MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Quantenstochastik und Quantenfeldtheorie

27.3. — 2.4.1994

Die Tagung fand unter der Leitung von L. Accardi (Roma) und W. von Waldenfels (Heidelberg) statt. Im Mittelpunkt des Interesses standen Aspekte der Quantenstochastik und der Quantenfeldtheorie, unter besonderer Betonung von Fragestellungen, die auf Gemeinsamkeiten abzielen.

Abstracts

L. Accardi:

The Stochastic Limit of Quantum Field Theory

In the last 15 years new mathematical techniques, born in the field of quantum probability, have begun to be applied to physical models. The idea that a more refined analysis of the basic principles of quantum theory should lead to a stochastic generalization of the Schrödinger equation, was substantiated in a mathematical theory of quantum noise (1986). In this theory the stochasticity is not jet by hands, as in the classical Langevin equation but, using some specific properties of quantum systems, it is deduced from the system itself. In this sense one might say that 'quantum systems are their own noises'.

In this theory one shows how quantum stochastic differential equations arise naturally in physics. But given a specific quantum system, how to determine the stochastic equation which describes it?

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The set of techniques developed to answer this question is now called 'the stochastic limit of quantum field theory'. The basic idea is to perform a detailed analysis of the long time cummulative behaviour of the effects of the interaction. The main results, obtained in the last years, are reviewed.

The collective quantum fields converge to quantum Brownian motions (1987). This phenomenon has some 'universal features' (such as independence of the initial state), and both Schrödinger and Heisenberg evolution converge to stochastic differential equations (1988). The universality includes non-linear interactions (1989). Also quantum Poisson processes emerge as stochastic limits of quantum Hamiltonian systems (1990-1991). The quantum noise emerging from quantum elektrodynamics without dipole approximation generalizes the so called 'free noise', emerging from Voiculescus 'free probability theory', and is neither Brownian nor Poisson (1991). The corresponding quantum stochastic equations require a 'quantum stochastic calculus on Hilbert moduli' developed by Y. G. Lu (1992). The stochastic limit of quantum field theory can be considered as a new semi-classical approximation, leading not to the usual smooth trajectories, but to the irregular trajectories of Brownian motion (1993). The stochastic limit of quantum chromodynamics can be achieved, even if with cut-off cut in a fixed gauge (1994).

The program for the next steps is: removal of cut-offs, gauge invariant stochastic limit of QCD, introduction of space (not only time) scaling, relativistic stochastic limit.

R. Alicki:

Quantum Dynamical Entropy II

The new definition of the quantum dynamical entropy based on the notion of operational partition of unity is applied to the quasi-free fermionic system at the invariant quasi-free state.

C. D'Antoni:

Aspects of AQF

Algebraic QFT provides a suitable framework to discuss structural properties of QFT in a model independent way. Two aspects are discussed in detail. The nuclearity property describing the 'essential' finite dimensionality of the phase space of local algebra. And the modular structure exploiting on the contrary, the (even locally) essential infinite dimensionality of quantum fields. Interplay between the two aspects is studied with emphasis on implementation of local and space-time symmetries and geometric interpretation of modular automorphisms.

I. Aref'eva:

Anisotropic Asymptotics in QFT

I have discussed anisotropic asymptotics in quantum field theory. This problem is related with high-energy behaviour of scattering amplitudes with small transferred momentum and with stochastic phase in QFT.

V. Belavkin:

We give an explicit stochastic Hamiltonian model of discontinuous unitary evolution for a system of quantum particles interacting with 'bubbels' which admit a continual counting observation. It allows to watch and follow with an unsharp position of a system like in cloud chamber by a sequential filtering of the spontaneous scatterings of the bubbles under the observation. This model leads to the continuous reduction and spontaneous localization theory as a result of the quantum filtering, i.e. the conditioning of the a priori quantum state by the measurement data. We show that in the case of the indistinguishable particles the Boltzmann type reduction equation rises. It coincides with the non-stochastic Schrödinger equation only in a mean field approximation while the diffusive type is appearing as the central limit of this equation.

G. Efimov:

The talk is devoted to the oscillator representation (OR) method and its application to research into the ground states of various quantum field theory and quantum mechanics models. The OR method is based on the conception of the representation of the total Hamiltonian in the *correct* form, i.e.

$$H = H_0 + H_I = \sum_i \omega_i a_i^\dagger a_i + g : H_I(a_i, a_i^\dagger):$$

where the free Hamiltonian contains quadratic operators in the normal form and the interaction Hamiltonian should be written in the normal form and $:H(\varphi): = O(\varphi^3)$ for $\varphi \to 0$. All possible phases of the system can be obtained by the canonical transformation under the requirement that the total Hamiltionian has the correct form in each representation.

