MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 19/1994

Buildings in Differential and Topological Geometry
1.-7. Mai 1994

The conference was organized by Rainer Löwen (Braunschweig), Helmut Salzmann (Tübingen) and Viktor Schroeder (Zürich). The intention of the meeting was to bring together people working in different fields and having a common interest in the theory of buildings. The participants were mathematicians working in differential geometry or in topological geometry, or in the abstract theory of buildings. Numerous stimulating talks showed the rôle of buildings and, in particular, topological buildings, in different geometric contexts. There was ample opportunity for discussions in smaller groups and for further exchange of ideas between the various groups of persons. As a result, everyone could form a clearer picture of some interrelations of those geometric theories, and some projects for joint papers emerged.

Vortragsauszüge

Rank and holonomy of Riemannian manifolds
Jost–Hinrich Eschenburg, Augsburg

A complete Riemannian manifold M has rank k if any geodesic lies in a k-flat and k is maximal with this property. A k-flat in M is an isometric totally geodesic immersion of $\mathbb{R}^k$ into M.

Theorem (C. Olmos, E.).
Let M be compact of rank k and suppose that the isometric group $I(M)$ acts transitively on the set of k-flats. Then M is locally symmetric. If in addition all k-flats are embedded flat tori, then M is globally symmetric.

The proof relies on local versions of the following theorems:

Theorem A. *Let M be analytic and irreducible with rank $k \geq 2$. Suppose that all k-flats are isometric tori and that there is a geodesic with not more than k linearly independent parallel Jacobi fields. Then M is locally symmetric.*

Theorem B. *Let M be complete with rank 1 such that all geodesics are congruent. Then M is symmetric of rank 1.*

The local version of Theorem A does not use analyticity. The proof of Theorem A uses the Theorem of Berger and Simons saying that an irreducible space is locally symmetric if its holonomy group has principal orbits of codimension ≥ 2.

In the proof of Theorem B one shows that M is two-point homogeneous.

Topological planes: an introduction

Rainer Löwen, Braunschweig

We consider stable n-spaces, that is, atomar, semimodular lattices of dimension n satisfying certain continuity and stability conditions as displayed by the subspace lattices of real projective or hyperbolic spaces. The topology is assumed to be locally compact and locally connected. If the geometric dimension n is at least 3, then Groh has shown that an embedding into some "classical" projective space $P_n\mathbb{F}$ over $\mathbb{F} \in \{\mathbb{R}, \mathbb{C}, \mathbb{H}, \mathbb{O}\}$ exists. Hence we concentrate on the planar case $n = 2$, where countless nonclassical examples exist. We describe the introduction of coordinates from a ternary field. Using them, one can show that lines are homology manifolds of dimension $l \in \{1, 2, 4, 8\}$. From the point of view of algebraic topology, each plane behaves like some (open subplane of) $P_2\mathbb{F}$. This fact (referred to as "classical domination") emerged from the work of Salzmann, Freudenthal, Breitsprecher, Dugundji, and Löwen. Algebraic laws satisfied by a coordinatizing ternary field imply the existence of special ("axial") automorphisms. Among other things, this leads to the theory of translation planes, which is to some extent a "linear" theory (i.e., uses vector space structures) and allows strong classification results (Betten, Hähl).

The automorphism group Σ of a plane is a locally compact group (Löwen). If it is transitive on the point set or on two line pencils, then the plane is a classical projective or affine or hyperbolic plane or a simple modification of the real hyperbolic plane (work of Salzmann and Löwen). The dimension $d = \dim \Sigma$ is considered as a measure of homogeneity of the plane. For a given number l, there is a critical value d_l such that only the classical projective plane has $d > d_l$, and all $2l$-dimensional projective planes with $d \geq d_l - 1$ are known; this is mainly due to Salzmann, Hähl, and Betten. Stable planes carrying a compatible structure of a symmetric space (non-Riemannian in general) may be treated via "infinitesimal models", like Lie groups. There are strong classification results, due to Löwen, Seidel, and Löwe.

Classification of compact, connected translation planes
Hermann Hähl, Kiel

H. Salzmann's classification program for compact, connected topological projective planes consists in determining all such planes whose automorphism group Σ has comparatively large dimension (cf. the talk by R. Löwen; Σ is a locally compact group of finite topological dimension). In this talk, the rôle of translation planes in the classification is outlined, and pertaining results for translation planes are given.

