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Mathematical Economics

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This was the sixth meeting on Mathematical Economics in Oberwolfach. Contrary to the last meetings, this time there was no focus on just a few selected topics. Instead, a very broad range of current research agendas of economic theory was covered. Most of the presented papers were not only interesting for their economic contribution, but also for their mathematical aspects. In this context, the active presence of some mathematicians was important and very stimulating.

Concluding, the last statement of the report on the preceding meeting on Mathematical Economics in Oberwolfach can be quoted: " ... the warm hospitality provided by the Mathematisches Forschungsinstitut combined with the excellent facilities created a stimulating atmosphere which was appreciated by all participants."

Vortragsauszüge

C.D. Aliprantis: Some Remarks on the Welfare Theorems

The fundamental theorems of welfare economics for convex economies were formulated by K. Arrow and G. Debreu as follows. First Welfare Theorem: Every allocation supported by prices is Pareto optimal. Second Welfare Theorem: Every Pareto optimal allocation can be supported by prices. The welfare theorems were established by Arrow and Debreu in the finite dimensional setting. In the infinite dimensional setting the theorems can be false or true depending upon the commodity-price duality employed to describe the economy. For instance, consider the two consumer exchange economy introduced by

L. Jones: Commodity space $X = L_1[0,1]$, initial endowments $\omega_1 = \omega_2 = \frac{1}{2}\chi_{[0,1]}$, and preferences represented by the utility functions $u_1(x) = \int_0^1 tx(t)\,dt$ and $u_2(x) = \int_0^1 (1-t)x(t)\,dt$. It turns out that the economy "satisfies" the welfare theorems with respect to the commodity-price duality $\langle L_1[0,1], C[0,1]\rangle$ while it does not satisfy them with respect to the duality $\langle L_1[0,1], C^1[0,1]\rangle$. In the infinite dimensional setting the welfare theorems are "approximately true". The following result gives the "flavor" of the approximate welfare theorems. In a convex exchange economy with commodity-price duality represented by a Riesz pair and with a weakly compact Edgeworth box $[0,\omega]$ (and the standard assumptions on preferences) every Edgeworth equilibrium is approximately supported by prices in the sense that for each $\varepsilon > 0$ there exists some price $p > 0$ satisfying $p \cdot \omega = 1$ and $x \succeq_i x_i$ implies $p \cdot x \geq p \cdot \omega_i - \varepsilon$.

The welfare theorems can be used to establish the existence of equilibria in growth models and overlapping generations models.

B. Allen: Supermechanisms

This paper examines the cooperative games with nontransferable utility that result when state-dependent allocations that agents can achieve in an exchange economy must satisfy an implementation requirement. An important distinction compared to the literature is that I do not restrict myself to a single mechanism because there may be no relation between a mechanism for the grand coalition and one for a subcoalition. To begin, I study the sets of state-dependent allocations that can be implemented by some randomized Bayesian incentive compatible mechanism. The mechanism can possibly depend on the allocation, although I show that this generalization doesn't enlarge the set beyond the implementable allocations for some "supermechanism" with sufficiently large message space. The set of superimplementable imputations is convex. Moreover, if an allocation is (super)implementable for a submarket, then it forms part of at least one (super)implementable allocation for the entire economy. The resulting game is superadditive and is well defined because the set of Bayesian incentive compatible direct mechanisms is compact. Existence of the superimplementable NTU value is shown. Implicitly, this provides a cooperative selection of mechanisms based on agents' marginal contributions with (noncooperative) strategic use of information which is equivalent to the selection of a noncooperative equilibrium for the supermechanism.

Y. Balasko: Expectational Stability of Walrasian Equilibria

This paper investigates the stability of competitive equilibria of two-period Arrow-Debreu economies by reinterpreting the period zero component as a temporary equilibrium associated with rationally expected period one prices. Any change of the period zero prices induces a change in the value of the expected period one prices. These revised expectations induce in turn new values for the period zero prices in order for the supply and demand of period zero to remain equal, a process that can go on indefinitely. This defines a dynamical system, the fixed points of which are the (period zero) temporary equilibria; local asymptotic stability for this dynamics, or expectational stability, captures the effect of expectations formation on the stability of equilibria. A Walrasian equilibrium of a two-period Arrow-Debreu economy is expectationally stable if the corresponding (period zero) temporary equilibrium is itself expectationally stable. We show the following properties of Walrasian equilibria: expectational stability is different from tatonnement and Hicksian stability; the set of expectationally stable equilibria has a non-empty interior that contains the set of no-trade equilibria; equilibria that satisfy the gross substitutability property are expectationally stable.