The method provides to investigate in QFT such non-perturbative phenomena as the strong coupling regime, the phase structure and phase transitions at arbitrary coupling constants and temperature.

OR is a kind of generalization of the variational technique , but in contrast to variational methods, it is applicable to QFT models with ultraviolet divergences in the highest perturbation orders and to theories with non-hermitian and complex actions (stochastic and dissipative processes). OR is characterized by a high accuracy of the lowest approximation and gives a regular prescription for calculations of the highest order corrections to the lowest approximation.

M. Fannes:

Quantum Dynamical Entropy I

In colaboration with R. Alicki, a new elementary construction of a dynamical entropy for quantum dynamical systems is proposed. This definition is shown to extend the Kolmogorov-Sinai invariant. Furthermore, the entropy of the shift on a quantum spin chain with arbitrary invariant reference state is computed.

G. W. Ford:

Weak Localizations in the Spin-Boson problem

Consider a single two-level (spin) system coupled to a boson field, with Hamiltonian

$$H = H_B + H_S, \quad H_S = \tfrac{1}{2}\hbar\omega_0(1+\sigma_x), \quad H_B = \sum_j \hbar\omega_j(a_j + c_j\sigma_z)^\dagger(a_j + c_j\sigma_z)$$

with the Pauli matrices $\sigma_x, \sigma_y, \sigma_z$ and the boson operators a_j. On special interest is the singular case where for a smooth test function $F(\omega)$

$$\sum_j |c_j|^2 F(\omega_j) \longrightarrow \frac{\alpha}{2}\int_0^\omega d\omega \frac{e^{-\frac{\omega}{\omega_c}}}{\omega} F(\omega).$$

Here α is a dimensionless coupling constant and ω_c is a cut-off frequency.

Suppose now the system is initially in a state consisting of the vacuum state Φ_0 and an eigenstate of σ_z with eigenvalue 1. The corresponding density matrix is $\rho_0 = \Phi_0\Phi_0^\dagger \frac{1+\sigma_z}{2}$. The system is therefore in a superposition of eigenstates of H_S. The time evolution $\rho(t)$ is a solution of the von Neumenn equation

$$i\hbar\frac{\partial\rho}{\partial t} = [H,\rho] \quad , \quad \rho(0) = \rho_0.$$

The approach to equilibrium is described by the after-effect function

$$P_z(t) = \mathrm{tr}\left(\rho(t)\frac{1+\sigma_z}{2}\right) \equiv e^{-\theta(t)}.$$

The expansion $\theta(t) = \theta^{(1)}(t) + \theta^{(2)}(t) + \cdots$ in powers of ω_0 has been calculated by master equation methods (Ford, O'Connell, Lewis, Raggio). $\theta^{(n)}$ vanishes for n odd. The second order term has been long ago obtained by path integral methods (Leggett, et al.).

$$\theta^{(2)}(t) = \frac{\omega_0^2}{\omega_c^2}\frac{1-\mathrm{Re}(1+i\omega_c t)^{-2(\alpha-1)}}{2(\alpha-1)(2\alpha-1)}.$$

Here we see the phenomenon of weak localization: for $\alpha > 1$ $P_z(t)$ remains finite for long times. In the critical case $\alpha = 1$ $\theta^{(2)}(t) = \frac{\omega_0^2}{\omega_c^2}\log(1+\omega_c^2 t^2)$. We find for the fourth order term

$$\theta^{(4)}(t) \sim \theta^{(2)}(t)^2 + \text{ less divergent terms.}$$

This leads to the suspition that the expansion in powers of ω_0 is not uniform.

R. L. Hudson:

On the DVISK Formula

The Dyson-Veretennikov-Isobe-Sato-Krylov formula

$$U_t = \sum_{n=0}^{\infty}\sum_{j_1,\ldots,j_n=1}^{m}\int_{0<t_1<\cdots<t_n<t} P_{t_1}V_{j_1}P_{t_2-t_1}V_{j_2}\cdots V_{j_n}P_{t-t_n}\,dK_{j_1}(t_1)\cdots dK_{j_n}(t_n)$$

for the solution of the stochastic differential equation

$$dU = U\left(\sum_{j=1}^{m} V_j\,dK_j + H\,dt\right), U_0 = 1.$$

where $P_t = e^{tH}$, enables us to estimate the partial trace over the initial space of the process U. Using the noncommutative tracial version of Hölderlin inequality we find that

$$|\mathrm{tr}\, P_{t_1} V_{j_1} P_{t_2 - t_1} V_{j_2} \cdots V_{j_n} P_{t - t_n}| \quad \leq \quad \mathrm{tr}\, P_t M^n$$

where $M = \max_{j=1,\ldots,m} \|V_j\|$. From this it follows from standard estimates of QSC that the partial trace over the initial space $\mathrm{tr}_{H_0} U$ exists. For a certain case this trace can be computed explicitly.