Compact, connected translation planes can be described as projective completions of affine planes with point set $\mathbb{R}^{2l}$, $l \in \{1,2,4,8\}$, whose lines are the elements of a spread $\mathcal{L}_0$ of l-dimensional linear subspaces of $\mathbb{R}^{2l}$ and their cosets; $\mathcal{L}_0$ is closed in the Graßmann manifold $G_l\mathbb{R}^{2l}$. Such affine planes are coordinatized by locally compact, connected quasifields. For $l = 1$, only the classical plane over $\mathbb{R}$ results. For $l \geq 2$, however, there is a plethora of such planes. Classification programs for these planes have been carried out for $l = 2$ by Betten and for $l \in \{4,8\}$ by myself.

In the talk, a specimen of such a classification result is given in the case $l = 8$. All 16-dimensional compact, connected translation planes satisfying $\dim \Sigma \geq 38$ are listed. Among them is the classical octonian plane $P_2\mathbb{O}$, with $\dim \Sigma = E_6(-26)$ of dimension 78. The other planes in the list satisfy $\dim \Sigma \in \{38, 39, 40\}$. For compact, connected 16-dimensional projective planes in general, it is conjectured that $\dim \Sigma \geq 38$ forces the plane to be a translation plane; then the classification result will cover the general case. For $\dim \Sigma \geq 40$, this has been proved by Hubig and in a different way by M. Lüneburg, on the basis of results by Salzmann.

Similar and even more complete classification results have been obtained for $l = 2$ by Betten and Salzmann and for $l = 4$ by Salzmann and myself.

Actions of Cartan subgroups, affine buildings and symbolic dynamics
Shahar Mozes, Jerusalem

Let G be a semisimple Chevalley group defined over $\mathbb{Q}_p$, $H < G$ a maximal split Cartan subgroup, $\Gamma < G$ a torsion free uniform lattice and μ the G-invariant Borel probability measure on $\Gamma\backslash G$. We are interested in studying the dynamical system $(\Gamma\backslash G, \mu, H)$ where H acts via translations $T_h : \Gamma\backslash G \to \Gamma\backslash G$, $T_h(\Gamma_g) = \Gamma_g h$.

Using the affine Bruhat-Tits Building Δ associated with G we construct a subshift of finite type (Ω, ν, H) which is a factor of $(\Gamma\backslash G, \mu, H)$. Let $\mathcal{A} \subset \Delta$ be the apartment on which H acts by translations. Let $\Omega = \{\omega : \mathcal{A} \to \Gamma\backslash\Delta |\ \omega$ is locally an isometry $\}$, note that $\Gamma\backslash\Delta$ is a finite complex. Let $M < H$ be the maximal compact subgroup of H. M is the stabilizer of $\mathcal{A}$ in G. The group H acts on Ω via its action on $\mathcal{A}$, thus the action factors through the action of $H\backslash M \cong \mathbb{Z}^d$ where $d = \operatorname{rank} G$. Let $\varphi : \Gamma\backslash G \to \Omega$ be defined by $\varphi(\Gamma g) = \pi \circ g|_{\mathcal{A}}$ where π is natural covering map from Δ to $\Gamma\backslash\Delta$. The map φ gives a surjective factor map commuting with the H action. Let $\nu = \varphi_*\mu$ be the induced measure on Ω. The fibers of φ are M orbits and φ induces a homeomorphism between $\Gamma\backslash G/M$ and Ω.

The subshift of finite type (Ω, ν, H) may be used to study various dynamical properties of $(\Gamma\backslash G, \mu, H)$.

A particular question is the structure of closures of H-orbits. It is conjectured (see Margulis ICM 90 lecture) that when $\operatorname{rank} G \geq 2$ and Γ satisfies some further conditions (every semisimple

$\gamma \in \Gamma$ is regular) then each closure of an orbit $\overline{\Gamma \times H}$ is itself an orbit of a, possibly, larger subgroup. We have the following result:

Theorem.
Let $G = PGL_2(\mathbb{Q}_p) \times PGL_2(\mathbb{Q}_l)$, $\Gamma < G$ an irreducible lattice and

$$H = \left\{ \left(\begin{pmatrix} * & 0 \\ 0 & * \end{pmatrix}, \begin{pmatrix} * & 0 \\ 0 & * \end{pmatrix} \right) \right\} \subset G.$$

If the closure $\overline{\Gamma_g H}$ contains a closed H-orbit then $\Gamma_g H$ is either dense or closed.

