MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 6/1994

# $C^*$-Algebren

6.-12.2.1994

Die Tagung fand unter der Leitung von J.Cuntz (Heidelberg),U. Haagerup (Odense) und L.Zsido (Rom) statt.Vorträge aus verschiedenen Zweigen der $C^*$-Algebrentheorie wie Klassifikation einfacher $C^*$-Algebren, von Neumannalgebren sowie Anwendungen in Geometrie und Physik standen im Mittelpunkt des Interesses. Dabei blieb Trotz vieler interessanter Vorträge Zeit für ausgedehnte Diskussionen und regen wissenschaftlichen Austausch.

## Vortragsauszüge:

### Composing finite depth subfactors

Dietmar Bisch joint with U.Haagerup

Suppose $N \subset P, P \subset M$ are inclusions of $II_1$-factors with finite depth.We study the "composed" inclusion $N \subset P \subset M$. Let $\alpha$ be the $N-P$-bimodule ${}_N L^2(P)_P$ and $\beta =_P L^2(M)_M$ ($P-M$-bimodule), then $\rho = \alpha \otimes_P \beta =: \alpha\beta$ gives the $N-M$-bimodule ${}_N L^2(M)_M$.Computing the $N-N, N-M, M-N, M-M$ bimodules associated to the inclusion $N \subset M$ amounts to decomposing $(\overline{\rho}\rho)^k, (\overline{\rho}\rho)^k\overline{\rho}, (\rho\overline{\rho})^k$ and $(\rho\overline{\rho})^k\rho$ into irreducible bimodules. Let $N \subset P \subset P_1$ be the Jones basic construction, then we have:

**Prop.:** Let $N \subset P, P \subset M$ be as above. Then $N \subset M$ has finite depht iff $\{({}_P L^2(M)_P e_P {}_P L^2(P_1)_P)^k, k \geq 1\}$ decompose into at most countably many irred. $P-P$ bimodules.

A nice class of subfactors of this type arises in the following way: Let $G, H$ be finite groups acting properly outerly on $M$, consider $M^H \subset M \subset M \rtimes G$ (e.g.: $M = R$, hyperfinte $II_1$-factor). Then

**Cor.:** $M^H \subset M \rtimes G$ has finite depth iff $K = \langle G, H\rangle$ (group generated in $Out(M)$) is a finite group.

Note that $M^H \subset M \rtimes G$ is irreducible iff $G \cap H = \{e\}$ (in $Out(M)$). It is even more interesting to study these examples when $K$ is infinite (2 extreme situations: $K = G \times H, K = G * H$). We show

**Thm.:** Let $M^H \subset M \rtimes G$ be as above with $G \cap H = \{e\}$. Then the inclusion $M^H \subset M \rtimes G$ is amenable (in the sense of Popa) iff the group $K = \langle G, K\rangle$ is amenable.

In the case where $H = \mathbb{Z}_2, G = \mathbb{Z}_3$,we can give explicitly the principal graphs for $R_2 \subset R \rtimes \mathbb{Z}_3$ for $K = \mathbb{Z}_2 * \mathbb{Z}_3$ (free product), $K = \langle a, b \mid a^2 = b^3 = (ab)^6 = 1\rangle$. In the latter case one gets a strongly amenable infinte depth (irred.) subfactor of $R$, the hyperfinite $II_1$-factor.

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# Generalized inductive limits of finitedimensional $C^*$-Algebras

Bruce Blackardar and Eberhard Kirchberg

We consider generalized inductive systems $(A_n, \varphi_{m,n})$ of $C^*$-algebras where the connecting maps $\varphi_{m,n} : A_m \to A_n$ are only assumed to be asymptotically $\star$-linear and multiplicative. The inductive limit can be defined as a $C^*$-subalgebra of $(\prod A_n)/(\bigoplus A_n)$.

It is particularly interesting to consider generalized inductive systems of finitedimensional $C^*$-algebras, especially when the $\varphi_{m,n}$ are $\star$-linear complete order embeddings.This construction is a noncommutative analog of the procedure of writing a compact metrizable space $X$ as $lim_{\leftarrow} X_n$, where $X_n$ is a polyhedron with specified triangulation,and so this can be regarded as giving a "combinatorial" describtion of the inductive limit.

**Theorem 1** A separable $C^*$-algebra can be written as $lim_{\rightarrow}(A_n, \varphi_{m,n})$ with $A_n$ finite-dimensional if and only if $A$ has an essential quasidiagonal extension by $\mathcal{K}$.Such an $C^*$-algebra is called MF-algebra.

**Theorem 2** A separable $C^*$-algebra $A$ can be written as $lim_{\rightarrow}(A_n, \varphi_{m,n})$ with $A_n$ finite-dimensional and each $\varphi_{m,n}$ completly positive if and only if the identitymap on $A$ can be approximated in the pointnorm by completly positive almost multiplicative contractions through matrixalgebras. $A$ is called an NF-algebra if this is satisfied.

**Theorem 3** A separable $C^*$-algebra $A$ can be written as $lim_{\rightarrow}(A_n, \varphi_{m,n})$ with $A_n$ finite-dimensional and each $\varphi_{m,n}$ a complete orderembedding if and only if the identity map on $A$ can be approximated in the point-norm topology by completly positive finite rank idempotent contractions.Such an algebra is called a strong NF-algebra.

It is possible that the classes of NF-, strong NF- and stably finite separable nuclear $C^*$-algebras coincide.Partial results are established in this direction.

