MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 7/1994

Funktionentheorie

13. - 19. 02. 1994

Die Tagung fand unter der Leitung der Herren G. Frank (Berlin), W. K. Hayman (York) und N. Steinmetz (Dortmund) statt. Von den insgesamt 40 in- und ausländischen Teilnehmern berichteten 30 in Vorträgen über ihre aktuelle Forschung.

Die Tagung hatte zwei Schwerpunkte: *Differentialgleichungen im Komplexen / Nevanlinnatheorie* und *Komplexe Dynamische Systeme*, es wurde aber auch vereinzelt über andere Themenkreise referiert. Die Vorträge fanden reges Interesse, was in eingehenden und lebhaften Diskussionen im großen und kleinen Rahmen zum Ausdruck kam.

Vortragsauszüge

Analytic Continuation of Dirichlet Sums

M. Anderson

If $f(z)$ is analytic near $z = 0$, the p-fold symmetrization is

$$f_p(z) = \frac{1}{p} \sum_{j=0}^{p-1} f(\omega^j z) , \quad \omega^p = 1 .$$

Now let $f(z) = \sum_{n=0}^{\infty} a_n \exp nz$ where $0 < \limsup_{n\to\infty} |a_n|^{1/n} = \delta < 1$. Then $f(z)$ is not entire, and $f_2(z)$ is not entire, but $f_p(z)$ can be entire for $p \geq 3$. In fact, $f_p(z)$ can be identically zero if $\delta \geq \delta_p = \exp(-\pi \operatorname{ctg} \frac{\pi}{p})$, but is not entire if $\delta < \exp(-\pi \operatorname{ctg} \frac{\pi}{p})$. (with D. Khavinson and H. Shapiro.)

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A somewhat simple proof of the subharmonicity property of the $\star$-function

Albert Baernstein

Let $A = \{z \in \mathbb{R}^2 : R_1 < |z| < R_2\}$, $u \in L^1(A)$. Define $u^* : A \cap (\operatorname{Im} z > 0) \to \mathbb{R}$ by $u^*(re^{i\theta}) = \sup \int_E u(re^{i\phi})\, d\phi$ where the sup is over all sets $E \subset [-\pi, \pi]$ with Lebesgue measure 2θ.

Theorem. Suppose $\Delta u = -\lambda$ in the sense of distributions, where λ is a signed Borel measure in A. Then $\Delta u^* \geq -\lambda^*$ in A^+, where λ^* is a certain measure associated to λ.

If $\lambda \leq 0$ then $\lambda^* \leq 0$, so a corollary of the theorem is that if u is subharmonic in A then u^* is subharmonic in A^+.

In the lecture I present a proof of the theorem which involves general principles about rearrangements and works when $\mathbb{R}^2$ is replaced by $\mathbb{R}^n$, A by an arbitrary domain, circular symmetrization by various types of other symmetrizations, and Δ by various other linear or nonlinear elliptic or parabolic operators.

A problem on factorization of meromorphic functions

I. N. Baker

All cases when

$$f(p) = f(q)\ ,$$

f meromorphic, p, q polynomial, are determined. The method involves the iteration of the algebraic function $p(q^{-1})$.

Boundary behaviour of polyanalytic functions

Mark B. Balk

Polyanalytic (p.a.) functions (which have important applications in plane elasticity) are solutions of the "generalized Cauchy-Riemann equation" $\partial^n w/\partial \bar{z}^n = 0$ in some region of the complex plane. Though such functions seem to be close to analytic functions the famous classical statements concerning boundary properties of analytic functions (such as the theorems of Fatou, Luzin-Privalov, Lindelöf a.o.) in their generally accepted formulations do not survive in the case of p.a. functions of arbitrary order $n > 1$. In this talk some recently obtained results about boundary behaviour of p.a. functions, as well as some important problems still awaiting their conquerers, will be discussed.

Multisummability - a method to compute a function from an everywhere divergent power series

Werner Balser

We shall define Ecalle's multisummability method and discuss some of its properties. For an explicit example, we indicate in which way one may use this method to analyze the Stokes' phenomenon of (non-linear) ODE in the complex domain.

