MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 10/1994

Mathematische Stochastik

6.3. bis 12.3.1994

Die Tagung fand unter der Leitung von M. Nussbaum (Berlin) und A. Wakolbinger (Frankfurt/Main) statt, es nahmen 54 Wissenschaftler aus 10 Ländern teil. Mit insgesamt 9 ursprünglich aus der früheren Sowjetunion kommenden Teilnehmern war diese Gruppe diesmal deutlich stärker vertreten als in früheren Jahren. Daß das Ziel der „Märztagung für Stochastik", die richtige Balance zwischen Spezialisierung und Gesamtschau zu finden, auch diesmal gut erreicht wurde, zeigten die regen Diskussionen, für die trotz des dichten Programms von 44 Vorträgen immer noch genügend Kraft und Interesse blieb. Die meisten Vortragenden haben nicht nur Inhalte vermittelt, die für die jeweiligen Spezialistenkollegen interessant waren, sondern haben auch zum Verständnis wichtiger Richtungen in der Stochastik über die Grenzen der engeren Fachgebiete beigetragen. Thematische Schwerpunkte waren:

- Teilchensysteme und deren Limiten
- Wechselwirkende Diffusionen und zufällige Medien
- Finanzmathematik
- Bilderkennung
- Entropiemethoden und Semiparametrik
- Kurvenschätzung und Wavelets
- Extremwerttheorie
- Statistik stochastischer Prozesse, insbesondere Bootstrap bei Zeitreihen.

Gefördert durch DFG Deutsche Forschungsgemeinschaft

ABSTRACTS

Large deviations and strong approximation

E. Berger
Göttingen

The talk is concerned with the interplay between large deviation expansions and strong approximation. It starts with certain improvements of the classical large deviation expansions obtained by Cramér (1938) and Richter (1957) - so-called "large deviation expansions with exponentially decreasing error terms." The emphasis here is on corresponding expansions for conditional distributions. It is indicated how these expansions can be utilized in order to systematically extend the Komlós-Major-Tusnády (1975, 1976) method of deriving almost sure invariance principles for empirical processes and partial sums of independent, identically distributed, real-valued random variables. The generalizations relate to partial sums of independent and not necessarily identically distributed random vectors, to partial sums with multi-dimensional time and to multivariate empirical processes and empirical processes indexed by sets and functions.

The effect of a hard wall on the free lattice field

E. Bolthausen
Universität Zürich

The free field $(X_i)_{i\in Z^d}$, $d \geq 3$, is defined to be the centered Gaussian random field whose covariances are given by the Green kernel of an ordinary symmetric random walk. We investigate the asymptotic behaviour of $P(A_n)$, where A_n is the event that $X_i \geq 0$ for all $i \in \{-n, n\}^d$, and the conditional law of (X_i) under A_n. The "random surface" (X_i) is repulled by this strong-wall-condition, and pusted to infinity (in the limit $n \to \infty$), an effect which is known under the name "entropy repulsion," and has first been observed by Lebowitz & Meas (1987). We give a quantitative description of the effect and prove

a) $\lim_{n\to\infty} n^{-d+2}(\log n)^{-1} \log P(A_n) = 2G(0)C(V)$,
where $G(0)$ is the Green kernel at 0, and $C(V)$ the Newtonian capacity of $V = [-1,1]^d$, and

b) $\lim_{n\to\infty} \mathcal{L}((X_i - 2\sqrt{G(0)\log n} \mid A_n) = \mathcal{L}(X_i)$

(joint work with J. D. Deuschel, TU Berlin & O. Zeitouni, Technion, Haifa)

A spectral domain bootstrap for time series

R. Dahlhaus
Universität Heidelberg

The properties of a spectral domain bootstrap for time series statistics based on the periodogram are discussed. The tapered periodogram is calculated and smoothed by a kernel estimate. The deviation of the periodogram from the kernel estimate is bootstrapped which leads to the bootstrap version of the periodogram. A correction is provided emulating the covariance structure of the periodogram. It is shown that this bootstrap is consistent for spectral mean estimates even in the case where the process is not Gaussian. Furthermore, we consider the bootstrap without correction for ratio statistics. Here we can prove that the bootstrap approximation is even better than the normal approximation. (joint work with D. Janas)

Data analysis and inference

L. Davis
Universität Essen

It was claimed that in traditional statistics (Bayes and Neyman-Pearson) there is a dichotomy between data analysis and inference. In particular the topologies used are different. Whilst data analysis uses a weak topology the whole of classical inference is based on density based concepts such as likelihood. One result of this is that many concepts of inference such as efficiency and sufficiency are pathologically discontinuous in the weak topology of data analysis. One further weakness of classical inference is that its definitions of optimality are based on a "truth" which is not visible at the level of the observations. It was argued that classical inference should be abandoned and replaced by an inference which explicitly recognizes the approximate nature of stochastic models. A concept of approximation was introduced based on the idea that a stochastic model is a good approximation to the data if the random samples generated by the model look very much like the actual sample. This idea was formalized using the concept of a data feature. Finally the problem of constructing functionals which are continuous in the weak topology of statistical inference was discussed and an example given.

