MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 15/1994

Arbeitsgemeinschaft zu einem aktuellen Thema: Von Neumann Algebren

04.04. bis 09.04.1994

The Arbeitsgemeinschaft was organized by Jean-Benoit Bost (Bures-sur-Yvette) and Joachim Cuntz (Heidelberg). The theory of von Neumann algebras ("noncommutative measure theory") has made tremendous progress since the advent of Tomita-Takesaki theory around 1970. It is by now a well-developed powerful machine which has found applications, besides the "classical" ones in representation theory, mathematical physics and ergodic theory, also in different branches of geometry and topology. The program covered a large part of the general theory as it stands now (with the exclusion of the important recent developments on subfactors and their indices) and some selected applications.

Von Neumann algebras were introduced and studied in the classical Murray-von Neumann papers in 1930's and 1940's. They identified factors (i.e. algebras with trivial center) as the basic building blocks of the theory and made a first classification of factors into type I, II or III. While type I factors are easily shown to be isomorphic to the algebra of all bounded operators on a Hilbert space, the classification of type II and III factors posed serious problems.

By associating von Neumann algebras to discrete groups and more generally to groups acting on measure spaces, Murray and von Neumann were able to construct factors of type II_1, II_∞ and a factor of type III. The type III factors (purely infinite case) seemed particularly intractable. The next breakthrough came in 1960's with the series of work of Powers, Araki-Woods and Krieger resulting in a wealth of type III factors. Powers' construction is an analogue of the construction of infinite products of measure spaces. By taking infinte tensor products of matrix factors $(M_2(\mathbb{C}), \phi)$, (ϕ a suitable state), one obtains a continuous family of pairwise nonisomorphic factors of type III: factors $R_\lambda, \lambda \in (0,1)$, of Powers. By analyzing the infinite tensor product construction, Araki and Woods were able to construct type III factors, nonisomorphic to the existing ones: e.g. the R_∞ factor. Krieger's factors were constructed using ergodic transformations of measure spaces.

The starting point of Connes' classification of amenable factors is Tomita's theorem: each faithful weight on a von Neumann algebra M gives rise to a canonical one-parameter group of automorphisms of M; the modular automorphism group. The modualr group is uniquely characterized by the so called KMS condition of quantum statistical mechanics. The first algebraic invariant, defined by Connes, for a type III factor M is Connes' spectrum

$$S(M) = \cap Sp(\Delta_\phi)$$

where Δ_ϕ is the generator of the modular group associated to ϕ and ϕ runs over all faithful weights over M. Now $S(M)\backslash\{0\}$ is a closed subgroup of the multiplicative group of positive real numbers. There are only three possibilities and one defines factors of type $III_0, III_\lambda (0 < \lambda < 1), and III_1$ accordingly. A crowning achievement of Connes' theory is his complete classification (except in the type III_1 case which was completed by Haagerup) of amenable factors. (Also called injective or hyperfinite, these algebras are generated by an increasing sequence of finite dimensional subalgebras). For example, the only amenable type III_λ factors are factors R_λ of Powers.

These developments and more were reported in this conference. In particular various applications of von Neumann algebras and their interplay with questions in geometry, topology and number theory were highlighted. Among them we may mention the covolume formula of a discrete series representation due to Atiyah and Schmid; the L^2-index theorem of Atiyah; Gromov's Kähler hyperbolicity and L^2-Hodge theory and finally the Connes-Bost approach to statistical theory of prime numbers and class field theory.