Singularities of inverse functions, value distribution, and iteration

(joint work with Alexander Eremenko)
Walter Bergweiler

Let f be a transcendental meromorphic function of finite order. Our main result is that if the inverse function of f has an indirect singularity over $a \in \widehat{\mathbb{C}}$, then a is a limit point of critical values of f. A corollary is that if f has only finitely many critical values, then f has only finitely many asymptotic values. Combining this result with results from iteration theory we can deduce that $f'f$ takes every finite non-zero value infinitely often. This answers (in the case of finite order) a question of Hayman. Our main theorem also provides a unified proof of recent results of Clunie, Eremenko, Langley, and Rossi on the existence of zeros of f' and f'/f for functions of small order.

Gromov hyperbolicity and the quasi-hyperbolic metric

Mario Bonk

Let X be a geodesic metric space. For $x, y \in X$ we denote by $[x,y]$ a geodesic joining x and y. Given $\delta \geq 0$ we call the space δ-hyperbolic, if for arbitrary points $x, y, z \in X$ and $u \in [x,y]$ we have $\mathrm{dist}(u, [x,y] \cup [z,y]) \leq \delta$. The space X is called (Gromov) hyperbolic, if it is δ-hyperbolic for some $\delta \geq 0$.
Consider a proper subregion $\Omega \subseteq \mathbb{R}^n$ equipped with the quasi-hyperbolic metric k_Ω. By ℓ_Ω we denote the internal euclidean distance in Ω, by $d(x,\Omega)$ the euclidean distance of a point $x \in \Omega$ to the boundary $\partial\Omega$ of Ω, by $\ell(\gamma)$ the euclidean length of an arc γ and by $\gamma(x,y)$ for $x, y \in X$ the subarc of γ joining x and y.
The following theorem gives a sufficient condition for (Ω, k_Ω) to be hyperbolic.
Theorem. Suppose Ω satisfies the following condition. There is a constant $K \geq 1$ such that two arbitratry points $x_1, x_2 \in \Omega$ can be joined by an arc γ in Ω with a) $\ell(\gamma) \leq K\ell_\Omega(x_1, x_2)$ and b) $\min_{j\in\{1,2\}} \ell(\gamma(x_j, x)) \leq Kd(x, \partial\Omega)$ for $x \in \gamma$. Then (Ω, k_Ω) is hyperbolic.

Polynomial Recursion and Iteration

Nikolai Busse

The consideration of polynomial recursion sequences of the form

$$p_{n+1}(z) = p_n(z)^d + Q(z, p_n, \ldots, p_{n-s+1})$$

with dominating first term leads to a division of $\widehat{\mathbb{C}}$ into an outer domain $A = \{z : |p_n(z)| \to \infty \text{ as } n \to \infty\}$, and a compact set $K = \mathbb{C}\backslash A$. Iterating each polynomial p_n gives a Böttcherdomain B_n , the attractor of ∞ . We prove the

Theorem. The outer domain A is the kernel of the domain sequence (B_n) , and $B_n \to A$ in the sense of Carathodory. This is, for example, valid in case of the Mandelbrot-sequence, where K is the Mandelbrot set.

In this example, on the other hand, there is no convergence of the filled-in Julia sets $K_n = \mathbb{C}\backslash B_n$ of p_n to the Mandelbrot set K in the Hausdorffmetric.

Vollständige Klassifizierung der komplexen linearen Differentialgleichungen nach dem Wachstum der Lösungen in ihren singulären Stellen.

Volker Dietrich

Das Wachstum der Lösungen einer linearen DGL im Komplexen ist in jeder ihrer singulären Stellen nach unten beschränkt durch Größen, die sich aus dem NP-Diagramm der DGL direkt berechnen lassen. Eine DGL, die in einer Umgebung jeder singulären Stelle in $\widehat{\mathbb{C}}$ ein Fundamentalsystem von Lösungen besitzt, welches genau das aus dem NP-Diagramm abgelesene Wachstum realisiert, heißt *(global) ausgezeichnet.* Wichtige DGLen, insbesondere die linearen DGLen der Mathematischen Physik zweiter Ordnung, zeichnen sich dadurch aus, daß sie in Umgebungen von singulären Stellen besonders schwach wachsende Lösungen zulassen. Damit läßt sich der Begriff "wichtige DGL" durch den Begriff *global ausgezeichnet* vollständig präzisieren. Die aus dem NP-Diagramm ablesbaren Wachstumsgrößen charakterisieren jeweils einen ganz speziellen DGL-Typ, so z.B. den DGL-Typ: *Konfluente hypergeometrische DGL.* Solche DGL-Typen lassen sich für jede Anzahl von singulären Stellen auf kanonische Weise ordnen und liefern damit eine vollständige Klassifizierung. Bis auf gewisse Ausnahmefälle sind dies DGL-Typen global ausgezeichnet, d.h. sie enthalten eine global ausgezeichnete DGL, die dann auch konstruiert werden kann. Diese Konstruktion beinhaltet u.a. eine Lösung des bekannten Umkehrproblems von Wittich unter sehr allgemeinen Voraussetzungen.