Refined Pickands estimators for the extreme value index

H. Drees
University of Cologne

Consider n i.i.d. r.v.'s whose d.f. F belongs to the weak domain of attraction of an extreme value d.f. Starting with the well-known Pickands estimator for the index of the extreme value d.f., one can improve the asymptotic efficiency by mixing an increasing number of Pickands estimators. Under very general conditions on F one can prove asymptotic normality of such mixtures if the number K_n of upper order statistics involved in estimation does not grow too fast. If K_n converges at a faster rate then a bias occurs and the estimator is not $\sqrt{K_n}$-consistent. Hence, in applications one main problem is to choose K_n appropriately. The usual method to tackle this problem is to determine an asymptotically sequence K_n depending on certain parameters of F and then to choose K_n corresponding to estimates of these parameters. Unfortunately, some of these parameters are very difficult to estimate and thus the results obtained by this procedure are often unsatisfying. Therefore a bias correcting term is introduced such that the resulting estimator is $\sqrt{K_n}$-consistent in this case, too. In a simulation study this estimator proved to be much less sensitive to an inappropriate choice of K_n than previously defined estimators for the extreme value index.

Simultaneous confidence sets for functions of a scatter matrix

L. Dümbgen
Heidelberg

Two particular confidence sets for an unknown covariance matrix Σ imply simultaneous confidence sets for various functions of Σ, for instance correlations (including partial, multiple and canonical correlations), eigenvalues and eigenspaces. Both sets are based on the extreme eigenvalues of $\Sigma^{-1}S$, where S is an estimator for Σ, and turn out to be superior to likelihood ratio type confidence sets.

SuperBrownian motion and blowing of a semilinear p. d. e.

A. Etheridge
University of Edinburgh, UK

SuperBrownian motion is usually characterised in terms of the semilinear heat equation

$$\frac{\partial u}{\partial t} = \Delta u + u^2 \qquad u(0,x) = \theta\phi(x)$$

where $\theta \in I\!R_+$ and ϕ is a test function. For $\phi \leq 0$ solutions of this equation are well-balanced, but for $\phi \geq 0$ in general the solution will blow up in a finite time. By investigating the genealogy of SuperBrownian motion we obtain a family of representations of the coefficients of the series expansion of the solution (as a power series in θ). At one extreme of the family lies a path-valued Markov process. Using techniques from large deviations and some elementary analysis we are able to recover from the study of this Markov process results on the blowup of the positive solutions of the p. d. e.

Von Mises conditions and δ-neighborhoods of generalized Pareto distributions

M. Falk
Eichstätt

The aim of this talk is to advocate the use of certain neighborhoods of generalized Pareto distributions in extreme value models.

These δ-neighborhoods, which arise quite naturally from von Mises conditions, turn out to have characteristic properties concerning the rates of convergence of extreme observations.

Estimating measures of sensitivity of initial values to nonlinear stochastic systems with chaos

J. Fan
University of North Carolina at Chapel Hill

A significant feature of nonlinear systems is that a small perturbation in initial conditions could possibly lead to a considerable divergence of the state of the system in the

short or medium time evolution. In this talk, we define two measures of sensitivity to initial conditions in the nonlinear time series. The notions give some insight into the relationship between the Fisher information in statistical estimation and initial-value sensitivity in dynamical systems. By using the locally polynomial regression, we develop nonparametric estimate of a conditional density function, its square root and its partial derivatives. The proposed procedures are innovative and of interests in their own right. They are also used to estimate the sensitivity measures. The asymptotic normality of the proposed estimations have been proved. We also propose a simpler and intuitively appealing method for choosing the bandwidths. Two simulated examples are used as illustrations.

Maximum likelihood in mixture models

S. van de Geer
University of Leiden

Let Λ denote the set of all probability measures F on a measurable space $(\mathcal{Y}, \mathcal{B})$, and let for all $y \in \mathcal{Y}$, $k(.,y)$ be a density on another measurable space $(\mathcal{X}, \mathcal{A})$, w.r.t a σ-finite measure μ. Consider i.i.d. observations $X_1, \ldots, X_n$ with density $p_0 = \int k(.,y)\, dF_0(y)$. The maximum likelihood estimator $\hat{p}$ is defined by

$$\hat{p} = p_{\hat{F}} = \arg\max_{p \in \mathcal{P}} \sum_{i=1}^{n} \log p(X_i),$$

where $\mathcal{P} = \{p_F = \int k(.,y)\, dF(y) : \ F \in \Lambda\}$.

In the first part, we review consistency results for $\hat{p}$ and $\hat{F}$, and we obtain rates of convergence for $\hat{p}$. The rates are deduced from the entropy of $\mathcal{P}$. (E.g. entropy results for convex hulls can be used.)

In the second part, we study functionals $\theta(F) = \int a\, dF$, where $a : \ \mathcal{Y} \to I\!R$ is given. We establish asymptotic normality of $\theta(\hat{F})$ under certain conditions involving the convex combination $\hat{F}_\alpha = \alpha \hat{F} + (1 - \alpha) F_0$, with $0 \le \alpha < 1$. Here, we make use of the fact that $\hat{F}_\alpha \gg F_0$.

