

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 17/1994

Designs and Codes

17. bis 23. April 1994

Die Tagung fand unter der Leitung von Herrn Jungnickel (Augsburg) und Herrn van Lint (Eindhoven) statt.

The 1994 meeting on Designs and Codes was the second one on this topic in Oberwolfach after the first conference in April 1990.

Design Theory and Coding Theory are two areas in Discrete Mathematics which are closely related and of course, both communities benefit from the opportunity to bring together leading researchers working in both areas, which was one of the main aims of this conference. There were 43 participants from 12 countries, among them 16 from North America.

Of the many interesting lectures, two talks provided particular highlights by solving major longstanding open problems. First, the puzzling formal duality between the Preparata and Kerdock codes has finally been explained by realizing that these non-linear binary codes are projections of linear \mathbb{Z}_4 -codes which are in fact duals. Secondly, a major breakthrough in the asymptotic existence of designs has been achieved by proving that orthogonal arrays of arbitrary strength t (with k rows and index 1) exist for all sufficiently large orders (for fixed k, t). This is the first asymptotic result for designs with $t \geq 3$.



Vortragsauszüge

R. Ahlswede:

Number theoretic correlation inequalities for Dirichlet densities.

For sets $A, B \subset \mathbb{N}$, the set of positive integers, consider the set of least common multiples $[A, B] = \{[a, b] : a \in A, b \in B\}$, the set of largest common divisors $(A, B) = \{(a, b) : a \in A, b \in B\}$, the set of products $A \times B = \{a \cdot b : a \in A, b \in B\}$ and the set of their multiples $M(A) = A \times N$, M(B), M[A, B], M(A, B), and $M(A \times B)$, respectively.

Our first discoveries are the inequalities

$$dM(A,B) \cdot dM[A,B] \ge dM(A) \cdot dM(B) \tag{1}$$

$$dM(A) \cdot dM(B) \ge dM(A \times B) \tag{2}$$

where d denotes the asymptotic density and A,B are finite. The first inequality is by the factor dM(A,B) sharper than Behrend's well-known inequality. The second inequality does not seem to have number theoretic predecessors.

The next discovery is that (1) can easily be derived from the Ahlswede/Daykin inequality via Dirichlet series. Actually, we found this way the much more general inequality

$$\underline{D}(A,B) \cdot \underline{D}[A,B] \ge \underline{D}A \cdot \underline{D}B, \tag{3}$$

where \underline{D} denotes the lower Dirichlet density and A,B are arbitrary subsets of \mathbb{N} . The applications of the AD-inequality give now correlation inequalities not only in Statistical Physics (Harris, FKG), Probability Theory (Hölley), Combinatorics (Kleitman, Seymour/Webster, Marica/Schönheim), but also in Number Theory.





E.F. Assmus, Jr:

Where does the Mattson-Solomon polynomial live?

We give an elementary proof of Berman's Theorem characterizing the Reed-Muller code $\mathcal{R}(n,m)$ as J^{m-n} where J is the Jacobson radical of the group ring $\mathbb{F}_2[G]$ where G is an elementary abelain 2-group of order 2^m . The essential new ingredient is the observation that $\mathcal{R}(n,m)$ is generated by the characteristic functions of the (m-n)-dimensional subspaces of the affine geometry of $G \approx \mathbb{F}_2^m$.

We contrast this very easy proof with the current more involved proof of Charpin's generalization to \mathbb{F}_p of Berman's Theorem. We make some progress in simplifying this discussion by describing in the most general case - an \mathbb{F}_q -algebra isomorphism between the \mathbb{F}_q -algebra

$$\mathbb{F}_q[x_1,\ldots,x_m]/(x_1^q-x_1,\ldots,x_m^q-x_m)$$

and the \mathbb{F}_q -algebra of fixed points of the Frobenius map, $\zeta \to \zeta^q$, of the algebra

$$\mathbb{F}_{q^m}[Z]/(Z^{q^m}-Z).$$

Th. Beth:

Designs, Codes and Puzzles.

Owing to the more practical problem of finding the algebraic classification of the solution for a special rather complicated toy puzzle, commercially available under the name DISCO, we consider the following

Pegging Problem: Let $N = \{w_0, \ldots, w_{d-1}\}$ be a transversal for the d = 14 orbits of the group $G = \mathbb{Z}_6$ acting regularly on the coordinates of $W = GF(2)^6$. By canonically embedding $W \simeq GF(2)[z]/(z^5-1)$ with the action $\mathbb{Z}_6 \ni i : w(z) \mapsto z^i w(z) \mod (z^6-1)$ the problem reads:

Find an arrangement $i_k \in G, j \in S_d$

$$A(z) = \left(z^{i_k} w_{jk}(z)\right)_{k=0}^{13} \mod (z^6 - 1)$$

of cyclically rotated words of N in such a way that every column A_i in $A(z) = \sum_{i=0}^{5} A_i z^i$ can be partitioned into disjoint words of 3 consecutive ones, forming the *pegs*.