P. E. T. Jorgensen:

Positive Representations of Wick Algebras

(Joint work with R. F. Werner and L. Schmitt)

Let H be a Hilbert space and $H^\dagger$ its conjugate Hilbert space. In the theory of quantum groups, in quantum probability, and in non-commutative differential geometry relations of the form

$$h \otimes k^\dagger \quad = \quad \langle h, k \rangle 1 + \tilde{T}(h \otimes k^\dagger)$$

arise where $\tilde{T} : H \otimes H^\dagger \to H^\dagger \otimes H$ is some linear operator. Positive representations of these relations are discussed. As examples the μ-CAR, μ-CCR, and $SU_\mu(2)$ are investigated.

J. Kupsch:

Fermionic White Noise and the Eucledean Quantum Field Theory for Fermions

Euclidean quantum field theory is a theory of linear functionals on test function algebras. In the bosonic/fermionic case this algebra is a symmetric/antisymmetric tensor algebra, and the evaluation of euclidean Green's functions (Schwinger functions) corresponds to the calculation of linear functionals on this algebra. For the usual bosonic theories this problem is equivalent to the construction of random fields and the Schwinger functions are expectations of these fields.

The main part of the lecture gives a construction of euclidean random fields also for fermions. In a first step complex white noise is supplemented by an antisymmetric tensor product (Grassmann product). That is possible either with a polynomial chaos expansion or with the homogeneous chaos. The resulting fermionic white noise is the building block for more complicated fields. Candidates for euclidean Dirac fields are given as example. The construction is essentially unique if the Markov property is demanded.

H. Leschke:

Classical and Quantum Dynamics by White Noise Hamiltonians

(Joint work with W. Fischer and P. Müller)

A class of random systems whose Hamiltonian can be written as a sum of a deterministic part and a random part is discussed. While the deterministic part is time independent and quadratic, the random part is time dependent and (its Weyl-Wigner symbol) is supposed to be a homogeneous Gaussian random field which is delta correlated in time and arbitrary but smooth in position and momentum. Exact expressions for the time evolution of both (mixed) states and observables averaged over randomness are obtained. Furthermore, the difference between the quantum and the classical case is clearly exhibited. As a special case it is shown that, if the deterministic part corresponds to a particle subject to a constant magnetic field, the averaged spatial variance of any initial state shows a diffusive behaviour in that it grows linearly in time.

M. Lindsay:

Y. G. Lu:

Hilbert Module and Free Stochastic Calculus

Hilbert module and free type quantum noise appear from the Markovian limit of QED. The construction of Hilbert module depends on the original physical model. If the system Hamiltonian has discrete spectral set, module is replaced by a direct sum of Hilbert spaces. If the system Hamiltonian is, up to a constant, p^2, then we need a Hilbert module on the von Neumann Algebra generated by the momentum operator. If the system Hamiltonian has, up to a constant, the form $p^2 + V$ (where $V \in L^1 \cap L^2$), then in order to describe the limit noise, we need a Hilbert module on a $*$-algebra generated by polynomials p and wave operator with respect to $(p^2 + V, p^2)$. Moreover, we introduce free type stochastic calculus on Hilbert module to study the limit of evolution.

A. Mohari:

On Stochastic Parallel Transport

M. Schürmann:

Direct Sums of Tensor Products and Non-Commutative Independence

For a set X we denote by

$$A(X) \quad = \quad \{(\epsilon_1 \cdots \epsilon_m) : m \geq 1, \epsilon_i \in X, \epsilon_i \neq \epsilon_{i+1}\}$$

the set of non-empty *finite*, ***alternating*** sequences in elements of X. For $M \subset A(\{1,2\})$ and $\epsilon \in A(\{1,2\})$ we denote by $\Delta^1_M(\epsilon)$ the subset of $A(\{1,2,3\})$ obtained from ϵ by replacing 2s by 3s and 1s by arbitrary element of M. Similarly, $\Delta^2_M(\epsilon)$ is the subset of $A(\{1,2,3\})$ obtained from ϵ by replacing 2s by arbitrary elements of

$$M_{+1} \; = \; \{(\epsilon_1 + 1, \ldots, \epsilon_m + 1) : (\epsilon_1 \cdots \epsilon_m) \in M\} \; \subset A(\{2,3\}).$$