Van der Laan's identity (with applications to nonparametric missing data problems)

R. Gill
Utrecht

Suppose i.i.d. observations come from a distribution P in a class of possible models P, and we want to estimate a real functional $\kappa(P)$. In many semiparametric models, in particular models which can be loosely described as nonparametric missing data models, we have the following special features: $\mathcal{P}$ is convex and κ is linear. Let $\hat{P}$ be the nonparametric maximum likelihood estimator of P and $\hat{\kappa} = \kappa(\hat{P})$ the corresponding estimator of κ. Van der Laan (1993, PhD thesis, Utrecht) showed that the following identity then often holds: $\hat{\kappa} = (P_n - P)(IC_{\hat{P}})$ where P_n is the empirical distribution of the data, and $IC_{\hat{P}})$ denotes the optimal influence curve for estimation of κ, evaluated at the estimate $\hat{P}$. The identity sometimes allows quick and effective analysis of the large sample behaviour of $\hat{\kappa}$ through empirical process theory. We give examples, and show how the identity can be extended to allow two forms of non-linearity, arising from the presence of nuisance parameters and from the presence of a normalizing constant. This allows analysis of such models as the random-truncation model and the 2-dimensional Laslett line-segment model.

Functional limit theorems in $\mathbb{R}^k$ and number theory

F. Götze
Bielefeld

We show that there is a close connection between the optimal rate of convergence in the CLT in $\mathbb{R}^k$ for ellipsoids (as well as other convex regions with lower and upper bounds on the curvature) and the lattice point problem in number theory. For sums of i.i.d. random vectors in $\mathbb{R}^k$ with finite fourth moment the rate of convergence in the CLT for ellipsoids is $O(n^{-1})$ for $k \geq 5$ provided the covariance is nondegenerate and either
a) the random vectors have independent components or
b) the quadratic form splits into two independent parts of rank larger or equal to three each or
c) the r.v. takes values in a lattice $\Lambda \subset \mathbb{R}^k$, $k \geq 5$ such that the quadratic form Q of the ellipsoid satisfies $\Lambda Q \Lambda \in GL(k, Q)$.

Random walks in random environments on a strip

I. Goldsheid
Universität Bochum

We consider the random walk (RW) in random environment (RE) on a strip of width m. This RW can be characterised by the sequence of transition probabilities $p_{i,j}(x,y) =$ probability to jump from the point (x,y) to $(x+i, y+j)$. We suppose that $p_{i,j}(x,y) = 0$ if $|i| \geq 2$ or, $y+j > m$, or $y+j < 1$. Here $-\infty < x < \infty$, $-1 \leq y \leq m$. The matrices $(p_{i,j}(x,y))$ are supposed to be i.i.d. random variables, $|x| < \infty$. The consideration of a RW in a one-dimensional lattice can be easily reduced to the previous model. We prove the analogue of Solomon-Key's condition for recurrency of the RW and establish that the recurrent RW exhibits sinai behaviour, which means that if ξ_t is the x-coordinate of the random walk at the moment t, $\xi_0 = 0$, then $\xi_t/(\ln t)^2$ converges in distribution.

On the second order minimax estimation of distribution functions

Y. K. Golubev and B. Y. Levit
Moscow and Utrecht

We describe the second order asymptotically minimax improvement of the L_2-risk of the empirical distribution function, against different smoothness properties of the underlying distribution function in $\mathbb{R}^1$. Our main results cover distribution functions admitting generalized derivatives up to order $m > 1$ as well as distribution functions admitting analytical continuation in the complex domain.

While the first class is quite familiar in nonparametric estimation, the latter, still being sufficiently broad, exhibits some essential features typical for the finite-dimensional parametric models, thus becoming quite attractive for possible statistical applications.

The second order minimax estimators with respect to ellipsoids in these classes turn out to be linear smoothers of the empirical distribution function with specified kernels.

Bahadur efficiency and the LAN expansion

P. E. Greenwood
University of British Columbia, Vancouver

Asymptotic efficiency is studied in the sense of Bahadur under local asymptotic normality of a statistical experiment. More exactly, we find a condition on the remainder in the LAN expansion under which a locally uniform Bahadur lower bound for the error in estimation can be proved in the same general context as the locally uniform Hajek-Le Cam asymptotic minimax efficiency bound. Under somewhat more restrictive conditions in terms of the LAN expansion a general formulation of the Bahadur lower bound itself is possible. Observation schemes to which the results apply include continuous time Gaussian examples and both i.i.d. and independent but not identically distributed examples. The efficiency bound is attained but under conditions which we suspect are too restrictive. (joint work with I. Ibragimov)

Qualitative properties of systems with infinitely many interacting components

A. Greven
Humboldt-Universität, Berlin

The purpose of this work is to explore the connection between multiple space-time scale behaviour for block averages and phase transitions, respectively formation of clusters, in infinite systems with locally interacting components. The essential object is the associated Markov chain (called interaction chain), which describes the joint distribution of the block averages at different time scales. A fixed-point and stability property of a particular dynamical system under a renormalization procedure is used to explain this pattern of cluster formation and the fact that the longterm behaviour is universal in entire classes of evolutions.
(joint work with D. Dawson, Ottawa)

On an integral equation, connected with interval censoring

P. Groeneboom
Delft

For the interval censoring problem, case 2, we consider Hellinger differentiable functionals. The (asymptotic) information lower bound for such functionals can be represented as the integral over a function ϕ. This function ϕ is the solution of an integral equation with a singular kernel and can be interpreted as the canonical gradient of the differentiable functional on the set of probability measures on the observation space. Properties of ϕ are discussed and it is shown that, under some condition on the underlying distributions, the maximum likelihood estimator leads to an estimator which attains the information lower bound.