# Tensor Products of $C(X)$-Algebras over $C(X)$

Etienne Blanchard

Given a Hausdorff compact space $X$ and two $C(X)$-algebras $A$ and $B$ (Kasparov), we define the two ideals of the algebraic tensorproduct $A \otimes_{alg} B$

$$I = \langle af \otimes b - a \otimes fb \mid a \in A, b \in B, f \in C(X) \rangle$$

$$J = \{\alpha \in A \otimes_{alg} B \mid \forall x \in X, \alpha^x = 0 \, in \, A^x \otimes_{alg} B^x\}$$

Then we can define a minimal and a maximal $C^*$-norm on $(A \otimes_{alg} B)/J$. Furthermore, as every semi-$C^*$-norm on $A \otimes_{alg} B$ which is zero on $I$ is also zero on $J$, in order to finish the study of $C^*$-norms on $(A \otimes_{alg} B)/I$ we only need to answer the following question of Elliot: When does $I = J$ holds ?

Giordano and Mingo proved that this is true if $C(X)$ is a von Neumann algebra.It is also true if $A$ is a separable continuous field of $C^*$-algebras, but there are counterexamples in the general case.

# Module maps and injectivity for von Neumann algebras

Erik Christensen joint with Allan Sinclair

Let $M$ be a von Neumann algebra on a Hilbertspace $H$. If $\rho$ is a completly bounded projection from the algebra $B(H)$ of bounded operators on $H$ onto $M$, then $M$ is injective. This result has also been obtained by Giles Pisier using the operator Hilbert space $O(H)$. The result has been extended by Haagerup , Pisier and Christensen , Sinclair to prove that if there exists a completly bounded projection of a $C^*$-algebra $\mathcal{A}$ onto a subalgebra $\mathcal{M}$ which is $W^*$ then there exists a projection of norm one of $\mathcal{A}$ onto $\mathcal{M}$. The methods used imply that for a pair $\mathcal{A} \supset \mathcal{M}$ as above, where $\mathcal{A}$ is $C^*$ and $\mathcal{M}$ is $W^*$ there exists a projection $P$ of the space $CB(\mathcal{A}, \mathcal{M})$ (of completly bounded mappings of $\mathcal{A}$ into $\mathcal{M}$) onto $B_{\mathcal{M}}(\mathcal{A}, \mathcal{M})$ the space of $\mathcal{M}$-bimodule mappings (bounded) of $\mathcal{A}$ into $\mathcal{M}$) such that $\| P \| \leq 1$. As a partial explanation of this the following result explains that $B_{\mathcal{M}}(\mathcal{A}, \mathcal{M})$ is non zero only if the inclusion of $\mathcal{M}$ in $\mathcal{A}$ is nice.

**Theorem:** Given $\mathcal{A} \supset \mathcal{M}$ a $C^*$-algebra containing a $W^*$-algebra $\mathcal{M}$.If $B_{\mathcal{M}}(\mathcal{A}, \mathcal{M}) \neq 0$ then there exists a central projection $z$ and a projection $\pi$ of $\mathcal{A}$ onto $\mathcal{M}_z$ of norm 1 s.t. $B_{\mathcal{M}}(\mathcal{A}, \mathcal{M}_{1-z}) = 0$.

# The long exact sequence in periodic cyclic cohomology

Joachim Cuntz

The subject of the talk was the following theorem obtained recently in collaboration with D. Quillen.

**Theorem** Let $0 \to J \to A \to 0$ be an extension of (non-commutative) algebras over $\mathbb{C}$. There is a six-term exact sequence, connecting the periodic cyclic cohomology groups $HP^*$ of $J, A$ and $B$, of the following form

$$\begin{array}{ccccc} HP^0(J) & \leftarrow & HP^0(A) & \leftarrow & HP^0(B) \\ \downarrow & & & & \uparrow \\ HP^1(B) & \rightarrow & HP^1(A) & \rightarrow & HP^1(J) \end{array}$$

This solves a natural open problem and gives the missing link between the formal properties of topological K-theory on the one hand and periodic cyclic cohomology on the other. The proof uses ideas from the formalism developped by Quillen and the author for treating cyclic homology/cohomology.

# Discrete and Compact Quantum Groups

## Alfons van Daele

There are different approaches to quantum groups. You have the approach of Drinfeld and Jimbo and the one of Woronowicz. For compact quantum groups there is the notion of Woronowicz and the one of Koernurudes. For discrete quantum groups there is a concept by Effros and Ruan, by Kirchberg and by myself. There is some confusion about all these concepts. Following the spirit of quantization, we should think of a $C^*$-algebra as a locally compact quantum space. A locally compact quantum group then as a $C^*$-algebra with a group like stucture. In the case of a locally compact group $G$ the corresponding $C^*$-algebra is $C_0(G)$ and the multiplikation on $G$ defines a $\star$-homomorphism $\Delta : C_0(G) \to C_b(G \times G)$. In general one considers a $\star$-homomorphism $\Delta : A \to M(A \otimes A)$ the multipieralgebra of the completed tensor product. It should be non-degenerate and satisfy coassociativity $(\Delta \otimes \iota)\Delta = (\iota \otimes \Delta)\Delta$ where $\iota$ is the identity map. Such a pair is a locally compact quantum semi-group.

In the above philosophy, a compact quantum group is such a pair where $A$ has an identity and $\Delta$ satisfies some extra properties. A discrete quantum group is such a pair where $A$ is a direct sum of full matrixalgebras and where $\Delta$ satisfies similar properties. In both cases, there exists Haar measures. There is also a nice duality between these two cases.