0		()	Several approaches to solving this problem algorithmi-
_	0	()	cally in spite of the huge searchspace of $(13!) \cdot 6^9 \cdot 3^2 \cdot 2$
1	1	()	are presented. By considering the patterns in W as
1	1	1	vectors of $GF(2^6)$ with respect to a normal basis over
α	1	1.	$GF(2)$ the canonical isomorphism $\mathbb{Z}_6 \simeq \mathbb{Z}_3 \times \mathbb{Z}_2$ allows
æ	0	1	a design type solution in an efficient algebraic defini-
α	N	.0	tion of sequences in $GF(4)^3$ with $GF(4) = \{1, \alpha, \alpha + 1\}$
0	α	α	$1 = \alpha^2, 0$. The solution reads as in the figure.
1	α	CY	From here it was similarly easy, to give an algebraic
1	0	α	construction for the modified Pegging Problems
1	α	α	(a) with $G = \mathbb{Z}_6$ on $W - \{1\}$
ά	α	α	(a) with $G = B_6$ on $W = \{1\}$ (b) with $G = D_6$ on W
α	α	0	` '
α	0	0	the latter one being the commercially available toy.

Open questions are posed as to which class of designs or codes the presented solutions belong.

I. F. Blake:

Decoding Reed-Solomon codes.

The Welsh-Berlekamp algorithm is an efficient means of decoding Reed-Solomon codes that does not involve the computation of syndromes. A brief description of this algorithm is given, showing in particular how the set of equations requiring simultaneous solution, is carried out. It is also shown the problem can be translated to an algorithm for the solution of polynomial congruences. A new module theoretic approach to the solution of such congruences is proposed. From this approach new and efficient parallel algorithms to solve the WB equations are derived.

A. Brouwer:

Correspondence between symmetric bilinear forms and alternating forms.

In a dual polar space (in the classical situations where maximal totally isotropic or totally singular subspaces have half the dimension of the surrounding space) the collection of t.i. (t.s.) subspaces disjoint from a given one can be naturally identified with a space of forms or matrices: one gets

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- (i) all alternating matrices (from D_n)
- (ii) all symmetric matrices (from C_n)
- (iii) all Hermitian matrices.

Noting that a nondegenerate hyperplane in a D_n geometry carries a B_{n-1} geometry, and that in even characteristic $B_{n-1} \cong C_{n-1}$, we find in even characteristic a one-to-one correspondence

$$P \longleftrightarrow \left(\begin{array}{cc} 0 & \sqrt{d}^T \\ \sqrt{d} & P + \sqrt{d}\sqrt{d}^T \end{array}\right)^{\frac{1}{4}}$$

between symmetric matrices of order m and alternating matrices of order order m+1. One of the consequences is a geometric explanation of a property noted by Calderbank.

A. A. Bruen:

(a) 10_3 configurations in Desarguesian planes.

(Joint work with J.C.Fisher)

- (b) Independent sets of points in projective and affine spaces. (Joint work with D.Wehlau and L.Haddad)
- (a) We briefly discuss a beautiful observation by G. Pickert, pursuant to a paper by Bruen and Fisher, on the embeddability of a certain 10₃ configuration in a projective plane which is Desarguesian but not Pappian.
- (b) In PG(n,q) let S be line-free. Then \overline{S} , the complement of S, intersects all lines. If \overline{S} does not contain a hyperplane we say that S is projective: otherwise, S is affine. If S is projective, each hyperplane contains at least one point of S. We elaborate on the following observation:

If |S| is large, S must be affine. In particular, in PG(n, 2), if $|S| > 5 \cdot 2^{n-3}$ then S must be affine.

We also desribe other intersection properties in PG(n, 2). Results similar to some of these have been obtained by E. Clark and by A.A. Davydov and L.M. Tombak.

The geometries AG(n,3) and PG(n,2) are Steiner triple systems and colourings of their points, such that no line is monochromatic, are of interest. These colourings give line-free sets. In AG(4,3) we show that a line-free set has at most 20 points and we describe geometrically the unique line-free set on 20 points. (Part of this result, at least, is due originally to R. Hill.) We also skirmish with line-free sets in AG(5,3).

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A.R. Calderbank:

Codes, Geometries and Extremal Sets of Euclidean Lines with Prescribed Angles.

We describe how Kerdock codes over the binary field \mathbb{Z}_2 and over the ring \mathbb{Z}_4 of integers modulo 4 determine extremal sets of lines in real and complex Euclidean space with only two angles. Extraspecial 2-groups serve as the bridge between discrete and Euclidean geometries. Classical geometric/group theoretic connections between binary symplectic and orthogonal geometries are expressed as a correspondence between binary symmetric $m \times m$ matrices and binary skew-symmetric $(m+1) \times (m+1)$ matrices.

G. Cohen:

Generalized Weights (upper bounds).

The generalized i-distance d_i of an [n,k] binary linear code C is the minimum size of the union of the supports of i independent codewords of C. We derive new upper bounds on d_i (Hamming, Plotkin, Elias). For the last one, which is the strongest asymptotically, we need a nonlinear extension of the concept of i-distance:

Definition: Let C be any binary code (subset of \mathbb{F}_2^n). Then set

$$d_i(C) := min[supp(c_1 + t) \cup supp(c_2 + t) \cup \ldots \cup supp(c_i + t)],$$

where the minimum is taken over all (i+1)-tuples (t,c_1,c_2,\ldots,c_i) such that $c_1+t,\,c_2+t,\ldots,\,c_i+t$ are linearly independent.