Minimum modulus of functions of order 1
David Drasin

In 1975 W. K. Hayman proved that it f is entire, order 1 with

(1) $$\log M(r) = O(r)$$

and

$$\log M(r) + \log L(r) = O(1)$$

($L(r)$= minimum modulus) then $f(z) = Ae^{Bz}$. Thus there are few extremals for the $\cos \pi\rho$ theorem when $\rho = 1$, a situation very different than for $\rho < 1$. Using a method developed by A. Fryntov, we give a family of other extremals, for functions of order 1 for which (1) fails.

Geodesics of Riemann surfaces
J. L. Fernandez

The lecture describes joint work with M. V. Melizin and D. Pestana.

Consider a Riemann surface R , endowed with its Poincar distance, $\rho = \rho_R$. Let f be a holomorphic function from the unit disc $\mathbb{D}$ into R . Then the set of bounded radial images under f has dimension at least $\delta(R)$=exponent of convergence of R ; more precisely:

(*) $$\text{Dim}\,\{\theta : \sup_{0<r<1} \rho(f(re^{i\theta}), f(0)) < +\infty\} \geq \delta(R) .$$

If we take for f the universal cover of R then one has equality in (*) and the result says that the dimension of the set of geodesics emanating from a given point of R and remaining at a bounded distance from p, is exactly $\delta(R)$.

On the coefficients of conjugating functions
Fritz v. Haeseler

We discuss several properties of the coefficients of Böttcher's function of a monic polynomial which has zero as a fixed point. In particular, we were concerned with the question if it is possible to decide from a knowledge of the coefficients whether the Julia set is connected.

Theorem Let $g(z) = z^d + \delta_{d+1}z^{d+1} + \cdots$ be a rational function such that $\infty \notin A^*(0)$. Then $g : A^*(0) \to A^*(0)$ is conjugate to $\xi \mapsto \xi^d$ on $\mathbb{D}$ if and only if the coefficients of $\varphi(\xi) = \xi + c_2\xi^2 + \cdots$, where $g(\varphi(\xi)) = \varphi(\xi^d)$, satisfy $|c_n| \leq n$ for all $n \geq 2$.

This Theorem has several consequences.

Corollary The Julia set J_p of $p(z) = z^d + \cdots + a_1 z$ is connected if and only if $|c_n| \leq n$, where $\varphi_p(\xi) = \xi + c_2\xi^2 + \cdots$ is the solution of $R(\varphi_p(\xi)) = \varphi_p(\xi^d)$

and $R(z) = p(z^{-1})^{-1}$.

Lemma If $\max\{|p'(z)||z \in J_p\} > d^2$ then J_p is not connected.

Lemma Let J_p be connected. Then the Hayman index of φ_p is zero if φ_p is not a rotation of the Koebe function.

Measure of Julia Sets

D. H. Hamilton

We proved a rigidity theorem for inner functions (special cases due to Sullivan and Schub): Let f be an inner function. Then there is a nontrivial conjugation $\varphi : \mathbf{T} \to \mathbf{T}$ which is absolutely continuous on $\mathbf{T}$ to inner $f^* = \varphi \circ f \circ \varphi^{-1}$ if and only if f is nonergodic.

This implies that if F is meromorphic with Julia set J a curve then J is a "circle-line" or $\dim J > 1$.

We discuss possible extensions.

Semigroups of rational functions

Aimo Hinkkanen

Let G be a semigroup of rational functions under compositon of functions. The speaker and G. J. Martin have developed the foundations for the Fatou–Julia theory of semigroups of rational functions. The set of normality $N(G)$ consists of those points on the Riemann sphere that have a neighbourhood U such that the restrictions of the elements of G to U form a normal family. The Julia set $J(G)$ is the complement of $N(G)$. The theory of rational semigroups forms a generalization of the iteration theory of a single rational function, but it also has unexpeced connections to the theory of moduli spaces of discrete groups.