# Quantum Spacetime

## Sergio Doplicher joint with K.Fredenhagen,J.E.Roberts

We propose operationally motivated uncertainity relations between different coordinates of events in spacetime, and discuss spacetime commutation relations which do imply them. The $C^*$-algebra describing the regular representation of the spacetime commutation relations is constructed and defines our Quantum spacetime. In the classical limit, when Planck's length tends to zero our Quantum Spacetime deforms to $\mathbb{R}^4$ times a ghost manifold $\{\pm\} \times TS^2$.

Starting with an ordinary interaction Hamiltonian, the pertubative approach to quantum field theory over quantum spacetime leads to the same theory an effective nonlocal Hamiltonian would define over ordinary Minkowski space.

# Classification of symmetries on some amenable $C^*$-algebras

## David E.Evans joint with H. Su

We consider a K-theoretic classification of certain $C^*$-algegra dynamical systems $(A, \mathbb{Z}_2, \alpha)$ where $\alpha$ is an action of $\mathbb{Z}_2$ on a certain infinite $C^*$-algebra $A$.

The classifaction of Elliott for AF-algebras by the ordered scaled group $K_0(A)$ was extended to classifying limit inner actions of compact groups on AF-algebras [Fack a. Marchel,Kishimoto,Handelman a. Rossman] (i.e. where $A = lim_{\rightarrow} A_n$, and acts by an inner unitary action on finite dimensional $A_n$) and to limitactions of $\mathbb{Z}_2$ [Elliot a. Su] using the following complete invariant

$i)$ $K_0(A)$, $[1]$, $\alpha_*$
$ii)$ $K_0(A \times \mathbb{Z}_2)$, $[special\ element]$, $\hat{\alpha}_*$
$iii)$ $K_0(A) \rightarrow K_0(A \times \mathbb{Z}_2)$.

where the special element is the projection for eigenvalue 1 of the complementary unitary, and $\hat{\alpha}$ is the dual action. The $C^*$-algebras involved in the present classification will be unital $C^*$-algebras that can be expressed as inductive limits of finte direct sums of matrix algebras over even Cuntz algebras. Rørdam showed that a complete invariant for such algebras is $P(A) \rightarrow K_0(A)$, $[1]$.

We restrict ourselfes to actions of the type $Ad(V) \otimes \beta$ on $M_k \otimes O_n$ where $V$ is a selfadjoint unitary and $\beta S_i = S_i, i \leq r_0$, $\beta S_i = -S_i, i > r_0$ for some $r_0 \leq n$. For such algebras and actions, the following is a complete invariant:

$i)$ $P(A) \rightarrow K_0(A)$, $[1]$
$ii)$ $P(A \times \mathbb{Z}_2) \rightarrow K_0(A \times \mathbb{Z}_2)$, $[special\ element]$, $\hat{\alpha}_*$
$iii)$ $P(A) \rightarrow P(A \times \mathbb{Z}_2)$

In particular $\mid n - 2r_0 + 1 \mid$ is a complete invariant for the action $\beta$, (as long as $r_0 \neq 0$). [for these algebras and actions $K_1(A) = 0 = K_1(A \times \mathbb{Z}_2)$]

## Invariants for Simple $C^*$-Algebras

George A. Elliot

Invariants where described for simple $C^*$-algebras, namely, $K_0$, as a pre-ordered group, $K_1$, as a group, and the topological convex cone $T^+$ of positive tracial functionals on the Peterson ideal, together with the natural pairing of this with $K_0$. The question arises whether, for separable amenable simple $C^*$-algebras, these invariants are complete. Furthermore, in analogy with the Effros-Handelman-Shen theorem, one might hope that all triples $(G_0, G_1, C)$ where $G_0$ is a simple pre-ordered countable abelian group, $G_1$ is acountable abelian group, and $C$ is a topological convex cone with a base which is a compact metrizable simplex, such that there is a pairing $G_0 \times C \rightarrow \mathbb{R}$, arise (together with the pairing) from a separable amenable simple $C^*$-algebra.

The problem was divided into three cases :

(1) $K_0^+ = 0$, $T^+ \neq 0$;
(2) $K_0^+ \cap -K_0^+ = 0$, $K_0^+ - K_0^+ = K_0$, $T^+ \neq 0$;

(3) $K_0^+ = K_0$, $T^+ = 0$.

Progress in the various cases, due to a large number of people, was surveyed.

## Commuting Squares

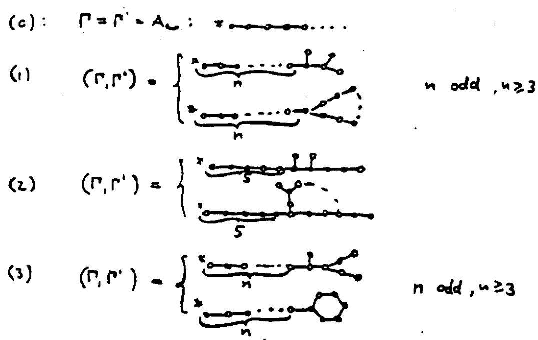
### Pierre de la Harpe, joint work with R. Bacher and V. Jones

Many examples of subfactors are constructed via commuting squares of multimatrix algebras. We investigate similar constructions for commuting squares in which the inclusions of algebras are not necessarily $\star$-inclusions, and this over fields possibly distinct from $\mathbb{C}$. There are numerous examples from combinatorics, involving e.g, Hadamard matrices, strongly regular graphs, symmetric designs or finitely generated linear groups. We present examples of constructions and of computations of higher relative commuting in this setting.