Abstracts of the talks

1. Basic notions in C^*-algebras and von Neumann algebras (I)
H.Knospe

A von Neumann algebra $M \subseteq B(H)$ is an ultraweakly closed $*$-subalgebra of the algebra of all bounded operators on a Hilbert space which contains the identity operator. Von Neumann's bicommutant theorem asserts that the topological condition on M is equivalent to M be equal to its bicommutant M''. (The commutant of a subset $S \subset B(H)$ is defined by $S' = \{T \in B(H) : TS = ST \forall S \in S\}$. The clssification of von Neumann algebras is up to algebraic isomorphism. A more restrictive notion of isomorphism is that of spatial isomorphism. A factor ("indecomposable algebra") is a von Neumann algebra with trivial center $M \cap M' = \mathbb{C}I$. Factors are building blocks of more general von Neumann algebras. Using the Gelfand representation theorem, one can describe abelian von Neumann algebras as algebras $L^\infty(X, \mu)$ of bounded measureable functions on a measureable space (X, μ). Every von Neumann algebra is algebraically isomorphic to a direct integral of factors

$$\int_X M(t)d\mu(t).$$

Finite-dimensional von Neumann algebras are exactly direct sums of matrix algebras.

2. Basic notions in C^*-algebras and von Neumann algebras (II)
E.Landvogt

A state on a C^*-algebra A is a positive linear functional on A of norm 1. The fundamental GNS-construction gives a 1-1 correspondence between cyclic representations of A on a Hilbert space (up to unitary equivalence) and states on A. Pure states correspond to irreducible representations. A normal state on a von Neumann algebra is a state which is σ-additive with respect to orthogonal families of projections. The noncommutative analogue of a probability space is a pair (M, ϕ), where M is a von Neumann algebra and ϕ is a faithful normal state on M. In the simplest example $M = M_n(\mathbb{C})$ every state ϕ is of the form (for $x \in M_n(\mathbb{C})$)

$$\phi(x) = Tr(\rho x),$$

where ρ is a positive matrix of trace 1.

3. Dimension function and type

A.Huber

Given a factor M on a Hilbert space H, one can introduce a notion of dimension relative to M (whose values are non-negative real numbers or ∞) on the set of all projections of M. The dimension function is unique up to an scaling factor. The properties of the dimension function lead to a first classification of factors into types I, II, III. Namely, let Δ be the image of the dimension function (up to an scale). Then M is of type I_n (resp. I_∞) if $\Delta = \{0, 1, \ldots, n\}$ (resp. $\Delta = \{0, 1, 2, \ldots\}$) and is of type II_1 (resp. II_∞), provided $\Delta = [0, 1]$ (resp. $\Delta = [0, \infty]$). Finally M is of type III if $\Delta = \{0, \infty\}$.

A factor of type I is isomorphic to $B(H)$, where $dimH = n$ (type I_n), or H is infinite dimensional (type I_∞).

4. Examples of factors of type II and of type III

R.Holtkamp

The group-von Neumann algebra $W^*(G)$ of a discrete group G is the completion of the group-algebra $\mathbb{C}G$ in its left regular representation over the Hilbert space $l^2(G)$. If G is an ICC group, that is all of its (nontrivial) conjugacy classes are infinte, then . $W^*(G)$ is a II_1 factor. Moreover, the corresponding factors for G locally finite are all isomorphic to each other. A non-isomorphic II_1 factor is obtained when G is the free group on two generators. The first example of a type III factor was constructed by Murray and von Neumann using the group-measure space construction. More generally, when a group G acts as an automorphism group $\alpha : G \longrightarrow Aut(M)$ of a von Neumann algebra M, one can construct the crossed product von Neumann algebra $R(M, G, \alpha)$. When M is abelian, G acts by measureable transformations on a measure space, and the corresponding crossed product algebra is the group- measure algebra. For $\Gamma = PSL(2, \mathbb{Z})$ acting by homographic transformations on $P_1(\mathbb{R})$, one obtain a factor of type III.

5. Finite factors and coupling constants

A.Meister

Let M be a finite factor and $P(M)$ the set of projections in M. Given a dimension function $D : P(M) \longrightarrow [0,1]$ (see 3. above) and a self-adjoint element h with spectral decomposition $\int \lambda de_\lambda$ in M, one can define the trace of h by $T(h) := \int \lambda dD(e_\lambda)$. The functional T has the trace property $T(uhu^*) = T(h)$ (u a unitary in M) and is linear on commutative subalgebras. The problem is to show that T is additive in general. Apart from Murray-von Neumann's original proof of this fact, we have also Yeadon's proof which acheives this via the Ryll-Nardzewski fixed poit theorem.

