoden der Nichtstandardanalysis in ihren vielfältigen Anwendungen sind. Schwerpunkte der Tagung waren die Theorie der stochastischen Prozesse und die Theorie der dynamischen Systeme. Deshalb präsentierten die meisten Vorträge neue Entwicklungen der Anwendung nichtstandard analytischer Methoden in diesen Theorien, allerdings wurden auch Vorträge gehalten, die die Tagungsteilnehmer über weitere aktuelle Entwicklungen der Nichtstandardanalysis und ihren Anwendungen informierten. Die Tagung hat auf sehr prägnante Weise die Lebendigkeit und Aktualität dieses schönen Zweiges der Mathematik gezeigt. Es sei insbesondere die Präsenz vieler junger dynamischer Mathematiker erwähnt, die auch zu sehr lebhaften stimulierenden Diskussionen zwischen den Vorträgen und in abendlichen Arbeitsgruppen geführt haben.

Die Tagung wurde dem Andenken zweier Mathematiker gewidmet, die das Gebiet der Nichtstandaranalysis stark geprägt haben, durch ihre grundlegenden Arbeiten und durch ihre menschliche Wärme und Begeisterungsfähigkeit: J. L. Callot und G. Reeb. An einem Abend fand unter der Leitung von W. A. J. Luxemburg eine Gedenkfeier statt. F. Diener und R. Lutz haben auf beeindruckende Weise an J. L. Callot und G. Reeb in ihren menschlichen Aspekten und als Mathematiker erinnert.

1994 ist auch der 20. Jahrestag des Todes von A. Robinson, dem Gründer der Nichtstandardanalysis. Robinson hatte 1970 persönlich an der ersten Tagung dieser Reihe in Oberwolfach teilgenommen. Ihm wurde auf der jetzigen Tagung ein Abend gewidmet, an dem sein Biograph J. W. Dauben einen eindrucksvollen Vortrag gehalten hat.

Vortragsauszüge

H. Akiyama:
APPLICATIONS OF NONSTANDARD ANALYSIS TO GENERALIZED SECTIONS OF VECTOR BUNDLES

A generalized section of a $\mathcal{C}^\infty$ vector bundle E over a compact (σ-compact) manifold M is defined as a continuous linear functional on the space of compactly supported $\mathcal{C}^\infty$ sections of $E^* \otimes |\Lambda_M|$, where E^* is the dual bundle of E and $|\Lambda_M|$ stands for the density bundle over M. It is shown that every generalized section T of E can be represented by a $*$-integral in the sense that there exists a ${}^*\mathcal{C}^\infty$ internal section β_T of the nonstandard extension ${}^*E$ of E such that $T(u) = \int_M \beta_T \cdot {}^*u$ for every compactly supported $\mathcal{C}^\infty$ section u of $E^* \otimes |\Lambda_M|$. As examples, a nonstandard delta function on a Riemannian manifold and the case of currents of de Rham are treated. Nonstandard representations are also obtained of linear maps either from the space of $\mathcal{C}^k$ sections of E or from its subspace consisting of compactly supported sections.

L. Arkeryd:
THE STATIONARY BOLTZMANN EQUATION IN A SLAB

We discuss a kinetic layer between two walls modelled by the Boltzmann Equation in a slab $-1 \leq x \leq 1$, for soft forces $\xi \delta_x f = Q(f,f)$ with boundary condition $f(j,v) = M_j(v) \int_{j\xi'>0} \xi' f(j,v')dv'$, $j\xi' < 0$, $j = \pm 1$. Here M_j is a Maxwellian normalized to give inflow equal outflow. Under some restrictions we prove that there is a solution in $L^1_{\xi^2}$. The proof uses NSA with several infinite scales to reduce from the entropy dissipation term that measure solutions first constructed, actually are Lebesgue absolutely continuous.

E. Benoît:
ASYMPTOTIC EXPANSIONS OF CANARDS WITH POLES. APPLICATION TO THE STATIONARY UNIDIMENSIONAL SCHRÖDINGER EQUATION

The central topic of this paper is the problem of the turning points. The paradigm is the stationary unidimensional Schrödinger equation, with various potentials. The first step is to transform the linear equation of second order into a Riccati equation. The non standard analysis and the theory of *canards* allows to compute the first eigenvalue and the corresponding solution. With a change of variables, it is possible to reduce the problem of the n-th energy level to the $(n-1)$-th. The first result (already proved by other methods) of the paper is an algorithm to compute the asymptotic expansion of the n-th energy level in powers of the small parameter h. The second (new) result is an algorithm to compute an expansion of the corresponding solution. This expansion is a fraction so that the singularity is *resolved*. For example it is possible to determine the zero of the eigenfunctions of the Schrödinger operator up to any power of h.