Theorem: Fix i. Let $R(\delta_i)$ be the largest possible asymptotic rate (k/n) of a code with normalized i-distance $d_i/n = \delta_i$. Then

$$R(\delta_i) \leq 1 - H(\lambda),$$

where $H(\cdot)$ is the binary entropy function and λ is the smallest root of

$$\delta_i = 1 - x^{i+1} - (1-x)^{i+1}.$$

M. Daberkow:

Computing invariants of algebraic number fields.

We gave a list of major problems in the field of constructive algebraic number theory, which is due to H. Zassenhaus:

For a given number field F we are interested in the computation of the following invariants of F:



- the ring of integers or
- the unit group U_F
- the class group Cl_F and the class number h_F
- the Galois group.

Starting with the Round-2 algorithm by H. Zassenhaus, we presented some algorithms for the construction of integral basis and introduced the idea of p-maximal overorders. Based on the Round-2 we showed some improvements using p-adic numbers. If the field $F = \mathbb{Q}(\alpha)$ is generated by a root of the polynomial $f(t) \in \mathbb{Z}[t]$, one can use a factorization of f modulo p and Hensel's Lifting to get polynomials f_1, \ldots, f_k , such that the p-maximal order of $\mathbb{Z}[\alpha]$ can be derived from the p-maximal orders of $G_i := \mathbb{Q}[t]/(f_i(t))$ $(1 \le i \le k)$.

The final part of the talk dealed with the problem of relative integral bases in extensions E/F with $\mathbb{Q} \subseteq F \subseteq E$. In the case [E:F]=2 solution of this problem has been given.

F. De Clerck:

On Generalized Quadrangles minus a Hyperplane.

P.J. Cameron raised the question of finding a characterization of partial quadrangles which have linear representations. An almost complete answer was given by R. Calderbank et al., the proof was a number-theoretic one. Here we discussed a more general class of geometries. S_p , those coming from generalized quadrangles S by deleting the set p^{\perp} , for a point p of S. S_p has the property that it is a (0,1)-geometry, but the graph is only strongly regular (hence S_p is a partial quadrangle) if $t = s^2$. S_p has the property (*):

If L and M are two disjoint lines of S_p , then there are either 0, s-1 or s lines of S_p concurrent to both L and M.

Our result (joint work with Hendrik Van Maldeghem) is:

If $T_n^*(K)$ $(n \ge 3)$ is a linear representation of a (0,1)-geometry satisfying (*), and if s > 2, then $T_n^*(K)$ is the partial quadrangle $T_3^*(O)$, O an ovoid.

J. Doven:

Lotto Numbers and Steiner Systems.

The Lotto number L(n, k, l, t) is defined as the smallest number of k-subsets (= Lotto tickets) of the set $N = \{1, 2, ..., n\}$ such that any l-subset of N meets at least one of them in at least t elements (this l-subset is interpreted as the set of winning numbers). We gave lower and upper bounds for the numbers L(42, 6, 6, t) and L(49, 6, 6, t) corresponding to the Lotto as played

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in Belgium and Germany. The upper bounds often use Steiner systems. For example, using Denniston's S(4,6,27), we get $1014 \le L(49,6,6,4) \le 13120$ (after the talk, this upper bound was lowered to 3784 by Andries Brouwer).

D. A. Drake:

A Theorem on Squeezes in the Game of Bridge.

Let p be the number of cards remaining per hand, and let r be the number of tricks which can run by North/South in a no-trump contract. Then North/South can only "squeeze" East/West out of another trick if r=1 and p=3 or if $p\leq 2r$. There are no squeezes with p<3, and the inequality is best possible for every p with $0\leq p\leq 1$. It is also true that the only no-trump squeezes with $0\leq r$ are triple squeezes.

T. Etzion:

Generalized designs and constant weight codes over finite alphabets.

We consider optimal constant weight codes over finite alphabets. These codes can be considered as generalization of Steiner systems. Other codes are reductions of orthogonal arrays into constant weight codes. Finally, in some of these codes two types of Steiner systems exist, generalized and non-generalized.

M. van Eupen:

An optimal ternary [69,5,45] code and related codes.

A ternary [69, 5, 45] code is constructed, thus solving the problem of finding the minimum length of a ternary code of dimension 5 and minimum distance 45. Furthermore, this code is shown to be a unique two-weight code with weight enumerator $1 + 210z^{45} + 32z^{54}$. It is also shown that a ternary [70, 6, 45] code, which would have been a projective two-weight code giving rise to a new strongly regular graph, does not exist. In order to prove the main results, the uniqueness of some other optimal ternary codes with specified weight enumerators is also established.

D. Hachenberger:

On Finite Elation Generalized Quadrangles with Symmetries.