This talk presents a survey of our results so far. There are many similarities to the iteration of a single function, for example, the repelling fixed points of the elements of G are dense in $J(G)$. Among the differences, one may note that $J(G)$ is, in general, only backward invariant under G , and $J(G)$ may have non-empty interior even if $J(G)$ is not the whole sphere. The dynamics of G on components of $N(G)$ can be complicated; we have obtained a complete description in the case when G is "nearly abelian". We have further shown that if G is finitely generated then $J(G)$ is uniformly perfect.

Sharp answer to Baker's problem concerning boundedness of stable domains

Xin-hou Hua

In this paper, we shall solve a problem due to I. N. Baker concerning boundedness of the stable domains of entire functions, some examples show that our result is best.

Theorem. Let $f(z)$ be a transcendental entire function with order λ less than $\frac{1}{2}$, then any component of the Fatou set $F(f)$ is bounded. Furthermore, for any number $\rho \geq \frac{1}{2}$, there exists an entire function f_ρ with order ρ such that at least one component of $F(f_\rho)$ is unbounded.

On the Convergence of Hyperbolic Components in Transcendental Families

Hartje Kriete

One of the questions in iteration of transcendental functions is, which results carry over from the rational case and which do not. An interesting approach to this question has been suggested by Devaney / Goldberg / Hubbard, namely to study certain dynamical properties of a family of entire functions by studying this properties for an approximating sequence of polynomial-families, and look how these properties carry over to the limit (e.g. $p_d(\lambda, z) = \lambda(1 + z/d)^d$ approximates λe^z). The main results are:

(a) An answer to the question whether or not a hyperbolic component of the limit family is kernel of a sequence of hyperbolic components in the parameter space of the approximating functions and/or vice versa.

(b) Description of the limits of attracting cycles.

(c) For any parameter value chosen from a hyperbolic component of the limit function the Julia sets of the approximating functions converge with respect to the Hausdorff metric to the Julia set of the limit function.

Second Order Linear Differential Polynomials

J. K. Langley

In 1959 Hayman conjectured that the only functions f meromorphic in the plane such that f and $f^{(k)}$ have no zeros for some $k \geq 2$ are of form e^{az+b} or $(az + b)^{-n}$, with $a, b \in \mathbb{C}$ and $n \in \mathbb{N}$. This was proved by Frank for $k \geq 3$

in 1976. Further, Steinmetz and Brüggemann have determined those f such that f and F have no zeros, where

$$F = L_k(f) = f^{(k)} + \sum_{j=0}^{k-1} a_j f^{(j)}$$

with $k \geq 3$ and the a_j polynomials. We consider the analogous problem for $k = 2$, with the a_j rational, and determine all f such that f and $F = L_2(f)$ have only finitely many zeros.

Factorization of meromorphic solutions of $(f')^n = R(z, f)$

Ilpo Laine

We consider factorizations of admissible meromorphic solutions of

$$(f')^2 = a_0(z)(f - \tau_1)(f - \tau_2)(f - q(z))^2 \tag{1}$$

and

$$(f')^3 = a_0(z)(f - \tau_1)^2(f - \tau_2)^2(f - \tau_3)^2 . \tag{2}$$

These differential equations appear via the Malmquist-Yosida-Steinmetz-von Rieth-He-Laine theorem as two of the possible six types of equations $(f')^n = R(z, f)$ with admissible solutions. Jointly with He Yuzan, we have proved:

(1) If a solution f of (1) can be factorized as $f = h \circ g$, where h is nonconstant meromorphic with simple τ_1-points (or τ_2-points) only and g ist transcendental entire, then $T(r, g) = O(\max T(r, a_0),\ T(r, q), 1)$.

(2) In the case of (2), if h is meromorphic and g is transcendental entire, then $T(r, g) = O(T(r, a_0))$ outside of a possible exceptional set E of upper logarithmic density less than one. To prove the case (2), we are using the Steinmetz-Gross-Osgood theorem and a weak form of admissibility, while for (1), the second main theorem is our essential device.

On Linear Combinations of Logarithmic Derivatives

J. Miles

Let $F_1, F_2, \ldots, F_L$ be entire functions of finite order and let $c_1, c_2, \ldots, c_L$ be complex numbers whose convex hull does not contain 0 . A lower bound in terms of the counting functions of the zeros of the F_j's is obtained for

$$\left| \sum_{j=1}^{L} c_j r e^{i\theta} F_j'(re^{i\theta}) / F_j(re^{i\theta}) \right|$$

valid for r in a set of positive logarithmic density and θ in a set $I_r \subset [0, 2\pi]$ of fixed positive measure.
This bound is used to extend a result of Bank and Langley concerning the exponent of convergence of the zero sequences of solutions of certain linear differential equations with entire coefficients.