On estimation of tail parameters for stable processes and related point process models

R. Höpfner
Universität Freiburg im Breisgau and Université Paris VI

We deal with joint estimation of the index α and a weight parameter ξ in a statistical model where one observes all jumps of a stable increasing process with height not less than Y_n up to time T_n, or more generally all points of a certain Poisson random measure μ on $(0,\infty) \times (0,\infty)$ in a window $[0, T_n] \times [Y_n, \infty)$.

For different type of asymptotic behaviour of T_n, Y_n as $n \to \infty$, we investigate local asymptotic normality (LAN) of the model at a true parameter value (α_0, ξ_0), and properties of maximum likelihood estimators at this point.

Whereas the maximum likelihood estimator for the pair (α, ξ) is seen to be efficient if the random measure μ is observed in windows where Y_n is deterministic and independent of n, it exhibits curious degeneracies as soon as Y_n increases to ∞. In this case, one-dimensional submodels through the true parameter value (α_0, ξ_0) allow for parameter estimation at essentially higher speed of convergence; at the same time, there exist "least favorable" sequences of one-dimensional submodels through (α_0, ξ_0) in which the maximum likelihood estimator has an optimality property.

For Y_n increasing to ∞, a similar phenomenon occurs in tail parameter estimation from point processes with intensity $\theta_G(ds, dx) = dsG(dx)$, $G \in \mathcal{G}_0$, where $\mathcal{G}_0$ is a certain class of measures with tails such that $G([y,\infty)) \sim \xi(G) y^{-\alpha(G)}$ as $y \to \infty$.

Superprocesses: single-level and multi-level

K. J. Hochberg
Bar-Ilan University, Israel

Superprocesses are measure-valued stochastic processes that arise as diffusion approximations to infinite particle branching diffusion systems. We give an introduction to the subject from both probabilistic and analytic viewpoints, concentrating on fixed-time and long-term properties of three examples that have been the object of much recent attention:

(i) Dawson-Watanabe (α, d, β_1) Superprocesses – the high density diffusion limit to an infinite system of index α in $\mathbb{R}^d$ and branching with parameter β_1 at random times. This process has values in $\mathcal{M}(\mathbb{R}^d)$, the space of bounded measures in $\mathbb{R}^d$.

(ii) $(\alpha, d, \beta_1, \beta_2)$ Super-2 Process – the high density limit of a hierarchically structural 2-level population in which, in addition to the diffusion and branching of individuals as in (i), groups of particles undergo another, independent, simultaneous branching with parameter β_2. This process takes values in $\mathcal{M}^d(\mathbb{R}^d) = \mathcal{M}(\mathcal{M}(\mathbb{R}^d))$.

(iii) Flewing-Viot Process – the diffusion approximation to a certain population genetic model. This process takes values in $\mathcal{M}_p(\mathbb{R}^d)$, the space of probability measures on $\mathbb{R}^d$.

Renormalization of hierarchically interacting diffusions

F. den Hollander
University of Utrecht

Dawson and Greven (Probab. Theory Relat. Fields 96, 1993, 435-473) have shown that a *multiple space-time scaling* analysis of an infinite system of hierarchically interacting diffusions taking values in $[0,1]$ can be described in terms of the iterates of a *renormalization* transformation acting on the diffusion function. We show that this transformation has a unique globally attracting stable orbit. We also obtain properties and rates of convergence to this orbit in an appropriate norm. The results imply that the scaling limit of the system has *universal* behavior independent of model parameters. (joint work with J.-B. Baillon, Ph. Clément and A. Greven)

On the normal form problem for random dynamical systems

P. Imkeller
Université de Franche-Comté, Besançon

The random coordinates ("normal coordinates") of a system of linear stochastic differential equations with respect to which it decomposes into a number of stochastic differential equations are found by choosing the initial vectors in the random Oscledets spaces. As a rule, this choice leads to non-adapted vectors. So one faces the problem of having to deal with a stochastic differential equation with non-adapted initial conditions. This problem is solved by an investigation of the regularity properties of parametrized Stratonovitch integrals.

Random dynamical systems with ordered state space

H. G. Kellerer
Universität München

Random dynamical systems $(X_u)_{u\geq 0}$ on an ordered topological space with minimal element (e.g. $\mathbb{R}^d_+$), given by i.i.d. isotone mappings F_1, $F_2, \ldots$, can be classified as Markov chains with a countable state space: according to transience, null recurrence and positive recurrence. In the recurrent case, existence and uniqueness of an invariant measure μ can be derived as well as mean and pointwise (ratio) ergodic theorems. In the positive recurrent case (i.e. μ finite), reversing the order of F_1, $F_2, \ldots$ yields almost sure convergence to a variable with the stationary distribution; moreover, the mean passage times corresponding to "ladder indices" turn out to be finite.