## Computing Principal Graphs of Subfactors

### Uffe Haagerup

Let $N \subset M$ be an irreducible inclusion of $II_1$ factors of finite index, and let $N \subset M \subset M_1 = \langle M, e \rangle$ be the first step in the basic construction. We consider the pair $(\Gamma, \Gamma')$ of the principal graphs for the inclusions $N \subset M$ and $M \subset M_1$, and prove that for $4 < [M : N] < 3 + \sqrt{3} \approx 4.732$ at most the following (unordered pairs) can occur:




In particular either $N \subset M$ has finite depth or has graph $A_\infty$ and is non-amenable in the sense of Popa.Of the possible finite depth subfactors we have so far only been able to realize the case (L) for $n = 3$.It follows that when $4 < [M : N] < 1/2(5 + \sqrt{13}) \approx 4.302$ all subfactors have graph $A_\infty$, and that there are precisely two conjugacy classes of finite depth subfactors of the hyperfinite $II_1$-factor of index $1/2(5 + \sqrt{13})$.

## Orbifold subfactors,central sequences, and the relative Jones invariant $\kappa$

Yasuyuki Kawahigashi

We study the structure of central sequences in a subfactor. Suppose that $N \subset M$ is an AFD subfactor of type $II$ with finite depth. Then Ocneanu's construction of the central sequence subfactor $N^\omega \cap M' \subset M_\omega$ can be regarded as an analogue of the quantum double construction for paragroups. By studying this subfactor more carefully, we prove the following theorem.

**Theorem** In the $SU(N)_\kappa$-orbifold construction with $N \mid \kappa$, the relative Jones invariant $\kappa$ or the relative Connes invariant $\chi(M,N)$ is trivial iff the resulting connection in the orbifold construction is flat.

## Discrete Quantum Groups

Eberhard Kirchberg

We study a class of Hopf $\star$-algebras which is characterised by the fact that each finite dimensional corepresentations is unitary. We present several equivalent properties of Hopf-algebras and argue, that Hopf-algebras fulfiling them are the quantum analogs of discrete gruops. We find that like in the case of a discrete group each unitary Hopf-$\star$-algebra is a quotient of a free product of certain universal unitary Hopf-$\star$-algebra which are the quantum analogs of the group $\mathbb{Z}$. These free quantum gruops are described and classified.

## Free Evolutions for Noncommutative Stationary Process

Burkhard Kümmerer

For the study of noncommutative processes it is essential to establish a "coupling representation". For a prosess with values in $M_n$ this amounts to decomposing a stationary automorphism group $T_t$ on a von Neumann algebra $M_n \otimes e$ with an invariant state $\varphi \otimes \psi$ into $T_t = Adu_t \circ (Id_{M_n} \otimes S_t)$ with $S_t$ a stationary automorphism group on $(\varphi, \psi)$, the free evolution of the process, and $(u_t)_t$ an adapted unitary cocycle for $T_t$, the coupling.

In discrete time the problem of establishing a coupling representation turns out to be equivalent to the problem of understanding "stable unitary equivalence" of states on von Neumann algebras.

**Theorem** If $C$ is semifinite and $\varphi \otimes \psi_1$ is unitarily equivalent to $\varphi \otimes \psi_2$, $\varphi$ some state on $M_n, \psi_1, \psi_2$ normal states on $C$,then $\psi_1$ is unitarily equivalent to $\psi_2$.

There are counterexamples in the Type III case. In continuous time there is an additional cohomological problem to which we can give answers in some cases. It is related to the problem of classifying continuous $\mathbb{R}$-actions on von Neumann algebras.

# $C^*$-Algebras Generated by Smoothable and Liftable Relations

## Terry Loring

The $C^*$-algebra $q\mathbb{C} = \{f \in C_0([0,1], M_2) \mid f(1) \in \mathbb{C}^2\}$ is universally generated by elements $h_1, h_2$ and $a$ subject to the following relations:

(*) $0 \leq h_i \leq 1$, $\| a \| \leq 1$, $a^2 = 0$, $h_2 a = a h_1$, $a^* a = h_1 - h_2^2$.

We prove these relations are smoothable, in the following sense: Given $h_1, h_2, a \in B$, where $B$ is any $C^*$-algebra with a "smooth" subalgebra $B_\infty$ satisfying (*) ($B_\infty$ is dense and closed under $C^\infty$-functional calculus).Then there are approximating elements $\overline{h}_i, \overline{a} \in B_\infty$ also satisfying (*).There is a similar lifting result, although not a complete lifting or there are K-Theory obstructions. Applications to inductive limits and K-Theory are given.

# Algebraic Indextheorems

## Ryszard Nest joint with B.Tsygan

We prove general index theorem for formal deformations of foliations. Given $(M, F)$ a symplectic foliation of a smooth manifold $M$ consider formal deformation $\mathbb{A}^h(M)$ of smooth functions $C^\infty(M)$ $[[h]]$,i.e. an associative product $\star$ satisfying:

$$f \star g = fg + h\{f,g\} = \Sigma_{n\geq 2} h^n P_n(f,g)$$

with $\{\ ,\ \}$ the Poisson bracket and $P_n$ bidifferential operators. Moreover $f \star g = g.f = fg$ if $f$ or $g$ is leafwise constant.