On iteration and composition sequences
Ch. Pommerenke

Let $\varphi_n : \mathbb{D} \to \mathbb{D}$ be analytic and consider the forward composition sequence $f_n = \varphi_n \circ \cdots \circ \varphi_2 \circ \varphi_1$. Unter a certain mild condition, it is shown that the normalized sequence

$$\frac{f_n(z) - f_n(0)}{1 - \overline{f_n(0)} f_n(z)} \, \frac{1 - |f_n(0)|^2}{f_n'(0)} = z + \cdots$$

converges locally uniformly in $\mathbb{D}$ as $n \to \infty$. This generalizes results about iteration (the case that φ_n does not depend on n). If $\varphi_n(z) \equiv f_\lambda(z) = z(\lambda + z)/(1 + \bar{\lambda} z)$ then, as $\lambda \to \lambda_0 \in \partial\mathbb{D}$ radially,

$$g_\lambda(z) \to z \ \text{ if } \ \lambda_0 = e^{2\pi i \alpha}, \ \alpha \notin \mathbb{Q}$$

but

$$g_\lambda(z) \to z(1 - (-z)^q)^{-2/q} \ \text{ if } \ \lambda_0 = e^{2\pi i p/q} .$$

Zeros of homogeneous differential polynomials
Martin Reinders

We give sharp upper and lower bounds for the number of zeros of certain homogeneous differential polynomials.
Theorem 1. Let g be an entire function and define φ by

$$\varphi = W(g^{(k_1)}, g^{(k_2)}, \ldots, g^{(k_n)})$$

where W denotes the wronskian determinant. If $\varphi \not\equiv 0$, then

$$N(r, \frac{1}{\varphi}) \le n N(r, \frac{1}{g}) + S(r, g) .$$

Theorem 2. Let g be an entire function which is not an exponential polynomial and let φ be one of the functions $W(g, g^{(k)})$, or $W(g, g'', g^{(4)}, \ldots, g^{(2n-2)})$ or $W(g, g''', g^{(6)}, \ldots, g^{(3n-3)})$. Then

$$N(r, \frac{1}{\varphi}) \ge N(r, \frac{1}{g}) + S(r, g) .$$

Asymptotic Values of Subharmonic Functions

P. J. Rippon

A subharmonic function u in the unit disc Δ belongs to the MacLane class A if it has asymptotic values at a dense set of points on the unit circle C. A theorem of Hornblower states that u belongs to A if and only if u has no Koebe arcs.
We prove a related result for functions which are merely continuous (or even fine continuous) in Δ, and this leads to an alternative proof of Hornblower's theorem. Our result holds also in the unit ball B in $\mathbb{R}^m$, and this suggests the possibility of characterizing the corresponding MacLane class for subharmonic functions in B.

Kœnigs functions, quasicircles and BMO

S. Rohde

Report on joint work with J. Heinonen
Consider the basin of attraction G of a rational function R, together with the Kœnigs function (linearizing map) f. The largest subdomain of G that is mapped univalently by f onto a disk is known to have several interesting properties. In the case that R is a Blaschkeproduct, we show that these domains are K-quasi-circles, where K depends only on the degree of R. We also show that a function f analytic in the unit disk preserves (real) BMO under composition if and only if f has bounded valency in disks of bounded hyperbolic radius. It turns out that Kœnigs functions belong to this class of functions. A generalization of the Fernandez-Heinonen-Martio subinvariance principle provides the link to the above geometric statement.

Asymptotic Functions for entire functions

John Rossi

Let f be an entire function. A function $a(z)$, entire, is called an asymptotic function if there exists a path Γ from 0 to ∞ such that $f(z) - a(z) \to 0$ as $z \to \infty$ on Γ. A classical theorem of Ahlfors shows that if f has n distinct asymptotic values (i.e. identically constant asymptotic functions) then its order is no less than $n/2$. A similar question can be asked about n distinct asymptotic functions. (Examples show that we must assume that the order of the asymptotic functions are less than $1/2$.)
A result of Fenton shows that if a function f has n distinct asymptotic functions then the order of f is at least $1/4$. Recent results of Hinkkanen and Rossi show how to improve the constant $1/4$ provided the asymptotic paths are not "too" far apart with respect to angular measure.