Let Ω be the subshift of finite type constructed for $(\Gamma \backslash G, \mu, H)$ as in this theorem then we can reformulate the theorem by:

Theorem'.
Every point $\omega \in \Omega$ such that the closure of its orbit contains a periodic orbit is either dense or periodic.

Let $p, l \equiv 1 \pmod 4$ be distinct primes. Let

$$\tilde{\Gamma} = \left\{ x = x_0 + x_1 i + x_2 j + x_3 k \;\middle|\; \begin{array}{l} x_i \in \mathbb{Z},\ x \equiv 1 \pmod 2,\ |x|^2 = p^* l^*, \\ x_0, x_1, x_2, x_3 \text{ relatively prime} \end{array} \right\}.$$

From $\tilde{\Gamma}$ one obtains an irreducible lattice Γ in $G = PGL_2(\mathbb{Q}_p) \times PGL_2(\mathbb{Q}_l)$. As a corollary of the theorem and its proof we obtain the following:

Corollary.
Let $\alpha, \beta \in \tilde{\Gamma}$ be such that $|\alpha|^2 = p^*$, $|\beta|^2 = l^*$ then either

1) $\alpha\beta = \pm\beta\alpha$ *or*

2) $\forall x \in \tilde{\Gamma}, \exists n > 0$ *s.t.* $\alpha^n\beta^n = uxv$ *for some* $u, v \in \tilde{\Gamma}$.

Minkowskian subspaces of non-positively curved metric spaces

Brian H. Bowditch, Southampton

Let (X, d) be a metric space. A *geodesic* in X is a path $\alpha : [0,1] \to X$ such that $d(\alpha(t), \alpha(u)) = k|t - u|$ for all $t, u \in [0,1]$, and for some constant $k \geq 0$. We say that X is *a Busemann space* if every pair of points are joined by a (unique) geodesic, and if, for any pair of geodesics $\alpha, \beta : [0,1] \to X$, the map $[t \mapsto d(\alpha(t), \beta(t))]$ is convex. Examples include Minkowskian n-spaces ($\mathbb{R}^n$ with metric d given by $d(x,y) = \Phi(x - y)$ for some norm $\Phi : \mathbb{R}^n \to (0, \infty)$) and CAT(0) spaces, including Hadamard manifolds. We show:

Theorem. *Suppose (X, d) is Busemann, and admits a discrete cocompact isometric action by a group Γ. Then either X is hyperbolic in the sense of Gromov (so that Γ is word hyperbolic), or else it contains an embedded totally geodesic Minkowskian plane.*

The special case where (X, d) is CAT(0) was stated by Gromov, and detailed proofs have been given independently by several people including Heber and Bridson. In this case a Minkowskian plane is necessarily Euclidean.

An important outstanding question is whether the existence of such a plane (in either the CAT(0) or Busemann cases) implies the existence of a rank 2 free abelian subgroup of Γ.

Symmetric planes

Harald Löwe, Braunschweig

The aim of my talk is the presentation of some old and some new results in the theory of symmetric planes. For definitions and the "elder" results we refer to the articles of R. Löwen (1979).

Theorem. *Every split symmetric plane P is a shear plane. The partial spread $\mathcal{P}$ defining P is a symmetric space in a very natural way. Conversely, any shear plane associated with such a "symmetric" partial spread is a split symmetric plane.*

Local theory. Every connected split symmetric plane is uniquely determined by its tangent translation plane.

Classification. Let P be the split symmetric plane defined by the "symmetric" partial spread $\mathcal{P}$. If $\mathcal{P}$ is not semisimple (regarded as a symmetric space), then P is said to have a huge radical. All split symmetric planes with huge radical are classified. Every such plane is an open subset of some topological dual translation plane.

Remark. There exist split "locally" symmetric planes (i.e. planes looking like an open subplane of some symmetric plane) with no global extension (i.e. no open subplane of these planes is isomorphic to an open subplane of any symmetric plane as a stable plane and a locally symmetric space). Moreover, some of these planes are not isomorphic to an open subplane of any topological projective plane.

Application. Let Q be a locally compact connected planar quasifield. Assume that $((ax)a)y = a(x(ay))$ holds for any $a, x, y \in Q$. Let $\mathcal{L}_0$ be the line pencil in 0 of the affine translation plane A_2Q. Consider the subset $\mathcal{P}$ of $\mathcal{L}_0$ consisting of all lines $y = s^2x$ with $s \in Q \setminus \{0\}$. Then $\mathcal{P}$ is a "symmetric" partial spread. Moreover, $\mathcal{P}$ is not a semisimple symmetric space. Therefore, the corresponding split symmetric plane is known. It turns out that Q is either an alternative real division algebra or one of the Kalscheuer nearfields.