Considering a representation $\phi : M \longrightarrow B(H)$ of a factor as an M-module structure on H, both $\phi(M)$ and its commutatnt $End_M(H)$ are factors. If M is a finite factor, then an M-module H is called finite if $End_M(H)$ is a finite factor. For a finite factor M there is a canonical finite M-module $L^2(M)$, the GNS representation with respect to the trace. Every M-module H can be considered as a submodule of $L^2(M) \otimes K$, where K is a Hilbert space. The dimension function D on $End_M(L^2(M) \otimes K)$ allows to compare the size of such a submodule to the size of the submodule $L^2(M)$, which defines the coupling constant $dim_M(H)$. It is a positive real number that characterizes the module completely.

If $N \subset M$ is a subfactor of a type II_1 factor M, then the index $[M : N]$ of N in M is defined as $dim_N(L^2(M)$. The possible set of values of this index, Σ, is given by an striking result of V.Jones:

$$\Sigma = \{4cos^2\frac{\pi}{n} : n \geq 3\} \cup [4, \infty).$$

6. Hilbert algebras and left Hilbert algebras

J.Michalicek

A Hilbert algebra is an involutive algebra with a compatible scalar product. The GNS-construction gives a connection between normal traces on a von Neumann algebra and Hilbert algebras . Associated with a Hilbert algebra are its left and right von Neumann algebra U and V. These algebras are commutants of each other and there is an operator J which carries U to V : $V = JUJ$. If G is a unimodular locally compact group, the left and right regular representations generate the left and right von Neumann algebra of a Hilbert algebra. If G is not unimodular, one obtains a "quasi-Hilbert algebra". and the modular function Δ defines an unbounded operator on the corresponding Hilbert space. The GNS-construction for a non-tracial faithful normal state or weight of a von Neumann algebra gives rise to, what is called a left (or generalized) Hilbert algebra. It is possible to define left and right von Neumann algebras U and V which are commutants of each other and operators J and Δ that generalize the corresponding operators in the case of the left and right regular representation of a group. The fundamental result is Tomita's theorem:

$$JUJ = V \, and \, \Delta^{it} U \Delta^{-it} = U$$

for each real number t.

7. Proof of Tomita's theorem

K.Kürsten

For the important special case of the GNS-construction for a faithful normal state (or, equivalently for a cyclic and separating vector), a proof of Tomita's theorem is sketched. Tomita's theorem is the basis for all of non-commutative integration theory and for the modern structure theory of von Neumann algebras.

8. The KMS (or modular) condition.

G.Illies

This condition (which has its origins in quantum statistical mechanics) can be viewed as a generalized trace condition for a state ω with respect to a one-parameter automorphism group τ_t of a C*- or von Neumann algebra A. One possible definition uses the extension of τ_t from $\mathbb{R}$ to $\mathbb{C}$ on the subalgebra of analytic elements and requires that, for a fixed $\beta \geq 0$,

$$\omega(yx) = \omega(x\tau_{i\beta}(y))$$

on a dense set of analytic elements x,y. Another way to express this condition uses complex analytic function theory.

Concerning modular theory, one has the following important result: let ω be a faithful normal state of a von Neumann algebra M. Then there exists a unique one-parameter group τ_t of automorphisms of M that satisfies the KMS condition with respect to ω and with $\beta = 1$, namely the modular group from Tomita-Takesaki theory.