The algorithms are given with *Maple* programs, and illustrated with a double symmetrical well as potential.

I. van den Berg:
APPROXIMATE EQUATIONS

We consider equations of the type

$$f(x) = \alpha$$

where f is an internal function, and the number α may or may not have some incertainty concerning its order of magnitude.

Typical examples are

1) Asymptotic equations; for instance, find an approximation for x such that

$$x + \log x = \omega$$

where ω is a fixed infinitely large number.

2) External equations, forinstance, if ω is a fixed infinitely large integer, find all x such that

$$e^x - \left(1 + \frac{x}{\omega}\right)^{\omega} \simeq 0$$

We derive certain theorems which are useful for the study of such equations. They include a substitution theorem, a solution theorem and an expansion theorem.

N. Cutland (with M. Capiński):
STOCHASTIC NAVIER-STOKES EQUATIONS

We show how hoeb space methods are used to solve Navier–Stokes equations of the form

$$du_t = [\Delta u_t - (u_t, \nabla)u_t + f(t, u_t)]dt + g(t, u_t)d\omega\ t \qquad (1)$$

where $u_t : D \to \mathbb{R}^n$ and $D \subseteq \mathbb{R}^n$ is a bounded domain. The process ω_t is an infinite dimensional Wiener process. The method of solution is to work in a hyperfinite dimensional space H_N and take standard parts.

The corresponding Foias equation can be derived heurictically, with solutions given by the distributions $(\mu_t)_{t\geq 0}$ of a solution to (1). An alternative method of solution was presented, using nonstandard densities to represent the measure μ_t. This approach to statistical solutions avoids stochastic analysis in favoir of PDE's of order 2.

H. G. Dales:
TOTALLY ORDERED FIELDS WITH ADDITIONAL STRUCTURE:
BETTER VERSIONS OF NON-STANDARD MODELS ?

In the theory of non-standard analysis, one uses as a model of ultrapowers $\mathbb{R}^k/_U$, where k is a cardinal and U is a free ultrafilter on k. These models are real-closed η_1-fields. We examine more general classes of real-closed η_1-fields: there are the ***super-real fields***, which are quotients of algebras $C(U)/_P$, where P is a prime ideal in $C(U)$, and large algebras of formal power series. We claim that, by going to these larger classes, we can find real-closed η_1-fields with varios interesting properties not possessed by any ultrapower. We develop a classification scheme that resolves various open problems. We present a particular field $\tilde{R}$ which seems to be the natural generalization of the real line $\mathbb{R}$ 'one cardinal higher'.

There is a natural structure theory of super-real fields as a model of a fragment analysis.

(This talk was to have been the first part of a pair, the second to be given by Woodin; unfortunately, Woodin was not able to attend.)

J. W. Dauben:
A BIOGRAPHY OF ABRAHAM ROBINSON

Abraham Robinson is best known for his discovery and development of non-standard analysis, and his succesful use of model theory in a wide variety of applications in modern mathematics. This lecture will cover Robinson's early history, his chilhood in Breslau, his emigration to Palestine from Nazi Germany in 1933, his early mathematical study at Hebrew University with Abraham Fraenkel, his brief study at the Sorbonne in 1940 and dramatic escape to England, where he spent World War II at the Royal Aircraft Establishment in Farnborough.

Thereafter, his work in mathematical logic, beginning with his Ph. D. on model theory at the University of London in 1949, began to dominate the aeronautical research he did after the war, first as a staff member teaching at the Cranfield Colloge of Aeronautics and then, beginning in 1951, in the Mathematics Department at the Univwersity of Toronto. His return to pure mathematics as Fraenkel's successor at Hebrew University in 1957 led to a joint appointment at Yale University in 1967, where he continued to develop his ideas on model theory and nonstandard analysis until his untimely early death in 1974.

This lecture is illustrated with more than 100 slides, mostly from photographs in t collectiuon of Mrs. Abraham Robinson, and will offer an overall evaluation of Robinson's life and career as a mathematical logician.

F. Diener:
CALLOT'S RELIEFS

During the seventies. Reeb popularized a very simple geometrical appeoach of slow-fast ODEs $\varepsilon\frac{dy}{dx} = f(x,y)$. based on NSA. It was the beginning of a lot of interesting results.