H. Bester: **A Bargaining Model of Financial Intermediation**

We study a financial market where investors have to search for investment projects. After identifying a project, the investor bargains with the entrepreneur about the financial contract. Alternatively, the investor may delegate search and bargaining to a financial intermediary. Delegated bargaining creates a precommitment effect. But, since the investor cannot monitor the intermediary's search behavior, intermediation may result in excessive risk taking. The tradeoff between these considerations determines whether direct or intermediated investment occurs in equilibrium.

B. Cornet: **A Remark on the Uniqueness of Equilibrium**

We consider a pure exchange economy $\mathcal{E} = (u_i, \mathbb{R}^\ell_+, e_i)_{i=1,\dots,m}$ where $e_i \in \mathbb{R}^\ell_+ \setminus \{0\}$ denotes the initial endowment of agent i and $u_i\colon \mathbb{R}^\ell_+ \to \mathbb{R}$ his utility function. We assume that, for every h, $\sum_i e_{ih} > 0$ and, for every i,

(A.1) u_i is continuously differentiable and $\nabla u_i(x_i) \in \mathbb{R}^\ell_+ \setminus \{0\}$.

(A.2) The matrix $e = (e_{ih})_{h=1,\dots,\ell}^{i=1,\dots,m}$ is not decomposable, in the sense that there does not exist a partition I, I' (non empty) of the set of agents $\{1,\dots,m\}$ and a partition H, H' (non empty) of the set of goods $\{1,\dots,\ell\}$ such that $e_{ih} = 0$ for $(i,h) \in I \times H' \cup I' \times H$.

(A.3) Equilibrium prices are positive (i.e. in the interior of $\mathbb{R}^\ell_+$).

Our main result states:

MAIN RESULT: Let (x,p) and (y,q) be two equilibria of the economy $\mathcal{E}$ such that, for every i, $u_i(x_i) = u_i(y_i)$. Then one has $p = \lambda q$ for some $\lambda > 0$ (i.e. uniqueness of prices up the multiplication by a positive scalar).

F. Delbaen: **Arbitrage and Change of Numéraire**
(joint work with W. Schachermayer)

If S is a semimartingale with the No Free Lunch Property (with Vanishing Risk) then $\exists\, \mathbb{P}$ a local martingale measure. If $\mathcal{K} = \{H \cdot S \mid H \text{ admissible}\}$ (H is admissible if $H \cdot S \geq -a$ for some $a \in \mathbb{R}_+$), then we prove that the following are equivalent:

1. $f > -1$, $f \in \mathcal{K}$, f is maximal ($f = (H \cdot S)_\infty$).
2. $V = 1 + (H \cdot S)$ has an equivalent martingale measure (not just a local martingale measure!).
3. $\frac{S}{V}$ has the No Arbitrage Property.
4. $\frac{S}{V}$ has an equivalent martingale measure.

5. $\exists Q$ local martingale measure for S ($Q \sim \mathbf{P}$), $\mathbf{E}_Q[f] = 0$.

As shown by examples the existence of $R \sim \mathbf{P}$ local martingale measure for S such that $\mathbf{E}_R[f] < 0$ cannot be excluded!

E. Dierker: **Profit Maximization Mitigates Competition**

We compare the Bertrand-Nash equilibria in case of utility maximization with those under the usual profit maximization. It turns out that profit maximization leads to less price competition than utility maximization and is thus beneficial for the owners as a whole. Furthermore, conditions are studied under which profit maximization and utility maximization yield approximately the same outcomes.

E. Eberlein: **Empirical Results on German Stock Returns**

Distributional assumptions for the returns of the underlying assets play a key role in valuation theories for derivate securities. Take for example the Black-Scholes formula, where normal returns are assumed. Based on a data set consisting of daily prices of 15 of the 30 DAX-values during a three-year-period we analyze the distributional form of compounded returns. After performing a number of statistical tests it becomes clear, that several of the standard assumptions cannot be justified. Some general variance models are discussed.

B. Grodal: **Strategic Behavior in Imperfectly Competitive Markets**

It is wellknown that in a Walras equilibrium only the relative prices of the commodities are determined. However, price normalization does not have real effects. Moreover shareholders unanimously instruct the managers of the firms to maximize profits. These features change drastically when markets are imperfectly competitive. As well in models with Cournot competition as Bertrand competition price normalization has real effect and there will be a continuum of equilibria. Even the set of limit points of Cournot-Walras equilibria depends on the normalization rule.