We proved that there are precisely 4 subsets M of $A(\{1,2\})$ satisfying the conditions

- $$(1), (2) \; \in \; M$$
- $$\Delta^1_M(M) \; = \; \Delta^2_M(M).$$

The 4 sets are

(A) $$M \; = \; \{(1), (2)\}$$

(B1) $$M \; = \; \{(1), (2), (12)\}$$

(B2) $$M = \{(1),(2),(21)\}$$

(C) $$M = A(\{1,2\})$$

We point out that the basically 3 devices A,B and C correspond to the 3 fundamental known notions of non-commutative independence in quantum probability. We believe that the fact that we have 2 B-cases corresponds to the existence of the large number of variations in the 'tensor independence' case known as 'twisted' or 'braided' independence.

M. Skeide:

The Lévy-Khintchine Formula for $SU_q(2)$

We present our results on a Lévy-Khintchine formula for Woronowiczs twisted $SU(2)$ and compare them with the classical Lévy-Khintchine formula.

A. Verbeure:

About the Goldstone Theorem

The Goldstone phenomenon in statistical mechanics and field theory is rigorously formulated.

A discussion of the Goldstone theorem is given in terms of quantum fluctuation theory distinguishing dynamical systems with short range and long range interactions. A rigorous formulation is found for the Anderson restauration of symmetry. This is a consequence of the theorem: the fluctuation of the generator of the broken symmetry and the fluctuation of the order parameter operator form a canonical pair.

W. von Waldenfels:

The Harmonic Oscillator in the Radiation Field

(Joint work with G. Efimov)

In order to study dissipation phenomena we investigate the analogue of Lamb's model (1900): An oscillator attached to an infinite string

$$H = \frac{p^2}{2} + \frac{1}{2}(q+B)^2 + \frac{1}{2}\int dk(P_k P_{-k} + \omega_k^2 Q_k Q_{-k})$$

with

$$[P_k, Q_{-k'}] = \frac{1}{i}\delta(k-k'), \quad Q_k = Q^\dagger_{-k}, \quad P_k = P^\dagger_{-k}, \quad \omega_k^2 = k^2 + \mu^2.$$

Introducing $P = \begin{pmatrix} p \\ P_k \end{pmatrix}$, $Q = \begin{pmatrix} q \\ Q_k \end{pmatrix}$ and assuming dipole approximation $B = G\int dk\, kiQ_k F(k^2)$ with $F(k^2) = e^{-k^2 a}$, we write

$$H = \frac{1}{2}(P^\dagger P + Q^\dagger M^2 Q) \quad \text{with} \quad M^2 = \begin{pmatrix} 1 & G\bar{v}_k \\ Gv_k & \omega_k^2\delta(k-k') + G^2 v_k \bar{v}_k \end{pmatrix} = D + |u)(u|$$

with

$$D = \begin{pmatrix} 0 & 0 \\ 0 & \omega_k^2\delta(k-k') \end{pmatrix}, \quad |u) = \begin{pmatrix} 1 \\ G\bar{v}_k \end{pmatrix}, \quad v_k = -GikF(k^2).$$

This special form allows to calculate the resolvent

$$\frac{1}{z-M^2} = \frac{1}{z-D} + \frac{1}{z-D}|u\rangle \frac{1}{C(z)} \langle u| \frac{1}{z-D}$$

with

$$C(z) = 1 - \left\langle u \left| \frac{1}{z-D} \right| u \right\rangle = 1 - \frac{1}{z} - G^2 \int \frac{|v_k|^2}{z - \omega_k^2}\, dk.$$

$C(z)$ may or may not have a real zero λ_0 corresponding to $\mu > \mu_c$ or $\mu < \mu_c$, where μ_c is a critical constant. The spectrum of M^2 consists of λ_0 and the intervall $[\mu^2, \infty[$. The eigenvectors can be calculated explicitly. From this we deduce a new vacuum $||0\rangle$. This allows to calculate an optimal constant a in $F(k^2)$. The vacuum $||0\rangle$ belongs to the initial Fock space $\Gamma(\mathbf{C} \oplus L^2(\mathbf{R}))$ if $\int \frac{|v_k|^2}{\omega_k^2}\, dk < \infty$.

R. F. Werner:

The Classical Limit of Quantum Theory

For general quantum observables we define convergence to a classical limit in norm. According to this notion, which is based on a generalization of inductive limits of normed spaces, the product of convergent sequences is convergent, and the quantum time evolution converges uniformly on finite time intervals to the classical Hamiltonian evolution.

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