Recently, Callot introduce a new idea, also geometric and very simple, on how to study slow-fast ODEs in the *complex domain*. Surely this idea will produce in the future deep results. I will explain it and present some applications.

Fan Ru-Zong:
HYPERFINITE DIRICHLET FORMS AND POTENTIAL THEORY

In this joint work with Prof. S. Albeverio we study the hyperfinite Dirichlet forms and their domains, resolvents and related Markov chains. We are also interested in exceptional sets, excessive functions, capacity theory and other issues on potential theory of hyperfinite Dirichlet forms. As the third part of our research, we consider the standard parts of hyperfinite Markov chains. Finally, we discuss the applications of the theory in infinite dimensional stochastic analysis.

R. Fittler:
FEYNMAN FORMALISM AND NSQED

In connection with Feynman diagrams the problems of the classical evaluations of asymptotic contributions (with respect to the limit $t \to \infty$) are demonstrated. A solution of the difficulties is suggested within nonstandard quantum–electrodynamics.

A. Fruchard:
COMPLEX STROBOSCOPY AND GHOST SOLUTIONS

Recent developments concerning general complex difference schemes have nice applications to the discretized logistic equation

$$\frac{y(x+\varepsilon)-y(x-\varepsilon)}{2\varepsilon} = 1 - y^2(x) \qquad (1)$$

The surprising behaviour of some solutions is related to an exponentially small splitting of separatrices. The crucial point is the construction of a complex solution of (1), which is analytic on the S–interior set of the strip $B = \{x \in \mathbb{C}; |\mathrm{Im} x| < \frac{\pi}{2}\}$.

E. I. Gordon:
HYPERFINITE APPROXIMATIONS OF TOPOLOGICAL GROUPS AND THEIR REPRESENTATIONS

The notion of hyperfinite approximation of a topological group was introduced and investigated for the case of abelian groups in the previous papers.

Here we present some results about noncommutative groups which have a hyperfinite approximation. For example if a compact group has it then every irreduceble unitary

representation of this compact group is a shadow of some S–continuous irreducible representation of a hyperfinite group. This result may be used for construction of the irreducible representations of such groups. A simple example of a group over p–adic numbers is considered.

The similar theorem holds for the regular representations of locally compact groups. Some applications of this theorem and connections with the ergodic theory will be presented in the talk.

D. Guido:
SINGULAR TRACES AND NON STANDARD ANALYSIS

Singular traces, i.e. traces on the algebras of operators on a Hilbert space which va[illegible] on finite rank operators, where introduced by J. Dixmier in 1966, and are a key tool in A. Connes non–commutative geometry.

The singular traces which are known up to now described by positive linear functionals on the sequence of partial sums of the eigenvalues of a compact operator, divided by a suitable normalizing sequence.

Then the problem of classifying these traces amounts to describe the class of normalizing sequences and the set of states, associazed with any such sequence, which give rise to a trace.

The first problem has been completely solved by Albeverio, Guido, Ponosov and Scarlatti in terms of an asymptotic condition called eccentricity, generalizing a result of Varga. When singular traces are studied at a local level, i.e. on principal ideals, the normalizing sequence can be taken as the sequence of the partial sums of eigenvalues of a positive operator generating the ideals.

The second problem may be solved in terms of decomposition of the admissible states in terms of the extremal ones, and here nonstandard analysis plays a role. In fact, at a local level, examples of singular traces can be given in terms of an easy non standard formula, therefore all singular traces are in the closed convex hull of the given traces.

At a global level, i.e. when traces are described on maximal ideals, the state needs to be 2–dilation invariant. Again a simple non standard formula, depending on two hyperfinite numbers, is available, and ergodic decomposition shows that extremal traces are described by this formula.

The previous results extends to the case in which the algebra is replaced by any t[illegible] II factor in the sense of Murray–von Neumann.

J. Harthong:
WAVE PROPAGATION IN MODULATED MEDIA

In this paper we propose mathematical methods for the study of the propagation of waves in a modulated medium. We call *modulated medium* a medium in which refractive index varies in such a manner that it is approximately periodic in small regions (containing however a large number of periods). *Periodic* media are particular cases of modulated

media: the surfaces along which the refractive index is constant (modulation surfaces) are theb the parallel planes. In non-periodic modulated media, the modulation surfaces are curved: they are the contour surfaces of a differentiable function.

Typical modulated media are holograms, because holograms record interference fringes, which are always contour surfaces.

The mathematical tools that we present in this paper are especially devoted to the analysis of the wave propagation in modulated media with a period of the same order magnitude as the wavelength.