One might therefore change the objective function of the firms and let firms maximize an increasing function of the shareholders utility levels f.ex. the weighted sum according to the share in the firm. This gives a well defined equilibrium concept.

Using this equilibrium concept it is essential that shareholders are served only through the Walrasian market. If we allow firms to withdraw commodities from the market and give them directly to the consumers, firms loose their strategic power. The Walras equilibria are the only equilibria, if shareholders do not restrict their net-trades on the Walrasian market. If on the other hand one allows shareholders to manipulate their demands on the Walrasian market in any desirable way i.e. allow the strategy sets $\mathcal{F}_i = \mathcal{F} = \{f : \Delta \to \mathbf{R}^\ell \mid f$ continuous and $pf(p) \leq 0\}$ then all feasible allocations which are consistent with a price system can be obtained as equilibrium (Nash) outcomes.

R. Guesnerie: **Coordination Failures due to Price-Flexibility: A Keynesian-like Argument**

A Rational Expectations Equilibrium is called strongly Rational whenever the actions taken by economic agents (at the equilibrium) are Common Knowledge i.e. derive from upstream - and somewhat more standard - common knowledge assumptions on the fundamentals of the economy. Three simple models are considered and it is examined whether their (unique) Rational Expectations Equilibrium is strongly Rational. These models are:

1. The Muthian model where farmers decide today on the size of crops that will be sold tomorrow.
2. A fixed-price model that retains the aggregate features of the elementary keynesian model of textbooks.
3. A flexible-price model that has the same aggregate features as the three-goods model of keynesian theory.

It is shown that the circumstances under which coordination failures occur in the third model - in the sense that the equilibrium is not strongly rational - relate on the one hand to supply and demand elasticities - along lines that are reminiscent of what happens in model no.1 - and of the keynesian multiplier the role of which is crucial in model no.2. It is also shown that wage flexibility has dubious effects - although rather negative - on coordination.

P. Hammond: **On Constructing Multiperson Hierarchies of Knowledge and Beliefs**

In a multi-person game, an epistemic state of the world describes what each player knows and believes about other players' knowledge and beliefs, including their knowledge and beliefs about other players, and so on ad infinitum. Here knowledge of an event is regarded as sufficient but not necessary for believing it with probability one. Then, by relying where necessary on information about prior beliefs, before players had any access to information affecting their posterior knowledge and beliefs, a procedure for constructing recursively a universal space of possible epistemic states is described. This procedure involves only a countable infinity of steps. Also, in the case where the intrinsic state spaces are all finite, the state space that is constructed to describe knowledge alone will be countably infinite, whereas that required to describe both knowledge and beliefs is a compact topological space.

T. Hens: **Excess Demand Functions with Incomplete Markets (joint work with J.M. Bottazzi)**

We characterize the structure of excess demand in a General Equilibrium Model, with Incomplete Markets and Real Assets. It is shown that as in the complete markets case (Arrow-Debreu-model) Continuity, Walras-Law and Homogeneity exhaust the structure of excess demand at non-critical prices.

M. Jerison: **A Discreet Characterization of Slutsky Symmetry (joint work with D. Jerison)**

A smooth demand function is generated by utility maximization if and only if its Slutsky matrix at each vector of income and prices is symmetric and negative semidefinite. The semidefiniteness is equivalent to a weak version of the weak axiom of revealed preference and hence is potentially refutable by two price-quantity observations. Slutsky symmetry is more problematic since Shafer (JET 1977) shows that for each $k > 0$ there is a demand function that violates Slutsky symmetry yet has no revealed preference cycles with fewer than k observations.

We characterize Slutsky symmetry by relating it to "antisymmetric revealed preference cycles". Let x_t be the demand vector chosen at price vector p_t with income fixed at 1. The sequence of price-quantity observations $\{p_t, x_t\}_{t=0}^{T}$ is an antisymmetric revealed preference cycle if $p_T = p_0$ and $p_{t-1}x_t > p_t x_{t-1}$ for $t = 1, \cdots, T$, so that real income growth from observation $t-1$ to t is larger than from observation t to $t-1$. Let A be the set of price vectors at which the Slutsky matrix is asymmetric. We show that p is in the closure of A if and only if every neighborhood of p contains an antisymmetric revealed preference cycle consisting of three observations. We also show how the rate of real income growth along such a cycle depends on the degree of Slutsky asymmetry and the extent of price variation.