A Covering Theorem for a Composite Class of Analytic Functions

Rudolf Rupp

Let S denote, as usual, the class of normalized schlicht functions and let $\mathcal{F}$ denote the class of analytic functions in the open unit disk $\mathbb{D}$, having no fixed point there. We prove the following covering

Theorem. For every function $f \cdot g, f \in S, g \in \mathcal{F}$, the image covers at least the open disk

$$D := \{w : |w| < 3.6 \cdot 10^{-10}\} .$$

As is shown in the proof, the class S can be replaced by a larger class S_0.

Repelling periodic points in the Julia set

Wilhelm Schwick

One of the most important characterizations of the Julia set J_g of a rational function with degree ≥ 2 or an entire function g is given by the following

Theorem. The Julia set J_g is the closure of the repelling periodic points.

The theorem is due to Fatou (1920) in the rational case. For entire functions it was proved by Baker (1968), who used Ahlfors' famous five island theorem. In later papers the question has been asked, whether there exists a more elementary proof in the entire case. Such a proof is the subject of the talk. Instead of Ahlfors' theorem we make use of Nevanlinna's second fundamental theorem and Zalcman's criterion to describe non-normality.

On the sectorial oscillation theory of $f'' + A(z)f = 0$.

Wang Shupei

Let $f(z)$ be a meromorphic function in $\mathbb{C}$. For any $\theta \in [0, 2\pi)$, we define the *radial exponent of convergence of zeros of* f as follows:

$$\lambda_\theta(f) = \lim_{\varepsilon \to 0+} \limsup_{r \to \infty} \frac{\log n^*_{\theta-\varepsilon,\theta+\varepsilon}(r, \frac{1}{f})}{\log r} ,$$

when $n^*_{\theta-\varepsilon,\theta+\varepsilon}(r, \frac{1}{f})$ denotes the number of zeros of f in $\{z = te^{i\varphi} : \theta - \varepsilon < \varphi < \theta + \varepsilon, 1 < t < r\}$, counting multiplicity. We obtain the following results.
Theorem 1. *Let* $A(z)$ *be a polynomial of degree* $n \geq 1$ *of the form* $A(z) =$

$a_n z^n + \cdots + a_0, a_n \neq 0$. *Denote* $\theta_j = (2\pi j - \arg a_n)/(n+2)$ *for* $j = 0, 1, \cdots, n+1$. *Let* f_1 *and* f_2 *be any two linearly independent solutions of the equation*

$$f'' + A(z)f = 0 .$$

Then there exists at least two adjacent rays θ_{j_0} *and* θ_{j_0+1} *say, such that* $\lambda_{\theta_{j_0}}(f_1 f_2) = \lambda_{\theta_{j_0+1}}(f_1 f_2) = (n+2)/2$.

Theorem 2. *Let* $P(z) = a_n z^n + \cdots + a_0$ *where* $a_n = \alpha + i\beta \neq 0$ *and* $n \geq 2$, *and let* $Q(z)$ *be a polynomial of degree* m *such that* $m < 2(n-1)$. *Set* $\delta(P,\theta) = \alpha \cos n\theta - \beta \sin n\theta$. *Then for any solution* $f \not\equiv 0$ *of the equation*

$$f'' + (e^{P(z)} + Q(z))f = 0,$$

$\lambda_\theta(f) = \infty$ *holds for any* θ *satisfying* $\delta(P,\theta) = 0$. *The same conclusion also holds if* $n = 1$ *and* Q *is identically zero.*
The proof of our results depends on the sectorial value distribution theory.

Some isoperimetric inequalities for polygons

Alexander Yu. Solynin

Several extremal problems for polygons are considered on the Euclidean and hyperbolic planes. In particular we shall prove the following theorem solving the Plya-Szegö problem for n-gons with a fixed area:

Theorem 1. Let D_n be an n-gon in the plane and $R(D_n)$ the maximal conformal radius of D_n; then

$$\frac{R^2(D_n)}{\text{area}(D_n)} \leq \frac{2^{\frac{4}{n}}\Gamma^2(1-\frac{1}{n})n\,\mathrm{tg}(\frac{\pi}{n})}{\pi\Gamma^2(\frac{1}{2}-\frac{1}{n})} .$$

Equality here is attained only for the regular n-gon D_n .
Also we shall prove that the regular hyperbolic n-gon has the maximal conformal radius among all hyperbolic n-gons with a given number of sides. So we get the complete solution of a problem posed by J. Hersch.