C. Ward Henson:
THE SITUATIONS OF GUARAII'S HOMOGENOUS, UNIVERSAL SEPARABLE BANACH SPACES

A *Guaraii space* is a separable Banach space X with the following property: for any finite dimensional spaces $E \subseteq F$, any linear isomorphism $T : E \to X$ (into) and any $\lambda > 1$, there is a linear isomorphism $S : F \to X$ such that $S(x) = T(x)$ for all $x \in E$, and $\|S\| \leq \lambda\|T\|$, $\|S^{-1}\| \leq \lambda\|T^{-1}\|$. W. Lusky showed that X is determined up to a isometry by this condition, and this unique space is a very natural but complicated object to study. We have used the methods of nonstandard analysis together with additional tools from the model theory of Banach spaces to obtain a number of new geometric results about this space, especially concerning the action of its automorphism group.

V. Kanovei, M. Reeken:
NEW SYSTEMS OF FOUNDATIONS FOR NONSTANDARD MATHEMATICS

A new theory of internal sets, *bounded set theory* BST, is presented. BST does everything that is available in IST, Nelson's internal set theory, and, in addition,

* provides the Extension principle for all formulas,
* makes it possible to convert every formula to the form $\exists^{st}\,\forall^{st}$ [internal],
* guarantees that every formula with standard parameters is provably equivalent to an internal formula,
* allows to parametrize all definable subclasses of sets by a single formula, and
* allows to obtain useful external enlargements of the universe of internal sete by definable classes.

H. Jerome Keisler (joint work with S. Fajaro):
NEOMETRIC SPACES AND APPLICATIONS

We introduce a method which produces a wide variety of applications of NSA to probability theory using conventional standard arguments. Given an adapted space $(\Omega, P, \nu_t)_{t\in\tau}$ and a complete separable metric space M, let $\mathcal{M} = L^0(\Omega, M)$ be the space of measurable functions $x : \Omega \to M$ with the metric of convergence in probability. A family of subsets of

$\mathcal{M}$ called *neocompact* sets is defined, which shares many properties with the compact sets but is much large. For examle, the sets of Brownian motions in $L^0(\Omega, C([0,1], I\!R))$ and of stopping times in $L^0(\omega, C[0,1])$ are neocompact. A nontrivial adapted space is *rich* if the intersection of any countable chain of nonempty neocompact sets is nonempty. *Theorem:* Rich adapted spaces exist. The only use of NSA here is in the proof of this theorem (via Loeb measures). Many optimization and existence theorems can be proved by standard methods in a rich adapted space. For example, for each $x \in L^0(\Omega, C([0,1], I\!R))$ there is a Brownian motion whose distance from x is minimal. Other examples include the strong existence theorems for stochastic d.e. 's in papers of Keisler, Hoover and Perkins, Cutland, Kosciuk and Lindstrøm, and the existence theorems of Cutland and Capinski for stochastic Navier–Stokes equations.

P. E: Kopp:
FROM DISCRETE TO CONTINUOUS STOCHASTIC CALCULUS

The martingale representation theorem for L^2–Brownian martingales provides the stochastic differential as the integrand occuring in the stochastic integral representation of the given martingale. In discrete time, martingales on the path spaces of random walks have analogous representations. Working on Wiener spaces one can formulate a natural convergence concept for discrete–time entries to their continuous counterparts which is preserved under *both* stochastic integration and stochastic differentiation. Thus invariance principle is stronger than the usual Donsker principle yielding weak convergence, by taking into account functional relationships as well as convergence of paths. We show that — modulo discretisation schemes — the convergence can be expressed in terms of the L^2–norm, and provide extensions to multi–dimensional Brownian martingales as well as the Ito–Wiener chaos decomposition. The ideas presented were motivated by question arising in the theory of option pricing. (Joint work with N. J. Cutland and W. Willinger.)

F. Koudjeti:
EXTERNAL PROBABILITY, AN CLASSICAL CENTRAL-LIMIT THEOREM

A classical construction of $I\!R$ is based on Dedekind cuts. The real number may be seen as the last upper bound of these cuts. The resulting completion property enables us build real analysis. If the cuts are external, then with a slight weakening of the notion of extrema, they appear to have least–upper bounds in the form of 'external numbers'. An external number is defined to be the sum of a real number and a neutrix, i.e. a convex (external) additive subgroup of $I\!R$.

This notion of (external) extrema turns out to be convenient, in many cases, for replacing the crossing of 'borders' of external sets using permanence principles by a simple analytical consideration using extrema's.