We consider the structure of finite groups which act as elation groups with symmetries on finite generalized quadrangles. Such a group is related to the translation group of a translation transversal design with parameters

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depending on that of the quadrangle. Using results on the structure of p-groups which act as translation groups on a transversal design, and results on the index of the Hughes subgroup of a finite p-group, we can show how restricted the structure of an elation group with symmetries of some finite generalized quadrangle is. One of our main results is the following:

If G is a finite group of even order s^2t which admits a 4-gonal family $(\mathcal{F}, \mathcal{F}^*)$ of type (s, t), and if one member of \mathcal{F} is a normal subgroup of G, then, necessarily, G is an elementary abelian 2-group?

W. Haemers:

Spreads in Strongly Regular Graphs.

Let G be a strongly regular graph with eigenvalues k, r and s (k > r > s). Delsarte proved that a clique has at most 1 - k/s vertices. Cliques with this size are called lines. A spread of G is a partition of the vertex set into lines. Such a spread gives rise to a 3-class association scheme. A necessary condition for the complement of G to have a spread is $kr \ge s^2$. Examples come from spreads and fans in (partial) geometries. An interesting example was found by Tonchev, who found spreads in the McLaughlin graph.

R. Hill:

"Mastermind" for four-year-olds.

We consider some variants of the game of "Mastermind" including the following version, which is suitable for play with 4-year-olds.

A coder chooses a codeword which is a permutation (i.e. no repetitions) of k colours from n. A guesser must determine the codeword by making guesses, each guess being a permutation. After each guess the coder says which colours are correct. Let f(n,k) be the smallest number of guesses needed to determine any codeword. Clearly $f(n,k) \leq n$ and we conjecture that f(n,k) = n for all n and k ($k \leq n$). The conjecture is easily proved for $k \leq 4$ (all n) and for n = k.

J.W.P. Hirschfeld:

Projective Geometry Codes.

Let M be a $b \times v$ incidence matrix of $v = |PG^{(0)}(n,q)|$ points and $b = |PG^{(r)}(n,q)|$ subspaces of dimension r for a finite projective space PG(n,q). Let C(r,n,q) be the code generated over \mathbb{F}_p , p prime where $q = p^h$, by the rows of M and let $C^*(r,n,q)$ be the dual code, which therefore represents sets of weighted points meeting each r-space in a multiple of p points. Hamada's formula says that



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$$\dim C^{\bullet}(r, n, q) = \frac{q^{n+1} - 1}{q - 1} - \sum_{\substack{(s_0, \dots, s_b) \ j = 0}} \prod_{j=0}^{h-1} \sum_{i=0}^{L(s_{j+1}, s_j)} (-1)^i \binom{n+1}{i} \binom{n+s_{j+1}p - s_j - ip}{n},$$

where the first sum is over all ordered sets (s_0,\ldots,s_h) of h+1 integers s_j such that

$$s_h = s_0, \ 0 \le s_i \le n - r, \ 0 \le s_{i+1}p - s_i \le (n+1)(p-1),$$

and

$$L(s_{j+1},s_j) = \lfloor (s_{j+1}p - s_j)/p \rfloor.$$

It is shown that, when h=1, this formula can be replaced by a single summation:

$$\dim \ C^{\bullet}(r,n,p) = \sum_{j=0}^{r-1} (-1)^{j} \left(\begin{array}{c} (r-j)(p-1)-1 \\ j \end{array} \right) \left(\begin{array}{c} n+(r-j)p-r \\ n-j \end{array} \right).$$

C.Y. Ho:

Regular collineation groups of a finite projective plane.

We mention the following results:.

- (1) A characterization of an abelian planar Singer group.
- (2) The multiplier group of a planar Singer group always fixes a line.
- (3) Let H be an abelian group of multipliers of a planar Singer group of planar order n. If |H| = n + 1, then $n^2 + n + 1$ is a prime. If |H| is odd, then |H| is at most n + 1.
- (4) For any complex number x, let $v(x) = x^2 + x + 1$. If p is any prime different from 3 and m is any positive integer, then $v(m^p) = \prod_{\sigma \in \langle \theta \rangle} v(m\sigma)$, where θ is a complex p-th root of unity. Also the greatest common divisor of v(m) and $v(m^p)/v(m)$ divides p.
- (5) Suppose an abelian Singer group is not normal in the collineation group. If the planar order n is not a square or $n=m^2$ with $m\equiv 2.3$ (mod 4), then the plane is Desarguesian.





J.D. Key:

Extending Steiner triple systems using codes.

We use the linear code associated with a design by forming the space spanned by the incidence vectors of the blocks over a finite field \mathbb{F}_p , where p is a prime, to give some partial answers to the following question: Can every Steiner triple system be extended?

Let \mathcal{D} be a 2-(v,3,1) design and let $C_p(\mathcal{D})$ denote its code over \mathbb{F}_p . Then if p=2 and $2^d-1 \leq v < 2^{d+1}-1$, $C_2(\mathcal{D}) \geq \mathcal{H}_d$, a Hamming code: if p=3 and $3^d-1 \leq v < 3^{d+1}$, $C_3(\mathcal{D}) \geq \mathcal{R}_{\mathbb{F}_3}(2(d-1),d)$, a generalized Reed-Muller code. From this we have:

- (i) If $v = 2^d 1$ and $\dim(C_2(\mathcal{D})) = 2^d d$, then \mathcal{D} can be extended and in such a way that the binary code of the extension is the extended code:
 - (ii) if $v = 3^d$ and $\dim(C_3(\mathcal{D})) = 3^d d$, then \mathcal{D} can be extended.