Let $\Omega^*_{hor}$ denote the deRham complex of horizontal (leafwise constant) differential forms. Then there exists a map of complexes

$$Tr_\Theta : CC^{per}_\star(\mathbb{A}^h(M)) \to \Omega^*_{hor}$$

In particular, if the foliation is transversally oriented, $\int Tr_\Theta$ is the Connes' transversal cocycle foliation. The following general result holds for $e \in proj\mathbb{A}^h(M)$, if $e = e_0 + O(h)$ then

$$\langle Tr_\Theta, e\rangle = \int_{M/F} ch(e_0)\hat{A}(M/F)$$

$\Theta$ is a characteristic class in $H^2(M, \mathcal{O}_{hor})$ canonically associated to the deformation. In the case when $\Theta$ is in the image of $H^2(M, \mathbb{C}[[h]])$, the deformed algebra $A^h(M)$ admits a horizontal connection $\nabla = d_{hor} + adA$ with curvature satisfying Bianchi identity $\nabla R = 0$. Given such a pair $(\nabla, R)$ the equivariant cochain $Tr_\Theta$ can be given as $ch(\nabla, R) = Tr(exp\nabla - R)$ and the index theorem becomes

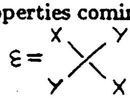
$$\langle ch(\nabla, e)\rangle = \Sigma_n(-1)^n(2n)!/n! \int_{t_0+\ldots+t_{2n}=1} Tr(ee^{-Rt_0}[\nabla, e]e^{-Rt_1}\ldots[\nabla, e]e^{-Rt_{2n}})$$

$$= \int_{M/F} ch(e_0)\, \hat{A}(M/F)\, e^{\Theta/h}$$

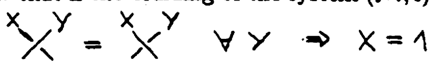
## Braiding and ghosts for subfactors

Adrian Oeneacnu

Given a subfactor $N \subset M$ construct a system of bimodules $\mathcal{M} = \{_M X_M\}$ by decomposing into irreducibles $_M(M \otimes_N M \otimes_N \ldots \otimes_N M)_M$. We introduce the notion of braiding $\epsilon$ on $\mathcal{M}$ as a distinguished element in $\bigoplus_{X,Y} Hom[X \otimes Y, Y \otimes X]$ which has the algebraic properties coming from the picture:




We show that if the braiding of the system $(\mathcal{M}, \epsilon)$ is nondegenerate, i.e.




,then going to the asymptotic inclusion $M_0 \cup M_0' \subset M_\infty$ where $N = M_{-1} \subset M_0 \subset M_1 \subset \ldots \subset M_\infty = \bigcup M_n$ ,the $M_\infty - M_\infty$ bimodules $\mathcal{M}_\infty$ are given by pairs $(X,Y)$ $X, Y \in \mathcal{M}$. In general, let $\mathcal{D} = \{X \in \mathcal{M}$: (crossing of $X,Y$) = (opposite crossing of $X,Y$) $\forall (X,Y) \in \mathcal{M}\}$ .Then one constructs an extension $\mathcal{M} \subset \mathcal{N}$ such that $\mathcal{N}$ is nondegenerate and we describe $M_\infty - M_\infty$ bimodules in terms of $\mathcal{N}$. We prove that for any system $\mathcal{M}$ there are at most finitely many braidings by showing that a braiding $\epsilon$ on $\mathcal{M}$ corresponds to an embedding of sets $\epsilon_* : \mathcal{M} \to \mathcal{M}_\infty$ and is characterized by the latter.

## $C^*$-Algebras constructed out of Hilbert bimodules

Michael Pimsner

To every $C^*$-algebra $A$ and to every Hilbert bimodule $E$ over $A$ -this is a (right) Hilbert module $E$ over$A$ together with an isometric $\star$-homomorphism

$\varphi : A \to \mathcal{L}(E)$-, we associate in a natural way a $C^*$-algebra $\mathcal{O}_E$. If $A$ is commutative and finite dimensional and if $E$ is projective and finitly generated these are the Cuntz-Krieger algebras while if $A$ is arbitrary but $E = A$, with left module structure given by an automorphism, we get the crossed product algebras by $\mathbb{Z}$. The $C^*$-algebras in this class satisfy universal properties and their K-theory is computed in terms of the K-theory of the $C^*$-algebra $A$ and of the class of the module $E$ in $KK$.

## Abstract Duality and Hopf Algebras

C.Pinzari,T.Ceccherini,S.Doplicher,J.Roberts

We defined model actions of locally compact quantum groups on $C^*$-algebras and discussed crossed product constructions. More specifically let $\mathcal{O}_d$ $d = 2, 3 \ldots, \infty$ be the Cuntz algebra of order $d$. Then every Hopf $C^*$-algebra $(\mathcal{A}, \delta)$ associated to a regular multiplicative unitary $V$ on a Hilbert space $H \otimes H$ (in the sense of Baaj-Skandalis) acts in a canonical way on $\mathcal{O}_d$ via it regular corepresentation. Let $\lambda$ denote this coaction. If $V$ contains the trivial corepresentation the fixedpoint algebra $\mathcal{O}_d^\lambda$ carries a natural action of a compact dual given by a faithful functor of tensor $C^*$-categories with conjugates from a f.d. representation category in the sense of Woronowicz' $C^*$-algebra $(\mathcal{A}, \delta)$ onto a full subcategory of $End(\mathcal{O}_d^\lambda)$, the endomorphism category of $\mathcal{O}_d^\lambda$ (this is a generalization of the Cuntz model). If $V$ is non discrete (i.e. it does not contain the trivial corepresentation) then the corresponding coaction is often ergodic.