Existence of extremal Teichmüller mappings with given asymptotic behaviour

Kurt Strebel

Let R be a compact Riemann surface and let φ be a rational quadratic differential of degree two on R . The leading coefficients a_j of its second order poles Q_j are supposed to be real; besides, φ can have first order poles P_i. For arbitrary $K > 1$, φ induces a Teichmüller mapping f of R, with complex dilatation $k\frac{\bar\varphi}{|\varphi|}$, $k = \frac{K-1}{K+1}$, onto a Riemann surface R'. Let z, w be local parameters at Q_j and $Q'_j = f(Q_j)$ respectively. Then, for $z \to 0$, f has the asymptotic behaviour

$$(*) \quad \tfrac{1}{B}\,|z|^{1/K} \le |w| \le |z|^{1/K} B \quad (a_j < 0)$$

(**) similar, with K instead of $\frac{1}{K}$ if $a_j > 0$,

for some fixed $B > 1$.
A necessary and sufficient condition is given, that there exists an extremal mapping with this behaviour. It is shown that if R is the Riemann sphere, it is a Teichmüller mapping.

On quasianalytic functions and a new unicity theorem for Borel-monogenic functions

J. Winkler

P. Turan's definition of quasianalytic classes of functions is as follows: A class F of functions is quasianalytic, if for any two functions with "local identical behaviour" follows that they have "global identical behaviour". This definiton came up from the question of Hadamard: What are the neccesary and sufficient conditions for an infinitely differentiable function f such that from $f^{(\nu)}(x_0) = 0$ for some point x_0 follows $f(x) = 0$ in whole the region of definition of f. E. Borel considered also the following (Borel monogenic-) functions: Let be $a_1, a_2, a_3, \dots$ a sequence of points from $\mathbb{C}$, $r_1, r_2, \dots$ and $g_1, g_2, \dots$ two sequences of reals with $0 < r_\nu < g_\nu$, $\sum g_\nu < \infty$ and $\sum r_\nu / g_\nu^n < \infty$ for all n. Then the functions defined and differentiable in each $C_p = G \setminus \bigcup_{\nu=1}^{\infty} \{z \,|\, |z - a_\nu| < 2^{-p} r_\nu\}$ are arbitrarily often differentiable in $C_p^* = G \setminus \bigcup_{\nu=1}^{\infty} \{z \,|\, |z - a_\nu| < 2^{-p} g_\nu\}$ (G any region $G \subset \mathbb{C}$). Borel proved that these functions are quasianalytic in the sense of Hadamard in $C = \bigcup_p C_p$ if $g_\nu = (\log\log \frac{1}{r_\nu})^{-1}$. In this lecture the result was given that these functions are quasianalytic in Hadamards sense if $g\nu = (\log \frac{1}{r_\nu})^{-1}$.

Berichterstatter: N. Steinmetz

Tagungsteilnehmer

Prof.Dr. James Milne Anderson
Dept. of Mathematics
University College London
Gower Street

GB-London , WC1E 6BT

Prof.Dr. Jochen Becker
Fachbereich Mathematik
Technische Universität Berlin
Straße des 17. Juni 136

D-10623 Berlin

Prof.Dr. Albert Baernstein
Dept. of Mathematics
Washington University
Box 11 46

St. Louis MO 63130
USA

Dr. Walter Bergweiler
Lehrstuhl II für Mathematik
(für Ingenieure)
RWTH Aachen
Templergraben 55

D-52062 Aachen

Prof.Dr. I. Noel Baker
Dept. of Mathematics
Imperial College of Science
and Technology
Queen's Gate, Huxley Building

GB-London , SW7 2AZ

Andreas Bolsch
Fachbereich Mathematik - FB 3
MA 8 - 2
Technische Universität Berlin
Straße des 17.Juni 135

D-10623 Berlin

Prof.Dr. Mark B. Balk
Department of Mathematics
The Smolensk Pedagogical Institute
Przhevalsky Str. 4

214000 Smolensk
RUSSIA

Mario Bonk
Department of Mathematics
University of Michigan

Ann Arbor , MI 48109
USA

Prof.Dr. Werner Balser
Abteilung für Mathematik V
Universität Ulm

D-89069 Ulm

Nikolai Busse
Fachbereich Mathematik
Universitat Dortmund

D-44221 Dortmund

Prof.Dr. James G. Clunie
Dept. of Mathematics
University of York

GB-Heslington, York YO1 5DD

Prof.Dr. David H. Hamilton
Dept. of Mathematics
University of Maryland

College Park MD 20742
USA

Dr. Volker Dietrich
Lehrstuhl II für Mathematik
(für Ingenieure)
RWTH Aachen
Templergraben 55