In our talk we will use this notion to define the measurability of an external set, and then to establish some elementary results of an external probability. We will deduce the

'external Chebysev equality' and use this later result well as an external calculus, to prove the classical Laplace's limit theorem stated as follows:

'Let S_n stand for the number of successes in a n Bernoulli vuals with probability p of success. Let $\kappa(k)$ be the probability of the event that $S_n = k$. Then: for any real number a and b in $\mathbb{R}$,

$$\lim_{t\to\infty}\left[p_2\left\{a \le \frac{S_n - np}{\sqrt{npq}}\right\} - \frac{1}{\sqrt{2\pi}}\int_a^b e^{-\frac{t}{2}}dt\right] = 0$$

S. Kuhlmann:
THE VALUATION THEORETIC INTERPRETATION OF EXPONENTIATION IN NON-ARCHIMEDIAN FIELDS

We study the stucture of non-Archimedian fields admitting an exponential function, that is an isomorphism of their additive groups onto their multiplicative group of positive elements. We give necessary conditions on the residue field and value group. We show that for a countable non-Archimedian field, a necessary and sufficient condition for the existence of an exponential is that the value groups is a Hahn sum over the rationals. Similarly, for power series field, it is sufficient that the value group satisfies the necessary condition, and is a Hahn product.

We then examine strong exponentials, that is exponentials satisfying growth and Taylor axioms. We show that the above characterization for countable and power series fields generalize in this case as well.

P. A. Loeb:
A GENERALIZATION OF RADIAL LIMITS AND THE BEST APPROACH TO BOUNDARIES

Jürgen Bliedtner and the speaker have recently shown that with a suitable normalization, there is a "best" family of filters for which the Fatou Boundary Limit Theorem holds. The normalization assigns to each positive harmonic function a set of boundary points at which that function must vanish. Known limits, such as those provided by the Lebesgue Differentiation Theorem, or the Fatou Radial Limit Theorem are used to force consistency in this assignment. The zero sets, in turn, are used in constructing the coarsest filter which produce those limits almost everywhere. This procedure is formulatd in terms of general potential theoretic settings, but the result is new even for harmonic functions on the unit disk in the compact plane. In this talk, couched in terms of harmonic functions on the unit disk, nonstandard analysis is used to extend to general domains the use of radial limits in the construction of zero sets.

G. Lumer:
NONSTANDARD ANALYSIS AND HEAT EXPLOSIONS

We consider the context of recent work on heat shocks (diffusions with shocks) in Banach space, X, formulation. [A replacing the 2nd order elliptic operators appearing in the classical application where $X = C(\bar{\Omega})$, Ω bounded $\subset I\!R^N$ with $\partial\Omega$ smooth; A is the generator of an irregular bounded analytic semigroup $Q(t)$ on X — for precise details and assumptions see the recent paper of the present lecture "Diffusion models with abrupt changes...", Evolution equations..., Lect. Notes in Pure and Appl Math. Vol. 155, Marcel Dekker, New York, 1994.]

Here we study via NSA the transition during an infinitesimal interval of time, say $[0, \eta]$, going from the conditions existing before a shock, at say time $t = 0$, to those prevailing afterwards. This study leads beyond shocks in particular to the discovery of new phenomena of "heat explosions" and to their adequate formulation in conventional mathematics as "initial boundary impulses", at say $t = 0$, $j \in X$. We also obtain an explicit formula for the effect (evolution) for $t > 0$ caused by such a heat explosion j at $t = 0$, which adds to the heat/diffusion flow that would have taken place without the explosion. In the simple situation just described, the effect of an isolated heat explosion (actually heat explosion on the boundary) would be $-Q'(t)j$ for $t > 0$.

R. Lutz:
ANWENDUNGEN DER NSA ZUR WAHRSCHEINLICHKEITSTHEORIE

We show that nonstandard analysis is a perfect tool to be used in finitely additive probability theory, due to the fact that the shadow of f.a. probability law is f.a too. This makes clear a lot of old problems which are of great interest in mathematical statistics, e.g. uniformity, null–probability, extension to larger event spaces or the signification of the laws of large numbers.

R. Lutz:
ABOUT MY MASTER GEORGES H. REEB

Georges Reeb was born the 12th of november 1920 at Saverne, a little town about 35 in the north–west of Strasbourg. Suddenly he has passed away on the 6th of november 1993.

He was proud of his origins as the son of a cooper, which certainly influenced his geometrical intuition.