Thus we introduced a generalized Cuntz algebra $\mathcal{O}_H$ associated covariantly to a Hilbert space $H$ and we generalized the above description to non discrete unitaries. An abstract $q$-dual is now described by a natural $C^*$-subalgebra $\mathcal{O}_V$ of $\mathcal{O}_H$ together with an endomorphism $\rho_V$.

We characterized algebraically pairs of the form $(\mathcal{O}_V, \rho_V)$ in the case where $\rho_V$, and hence $V$, has a braided symmetry $\epsilon$. We obtained in particular a duality result for locally compact groups just requiering $\epsilon$ to be a permutation symmetry (i.e. $\epsilon$ factors through a representation of the permutation group).

## $C^*$-norms on $B(H) \otimes B(H)$

Giles Pisier joint with M.Junge

We prove the following (here $H$ is a Hilbert space,$dim H = \infty$):

**Main Theorem** : There is more than one $C^*$-norm on $B(H) \otimes B(H)$ ,or equivalently $B(H) \otimes_{min} B(H) \neq B(H) \otimes_{max} B(H)$.

Following an approach proposed by Kirchberg it suffices to prove the non-separability of a certain metric space, as follows.

Let $n \geq 1$ be fixed. Consider $OS_n = \{E \subset B(H) \mid dimE = n\}$. This is the set of all $n$-dimensional operator spaces. We equip this set with the metric $\delta(E,F) = log\, d_{cb}(E,F)$ where $d_{cb}(E,F) = inf\{\| u \|_{cb} \| u^{-1} \|_{cb} | u : E \to F\}$. (we consider that $E = F$ when $E$ and $F$ are completly isometric, or equivalently when $\delta(E,F) = 0$.) It is easy to check that $(OS_n, \delta)$ is a complete metric space, but it is not compact for $n > 2$ and more importly we have

**Theorem** $(OS_n, \delta)$ is non separable if $n > 2$.

## Multipliers of Bimodules and Morita Equivalence of Crossed Products

Ian Raeburn joint with S.Echterhoff

Just as each $C^*$-algebra $A$ has a multiplier algebra $M(A)$, every imprimitivity bimodule ${}_AX_B$ has a natural multiplier bimodule $M(X)$. Two coactions $\delta_A, \delta_B$ of a locally compact group $G$ on $C^*$-algebras $A, B$ are Morita equivalent if there is an imprimitivity bimodule ${}_AX_B$ carrying a compatible "coaction" $\delta_X : X \to M(X \otimes C^*(G))$. A theorem of Baaj-Skandalis says that Morita equivalent coactions give Morita equivalent crossed products, and we discussed a new proof of this result which uses spatial realizations of the crossed products and the multiplier bimodule $M(X \otimes C_r^*(G))$. As a corollary,we obtain a simplified proof of the corresponding result for twisted crossed products, due to Bui (1991).

## Classification of some Infinte Simple $C^*$-Algebras

Mikael Rørdam

Georges Elliot's classification conjecture for simple infinite $C^*$-algebras asks the following: Suppose that $A$ and $B$ are unital, separable, nuclear, purely infinite and simple $C^*$-algebras so that the invariants $(K_0(A), [1], K_1(A))$ and $(K_0(B), [1], K_1(B))$ are isomorphic (as abelian groups with a distinguished element $[1]$ of $K_0$). Does it follow that $A$ and $B$ are isomorphic ?

We have defined a natural subclass $\mathcal{C}$ (the classifiable $C^*$-algebras) of purely infinite, simple $C^*$-algebras inside which K-theory is a complete invariant, also in the sense that the range of the invariant is complete: For every triple $(G_0, g_0, G_1)$ where $G_0, G_1$ are countable abelian groups and $g_0 \in G_0$ there is a unital $C^*$-algebra $A$ in $\mathcal{C}$ with $(K_0(A), [1], K_1(A)) \cong (G_0, g_0, G_1)$.The class $\mathcal{C}$ is closed under inductive limits,and it contains the Cuntz-Krieger algebras $\mathcal{O}_A$ with $K_0(\mathcal{O}_A)$ of odd order.

It is my hope that the classifiable class $\mathcal{C}$ is closed under all "amenable" operations, so that every simple infinite $C^*$-algebra constructed by such operations will belong to $\mathcal{C}$.

# Novikov's Conjecture for "Bolic" Groups

Georges Skandalis joint with G.Kasparov

The Baum-Connes conjecture for discrete groups asserts that a natural map $\mu$ is an isomorphism from a group of topological $K$-theory onto the $K$-Theory of the associated reduced $C^*$-algebra.The rational injectivity of $\mu$ implies Novikov's conjecture.

We define a geometric property of a metric space called "bolicity".