I. Landgev:

Nonexistence of some Quaternary Codes.

We demonstrate the nonexistence of quaternary codes with parameters [56, 4, 41], [104, 4, 77].

This result violates also codes with parameters [57.4.42], [105.4.78], [106.4,79], [107.4.80].

Thus the only value of d, for which $n_4(4,d)$ remains unknown is d=37.

S.L. Ma:

Regular Automorphism groups on partial geometries.

Suppose a partial geometry $pg(s+1,t+1,\alpha)$ admits an automorphism group G acting regularly on the points so that the points of the geometry can be identified with the elements of G. Let L_0, L_1, \ldots, L_t be the lines passing through e. I find that G is a direct product of two groups of relatively prime orders and if $L_i = L_i^{(-1)}$ for all i, then the geometry is a translation net. Applying the result to the case when s = t and G is abelian, I find that either the geometry is a translation net or all the lines of the geometry are of the form $gL_0, g \in G$. Also, for the latter case, all known examples have parameters $\alpha = s$ or s+1 except for the pg(6,6,2) obtained by van Lint and Schrijver which has $(s,\alpha) = (5,2)$. I have checked that this is the only example if $s \leq 500$.



S. S. Magliveras:

Block Transitive Resolutions of t-designs and Room Rectangles.

(Joint work with R.A. Liebler and S.V. Tsaranov)

We call a partition of the trivial design $\binom{K}{k}$ of all k-subsets of a v-set X into $t-(v',k,\lambda)$ designs, $v' \leq v$, a resolution of t-designs. A resolution of t-designs with v=v' is also called a large set of t-designs. A Room rectangle R, based on $\binom{K}{k}$ is a rectangular array whose nonempty entries are k-sets. This array has the further property that taken together the rows (columns) form a resolution of t_1 - (t_2) -designs. One of these structures admits G as a block transitive automorphism group if G is a permutation group on the set X leaving invariant the structure and the k-sets of X fall in a single G-orbit. Some examples of block transitive resolutions of nontrivial t-designs $t \geq 2$ are:

- (1) an M_{11} -invariant set of 3 (10, 4, 1) designs,
- (2) an M_{12} -invariant set of 4 (11, 5, 1) designs,
- (3) an M_{24} -invariant set of 2-(21,5,1) designs,
- (4) a $P\Gamma L_2(2^s)$ -invariant set of $3 (2^s, 4, 1)$ designs (s = 3 or 5),
- (5) a $P\Gamma L_2(32)$ -invariant set of 2-(16,4,1) designs, and
- (6) a variety of PSL₂(q)-invariant sets of 2-designs with k = 3.
 We show that this is a complete list. In particular there are no block tran-

We show that this is a complete list. In particular there are no block transitive large sets of t-designs. Moreover, if $1 \neq a < b < c$ are odd integers such that gcd(a,b) = 1 and ab|c, then we construct a block transitive Room rectangle based on the 3-subsets of a $7^c + 1$ -set whose rows (columns) are Steiner triple systems on 7^a (7^b) point.

K. Metsch:

Characterization of certain distance regular graphs.

We present some recent results that characterize the following distance regular graphs in terms of their parameters.

- (1) The folded Johnson graphs $\overline{J}(2m, m)$, $m \geq 6$.
- (2) The folded halved cubes of diameter $d \geq 8$.
- (3) The Grassmann graphs $\Gamma(e,q,n)$ for $q \ge 4$, $2 < e < \frac{n-1}{2}$.

G. E. Moorhouse:

New distance regular graphs related to Preparata codes.

(Joint work with D. de Caen and R. Mathon.)

We present a new family of antipodal distance regular graphs $\Gamma(q,\sigma)$ of diameter three, related to the classical Preparata codes. Here $q=2^{2t-1}$ and

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 $\langle \sigma \rangle = \operatorname{Aut}(GF(q))$. The graph $\Gamma(q,\sigma)$ is a q-fold cover of K_{2q} . For $q \geq 2^5$, we have $|\operatorname{Aut}\Gamma(q,\sigma)| = 2q(q-1)(2t-1)$, and $\Gamma(q,\sigma') \cong \Gamma(q,\sigma)$ iff $\sigma' = \sigma^{\pm 1}$. These results are analogous to those of Kantor (1983) for extended Preparata codes. Moreover, the Preparata codes may be synthetically produced from our graphs. The three-class association scheme for $\Gamma(q,\sigma)$ is formally dual to the scheme for the known systems of linked symmetric designs arising from Kerdock sets. This formal duality appears to be related to the formal duality between Preparata and Kerdock codes. It is hoped that the algebraic duality between \mathbb{Z}_4 -linear Preparata and Kerdock codes (recently discovered by Sloane, Calderbank et al.) may help to explain this.

R. Mullin:

The Structure of Normal Bases over Finite Fields and some Specific Completely Normal Polynomials.