U. Kamecke: **Dominance or Maximin: How to Solve an English Auction**

It is widely believed that the English auction is solved after the dominated strategies are eliminated. This opinion is formally confirmed only for one "ascending clock" auction and can not be generalized for most other English auctions. In this paper I show that the formal argument which is usually given in this context can not be derived from the dominance criterion alone, but that it can be used to prove that competitive bidding is the unique maximin strategy of a bidder who believes that his opponents do not play dominated strategies. Since the competitive bids also form a Nash-equilibrium of the auction game this result gives an unfamiliar but strong support for the established solution.

A. Kirman: **Formation of Markets: Evolving Networks**

Economic models in the general equilibrium tradition typically ignore the problem of direct interaction between agents. The latter interact only through the price mechanism. In game theoretic models agents take account of the actions of all the other individuals and know that the others do the same. In real economies some strong links exist between agents or they are absent. It is suggested that this may be modelled by using a random graph which evolves depending on the profitability of the links. Thus agents act so as to increase the probability of using those links which have proved most satisfactory in the past. Empirical evidence from the Marseille fish market and a simple simulated model of an artificial market suggest that very asymmetric networks may arise from symmetric initial conditions.

H. König: On the Foundations of Measure Theory (a very long discussion remark)

The first part summarizes the contribution of the present author to the new theories of construction of measures from primitive set functions, defined on lattices of subsets, as initiated by Topsoe, Kelley & Srinivasan, Ridder, Sion, ... since 1970. Our contribution permits to characterize those set functions which can be extended to measures with certain prescribed regularity and smoothness properties. The basic tool is a little collection of new envelopes for set functions, of σ (= sequential) or τ (= nonsequential) and inner or outer type, which are relatives of the Carathéodory outer measure but quite different.

As a direct application of the above theory and in the same spirit the second part then obtains a new version of the Daniell-Stone representation theorem for certain lattice cones of $[0,\infty[$-valued functions, and in σ and τ versions. The τ theorem contains a new version of the Riesz representation theorem on Hausdorff topological spaces. The latter result characterizes those elementary integrals, defined on certain lattice cones of upper semicontinuous $[0,\infty[$-valued functions concentrated on compact subsets, which come from Radon measures.

H. König: The Classical Minimax Theorem

The talk describes the development from the minimax theorem of Sion 1958 to the topological minimax theorem of the present author (Arch. Math. 59 (1992), 55-64) formulated below. For X and Y nonvoid sets and $F : X \times Y \to \overline{\mathbb{R}}$ define

$$F_{\bullet} := \mathrm{Sup}_{x\in X}\, \mathrm{Inf}_{y\in Y}\, F(x,y) \leqq \mathrm{Inf}_{y\in Y}\, \mathrm{Sup}_{x\in X}\, F(x,y) =: F^{\bullet}.$$

The minimax relation is $F_{\bullet} = F^{\bullet}$.

THEOREM: Let X and Y be topological spaces such that

$\forall y \in Y$ the function $F(\cdot, y) : X \to \overline{\mathbb{R}}$ is upper semicontinuous;

$\forall x \in X$ the function $F(x, \cdot) : Y \to \overline{\mathbb{R}}$ is lower semicontinuous.

Assume that

$\forall T \subset Y$ nonvoid and $t > F_{\bullet}$: $I(T,t) = \bigcap_{y\in T} [F(\cdot, y) \geqq t] \subset X$ is connected;

$\forall S \subset X$ finite $\neq \emptyset$ and $t > F_{\bullet}$: $II(S,t) = \bigcap_{x\in S} [F(x,\cdot) \leqq t] \subset Y$ is connected.

Assume in addition that Y is compact. Then $F_{\bullet} = F^{\bullet}$.

Another development led from the minimax theorem of Ky Fan 1952 to extended versions in terms of mean functions. It seems that the two lines went into very different directions.

S. Krasa: **Fiat Money in Differential Information Economies (joint work with M. Huggett)**

The paper applies mechanism design methods to analyze the welfare role of fiat money. The economies under consideration are generalizations of the Townsend "turnpike model" that include limited commitment and differential information. We show that in the Townsend turnpike model fiat money is not essential unless there is limited commitment. In the presence of differential information fiat money is essential in overcoming incentive problems. This is the case even if there are positive returns to storage.

W. Leininger: **The All-Pay Auction with Incomplete Information: Existence and Uniqueness of Bayesian Equilibrium**

A family of (sealed-bid) all-pay auctions is modelled as Bayesian games. Existence, uniqueness and closed-form characterization of (Bayesian) Nash equilibrium is established for arbitrary regular prior distributions of types. The famous 'war of attrition' is obtained as a limit of auctions taken from this family and the – drastically – different solution theory of the latter is accounted for via properties of the limit process.