Adaptivity in density estimation via wavelets

G. Kerkyacharian and D. Picard
Universités d'Amiens et de Paris VII, France

Let $X_1, \ldots, X_n$ be n iid observations of a density f. A density estimator f^* will be called adaptive for a class $(C(\alpha), \alpha \in A)$ of balls of functional spaces, and the L_p risk if,

$$\forall \alpha, \quad \forall f \in C(\alpha), \qquad E\|f^* - f\|_p^p \leq c \inf_{f \in C(\alpha)} \sup_{\hat{f}\,\text{estim.}} E_f\|\hat{f} - f\|_p^p \qquad (*)$$

Let ϕ be the scaling function of a multiresolution analysis of regularity r_0, ψ the associated wavelet.

Consider the following estimators:

$$- \quad \hat{f}^{(1)} = \sum_k \hat{\alpha}_{0k}\phi_{0k} + \sum_{j=0}^{j(n)} \sum_k \tilde{\hat{\beta}}_{jk}\psi_{jk},$$

$$\text{where} \quad \hat{\alpha}_{0k} = \frac{1}{n}\sum_{i=1}^{n} \phi_{0k}(X_i), \; 2^{j(n)} \asymp n,$$

$$\hat{\beta}_{jk} = \frac{1}{n}\sum_{i=1}^{n} \psi_{jk}(X_i), \; \tilde{\hat{\beta}}_{jk} = \hat{\beta}_{jk} \text{ if } |\hat{\beta}_{jk}| > c\sqrt{\frac{\log n}{n}}, \; 0 \text{ otherwise}$$

$$- \quad \hat{f}^{(2)} = \sum_k \hat{\alpha}_{0k}\phi_{0k} + \sum_{j=0}^{j(n)} \hat{\lambda}_j^{1,p} \sum_k \hat{\beta}_{jk}\psi_{jk},$$

$$- \quad \hat{f}^{(3)} = \sum_k \hat{\alpha}_{0k}\phi_{0k} + \sum_{j=0}^{j(n)} \hat{\lambda}_j^{2,p} \sum_k \hat{\beta}_{jk}\psi_{jk},$$

$$\hat{\lambda}_j^{1,p} = 1 \text{ if } \sum |\hat{\beta}_{jk}|^p > \frac{2^j}{n^{p/2}}, \; 0 \text{ otherwise},$$

$$\hat{\lambda}_j^{2,p} = \frac{(\sum |\hat{\beta}_{jk}|^p)^{\frac{1}{p-1}}}{(2^j/n^{p/2})^{\frac{1}{p-1}} + (\sum |\hat{\beta}_{jk}|^p)^{\frac{1}{p-1}}} \qquad (p > 1)$$

If f is compactly supported and $\mathcal{F}_{spq}(M)$ is a ball of radius M, around 0, in the Besov space B_{spq}, then $\hat{f}^{(2)}$, $\hat{f}^{(3)}$ are adaptive for the class $\{\mathcal{F}_{spq}(M),\ 1/p < s < r_0,\ 1 \leq q \leq +\infty,\ 0 < M < +\infty\}$ and the L_p risk (K. P. Tribouley).

$\hat{f}^{(1)}$ is almost adaptive ($(*)$ is attained up to a log term) for the class $\{\mathcal{F}_{spq}(M),\ 1/p < s < r_0,\ 1 \leq q \leq +\infty,\ 0 < M \leq M_0, 1 \leq p \leq p'\}$ and the $L_{p'}$ risk (Donoho, Johnstone, Kerkyacharian, Picard).

Generating random trees from point processes

G. Kersting
Universität Frankfurt/Main

In this talk we discuss two ways of constructing random branching trees. One is the familiar representation via random walk excursions ("depth-first search"). Another possibility is to construct certain branching branching trees from Poisson point processes in the upper positive quadrant of the plane. In a certain sense both approaches are complementary to each other. Depending on whether the second moment of the offspring distribution is finite or infinite, either the random walk or the Poisson process construction is more useful. We explain this in deriving the asymptotic contour of such branching trees. The limit is a Brownian excursion, without or "with built-in barriers."

On stability index for stochastic differential equations

R. Khasminskii
Wayne State University, Detroit

Stability index was introduced by P. Baxendale in 1987 for linear SDE. It is proved that this index has some robustness with respect to small nonlinear perturbations. (joint work with L. Arnold)

Image reconstruction theory: Dynamical images

A. Korostelev
Institute for System Analysis, Moscow

The talk is after the joint paper with M. I. Freidlin "Image processing for growing domains: change-point problems for domain's area" (Preprint TR 94-01, University of Maryland, College Park, Jan. 1994).

The image models are considered for growing domains in Gaussian white noise. The "jump" and "kink" change-point problems are studied for domain's area. When noise becomes small the asymptotics of the minimax rates of convergence is found for Markov stopping rules in different classes of domain evolutions.

The peculiarity of the problem is that the domain's shape is unknown and plays the role of an infinite-dimensional "nuisance parameter" which influences strongly upon the rates of convergence and the efficient stopping rules.