Smooth stable planes and their automorphism groups

Richard Bödi, Tübingen

A *smooth stable plane* $\mathcal{S} = (P, \mathcal{L})$ is a linear space $(P, \mathcal{L})$ containing a quadrangle, where P and $\mathcal{L}$ are smooth $(=C^\infty)$ manifolds in such a way that the join map $\vee : P \times P \setminus \mathrm{diag}_P \to \mathcal{L}$ and the intersection map $\wedge : \mathcal{O} \to P$ are smooth, and $\mathcal{O}$ is an open subset of $\mathcal{L} \times \mathcal{L}$. A smooth stable plane $\mathcal{S}$ is called a *smooth projective plane*, if $(P, \mathcal{L})$ is a projective plane (as an incidence structure). Identifying a line with the set of those points that are incident to it, it can be shown that every line is a closed smooth submanifold of the point space P, and dually, every line pencil $\mathcal{L}_p$ is a closed smooth submanifold of the line set $\mathcal{L}$. Moreover, the flag space $\mathcal{F}$ (= set of all incident point/line pairs) is a closed smooth submanifold of the product manifold $P \times \mathcal{L}$.

A *collineation* γ of a smooth stable plane $\mathcal{S}$ is a pair $(\gamma_P, \gamma_{\mathcal{L}})$ of continuous bijections $\gamma_P : P \to P$ and $\gamma_{\mathcal{L}} : \mathcal{L} \to \mathcal{L}$ such that $\gamma(\mathcal{F}) = \mathcal{F}$. A collineation γ is called *smooth*, if the mappings γ_P and $\gamma_{\mathcal{L}}$ are smooth.

Theorem. *Every collineation of a smooth stable plane is smooth. In particular, every stable plane admits at most one smooth structure on P and $\mathcal{L}$ such that it becomes a smooth stable plane.*

As an immediate consequence, we have that the group Γ of all (continuous) collineations of $\mathcal{S}$ is a Lie transformation group on both the point set P and the line set $\mathcal{L}$ with respect to the compact-open topology. In order to investigate the structure of Γ, we consider for every point $p \in P$ the set $\mathcal{S}_p = \{ \mathrm{T}_p L \mid L \in \mathcal{L}_p \}$ of all tangent subspaces of lines through p.

Theorem. *The set $\mathcal{S}_p$ is a compact spread (with respect to the Grassmannian topology) in the tangent space $\mathrm{T}_p P$ and thus defines a compact projective translation plane $\mathcal{P}_p$.*

Moreover, every collineation $\gamma \in \Gamma_p$ (= stabilizer of some point p) induces a collineation $\mathrm{D}_p\gamma$ (= derivative of γ at p) of $\mathcal{P}_p$ which fixes the origin o and the translation line L_∞ of $\mathcal{P}_p$. The main tool for studying the stabilizers Γ_p is the derivation mapping $\mathrm{D}_p : \Gamma_p \to \mathrm{GL}(\mathrm{T}_p P)$ which can be shown to be continuous. Its kernel consists of all central collineations with center p (i.e. collineations that fix every line through p) whose axes, if they exist, are incident to p. In case of a smooth projective plane this means that $\ker \mathrm{D}_p = \Gamma_{[pp]}$ (= group of elations with center p). Using the structure of the stabilizer of a suitable triangle in the tangent translation plane $\mathcal{P}_p$ (which is a result of H. Hähl) it is possible to prove the following result.

Theorem. *Let $\mathcal{S}$ be a smooth stable plane of dimension $2l$ with Γ as its group of automorphisms. Then the following statements hold:*

a) The elation group $\Gamma_{[pp]}$ is a simply connected solvable Lie group for every point p.
b) Every stabilizer of a triangle or of an antiflag is a linear Lie group.
c) A Levi subgroup of the stabilizer of some flag is compact.
d) If $l \leq 2$ then the stabilizer of a flag is solvable.
If $l \geq 4$ then every Levi subgroup of the stabilizer of some flag is isomorphic to some subgroup of the Levi subgroup found in the classical planes of the respective dimension.
e) Every stabilizer of some point or of some line has a linear Levi subgroup.