M. Magill and M. Quinzii: **Infinite Horizon Incomplete Markets**

Let $\mathcal{E}(l_\infty^+(\mathbf{D}), \succeq, \omega, A)$ denote an exchange economy with a finite number of agents ($i \in I$) with commodity space $l_\infty(\mathbf{D})$ over an infinite event tree $\mathbf{D}$. At each node $\xi \in \mathbf{D}$ agents trade on spot markets (with spot prices $p = (p(\xi), \xi \in \mathbf{D})$) and on financial markets (with prices $q = (q(\xi), \xi \in \mathbf{D})$ and payoff structure $A = (A(\xi), \xi \in \mathbf{D})$). At each node an agent must satisfy the budget equation $p(\xi)(x^i(\xi) - \omega^i(\xi)) = p(\xi)A(\xi)z^i(\xi^-) - q(\xi)z^i(\xi)$ and to prevent Ponzi schemes one of the three additional constraints:

(a) $q(\xi)z^i(\xi) \geqq -M$, $\forall \xi \in \mathbf{D}$

(b) $(qz^i) = (q(\xi)z^i(\xi), \xi \in \mathbf{D}) \in l_\infty(\mathbf{D})$

(c) $\lim\limits_{T \to \infty} \sum\limits_{\xi' \in \mathbf{D}_T(\xi)} \pi^i(\xi')q(\xi')z^i(\xi') = 0$

Condition (a) leads to the concept of an equilibrium with explicit debt constraint M; (b) to an equilibrium with implicit debt constraint (DC); (c) to an equilibrium with transversality condition (TC), by requiring that agents optimize over their budget set and markets clear ($\sum_{i \in I} x^i = \sum_{i \in I} \omega^i$, $\sum_{i \in I} z^i = 0$). The following assumptions are made. A1: the branching number of $\mathbf{D}$ is finite. A2: $0 < m < \omega^i(\xi) < m'$, $i \in I$. A3: $\succeq_i$ is transitive, reflexive, complete, monotonic, has convex preferred sets and is continuous in the Mackey topology $\tau(l_\infty, l_1)$. A4: there exists $\beta < 1$ such that for all x^i with $0 \leq x^i(\xi) < m'I$, $\forall \xi \in \mathbf{D}$, $x^i\chi_{\mathbf{D}\setminus\mathbf{D}^+(\xi)} + e_1^\xi + \beta x^i \chi_{\mathbf{D}^+(\xi)} \succ_i x^i$ for all $\xi \in \mathbf{D}$. A5: each security is short-lived and has payoff in good 1. A6: there exists a riskless bond at each node.
Theorem. (i) Under A1–A6 there exists a TC equilibrium

(ii) the TC equilibria and DC equilibria coincide.
Corollary. If $\mathcal{E}(l^+_\infty(\mathbb{D}), \succeq, \omega, A)$ is an economy satisfying A1–A6 then there exists a bound M such that the economy has an equilibrium with explicit debt constraint M which is never binding.

A. Mas-Colell: **Two Remarks on Bimatrix Games (joint work with L. Corchón)**

Remark 1: Proposition: Let $\mathcal{U}$ be an affine space of $m \times n$ matrices. Then there is a set $\mathcal{U}' \subset \mathcal{U}$, which complement has null measure, and such that for any $V_1 \in \mathcal{U}'$ and $V_2 \in \mathcal{U}'$ the bimatrix game with payoff matrix (V_1, V_2) has a finite number of equilibrium payoffs. This result is motivated by the fact that typically entries of a payoff matrix represent final positions of play and so, the range of allowable perturbations may be generally restricted. The result does not generalize to $n > 2$ (McLennan).
Remark 2: Suppose that the strategy sets are $S_1 = S_2 = [0, 1]$ and that the two players best reactions $f_1(x_2)$, $f_2(x_1)$ are C^1 functions. Consider the Cournotian adaptive dynamics $\dot{x}_1 = f_1(x_2) - x_1$, $\dot{x}_2 = f_2(x_1) - x_2$. Then for any initial $(x_1(0), x_2(0))$ the trajectory $(x_1(t), x_2(t))$ must converge to some equilibrium. (Proof: Liouville's Theorem.) The result does not generalize to $n > 2$.