D-52062 Aachen

Prof.Dr. Walter K. Hayman
Dept. of Mathematics
University of York

GB-Heslington, York YO1 5DD

Prof.Dr. David Drasin
Dept. of Mathematics
Purdue University

West Lafayette , IN 47907-1395
USA

Prof.Dr. Simon Hellerstein
Department of Mathematics
University of Wisconsin-Madison
480 Lincoln Drive

Madison WI, 53706-1388
USA

Prof.Dr. Jose L. Fernandez
Departamento de Matematicas
Universidad Autonoma de Madrid
Ciudad Universitaria de Cantoblanco

E-28049 Madrid

Prof.Dr. Aimo Hinkkanen
Dept. of Mathematics
University of Illinois
1409 West Green Street

Urbana , IL 61801
USA

Dr. Fritz von Haeseler
Institut für Dynamische Systeme
Universität Bremen
Postfach 330440

D-28334 Bremen

Prof.Dr. Xinhou Hua
Fachbereich Mathematik
TU Berlin
Sekr. MA 8-2
Straße des 17. Juni 135

D-10623 Berlin

Prof.Dr. Gerhard Jank
Lehrstuhl II für Mathematik
(für Ingenieure)
RWTH Aachen

D-52056 Aachen

Prof.Dr. Erwin Mues
Institut für Mathematik
Universität Hannover
Postfach 6009

D-30060 Hannover

Dr. Hartje Kriete
Lehrstuhl II für Mathematik
(für Ingenieure)
RWTH Aachen
Templergraben 55

D-52062 Aachen

Prof.Dr. Johannes Nikolaus
Fachbereich 6 Mathematik
Universität/Gesamthochschule Siegen

D-57068 Siegen

Prof.Dr. Ilpo Laine
Department of Mathematics
University of Joensuu
P. O. Box 111

SF-80101 Joensuu 10

Prof.Dr. Christian Pommerenke
Fachbereich Mathematik
Technische Universität Berlin
Straße des 17. Juni 135

D-10623 Berlin

Prof.Dr. James Kevin Langley
Department of Mathematics
University of Nottingham

GB-Nottingham NG7 2RD

Dr. Martin Reinders
Institut für Mathematik
Universität Hannover
Postfach 6009

D-30060 Hannover

Prof.Dr. Joseph B. Miles
Department of Mathematics
University of Illinois
273 Altgeld Hall MC-382
1409, West Green Street

Urbana , IL 61801-2975
USA

Prof.Dr. Philip J. Rippon
Faculty of Mathematics
The Open University
Walton Hall

GB-Milton Keynes , MK7 6AA

Steffen Rohde
Department of Mathematics
University of Michigan

Ann Arbor , MI 48109
USA

Prof.Dr. Alexander Solynin
St. Petersburg Branch of Steklov
Mathematical Institute - POMI
Russian Academy of Sciences
Fontanka 27

191011 St. Petersburg
RUSSIA

Prof.Dr. John F. Rossi
Dept. of Mathematics
Virginia Polytechnic Institute and
State University
460 McBryde Hall

Blacksburg , VA 24061-0123
USA

Prof.Dr. Norbert Steinmetz
Fachbereich Mathematik
Universität Dortmund

D-44221 Dortmund

Dr. Rudolf Rupp
Mathematisches Institut I
Universität Karlsruhe

D-76128 Karlsruhe

Prof.Dr. Kurt Strebel
Mathematisches Institut
Universität Zürich
Winterthurerstr. 190

CH-8057 Zürich

Dr. Wilhelm Schwick
Fachbereich Mathematik
Universität Dortmund

D-44221 Dortmund

Prof.Dr. Shupei Wang
Department of Mathematics
University of Joensuu
P. O. Box 111

SF-80101 Joensuu 10

Prof.Dr. Daniel F. Shea
Department of Mathematics
University of Wisconsin-Madison
480 Lincoln Drive

Madison WI, 53706-1388
USA

Prof.Dr. Jörg Winkler
Fachbereich Mathematik - FB 3
MA 8 - 2
Technische Universität Berlin
Straße des 17.Juni 135

D-10623 Berlin