9. Modular invariants for type III factors and examples of amenable factors

W.Gubler

The starting point for Connes' analysis of type III factors is his "2×2-matrix trick" which shows that up to inner automophisms. the modular automorphism group σ^{ϕ} of a type III factor M does not depend on the choice of the faithful normal state ϕ. Thus the essential Connes spectrum $\Gamma = \Gamma(\sigma^{\phi})$ is an invariant of the factor M. It is a closed subgroup of $\mathbb{R}$ and M is said to be of type III_0, III_λ with $0 < \lambda < 1$, or III_1, according to wether $\Gamma = \{0\}$, $\Gamma = (log\lambda)\mathbb{Z}$ or $\Gamma = \mathbb{R}$. An important class of examples of factors is obtained from an ergodic transformation T of a standard Borel probability space with measure μ. Here, the invariant Γ can be determined from the local scaling properties of the measure μ under T.

10. Classification of amenable factors

H.Hofmeier

Amenable factors are also called injective, matricial, approximately finite or hyperfinite in the literature. Many of the factors appearing in applications and arising from seemingly very different constructions are amenable. The definition of amenability has a cohomological nature. A von Neumann algebra M is said to be amenable if for every normal dual Banach bimodule X over M, the derivations of M with coefficients in X are all inner. Most of amenable factors can be obtained by taking infinite (algebraic) tensor products of finite-dimensional factors and then completing in the GNS-representation with respect to an appropriate state (factor state) on the tensor product algebra (ITPFI-factors). A tensor product state ϕ on an infinite tensor product of 2×2-matrix factors is a factor state and the invariant Γ can be easily computed . For appropriate choices of ϕ one obtains examples of amenable factors of type III_λ or III_1. The complete classification of amenable factors (due to Connes) is sketched.

11. Discrete series representations and von Neumann dimension

A.Deitmar

The goal of this talk is the proof of the following formula, due to Atiyah and Schmid :

$$dim_{W^*(\Gamma)} H = covol(\Gamma) d_\pi$$

for the coupling constant of a discrete series representation $\pi : G \to U(H)$ of a semisimple real Lie group G with respect to the factor $W^*(\Gamma)$ defined by a discrete ICC-subgroup Γ of G (d_π is the formal degree of π and $covol(\Gamma)$ the covolume of Γ). This result can be generalized as follows. Let $S \subset \hat{G}$ be a measureable set of irreducible representations of G and $H := \int_S H_\pi d\mu(\pi)$, where μ is the Plancherel measure. Then

$$dim_{W^*(\Gamma)} H = \mu(S) covol(\Gamma).$$

12. An L^2-index theorem

T. Schick

This talk is devoted to the proof of the following L^2-index theorem of Atiyah:

Theorem. Let M be a closed smooth manifold with vector bundles E and F over M and $D : C^\infty(E) \longrightarrow C^\infty(F)$ an elliptic differential operator. Let $\tilde{M}$ be a normal covering of M with covering group Γ. Lift E, F and D to $\tilde{E}, \tilde{F}$ and $\tilde{D} : C^\infty(\tilde{E}) \longrightarrow C^\infty(\tilde{F})$. Then,

$$index D = index_\Gamma \tilde{D}.$$

Here, $index D := dim ker D - dim coker D$ is the usual index of Fredholm operators. To define $index_\Gamma \tilde{D}$, we note that $ker \tilde{D} := \{f \in L^2(\tilde{E}) : \tilde{D} f = 0\}$ is a Γ-invariant subspace of $L^2(\tilde{E})$, and similarly for $coker \tilde{D}$. For a Γ-invariant subspace H of $L^2(\tilde{E})$, define $dim_\Gamma H = Tr_A(pr_H)$, where $A = W^*(\Gamma) \otimes B(L^2(E))$ and Tr_A is induced by the traces on $W^*(\Gamma)$ and $B(L^2(E))$. Using this, one defines $index \tilde{D} := dim_\Gamma ker \tilde{D} - dim_\Gamma coker \tilde{D}$.