K. Miltersen: **An Arbitrage Theory of the Term Structure of Interest Rates**

In the setting of the Heath-Jarrow-Morton model this paper presents sufficient conditions to assure that the stochastic forward rates are strictly positive while maintaining the martingale property of the discounted bond price processes in the case where the stochastic forward rates are described as stochastic differential equations with explicitly state dependent stochastic volatility. Moreover, the stochastic development of the term structure of interest rates is generalized to be described by a class of continuous local martingales instead of Wiener processes. An example showing that this is a true extension of the Heath-Jarrow-Morton model is provided.

S. Müller: **Underpricing of Initial Public Offerings: A Common Value Model**

This paper presents a theoretical model, where average underpricing of an initial public offering results from the strategic interaction of investors and the seller (the investment bank) in a situation of incomplete information. The model uses a priority pricing set-up as in Parsons/Raviv (1985). However, in contrast to earlier models a common value set-up is employed. It is shown that

(i) the expected revenues from the sale in the primary market are smaller than the expected revenues accruing via the opening price in the secondary market, i.e. there is underpricing on average,
(ii) complete revelation is the optimal information policy of the seller,
(iii) average underpricing decreases with an increasing number of investors.

M. Nermuth: **Equilibrium in Markets with Imperfectly Informed Buyers**

The Prékopa-Borell theorem is used to show existence of Nash-equilibrium in pure strategies in an oligopolistic market where the firms set their prices, and the households have imperfect information about these prices in the sense that they cannot observe the prices directly, but only signals which are randomly distorted or noisy representations of the true prices. If the error terms in these signals are distributed with a density which is log-concave, then the firms' profit functions are quasiconcave and an equilibrium exists.

K. Podczeck: **Equilibria in Markets with Infinitely Many Commodities and Agents with Non-Convex Preferences**

It is wellknown that for exchange economies with finitely many commodities equilibrium existence can be demonstrated without requiring individual preferences to be convex if the set of agents is given as an atomless measure space. This is not the case for an infinite dimensional commodity space. It is the purpose of this paper to show that also for infinitely many commodities the existence of an equilibrium can be established without postulating convex preferences if one makes the hypothesis that there are many agents of every type that appears in a given economy.

K. Sandmann: **A Lognormal Term Structure Model and the Pricing of Interest Rate Derivatives (joint work with K. Miltersen and D. Sondermann)**

The lognormal distribution assumption for the term structure of interest is the most natural way to exclude negative spot and forward rates. However, imposing this assumption on the continuously compounded interest rate has a serious drawback: expected rollover returns are infinite even if the rollover period is arbitrarily short. As a consequence such models cannot price one of the most widely used hedging instruments on the Euromoney market, namely the Eurofuture contract.

We show that the problem with lognormal models results from modelling the wrong rate, namely the continuously compounded rate. If instead one models the effective annual rate the problem disappears; i.e. the expected rollover returns are finite. Let $f_e(t, \mathcal{T}, \alpha)$ be the effective annual forward rate at time t for the time period $[\mathcal{T}, \mathcal{T} + \alpha]$ we assume the following family of processes: $\forall \mathcal{T} \in [0, T]$, $\forall t \in [0, \mathcal{T}]$, $\forall \alpha \in]0, 1]$

$$df_e(t, \mathcal{T}, \alpha) = \mu(t, \mathcal{T}, \alpha) f_e(t, \mathcal{T}, \alpha)\, dt + \sigma(t, \mathcal{T}, \alpha) f_e(t, \mathcal{T}, \alpha)\, dW(t)$$

where $\{W(t)\}$ is a Wiener-process. Under this assumption we study the dynamics of the continuously compounded rates, i.e.

$$f_C(t, \mathcal{T}, \alpha) := \ln(1 + f_e(t, \mathcal{T}, \alpha))$$
$$r_C(t) := \ln(1 + r_e(t)) := \ln(1 + f_e(t, t, 1))$$

These dynamics are neither normal or lognormal. Furthermore we derive analytic close form solutions for European put and call option on zero coupon bonds and for interest rate caps and floors relative to the proposed lognormal model of the effective annual interest rates.

W. Schachermayer: A General Version of the Fundamental Theorem of Asset Pricing (joint work with F. Delbaen)

The "FUNDAMENTAL THEOREM OF ASSET PRICING" states that the no arbitrage condition is "essentially" equivalent to the existence of an equivalent martingale measure.

We formulate a general version of this theorem giving a precise meaning to the word "essentially".