A normal basis of a finite field $GF(q^n)$ over GF(q) can be defined to be a set S of zeroes of an irreducible polynomial of degree n over GF(q) which has the additional property that S is linearly independent. Such bases have applications in certain aspects of Communications Engeneering. In this talk, an example of such an application will be used to motivate the investigation of normal bases. A structural characterization of such bases will be given, and this will be used to construct infinite families of completely normal polynomials, that is, polynomials whose zeroes contain normal bases for all subfields between GF(q) and $GF(q^n)$.

S. E. Payne:

New Hyperovals in PG(2,q).

Let $q = 2^e$, F = GF(q). A q-clan is a set

$$C = \{A_t = \begin{pmatrix} x_t & y_t \\ 0 & z_t \end{pmatrix} : t \in F\}$$

of $q \ 2 \times 2$ matrices over F for which $A_s - A_t$ is anisotropic for all $s, t \in F$, $s \neq t$. Since $q = 2^e$, a q-clan gives a generalized quadrangle of order (q^2, q) with subquadrangles of order q associated with ovals in PG(2, q). As for all prime powers q there is also a flock of a quadratic cone, a line spread of PG(3, q) and a translation plane. For each $q = 2^e$ there is a new so-called Subiaco q-clan. When $e \not\equiv 2 \pmod{4}$, i.e., 5 does not divide q+1, there is, up to projective equivalence, just one oval with a stabilizer of order 2e. When $e \equiv 2 \pmod{4}$, there are two Subiaco ovals, one with stabilizer isomorphic



to $C_5 \rtimes C_{2e}$, the other with stabilizer isomorphic to $C_5 \rtimes C_{e/2}$. We give here the one with the larger group, since it is the only case where the description is rather easy to give. Here $\mathrm{GF}(4) \leq F$, so let $w \in F$ satisfy $w^2 + w + 1 = 0$. Then the oval has the form $\mathcal{O} = \{(1,t,f(t)): t \in F\} \cup \{(0,1,0)\}$, with nucleus (0,0,1), where $f(t) = \frac{w^2(t^4+1)}{(t^2+wt+1)^2} + t^{\frac{1}{2}}$.

K. Phelps:

Perfect Codes and Quadruple Systems.

We consider the question of whether every Steiner triple system of order 15 occurs as the words of weight 3 of some nonlinear perfect binary code of length 15. We review related results and describe an approach to answering this question. We also present some new discoveries.

V. Pless:

Generators for quadratic residue codes over various fields.

A class of cyclic codes called quadratic residue (Q.R.) codes over GF(q) exist at all prime length p where q is a square (mod p) and $\gcd(p,q)=1$. For such a pair of p and q we define $Q(x)=\sum x^i$ where i ranges over the quadratic residues in GF(p) and $N(x)=\sum x^i$ for i a non-residue. The idempotent generator of any Q.R. code is of the form $a_0+a_1Q+a_2N$. We give explicit values of a_0,a_1,a_2 for Q.R. codes over fields of characteristic 2 and 3 and also for GF(q) where q is odd, the length of p is $\equiv 3 \pmod 4$, and q divides p+1.

A. Pott:

A new class of symmetric designs.

(Joint work with D. Jungnickel.)

We describe a construction of symmetric (v,k,λ) -designs with parameters

$$v = p^{s} \frac{q^{2m}-1}{q-1}, k = p^{s-1} q^{2m-1} \text{ and } \lambda = p^{s-1} q^{2m-2} \frac{p^{s-1}-1}{p-1}$$

of order $p^{2s-2}q^{2m-2}$ provided that p is a prime and q is a prime power with $q = \frac{p'-1}{p-1}$.

M. J. de Resmini:

Hyperovals in Figueroa Planes.

(Joint work with Nicholas Hamilton, U.W.A. Perth.)

We construct Hyperovals in the Figueroa planes of order q^3 , q a power of 2, which are inherited from regular hyperovals in the Desarguesian plane



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 $\operatorname{PG}(2,q^3)$ which are stabilized by the automorphism α of order 3. $\langle \alpha \rangle = \operatorname{Aut}_{\mathbb{F}_q}(\mathbb{F}_{q^3})$ which provides the construction of the Figueroa plane.

M.A. Shokrollahi:

Stickelberger Codes.

Let p be an odd prime. The Stickelberger ideal S_p is an ideal in the integral group ring $\mathbb{Z}[G]$, G being the Galois group of the cyclotomic field $\mathbb{Q}(\zeta_p)/\mathbb{Q}$. S_p annihilates the class group of $\mathbb{Q}(\zeta_p)$. Let q be another prime which is not a divisor of p-1 (q=p possible!). We study the cyclic code (S_p mod $q) \leq \mathbb{F}_q[G]$ of length (p-1) and show that its dimension is closely related to the relative class number of $\mathbb{Q}(\zeta_p)$.

G. Simonyi:

Entropy and Uniform Hypergraphs.

Hypergraph entropy is an information theoretical functional on a hypergraph with a probability distribution on its vertex set. It is sub-additive with respect to the union of hypergraphs. In case of simple graphs, exact additivity for the entropy of a graph and its complement with respect to every probability distribution on the vertex set gives a characterization of perfect graphes.