The blockwise bootstrap for general parameters of a time series

H. R. Künsch
ETH Zürich

We consider a stationary time series $(X_t)_{t\in Z}$ with distribution F^∞ and marginal distributions F^m. Let T_n be an estimator of a general parameter $T(F^\infty)$ of the form

$$T_n = T^m\left((n-m+1)^{-1}\sum_{t=0}^{n-m}\Delta(\tau_t(X_1,\ldots,X_m))\right).$$

Here Δ denotes a point mass, τ_t is the shift operator, $(T^m(F^m))$ is a sequence approximating $T(F^\infty)$ and $m = m(n) \to \infty$ slowly. Main examples are various spectral estimators. In order to bootstrap T_n we select blocks of shift indices at random:

$$T_n^* = T^m\left((n-m+1)^{-1}\sum_{j=1}^{b}\sum_{u=1}^{l}\Delta(\tau_{s_j+u}(X_1,\ldots,X_m))\right).$$

where l is the block length, $b=(n-m+1)/l$ and $S_1,\ldots,S_b$ are i.i.d. uniform on $\{0,\ldots,n-m-l+1\}$. The proof of consistency of this bootstrap depends on properties of the linearization of T_n.

Minimum distance approach in parametric estimation for diffusion-type observations

Yu. Kutoyants
Le Mans

We consider several estimators of the parameter of diffusion type processes based on the "model fitting" approach - minimizing some functionals of the observations providing the fitting of the parametric model to the observed process. These estimates called minimum distance estimates are in regular cases consistent, asymptotically normal and in certain minimax sense local asymptotic minimax. The asymptotic corresponds to "small noise" and $T \to \infty$ limits.

Local adaptivity to inhomogeneous smoothness. Resolution level

O. V. Lepskii and V. G. Spokoiny
Moscow

We consider the problem of nonparametric estimation of functions of inhomogeneous smoothness. Our goal is to define the notion of local smoothness of a function $f(.)$, to evaluate the optimal rate of convergence of estimators (depending on this local smoothness) and to construct an asymptotically efficient estimator.

We define the local smoothness of a function at a given point as a smoothness of this function being restricted on a small interval (of length δ) around this point. The less is this value δ the more accurate is our local resolution analysis. But the natural question arises here. How small can this resolution level be chosen and how to construct the corresponding locally adaptive procedure. We show that the answer on this question strongly depends on the upper considered smoothness β^*, what we wish to attain, and also depends on the level of noise ε.

The main result claims that one cannot take the level of "locality" δ_ε less (in order) than $\delta_\varepsilon^* = \varepsilon^{\frac{2}{2\beta^*+1}}$. This value is called the resolution level. As more is the accuracy of our local procedure described by the upper smoothness β^* as more rough is the local resolution analysis of this procedure.

Estimation of decreasing functions

M. Low
University of Pennsylvania

Estimators of decreasing functions were constructed which are simultaneously within a constant factor being minimax over Lipschitz classes $f : |f(x) - f(y)| \leq M|x - y|^\alpha$ for $M > 1$ and $0 \leq \alpha \leq 1$ when it is known that f is decreasing. This result is for estimating the function at a particular point with loss $l(f, a) = |f(x) - a|^p$. This result is then applied to yield minimax rates of convergence under L_p loss over the class of all decreasing functions. Throughout the talk we assume that we observe a white noise model $dY(t) = f(t) + \sigma dW(t)$.

Least squares splines with total variation penalty terms

E. Mammen
Berlin

A new procedure is proposed which works for the estimation of functions with spatially varying smoothness. This approach is based on penalized least squares with total variation of a derivative as penalty term. It is shown that this estimate is based on thresholding of empirical spline coefficients and data adaptive location of knot points. Asymptotic properties (rates of convergence and local pointwise asymptotic distribution theory) are discussed. (joint work with S. van de Geer)

Edge preserving smoothers for image processing: An M-estimation approach

J. S. Marron
University of North Carolina

Classical smoothers have limited usefulness in image processing, because sharp "edges" tend to be blurred. There is a literature on edge preserving smoothers, but these all require moderately large "smooth stretches." Here we discuss an approach to this problem called "sigma filtering," and propose an improvement, which is based on running M-estimation. Both computational and theoretical aspects are developed. For image processing, the methods have a niche in between the popular methods of mathematical morphology, and Bayes-Markov random field analysis. (joint work with C. K. Chu, I. Glad, F. Godtliebsen)

Approximation theory of Monte Carlo methods

P. Mathé
IAAS, Berlin

We investigate approximative properties of *vector valued* Monte Carlo methods, i.e., random numerical methods are used to approximate given *deterministic* problems. After reviewing some of the basic concepts within this field we turn to the question, for what kind of problems (operators between Banach spaces) there exist *unbiased* estimates obtained from finite rank Monte Carlo methods. We provide necessary and sufficient conditions for special situations.

Single index models with mixed discrete-continuous explanatory variables

M. Müller
Humboldt University, Berlin

The talk presents possibilities to estimate single index models with mixed discrete-continuous explanatory variables in the special case of one binary discrete variable. This model leads to a two-sample semiparametric problem with a shift parameter related to the discrete variable.

The talk gives the results of a joint work with A. Korostelev and applications to data. The minimal Fisher information is found in a class of nonparametric link functions. Two root-n consistent estimators of the shift parameter are proposed basing on the comparison of the link functions in "vertical" and "horizontal" direction. The limiting variances of these estimators show that properly weighted "horizontal" shifts guarantee asymptotical efficiency. Some simulations show the applicability of these methods.