Compact polygons

Linus Kramer, Tübingen

A *generalized n-gon* is an incidence structure $(\mathcal{P}, \mathcal{L}, \mathcal{F} \subset \mathcal{P} \times \mathcal{L})$, subject to the following three axioms:

(n-gon1) Every element $x \in \mathcal{P} \cup \mathcal{L}$ is incident with at least three other elements.
(n-gon2) Any two elements $x, y \in \mathcal{P} \cup \mathcal{L}$ can be joined by a chain of length $\leq n$. The distance $d(x, y)$ is the length of a *minimal* chain joining x and y.
(n-gon3) If $d(x, y) = k < n$, then there is precisely one k-chain joining x and y.

The generalized triangles are precisely the projective planes, e.g. a generalized n-gon $\mathcal{P}$ is called a *compact n-gon*, if $\mathcal{P}$ and $\mathcal{L}$ are compact Hausdorff spaces, and if the $(n-1)$-chain determined by elements x, y with $d(x,y) = n-1$ depends continuously on x and y. This is equivalent to the fact that the flag space $\mathcal{F} \subset \mathcal{P} \times \mathcal{L}$ is closed. If in addition the (covering) dimension of $\mathcal{P}$ and $\mathcal{L}$ is finite and positive, then the following is true (K. '94):

- $\mathcal{P}$ and $\mathcal{L}$ or ANRs and generalized manifolds (over every PID).
- Every point row $L \subset \mathcal{P}$ and every pencil of lines $\mathcal{L}_p \subset \mathcal{L}$ is a homotopy sphere (of dimension m and m', say).
- $pr_1 : \mathcal{F} \to \mathcal{P}$ and $pr_2 : \mathcal{F} \to \mathcal{L}$ are locally trivial bundles.
- The double mapping cylinder $\mathrm{D}\mathcal{F}$ over pr_1, pr_2 is a homotopy $(\dim \mathcal{F} + 1)$-sphere. The embedding $\mathcal{F} \hookrightarrow \mathrm{D}\mathcal{F}$ is the *topological Veronese embedding* of the n-gon.

Theorem (Knarr '90, K. '94). *Under the above hypothesis, we have $n \in \{3,4,6\}$, and there are restrictions on the numbers m, m'.*

This leads to the following classification result:

Theorem (Grundhöfer, Knarr, K. '94). *Let Σ denote the group of all continuous automorphisms of the n-gon $\mathcal{P}$. If Σ is transitive on the set $\mathcal{P}$ of points, and if $n \neq 4$, or if $m = m'$, then $\mathcal{P}$ is one of the following Moufang n-gons:*

(n=3): *the projective plane* $\mathrm{P}_2\mathbb{F}$, *where* $\mathbb{F} = \mathbb{R}, \mathbb{C}, \mathbb{H}, \mathbb{O}$
(n=4): *the symplectic quadrangle over* $\mathbb{R}, \mathbb{C}$
(n=6): *the hexagon associated to* $G_{2(2)}$, $G_2^{\mathbb{C}}$.

There are examples of quadrangles $\mathcal{P}$ with $m \neq m'$, where Σ is transitive on $\mathcal{P}$, but where $\mathcal{P}$ is not Moufang (Ferus - Karcher - Münzer '81, Thorbergsson '92). If one puts additional structure on $\mathcal{P}$ one gets stronger results, e.g.:

Theorem. *If $\mathcal{P}$ is smooth, the* Auttop($\mathcal{P}$) *is a smooth Lie transformationtion group (Bödi, K. '94). If $n = 3$, then m determines $\mathcal{P}$ up to homeomorphism (K. '94). If $\mathcal{P}$ is holomorphic, then $\mathcal{P}$ is Moufang (K. '94).*

Homogeneous spaces and buildings at infinity

Jens Heber, Augsburg

Let H^n denote a complete, 1-connected Riemannian manifold of sectional curvatures $K \leq 0$. The set of equivalence classes of asymptotic geodesics, $H(\infty)$ ("points at infinity"), carries a natural $(n-1)$-sphere topology. In the case of a *symmetric space of noncompact type* and rank $k \geq 2$, any k-flat F (= totally geodesic copy of $\mathbb{R}^k$) in H is bounded by a $(k-1)$-sphere $F(\infty) \subseteq H(\infty)$. The family of all $F(\infty)$ has the intersection pattern of apartments in (the geometric realization of) the spherical Tits building of G.