Here we investigate uniform hypergraphs with an analogous behaviour of their entropy. The main result is the characterization of 3-uniform hypergraphs having this entropy splitting property. Partitioning the edge set of the complete uniform hypergraph into more than two parts with similar criteria is also discussed. For k larger than 3 it turns out that no non-trivial k-uniform hypergraph splits entropy in the above manner.

E. Spence:

Hadamard Matrices of order 28.

We report on an independent verification of a recent result of Kimura that there are precisely 487 pairwise non-isomorphic Hadamard matrices of order 28. We consider also the classification of skew Hadamard matrices of order 28 (in total there are 54) and mention an application of these to the possible construction of a symmetric (81, 16, 3) design, using an idea of Tonchev.

T. Szönyi:

Cyclic caps.

Consider a cyclic Singer group S of a Desarguesian projective space PG(n,q).





and let H be a subgroup of S. We concentrate on the following question:

When are the point-orbits of H caps (complete caps)?

Using elementary properties of fields we give a short proof of some theorems by Ebert. Keeping |H| fixed, these orbits are always caps if the characteristic of GF(q) is large enough compared to |H|.

J.A. Thas:

Symplectic spreads in PG(3,q), inversive planes and projective planes.

A (line) spread in PG(3,q) is any set of $q^2 + 1$ disjoint lines in PG(3,q). The spread S is called symplectic if all lines in S are totally isotropic for some symplectic polarity ζ of PG(3,q). An ovoid of PG(3,q), q > 2, is a set of $q^2 + 1$ points, no three of which are collinear; an ovoid of PG(3,2) is the same as an elliptic quadric. An ovoid of the nonsingular quadric Q(4,q) of PG(4,q) is any set O of points of Q(4,q) which has exactly one point in common with each line of Q(4,q). Symplectic spreads of PG(3,q) and ovoids of Q(4,q) are equivalent objects, and, for q even, also ovoids of Q(4,q) and ovoids of PG(3,q) are equivalent objects.

With each symplectic spread of PG(3,q) there corresponds a plane of order q^2 . A $3-(q^2+1,q+1,1)$ design is called an inversive plane of order q. With each ovoid of PG(3,q) there corresponds an inversive plane of order q.

Here we survey some important characterizations of inversive planes, the inversive planes of small order, the planes of small order arising from symplectic spreads of PG(3,q), and all known classes of ovoids of PG(3,q) resp. Q(4,q).

V.D. Tonchev:

- (a) Preparata Codes and a Class of 4-Designs.
- (b) The Existence of extremal self-dual [50,25,10] codes and quasi-symmetric 2 (49,9.6) designs.
- (a) An extension theorem for t-designs is proved. As an application, a class of $4 (4^m + 1, 5, 2)$ designs is constructed by extending designs related to 3-designs formed by the minimum weight vectors in the Preparata code of length $n = 4^m$, $m \ge 2$. The derived designs are doubles of AG(2m, 2). Although these 4-designs contain repeated blocks, they provide an infinite class of 4-designs with the smallest known fixed λ .
- (b) All extremal binary self-dual [50, 25, 10] codes with an automorphism of order 7 fixing one coordinate are enumerated. The minimum weight



codewords yield (previously unknown) quasi-symmetric 2-(49, 9, 6) designs.

S.A. Vanstone:

Graphs, Codes and Difference Sets.

Let G be a finite graph having p vertices, q edges and girth g. It is well known that the cycle space of G gives rise to a binary [q,q-p+1,g]-code C. In this talk we discuss various properties of these codes. For example, when p is even and $p \geq 2g$ (p odd and $p \geq 2g+1$) then one can always embed C in a binary [q,q-p+2,g]-code. This is accomplished by showing that every connected graph with an even number of vertices contains an odd spanning subgraph. We also determine when the code C coming from the complete graph is contained in a Hamming code. This result relies on the existence of certain difference sets in the elementary abelian 2-group. We conclude the talk with the discussion of constructing ternary codes from directed graphs and various open problems.

W. Willems:

Radical Codes.

By results of Berman and Charpin, Reed-Muller Codes over the prime field \mathbb{F}_p may be considered as powers of the Jacobson radical of an elementary abelian p-group C_p^m . From this point of view, one easily sees that the affine general linear group $\mathrm{AGL}(m,p)$ acts on the radical powers $J^r(\mathbb{F}_pC_p^m)$. Moreover, via this action $\mathbb{F}_pC_p^m$ becomes a uniserial $\mathbb{F}_p\mathrm{AGL}(m,p)$ -module. As a consequence we have the Theorem (Knörr, Wi.):

Let C be a linear code over \mathbb{F}_p of length p^m . Then $\mathrm{AGL}(m,p) \leq \mathrm{Aut}(C)$, if and only if C is a Reed-Muller Code.