Wavelet methods in non-Gaussian models

M. H. Neumann
IAAS, Berlin

At present it is known in the framework of i.i.d. Gaussian regression and density estimation that nonlinear wavelet methods outperform traditional linear methods in situations where the unknown function has a considerable inhomogeneity in its smoothness properties. This was impressively demonstrated by minimax results in Besov spaces due to Donoho, Johnstone, Kerkyacharian and Picard. The aim of this work is to show how these results can be transferred to a wide variety of other estimation problems.

In the regression case with independent, not necessarily identical error distributions it was shown in a joint work with V. Spokoiny that the minimax risk under Besov constraints can be bounded from below by the minimax risk in a similar Gaussian model. Further we show that, in spite of the nonlinear structure of these estimation rules, hard and soft thresholded wavelet estimators are asymptotically L_2-risk equivalent to the same estimators in a corresponding Gaussian model. This equivalence is based on large deviation results for the empirical wavelet coefficients, which state their asymptotic Gaussianity in an appropriate way. Hence, properly thresholded wavelet estimators keep all their appealing properties that are known from the case of Gaussian regression.

Wavelet methods as highly spatially adaptive methods are an interesting alternative to usual linear methods in spectral density estimation, since the recognition of peaks is of great importance in this context. We define empirical wavelet coefficients on the basis of the ordinary and the tapered periodogram and show via cumulant techniques that they satisfy again large deviation properties similar to the regression case. Having this, we can immediately conclude the risk equivalence for monotone estimators to the Gaussian case, which means that thresholded wavelet methods keep all their advantages in the framework of spectral density estimation, too.

In view of these two examples it is clear that wavelet methods can be applied in the same way to other nonparametric curve estimation problems, as e.g. regression or density estimation with dependent observations under appropriate mixing conditions.

Large deviations via subadditivity

P. Ney
University of Wisconsin, Madison

Let $\{X_n\}_1^\infty$ be an irreducible Markov chain (aperiodic) on a general state space $(S, \mathcal{S})$, $f : S \to E$ = a separable Banach space, f bounded. Let $S_n = \sum_1^n f(X_i)$, and assume that $\{X_n\}$ is irreducible and aperiodic. Then $\{X_n\}$ admits a regeneration structure with regeneration measure ϑ and a "small" set C. Let $a_n = \partial P_\vartheta \left\{ \frac{S_n}{n} \in B,\ X_n \in C \right\}$ where B is a ball and $\delta > 0$ sufficiently small. One shows that $\{-\log a_n\}$ is sub-additive, and hence that $\lim_{n\to\infty} \frac{1}{n} \log P_\vartheta \left\{ \frac{S_n}{n} \in B,\ X_n \in C \right\}$ exists. From this fact one goes on to prove that the sequence of measures $P_\vartheta\left\{\frac{S_n}{n} \in B,\ X_n \in C\right\}$ satisfies the large deviation principle with rate $I(x) = \Lambda^*(x)$, where $\Lambda(\alpha) = \lim \frac{1}{n} \log E_\vartheta\left[e^{(\alpha, S_n)}; X_n \in C\right]$, $\alpha \in E^*$. This is turn leads to a simple proof of the Donsker, Varadhan theorem, i.e. the LDP for the empirical measure of a Markov chain.

Regularity properties of interacting many-particle systems

K. Oelschläger
Universität Heidelberg

We demonstrate that for certain interacting many-particle systems the empirical processes $\underline{X}_N$ belonging to the system with N particles exhibit a similar regularity behaviour as the solutions of the respective limit dynamics. For that purpose we deduce upper bounds, which hold uniformly in N for the Sobolev norms $\|\underline{X}_N(t) * \phi_N\|_{(\vartheta)}$, $t \geq 0$, $\vartheta = 0, 1, 2, \ldots$, where ϕ_N is a mollifier tending to δ_0 as $N \to \infty$. Our examples include systems of weakly interacting diffusions and Newtonian systems with long-range interaction. In these cases the limit dynamics is given by a parabolic equation, or a strongly hyperbolic system of 1*st* order partial differential equations, respectively.

Residual life functionals

R.-D. Reiss
Siegen

The mean residual life (MRL) function is the mean of the conditional distribution $P(X - t \in . \mid X > t)$ as a function in t; thus, it is the expected remaining life given survival at age t. First, it is shown that the empirical MRL function is an inaccurate estimator of the Pareto MRL function when the shape parameter is close to 1. Therefore, other parameters of the conditional distribution such as the median and certain trimmed means are investigated. The median and the trimmed mean residual life functions possess again the property of being linear in t and can thus be used for a visual model selection. We will prove the asymptotic normality of these statistics at increasing ages t within a framework of residual life functionals. (joint work with H. Drees)

Asymptotic turnpike results in optimal portfolio theory

L. C. G. Rogers
University of Bath

In the simplest optimal investment problems in continuous time, the aim is to optimise $EU(X_\tau)$, where U is a strictly increasing, strictly concave utility function, X is the wealth process obtained by investing in a single risky asset and a riskless bond in proportion to be decided by the investor. Simple closed-form solutions are rare, the main examples being where U is a power-law, and the risky asset is a log-Brownian motion. The question that arises is that if U_0 and U_1 are two utilities which are asymptotically equivalent in some sense, do the optimal policies for the two utilities look similar as $T \to \infty$? We provide some fairly complete answers to this question. (joint work with Kerry Back, Phil Dybvig)