**Def.** A metric space $(X,d)$ is said to be $\delta$-"bolic" if

a) $\forall r > 0 \exists R > 0$ such that $\forall a,b,x,y \in X$ such that $d(a,b)+d(x,y) \leq r$ and $d(a,x)+d(b,y) \geq R$ then $d(a,y)+d(b,x) \leq d(a,x)+d(b,y)+2\delta$

b) There exists a "middle point" map $m : X \times X \to X$ such that

$$\forall a,b,x \in X : d(x,m(a,b)) \leq [d(a,x)^2/2 + d(b,x)^2/2 - 2d(a,b)^2]^{1/2} + \delta$$

. Our main theorem is

**Theorem** : Let $\Gamma$ be a discrete group acting properly on a weakly-$\delta$-geodesic, uniformly locally finite, (weakly) bolic metric space. Then the correponding Baum-Connes map is injective (hence Novikov's conjecture holds for $\Gamma$)

# Dimension groups and minimal dynamical systems

Christian Skau joint with T.Giordano a.I.Putnam

Let $(X,\varphi)$ be a minimal Cantor system i.e. $\varphi$ is a minimal homeomorphism of the Cantor set $X$.The group $K^0(X,\varphi) = C(X,\mathbb{Z})/B_\varphi$, where $B_\varphi = \{f - f \circ \varphi \mid f \in C(X,\mathbb{Z})\}$, endowed with the natural ordering, is a simple dimension group. Moreover, all simple dimension groups arise in this way. Let $C_\varphi = \{f \in C(X,\mathbb{Z}) \mid \int_X f d\mu = 0, for\ all\ \ \varphi - inv.prob.meas.\ \mu\}$. Then $B_\varphi \subset C_\varphi$ and $C_\varphi/B_\varphi$ is equal to the infinitesimal subgroup $InfK^0(X,\varphi)$ of $K^0(X,\varphi)$.

**Theorem** Let $(X_i,\varphi_i), i = 1,2$ ,be Cantor systems.The following are equivalent:

(i) $K^0(X_1,\varphi_1)/InfK^0(X_1\varphi_1) \cong K^0(X_2,\varphi_2)$ as ordered groups with distinguished

order units.

(ii) There exists a homeomorphism $F : X_1 \to X_2$ carrying the $\varphi_1$-invariant prob.measures onto the $\varphi_2$-invariant prob. measures.

(iii) $(X_1,\varphi_1)$ and $(X_2,\varphi_2)$ are (topologically) orbit equivalent, i.e. there exists

$\Gamma : X_1 \to X_2$ so that $\Gamma(orbit_{\varphi_1}(x)) = orbit_{\varphi_2}(x)$ , $\forall x \in X_1$.

**Corollary** Let $(X, \varphi)$ be a uniquely ergodic Cantor system, i.e.$\varphi$ has a unique inv.prob.measure. Then $(X, \varphi)$ is either orbit equivalent to an odometer system or to a Denjoy system.

(Recall: A Denjoy system is an aperiodic homeomorphism of the circle that is not conjugate to a pure rotation - restricted to the support of the unique invariant prob.measure.)

# From Trace States to Trace States of the $K_0$-Group

Klaus Thompson

For any unital $C^*$-algebra $A$ we have naturally associated two compact convex sets the tracial state space $T(A)$ which is always a Choquet simplex and the state space $SK_0(A)$ of the $K_0$ group of $A$. There is also a wellknown way to define an element $r_A(\omega) \in SK_0(A)$ from a trace state $\omega \in T(A)$ and we get in this way a continuous affine map $r_A : T(A) \to SK_0(A)$. The talk was a review of what is known about this map in case of a simple separable $A$ and a presentation of the following new result:

**Theorem** Let $\Delta$ be a metrizable compact Choquet simplex, $\Gamma$ a metrizable compact convex set and $r : \Delta \to \Gamma$ a continuous affine surjection.There exists a unital separable nuclear $C^*$-algebra $A$ such that $(T(A), r_A, SK_0(A)) = (\Delta, r, \Gamma)$

# Toeplitz $C^*$-Algebras and Quantization

Harald Upmeier

For a $K$-circular domain $\Omega = K\ expV$ associated with a compact Lie group $K$ and a cone $V \subset ik$, one can define a Toeplitz $C^*$-algebra $\mathcal{T}_\Lambda(K)$, where $\Lambda \subset \hat{K}$ is the dual cone. Similarly, a discrete series representation $\lambda \in \hat{G}$ of a semisimple Lie group $G$ defines a Toeplitz $C^*$-algebra $\mathcal{T}_\lambda(G/K)$ over the symmetric space $G/K$. We analyse the spectrum and composition series of these $C^*$-algebras.

# Free Entropy

Dan Voiculescu

We introduce a free entropy function $\chi(X_1, \ldots, X_n)$ for $N$-tuples of selfadjoint noncommutative random variables based on volumes of matrix approximants.Up to constants, for $n = 1$, this entropy is minus the logarithmic energy of the distribution of the random variable. Moreover it has the right transformation properties under noncommutative functional calculus and in the case of free random variables $\chi(X_1, \ldots, X_n) = \chi(X_1) + \ldots + \chi(X_n)$

We also introduce a free entropy dimension function $\delta(X_1,\ldots,X_n)$ related to $\chi$ in a similar way Minkowski dimension is related to Lebesgue measure in geometric measure theory. We use it to prove results, which can be roughly stated:

If a $II_1$-factor can be generated by selfadjoint elements $X_1,\ldots,X_n$ and there is also a semicircular system $Y_1,\ldots,Y_m$ of generators, which are "smooth non-commutative functions of $X_1,\ldots,X_n$" then $n \geq m$.