Reeb was an outstanding mathematician, but also a bright man in his familial and social life. He married Trudy in 1945 and they got two girls Christiane and Elisabeth. For his numerous students, he was a sort of a modern "Socrates", everywhere dense in the corridors of the "Institut de Mathématiques" at Strasbourg. He always warmly encouraged them and asked them ten times a day his legendary "qoui de neuf?".

He brought up in the mechanicqal and topological tradition of Henri Poincaré, Paul Painlevé and Elie Cartan. Under the advisorship of his Master Charles Ehresmann he created during the late 40's the famous foliation theory, a "nonstandard" idea of global topology at a time where local problems seemed to be the important ones. After 11 years as a professor at Grenoble, he came back to Strasbourg and founded his own school of differential topology, based on the "Séminaire trajectorien", which emphasize on foliation theory, differential forms and equations, contract geometry... In the early 70's, he learnt about Robinson's NSA and claimed at every mathematician he could meet (the most of whom flew away in a hurry !) that a kind of revolution had come up in mathematics, the most important in this century...

This was the starting point of the second school he created, which grew up in the philosophy of using NSA in everyday mathematics as a tool to get simple and natural proofs and to detect new mathematical phenomena. Some sort of "green" mathematics, indeed. The axiomatic presentation of NSA by E. Nelson within IST in 1977 gave a second impulse to the alsacian school: it was partly because this formal setting points out Reeb's philosophical conviction that infinitesimals where an unexpected benefit of the impossibility to formalize completely the intuitive feeling that all natural numbers, being obtainable by the successive addition of one, are of the same kind. His question "Les entires naifs remplissent-ils N?" became famous in France and more widely in Europe, where he is considered as a subbtle and efficient propagandist of NSA.

He was an enthusiastic man, highly enjoying the deep affection that he shared with his students.

M. Oberguggenberger:
GENERALIZED STOCHASTIC PROCESSES

In this communication a new approach to generalized stochastic processes is presented, geared to solving nonlinear differential equations involving distribution valued processes. Employing ideas from the theory of multiplication of distributions as well as nonstandard analysis, differential algebras of stochastic processes are constructed which contain all processes with continuous paths together with their generalized derivatives. The method is based on factor algebras in the algebra of sequences of processes with smooth paths. Two special choices are discussed and compared with each other: an ultrapower construction ${}^*\mathcal{C}^\infty P$ and a Colombeau-type algebra GP. Nonlinear stochastic differential equation are shown to admit generalized solutions in both algebras.

Related work has recently undertaken by S. Albeverio, Z. Haba, T. Lindstrøm, B. Øksendal, S. Pilipović, Ya. V. Radyno, F. Russo and co-workers.

J. Oikkonen:
PLAYFUL (HAHN) POWERS

Hahn powers have been considered during this Tagung in connection to fields of power series. We discuss two other applications of this idea.

1) Suitable games can be used to give meaning to the familiar equations $(1+\eta)^0 = 1$, $(1+\eta)^{\theta+1} = (1+\eta)^\theta \cdot (1+\eta)$ and $(1+\eta)^\theta = \sup_i (1+\eta)^{\theta_i}$ for $\theta = \sup_i \theta_i$ when η and θ are linear orders. These games inspect linear ordings in terms of their initial segments. (see J. Oikkonen, A recursion principle for linear ordings, JSL, 1992)
2) Brownian motion inspects a domain in a analogous way leading to a definition of Harmonic measure. This analogy is especially strong in connection to a hyperfinite representation. (see J. Oikkonen, Harmonic analysis and nonstandard Brownian motion in the plane, Math. Scand., 1985)
3) It is shown in A. Mekkler and J., Oikkonen, Abelian p–groups with no invariants, JPAA, 1993, that (torsion subgroups of) suitable Hahn powers H^η of Abelian p–groups lend to groups with no invariants in a strong sense formulated in terms of approximations of Ehrenfeucht–Fraissé games.

H. Osswald:
ON LINEAR CONTINUOUS FUNCTIONALS ON THE HARDY SPACE H_1

The notion "Hardy space" comes from the theory of analytic functions in connection with Potential Theory, and can be transformed to spaces of martingales via hitting time of Brownian motion.

Fix an adapted Loeb space $(\Lambda, (b_t)_{t\in[0,1]}\hat{Y})$ where $(b_t)_{t\in[0,1]}$ is the standard part of an internal filtration $(a_t)_{t\in T}$, where T is a hyperfinite discretization of $[0,1]$. The space H_1 is the space of (b_t)–martingales such that $\sup_{t\in[0,1]} \mathrm{Im}\,(\cdot,t)$ is integrable. We prove the existence of integral representations of linear continuous functionals and use them to construct H_1–valued measures.