This explains why the Reed-Muller Codes $J^{k(p-1)}(\mathbb{F}_pC_p^m)$ $(1 \le k \le m)$ are exactly the duals of the affine geometry codes. Now, from the point of view of group theory, there is no reason to restrict ourself to elementary abelian p-groups and the prime field. So let K be any finite field of characteristic p and let G be a finite p-group. Let U be a subgroup of G with $|G:U|=p^m$ and consider the radical powers $1_G^UJ^r(KG)$ of the permutation module 1_G^U endowed with the natural basis $\{Ug|g\in G\}$. With this notation, a student of mine, A. Faldum, proved:

Theorem: (a) If dim $1_U^G J^r(KG) \stackrel{>}{\geq} \dim J^s(KC_p^m) \geq 1$, then for the minimum distances we have $d(1_U^G J^r(KG)) \stackrel{<}{\leq} d(J^s(KC_p^m))$.

(b) Suppose equality holds everywhere and $\dim J^{s}(KC_{p}^{m}) \notin \{0,1,p^{m} - 1\}$



1, p^m }. Then either (1) $U \subseteq G$ and $G/U \cong C_p^m$ or (2) p=2, r=2s-1 and some conditions on the group G.

Remark: The Code ${}^{G}_{U}J^{r}(KG)$ in the latter case is equivalent to $J^{s}(KC_{p}^{m})$ for r=2s-1.

Summarizing the above results we see that in the large class of radicals of permutation modules over p-groups, the radical powers of a group-algebra over an elementary abelian p-group (which are the Reed-Muller Codes for $K = \mathbb{F}_p$) are optimal.

R.M. Wilson:

Two-error-correcting codes and absolutely irreducible polynomials over GF(2).

Let $n=2^r+1$, let ω be a primitive element in $GF(2^r)$, and let C_t denote the set of binary polynomials f(x) of degree $\leq n$ so that $f(\omega)=f(\omega^t)=0$. That is, C_t is the binary cyclic code of length n generated by $m_1(x)m_t(x)$.

Words of weight ≤ 4 in C_t correspond to zeros of the projective plane curve

$$g_t(x, y, z) = \frac{x^t + y^t + z^t + (x + y + z)^t}{(x + y)(x + z)(y + z)}$$

with distinct coordinates over $GF(2^r)$. Factorizations of $g_t(x,y,z)$ for $t=2^i+1$ and $t=2^{2^i}-2^i+1$ can be used to prove that such words do not exist, i.e. that C_t is two-error-correcting, when t has either of these forms with (i,r)=1.

We conjecture that $g_t(x,y,z)$ is absolutely irreducible if t does not have one of these two forms. If $g_t(x,y,z)$ is absolutely irreducible, then the codes C_t are two-error-correcting for at most finitely many values of $n=2^r-1$. We prove the absolute irreducibility of $g_t(x,y,z)$ for $t\equiv 3 \pmod 4$, t>3. This is joint work with H. Janwa and G. McGuire.

R.M. Wilson:

Blanchard's theorem on asymptotic existence of transversal designs of strength $t \ge 3$.

This talk describes a recent result of John Blanchard. A transversal design TD(t, k, n) consists of a set X of kn points partitioned into k groups G_1, G_2, \ldots, G_k of size n and a family A of transverse k-subsets so that every transverse t-subset of X is contained in exactly one member of A. Here, a transverse subset of X is one that meets each group G_i in at most one point.



Transversal designs $\mathrm{TD}(2,k,n)$ are equivalent to k-2 pairwise orthogonal Latin squares of order n and the Chowla-Erdös-Straus theorem asserts that these exist for all n sufficiently large with respect to k. Blanchard's theorem is that for any strength t and block size $k, t \leq k$, transversal designs $\mathrm{TD}(t,k,n)$ exist for all n > n(k).

The proof combines recursive constructions and direct constructions involving finite fields. We specifically discuss a technique that may be called 'spreading blocks' where a family of subsets A of a set U that covers some t-subsets q times and others not at all may be 'lifted' to a family of transverse subsets of $U \times V$ that covers uniquely those and only those transverse t-subsets that project onto covered subsets of U, when q is a prime power and where V is a vector space over $\mathrm{GF}(q)$ of dimension $d \geq \binom{|U|}{r}$.

V.A. Zinoviev:

On Preparata-like Codes and 2-Resolvable Steiner Quadruple Systems.

(Joint work with A.R. Calderbank.)

A binary code with length $n=4^m$, $m=2,3,\ldots$, minimal distance d=6 and cardinality $N=2^{4^m-4m}$ we call (extended) Prepatrata-like code and denote it by P. A binary code with parameters $n=2^n$, d=4, $N=2^{n-1-n}$ we call (extended) Hamming-like code and denote it by H. We have several new statements.

Theorem 1: Let P be any Preparata-like code of length $n=4^m$. Then the Hamming-like code H which contains P (for any P there is always a H such that $P \subset H$) is partitioned into P and its translates. This partition is completely regular in this Hamming code H.

Theorem 2: Let P be any Preparata-like code of length n. Then this code implies the existence of a 2-resolvable Steiner system S(3,4,n).

Theorem 3: Let P and P' be any two known nonisomorphic Preparatalike codes of length n, $n = 4^m$, m = 3.4,... Then these codes induce nonisomorphic 2-resolutions of the same Steiner system S(3.4,n) (which are planes of the affine geometry AG(2m.2)).

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