(Note: replacing "smooth" by $L^\infty$ in the above statement would yield the non-isomorphism of the free group $II_1$-factors)

## Connes Fusion in Conformal Field Theory

Anthony Wassermann

A precise and mathematically rigorous link is established between operator algebras and unitary conformal field theory.Three related aspects of the multiplicative theory of von Neumann algebras are needed:V.Jones' theory of subfactors; Connes' theory of bimodules or correspondences; and the algebraic approach to quantum field theory of Haag, Doplicher and Roberts. In the WZW or minimal models there are always generating primary fields $\phi(z)$ of small conformal dimension which when smeared,turn out to be bounded just like fermions: $\| \phi(z) \| \leq A \| f \|_2$.Smearing,i.e. the passage from a holomorphic theory to a unitary 'boundary' theory,translates formal algebraic properties of fields $\phi(z)$, such as braiding, into concrete facts about bounded intertwiners. Connes' fusion,a tensor product operation on bimodules over type $III$ von Neumann algebras, leads to a manifestly unitary way of fusing two positive energy representations $H_i, H_j$ to yield a third $H_i \otimes H_j$ which is defined and computed using four point functions of bounded interwiners. The $H_i$'s then become a braided tensorcategory. The finite dimensional $\star$-algebras $End_{LG}H^{\otimes n}$ describe quantum invariant theory without recourse to quantum groups at roots of unity. They lead to a non combinatorial construction of the type $II_1$ braid group subfactors of Jones and Wenzel: $N_0 = (\bigcup \mathbb{C} \otimes End_{LG}H^{\otimes n})'' \subset (\bigcup End_{LG}H^{\otimes (n+1)})'' = M_0$ These also arise in a simple but less obvious way from the type $III_1$ subfactors defined jointly with Jones as a measure of the failure of Haag duality in non vakuum representations $\pi$: $N = \pi(L_I G)'' \subset \pi(L_{I^c} G)' = M$, where $L_I G$ denotes the subgroup of loops supported in an interval $I \subset S^1$ and $I^c = S^1 \setminus I$. Using work of Popa on finite depth subfactors,a localized endomorphism $\rho : M \to M$ can be constructed with $N = \rho(M)$ such that,if $M_1 = (\cup \rho^k(M)' \cap M)''$ , $N_1 = \rho(M_1)$ and $M^\rho$ is the fixed point algebra of $\rho$,then $M = M_1 \otimes M^\rho$ and $N = N_1 \otimes M^\rho$. Moreover the inclusion $N_1 \subset M_1$ is isomorphic to $N_0 \subset M_0$, confirming a joint conjecture with Jones. Other finite depth subfactors can be manufactured from the above subfactors using conformal and Goddard-Kent-Olive inclusions,for example the Jones subfactor of index $3 + \sqrt{3}$.

# Exact $C^*$-algebras and Continuous Bundles of $C^*$-algebras

Simon Wassermann joint with E.Kirchberg

A $C^*$-bundle $\mathcal{A} = \{X, A, \pi_x : A \to A_x\}$ is a triple consisting of a locally compact Hausdorff space $X$,a $C^*$-algebra $A$ (the bundle algebra) and $*$-homomorphisms of $A$ onto fibre algebras $A_x$ such that $\| a \| = sup_{x\in X} \| \pi_x(a) \|$ for each $a \in A$ and such that for $f \in C_0(X), a \in A$ there is an element $f_a \in A$ satisfying $\pi_x(f_a) = f(x)\pi_x(a)$ $x \in X$.(i.e. $A$ is a $C_0(X)$-module). If each of the functions $x \mapsto \| \pi_x(A) \|$ is in $C_0(X)$ , $\mathcal{A}$ is a continuous $C^*$-bundle. If $B$ is a fixed $C^*$-algebra and $\mathcal{A}$ a continuous $C^*$-bundle, the triple $\mathcal{A} \otimes B = \{X, A \otimes_{min} B, \pi_x \otimes id : A \otimes_{min} B \to A_x \otimes_{min} B\}$ is a $C^*$-bundle.We consider the

**Problem** When is $\mathcal{A} \otimes B$ continuous ?

The following theorems answer this (to a certain extend)

**Theorem A** Let $B$ be a $C^*$-algebra.Then the following conditions are equivalent:

(i) $B$ is exact

(ii) For any continuous $C^*$-bundle $\mathcal{A}$ ,$\mathcal{A} \otimes B$ is continuous.

(iii) For any continuous $C^*$-bundle $\mathcal{A}$ of the form $\{\hat{\mathbb{N}}, A, A_t\}$ with $A$ separable, $\mathcal{A} \otimes B$ is continuous.

**Theorem B** Let $\mathcal{A} = \{X, A, A_x\}$ be a continuous $C^*$-bundle such that each $A_x$ is exact.Then $A$ is exact iff $\mathcal{A} \otimes B$ is continuous for any $C^*$-algebra $B$.

Since there exist inexact $C^*$-algebras, e.g.$C^*(\mathbb{F}_2)$, it follows by Theorem A that there are examples of $B$ and continuous $\mathcal{A}$ such that $\mathcal{A}\otimes B$ is not continuous.

We give an example of $\mathcal{A}$ as in Theorem B such that the bundle algebra $A$ is not exact.

(Note: The notions of continuous bundle and continuous field of $C^*$-algebras are essentially equivalent).

# The flow of weights on a subfactor

Carl Winsløw

The flow of weights on a factor of Type $III$ was defined by Connes and Takesaki in 1973 and has been extensively studied as a tool to classify factors of Type $III$ and their automorphisms. The purpose of this talk is to explain how to adapt this approach to the classification of subfactors, in particular subfactors of Type $III$ and their automorphisms. In survey form, the talk will cover

- Popa's classification of subfactors by the standard invariant
- The flow of weights on a subfactor and its computation
- The fundamental homomorphism in subfactor theory

- Approximately inner and centrally free automorphisms
- Classification of strongly free actions on subfactors

Throughout, we shall observe the similarities - and differences - with the single factor case.

Berichterstatter: J. Zacharias

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