M. Schweizer: Risk-Minimizing Hedging Strategies under Restricted Information

We construct risk-minimizing hedging strategies in the case where there are restrictions on the available information. The underlying price process is a d-dimensional $\mathbb{F}$-martingale, and strategies $\varphi = (\xi, \eta)$ are constrained to have ξ $\mathbb{G}$-predictable and η $\mathbb{G}'$-adapted for filtrations $\mathbb{G} \subset \mathbb{G}' \subset \mathbb{F}$. We show that there exists a unique $(\mathbb{G}, \mathbb{G}')$-risk-minimizing strategy for every contingent claim H in $\mathcal{L}^2(G'_T, P)$ and provide an explicit expression in terms of $\mathbb{G}$-predictable dual projections. Previous results of Föllmer/Sondermann (1986) and DiMasi/Platen/Runggaldier (1993) are recovered as special cases. Examples include a Black-Scholes model with delayed information and a jump process model with discrete observations.

W. Shafer: The Structure of the Pseudo-Equilibrium Manifold

It is shown that the Pseudo-Equilibrium manifold for a General Equilibrium model with incomplete markets has the topological structure of a vector bundle over a Grassmann manifold, which is orientable if and only if the number of states and the number of assets have the same parity. Similar results are obtained in the case of a fixed asset structure consisting of commodity forward contracts.

B. Sloth: Nash Equilibrium with Lower Probabilities (joint work with E. Hendon, H. J. Jacobsen, T. Trances)

A Nash equilibrium is a common theory held about each player's action. It is required that the theory is consistent with each player choosing an optimal response to the theory. It is usually required that the theory takes the form of a probability measure on the players' strategies. We analyze the effect of relaxing this requirement allowing the theory to take the form of a lower probability measure, also called a belief function. In particular this allows for a strategy which is never a best reply against probability measures on other players, to be part of an equilibrium. We discuss the concept of a perfect equilibrium in this setting. Further we show that for some attitudes towards uncertainty it is possible that players cooperate in one shot prisoners' dilemma. Finally we demonstrate how uncertainty may arise or be resolved if players reason about the game à la fictitious play.

D. Sondermann: **Different Dynamical Specifications of the Term Structure of Interest Rates and their Implications (joint work with M. Musiela)**

Alternative ways of introducing uncertainty to the term structure of interest rates are considered. They correspond to the different expectation hypotheses. The dynamics is analyzed in a framework of stochastic equations in infinite dimensions. Exploiting the FX-analogy and Girsanov's transformation the arbitrage-free dynamics are easily derived for the spot- and x-forward markets. They correspond to the LEH- resp. RTM-Hypothesis resp..

H.G. Tillmann: **Köthe-Riesz-Spaces in Mathematical Economics**

If $\mathcal{E} = (X^i, P^i, a^i; Y^j, \alpha_{ij})$ shall be a model of an economy with infinite time horizon, the following framework is used:
$\langle E_n, F_n\rangle$, $F_n = E_n'$, is a symmetric dual pair of Riesz-spaces for each time period n, $E = \prod E_n$, $F = \prod F_n$. Commodity space $\Lambda \subset E$, Price space $\mathbb{P} \subset F$. $\Lambda^* = \{p = (p_n) \in F : \sum |p_n| \cdot |x_n| < \infty\} = \mathbb{P}$, $\Lambda = \mathbb{P}^* = \Lambda^{**}$ is a perfect Köthe-Riesz-space.
Thm 1: If $\langle \Lambda, \Lambda^* \rangle$ is a pair of perfect Köthe-Riesz-spaces $|\sigma|$ and $|\sigma^*|$ are admissible topologies for the duality $\langle \Lambda, \Lambda^* \rangle$. This implies: All order intervals in Λ and Λ^* are weakly compact.
Myopic topologies in the sense of Brown-Lewis, Econometrica 1981, can be considered in this framework.
Prop. 2: A topology τ on Λ is myopic iff $\hat{z}^{(n)} \xrightarrow{\tau} 0$, $\hat{z}^{(n)} = (0, \ldots, 0, z_{n+1}, z_{n+2}, \ldots)$. There exists a strongest (regular) topology τ_{SM} on Λ.
Under conditions which generalize those of Araujo, Econometrica 1985, existence of individual rational, Pareto-optimal states and of equilibria can be proved for exchange economies and economies with production. If E_n, F_n are separable Banach Lattices, an inverse result can be proved:
Thm 3: Individual rational Pareto optima (resp. equilibria) exist for all $\mathcal{E}$ with conditions A and P iff τ is a myopic topology. The myopic topologies $\tau \supset \sigma = \sigma(\Lambda, \Lambda^*)$ are admissible for the duality $\langle \Lambda, \Lambda^* \rangle$: $\tau_{MA} \supset \tau_{SM} \supset \tau_{|MA|} \supset \sigma$.
Ref.: Cherif-Deghdak-Florenzano, Econ. Theory, 1994, for exchange economies.