13. Kähler hyperbolicity and L^2-Hodge theory

W.Lück

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A Kähler manifold is a complex anlytic manifold M equipped with a hermitian metric such that the associated fundamental 2-form ω is closed. Let $\tilde{\omega}$ be the canonical lift of ω to the universal cover $\tilde{M}$. M is called Kähler hyperbolic iff $\tilde{\omega}$ is the boundary of a bounded 1-form. Compact Kähler manifolds such that $\pi_2(M) = 0$ and $\pi_1(M)$ is word - hyperbolic, are Kähler hyperbolic. $\mathbb{C}P^n$ is Kähler but not Kähler hyperbolic. Let $H^r(\tilde{M})$ be the space of harmonic L^2-r- forms on $\tilde{M}$ and $H^{p,q}(\tilde{M})$ the harmonic p,q-forms. The following theorem is due to Gromov.

Theorem. Let M be compact and Kähler hyperbolic of complex dimension m. Then

$$H^{p,q}(\tilde{M}) = 0 \Longleftrightarrow p + q \neq m.$$

From this, it follows that $(-1)^m \chi(M) > 0$.

The proof is based on an L^2- version of Lefschetz theorem, a perturbation technique of Vafa-Witten and the L^2- index theorem of Atiyah.

14. Euler products and type III factors

M.Spieß

Let Γ be a discrete group and $\Gamma_0 \subset \Gamma$ a subgroup which is almost normal, that is the orbits of Γ_0 acting on the left on Γ/Γ_0 are finite. One defines the Hecke algebra $\mathcal{H}(\Gamma, \Gamma_0)$ as the convolution algebra of $\mathbb{C}$-valued functions with finite support on $\Gamma_0\backslash\Gamma/\Gamma_0$. The C^*-algebra $\bar{\mathcal{H}} = C^*_r(\Gamma, \Gamma_0)$ is the completion of $\mathcal{H}$ in its regular representation in $l^2(\Gamma_0\backslash\Gamma)$. One has a canonical one-parameter group of automorphisms of $\bar{\mathcal{H}}$ (A C^*-dynamical system) defined by

$$\sigma_t(f)(\gamma) = \left(\frac{L(\gamma)}{R(\gamma)}\right)^{it} f(\gamma),$$

where L and R are certain Γ_0 bivariant functions on Γ. Let us now consider the Hecke algebra for the groups $\Gamma = P_{\mathbb{Q}}$ and $\Gamma_0 = P_{\mathbb{Z}}$ where P is the group of invertible 2×2-matrices of the form $\begin{pmatrix} 1 & b \\ 0 & a \end{pmatrix}$. The following theorem of Connes and Bost shows that the C^*-dynamical system associated to $\bar{\mathcal{H}}$ has a phase transition with spontaneous symmetry breaking.

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Theorem. Let (A, σ_t) be the C^*-dynamical system associated to the almost normal subgroup $P_{\mathbb{Z}}$ of $P_{\mathbb{Q}}$. Then:

a) For $0 < \beta \leq 1$ there exists a unique KMS_β state ϕ_β. Each ϕ_β is a factor state and the associated factor is the hyperfinite factor of type $III_1 : R_\infty$.
b) For $\beta > 1$ the KMS_β states form a simplex whose extreme points are parametrized by imbeddings $\chi : K \longrightarrow \mathbb{C}$ of the field $K = \bar{\mathbb{Q}}_{ab}$ and whose restriction to $C^*(\mathbb{Q}/\mathbb{Z})$ are given by the formula

$$\phi_{\beta,\chi}(\gamma) = \zeta(\beta)^{-1} \sum_{n=1}^{\infty} n^{-\beta} \chi(\gamma)^n.$$

These states are type I_∞ factor states.

The normalization factor is the inverse of the Riemann ζ function evaluated at β.

The next step is to study the statistics of prime numbers and class field theory using C^*-algebraic formulation of quantum statistical mechanics. To this end one constructs, via a Bosonic Fock space (second quantization) a selfadjoint operator whose spectrum is the set of prime numbers and the corresponding C^*-algebra (obtained by polar decomposition) from creation and annihilation operators. The algebra turns out to be the infinite tensor product of Toeplitz C^*-algebras .

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