The stochastic integral is exact the Pettis integral w. r. to these measures.

M. Munoz de Ozak:
LIFTING THEOREMS FOR TWO PARAMETER MARTINGALES

Following the paper "Nonstandard constuction of the stochatic integral and applications to stochastic differential equations" D. N. Hoover and E. Perkins, I give the two parameter version of some theorems, in order to show that if X is a Δt–martingale, then $st(X)$ is a large martingale and if χ is a large martingale, then there is a Δt martingale lifting for some Δt in T.

Y. Peraire:
OSCILLATORY SYSTEMS WITH *RELATIVE* NONSTANDARD ANALYSIS TOOLS

The general framework of my speach would be Relative Internal Set Theory (RIST).

The language of RIST contains a binary predicate "$.st.$" and this permits to define levels in the universe of the sets: a level is a class of equivalence for the relation of "equistandardicity" defined by "x is equistandard to y if x st y and y st x". Relatively to each level corresponds principles of transfer, idealization and *standardization* analogous to the Nelson's axioms, but "transversal principles" (using simultaneously several levels) exist too. In my account, I shall emphasize only two points:

1) I shall investigate the possibility to oper transfers only on a subformular of a given formula (partial transfer) and give an application.
2) I shall prove a theorem of idealization relative to $^\mu$ galactic formulas (of the form $\exists^\mu x\ \Phi(x)$, with $\Phi(x)$ internal) and give an application.

M. Reeken:
DYNAMICAL SYSTEMS FROM THE NONSTANDARD POINT OF VIEW

As in any field of some complexity looking at it in the language of BST opens up new perspectives on the fundamental notion of the field and their interdependence.

The field of topological dynamical systems is an interesting example of this kind.

S. Samborski:
FAST CHANGEMENTS OF VISCOSITY SOLUTIONS FOR HAMILTONIANS DEPENDING OF PARAMETERS

The boundary value problem

$$\varepsilon \Delta y + H(x, Dy, a) = 0 \qquad y/\Gamma = g$$

where $x \in \Omega^* \subset (\mathbb{R}^n)^*$, $\Gamma = \partial\Omega$, D–gradient, $a \in \mathbb{R}^*$ is studied in nonstandard domain with infinitesimal ε and standard Hamiltonian H depending on a nonstandard parameter. Let us say that a "fast changement" occurs if for two values a_1 and a_2 of the parameter a, $a_1 \sim a_2$ the Hausdorff distance between the shadows of corresponding solutions $y(a_1)$ and $y(a_2)$ is perceptible. The main theorem describes the unique possible scenario of this "fast changement" in a generic situation. Applications are connected with "catastrophes" for the value function of control problems.

K. Stroyan:
FINITE DIFFERENCE APPROXIMATION OF HIGHER DERIVATIVES

We prove the following discrete form of Taylor's Theorem and its converse and discuss generalizations.

THEOREM: A real-valued eigenfunction is n -times continuously differentiable if and only if the polynomical interpolated at $(n+1)$ infinitely close finite points is close to a polynomical dependent only on the common standard part of the interpolating points.

T. Todorov:
A GENERAL EXISTENCE RESULT FOR LINEAR PARTIAL DIFFERENTIAL EQUATIONS WITH SMOOTH COEFFICIENTS

We prove a general existence result for essentially all linear partial differential equations with smooth coefficients in an algebra $\mathcal{A}(\Omega)$ of localizable generalized functions. The algebra $\mathcal{A}(\Omega)$ is defined as a factor space of the class of nonstandard internal smooth functions. In particular, we show that H. Lewy's equation has solution whenever its right hand side is a classical smooth function. There is a strong similarity between the algebra of generalized functions $\mathcal{A}(\Omega)$ and the variety of algebras of generalized functions introduced recently by Jean F. Colombeau and other authors in the framework of standard analysis. For that reason we view this work as an attempt to establish a connection between the nonstandard analysis and the nonlinear theory of generalized function with the belief the interaction between them will prove fruitful for both.

F. Wattenberg:
ECONOMIC MODELS BASED ON THE IDEAS BEHIND SIMULATED ANNEALING

The behavior of optimization algorithms based on the simulated annealing is strikingly similar in many ways to the behavior of economic aglnis responding to a combination of centralized deterministic influences and decentralized, individual, random influences. In this talk we are particulary interested in the way that changing the connectinity or topology of the state or behavior space by introducing new neighbor relationships can speed up convergence in the equilibrium without sacrifying the efficiency of the economy at equilibrium. Working with nonstandard models we deal with state spaces parts of which are manifolds and parts on which are not.