On some optimization problems from financial mathematics

M. Schweizer
Universität Göttingen

Let X be a semimartingale and Θ the space of all predictable X-integrable processes ϑ such that $G(\vartheta) = \int \vartheta \, dX$ is in the space $\mathcal{S}^2$ of semimartingales. For a fixed time horizon $T > 0$ and a given $\mathcal{F}_T$-measurable random variable $H \in \mathcal{L}^2$, we consider the problem of minimizing $\|H - c - G_T(\vartheta)\|_{\mathcal{L}^2}$ over all $(c, \vartheta) \in I\!R \times \Theta$. Under the assumption that X has the form $X = X_0 + M + \int \alpha \, d\langle M \rangle$, we show that $G_T(\Theta)$ is closed in $\mathcal{L}^2$ if the mean-variance tradeoff process $\widehat{K} = \int \alpha^2 \, d\langle M \rangle$ is bounded. In the case where $\widehat{K}$ is deterministic, the optimal $\mathcal{F}$ can be given fairly explicitly in feedback form. The optimal constant $\tilde{c}$ can be written as $\tilde{c} = \tilde{E}[H]$, where $\tilde{P}$ is the so-called variance-optimal signed martingale measure for X, i.e., $\frac{d\tilde{P}}{dP}$ minimizes $\|D\|_{\mathcal{L}^2(P)}$ over all $D \in (G_T(\Theta))^{\perp}$ with $E[D] = 1$. In general, $\tilde{P}$ is a signed measure, but if X has continuous trajectories, we can show that $\tilde{P}$ is actually a measure.

On the term structure of interest rates

W. Stummer
University of Bath

In this talk we take a real data set of zero coupon bond prices. We fit/apply two well-known stochastic process models - the Vasicek model resp. the Cox/Ingersoll/Ross model - and compare them. The comparison includes daywise fits versus "overall-fits," different parameter-constraints, MLE with transition densities and degeneracies. It is shown that even over a pricing period of two years, one can perform a very good "overall-fit." Diffe-rent parameters lead to different qualitative theoretical properties (like positivity) of the underlying interest rate process. However, it is shown that the corresponding parameter constraints all lead to approximately the same regression results; there is also not much change if one incorporates transition densities. Finally, one particular parameter point is presented which

- is "completely" different to the optimal parameter-point
- corresponds to the case where the spot rate process is almost deterministic
- nevertheless leads to a RSSQ close to the optimal one. (joint work with L.C.G. Rogers, Bath)

M-estimation and empirical processes

A. W. van der Vaart
Amsterdam

Empirical process methods are useful both for obtaining rates of convergence and establishing asymptotic distributions of M-estimators. Two types of maximal inequalities for the supremum of the empirical process were presented. The first type is driven by the envelope function of the indexing class of functions. We presented a new application of this inequality to establishing a root-n rate for the finite dimensional parameter in semi-parametric models. The second type of maximal inequality depends stronger on the entropy of the indexing class. We reviewed its application to obtaining rates of convergence for maximum likelihood estimators for densities. Derivations of limit distributions of estimators of infinitely dimensional parameters are usually based on equations. Then Donsker classes of functions come in to take care of remainder terms. This was illustrated with a particular semi-parametric model.

Estimation in Gibbsian texture models

G. Winkler
LMU, München

Markov random field models in texture analysis (and other fields of imaging) depend on high-dimensional parameters characterizing different types of texture. These parameters critically determine the ability of algorithms to segment and label. Similarly, neural networks are tuned for specific applications by the appropriate choice of parameters, called synaptic weights.

'Learning' parameters frequently amounts to maximum likelihood estimation. Conventional computation of ML estimates is barred by the size of spaces: The cardinality of sample spaces typically is of order 10^{100000} and the the dimension of parameter spaces can be of order 10^4 or even more.

There are two possible strategies to overcome these difficulties: Either, maximum likelihood is replaced by computationally feasible variants like the pseudolikelihood method in imaging. Or, classical optimization techniques are replaced by new heuristics. Recently, stochastic gradient algorithms received considerable interest. Their foundations were laid by Métevier and Priouret around 1987.

We shortly review these techniques and give indications of proofs. These and related topics are treated in some detail in the author's monograph on Bayesian Image Analysis and Dynamic Monte Carlo Methods (Springer Verlag, to appear autumn 1994).

Estimating L^1 error of the kernel estimator based on Markov samplers

Bin Yu
University of California at Berkeley

In many Markov chain Monte Carlo problems, the target density function is known up to a normalization constant. In this talk, we take advantage of this knowledge to facilitate the convergence diagnostic of a Markov sampler by estimating the L^1 error of a kernel estimator. Theoretical properties of the estimated L^1 error are given and two sequential plots are proposed for convergence monitoring purpose. Different Markov samplers aiming at the same target density can be compared through these plots. A simulation study is also given.

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Gefördert durch
DFG Deutsche Forschungsgemeinschaft