W. Trockel: **On Nonconvergence of the Bargaining Set (joint work with R. Anderson and L. Zhou)**

Mas-Colell (1989) had proved that in a continuum atomless exchange economy the Bargaining Set coincides with the set of Walras allocations. We give an example of a replica sequence for a base exchange economy with two commodities and two types of agents in which the measure of equal treatment Pareto optimal individually rational allocations not in the Bargaining set tends to zero with $n \to \infty$.

F. Vega–Redondo: Three Evolutionary Tales

Evolutionary models of equilibrium selection in games can be very dependent on the specific formulation of the selection dynamics. I explore three possibilities in this respect: imitation dynamics, best-response dynamics based on static expectations, best-response dynamics based on expectation updating. The implications are seen to be quite different in certain class of games. Specifically, they range from efficient equilibrium in the first context, risk-dominant equilibrium in the second, volatility access equilibria in the third. All this points to the need of allowing for a wide range of behavior in the "primitives" of the model, having the evolutionary model itself select endogenously the long-run pattern of behavior which will eventually prevail.

K. Vind: Uncertainty as a Foundation for Statistics

The simplest case of uncertainty is $(X, \mathcal{A}, P)$ where $\mathcal{A}$ is a σ-algebra on the arbitrary set X and P is a relation on $\mathcal{A}$. If P is independent, then there exist two measures $\lambda, \mu : \mathcal{A} \to \mathbb{R}_+$ such that $A \in P(A') \Leftrightarrow \lambda(A) - \lambda(A') > \mu(A \triangle A')$. Uncertainty on products and conditional uncertainty is introduced and a foundation for statistics is suggested as a two level uncertainty. First there is uncertainty on a parameter space, and each element in the parameter space is itself uncertainty on the outcome space.

J. Werner: Diversification and Equilibrium in Securities Markets

Diversification is a strategy of investing small fractions of wealth into large number of securities in order to reduce investment's risk. In this paper we study implications of diversification for prices of securities in an equilibrium. We consider markets with a countably infinite collection of securities. The main modeling issue is description of a set of portfolios from which an investor can choose. Our objective is to accommodate both finite portfolios, and portfolios with returns which arise from diversification of risk. The set of feasible portfolios (portfolio space) in our model is the space *ba* of all finitely additive measures (set functions) on the set of natural numbers (index set of securities). We study the existence of a securities market equilibrium, and provide a characterization of equilibrium prices of securities.

H. Wiesmeth: The Equivalence of Core and Lindahl Equilibria in an Economy with Semi-Public Goods (joint work with V. Vasil'ev and S. Weber)

This paper examines a model of an infinite production economy with a finite number of types of agents and semi-public goods, which are subjected to crowding and exclusion. The utility of an agent depends not only on the vector of public commodities produced by the coalition to which this agent belongs, but also on the mass of agents of his type, who are the members of this coalition. The main purpose of the paper is to derive necessary and sufficient conditions on the local degrees of congestion, which would guarantee the equivalence between the core and the set of equal treatment Lindahl equilibria. It is shown that this equivalence holds if and only if there are constant returns to group size for each type of agents. It implies that linearity of each agent's congestion function with respect to the mass of her own type is necessary for the core equivalence to hold.

N.C. Yannelis: **Learning in Differential Information Economies with Cooperative Solution Concepts: Cores and Values**

The core and the Shapley value are examined in an economy with differential information, i.e., an economy where each agent is characterized by a random utility function, a random initial endowment and a private information set.

Since the core and the value allow for cooperation, one would like to know how agents in a coalition share their private information.

It is shown that core and value allocations are incentive compatible in the sense that agents don't lie about their private information. Moreover, it is shown that agents with informational superiority are rewarded. This can happen even if an agent has zero initial endowments but his/her superior information allows him to make a Pareto improvement for the economy as a whole.

Finally, the two above solution concepts are examined in a dynamic framework. In particular, it is shown that if $\{\mathcal{E}^t : t = 1, 2, \ldots\}$ is a sequence of differential information economies and $\{x^t : t = 1, 2, \ldots\}$ is a sequence of core (value) allocations for $\mathcal{E}^t$, we can extract a subsequence which converges to a limit full information core (value) allocation for a one shot limit full information differential information economy. Moreover, it is shown that given a limit full information core (value) allocation for the one shot limit full information differential information economy, we can find a sequence of approximate or ε^t-core (ε^t-value) allocations which converges to the one shot limit full information differential information economy.

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