B. Wietschorke:
EXTERNAL EQUIVALENCE RELATIONS IN TOPOLOGY AND FUNCTIONAL ANALYSIS

In the context of Nelson's Internal Set Theory (IST) we use the concept of external Equivalence relations developed by Reeken in order to give external characterizations of some classes of locally convex spaces. For this on the dual of a given locally convex space X several external relations are defined. The coincidence of two such relations leads to an equivalent standard condition giving some informations about the topology on X. In that way we are able to describe locally convex spaces with invariant nonstandard hulls ((HM)-spaces), quasinormable and Schwartz spaces. The result concerning Schwartz-spaces confirms a conjecture posed by Grainger.

As an application we give an improvement of a result due to Benninghofen, Richter and Stroyan, by showing that the locally convex topology associated to a certain kind of infinitesimals is always a Schwartz topology.

W. Willinger:
ON HOW TO MAKE MONEY IN THE STOCKMARKET USING NSA

Empirical evidence from actual stock price data suggest the use of stochastic models in finance that are not covered by the traditional semimartingale framework. In particular, this empirical evidence suggests that "long-range dependence" (also called the "Joseph Effect") be accounted for when modeling stock price movements.

In this talk, a "fractional" version of the widely used Black-Scholes option pricing model is presented that (i) is based on fractional Brownian motion, (ii) accounts for long-range dependence and is therefore inconsistent with the semimartingale setting, and (iii) allows for "orbitrage" oppertunities ("free lunches"). Mathematical problems of practical importance to finance for this class of fractional Black-Scholes models are dealt with using methods from nonstandard analysis. (This is joint work with N. Cutland and P. E. Kopp, Univ. of Hull, England.)

M. Wolff:
SUPERSTABLE OPERATORS AND SEMIGROUPS

Nagel and Rübiger defined a contraction T on the Banach space E to be superstable if the semigroup $\{\hat{T}^n : n \geq 0\}$ of the extension $\hat{T}$ on every ultrapower $\hat{E}$ is strongly relatively compact. We give an intrinsic definition of this notion and generalize the characterization of Nagel and Rübiger. Moreover we treat the case of strongly continuous semigroups in the same manner generalizing a result of Rübiger and Huong. Our main result reads as follows: If E is superreflexive and the peripheral spectrum of T is countable then T is superstable. The converse holds for arbitrary Banach spaces.

Jiang-Lun Wu:
ON GENERALIZED BOCHNER THEOREM ON LOCALLY CONVEX SPACES

This talk presents a nonstandard approach to Radon extension of cylindrical measures on weak dual spaces of locally convex spaces. The results obtained are applied to generalize the (classic) Bochner theorem to the case that Radon probability measures are defined on the weak dual spaces of locally convex spaces. We also compare our result with other topological descriptions of characteristic functionals of probability measures on other infinite dimensional spaces.

B. Zimmer:
FINDING LOST RADON-NIKODYM DERIVATIVES

Radon-Nikodým derivatives of absolutely continuous vector measures of bounded variation are usually obtained through a limiting process over finer and finer partions. If the Banach

space lecks the Radon -Nikodým property, these limits may not exist. Nonstandard Analysis allows us to "fatten" the measure space and the Banach space such that we can recover those lost derivatives. P. A. Loeb showed that there is a hyperfinite internal partition of the nonstandard extension $^*[0,1]$ of the unit interval, which is finer than any standard Lebesgue measurable partition. This fine nonstandard partition $\{A_1, \ldots, A_H\}$ allows us to recover those lost derivatives by defining a function $\varphi = \sum_{i=1}^{H} \frac{^*\nu(A_i)}{^*\lambda(A_i)} \cdot \forall A_i$ which defines an internal S-integrable function from $^*[0,1]$ to the set of elements of finite norm in the nonstandard extension of the Banach space. Composing this map φ with the projection on the nonstandard hull, we get a function which we can integrate with an appropriate integral. This way we obtain that $\nu(A) \approx \int_A \varphi d^*\lambda$ or, after composing φ with the projection onto the nonstandard hull, that $\nu(A) = \pi\left(\int_A \varphi d^*\lambda\right)$. If the Radon–Nikodým derivative exists, then $\pi \circ {}^* \left(\frac{d\nu}{d\lambda}\right) = \pi \circ \varphi$, i.e. our construction generalizes the Radon–Nikodým derivati

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Gefördert durch
DFG Deutsche Forschungsgemeinschaft