For any Iwasawa decomposition $G = \mathcal{K} \cdot \mathcal{A} \cdot \mathcal{N}, \mathcal{K} = G_p$ for some $p \in H$, $A := \mathcal{A} \cdot p$ is a k-flat; the Tits building can be constructed from the apartments $n \cdot \mathcal{A}(\infty), n \in \mathcal{N}$, alone, i.e. from the solvable group $S = \mathcal{A} \ltimes \mathcal{N}$. It is natural to ask for a richer class of *solvmanifolds* $(S, <,>)$ (with $<,>$ left invariant) with a natural notion of infinity and a comparable combinatorial structure.

In the sequel, let H denote a *homogeneous space of* $K \leq 0$. Every such H is known to be isometric to a solvmanifold $(S = \mathcal{A} \ltimes \mathcal{N}, <,>)$, $\mathcal{N} = [S,S]$. It can be shown that $H(\infty)$ carries a natural *polyhedral* structure whose polyhedra are isometric to polyhedra in (S^{k-1}, can), $k = \dim \mathcal{A}$, when equipped with Gromov's *Tits metric*. This structure is used in an essential way to prove the following results:

Theorem A. *If H is irreducible, then either* $\text{rk}(H) = 1$ *($\exists$ geodesics, not contained in any 2-flat) or H is symmetric of rank at least 2.*

Theorem B. *The geodesic symmetries of H are volume-preserving, iff*

(i) $\dim \mathcal{A} \geq 2$ *and H is symmetric of rank ≥ 2,*
or (ii) $\dim \mathcal{A} = 1$ *and H is a harmonic space.*

Theorem A has been proved by Ballmann, Burns-Spatzier for H which cover a finite volume quotient. The proofs do not carry over to the homogeneous case where finite volume quotients exist iff the space is symmetric.

Many classes of homogeneous spaces with volume-preserving geodesic symmetries are known (e.g. distance spheres in $\mathbb{F}H^n, \mathbb{F}P^n$, all naturally reductive spaces, some nilmanifolds), so according to Theorem B, the situation is much more rigid in nonpositive curvature.

Arithmetic quotients of Tits buildings

Enrico Leuzinger, Zürich

Let X be a Riemannian symmetric space of non-compact type and rank ≥ 2 and let Γ be an irreducible non-uniform lattice in $\text{Is}(X)^0$. By the arithmeticity theorem of Margulis Γ is arithmetic. This roughly means that Γ is commensurable with the group $G(\mathbb{Z})$ of integer points of a semi-simple linear algebraic group G defined over $\mathbb{Q}$. We are interested in the geometric shape of the ends of the locally symmetric quotient $V = \Gamma \backslash X$. Now reduction theory for arithmetic groups provides fundamental sets for Γ; and we use such a set $\Omega \subset X$ together with a quotient $|T_\Gamma|$ of a geometric realization of the Titsbuilding of $G(\mathbb{Q})$ modulo Γ to describe the geodesic rays in V. Let $V(\infty)$ denote the set of equivalence classes of asymptotic geodesic rays.

Theorem A. *The points in $|T_\Gamma|$ correspond bijectively to the equivalence classes of asymptotic geodesic rays in V; i.e. $V(\infty) \cong |T_\Gamma|$.*

That such rays exist was shown by the author and that there are no others by Ji and Mac Pherson.

We also determine the asymptotic cone of V. The latter is defined as the pointed Hausdorff-limit $\text{Cone}_\infty V := \mathcal{H} - \lim_{t\to\infty} (V, v, \frac{1}{t} g)$ where $v \in V$ and g ist the Riemannian metric on V. To describe this limit we paste together appropriate $\mathbb{Q}$-Weyl chambers and get a cone $C|T_\Gamma|$ with distinguished vertex 0 and with a polyhedral metric d which equals the euclidean metric on the chambers.

Theorem B. *For every point $v \in V$ the asymptotic cone* $\text{cone}_\infty V$ *is quasi-isometric to* $(C|T_\Gamma|, d)$.

Topological spherical buildings

Regina Kühne, Braunschweig

A spherical building is called a topological spherical building, if each set of vertices of type i carries a non-trivial Hausdorff topology and the following three axioms hold:

(φ) $\varphi : \Delta_I^{2op} \to \Delta_I^w : (C, \bar{C}) \mapsto \text{Cham}(C, \bar{C})$ is continuous.
(Vop) The set of pairs of opposite vertices is open in the set of pairs of vertices.
(Π) The canonical projection $\Pi_i : \Delta_I \to V_i : c \mapsto x_i$ is open for all $i \in I$.

Axiom (φ) requires a continuous action of the Weylgroup W on the set Δ_I^{2op} of pairs of opposite chambers.

In the case of buildings of type A_n this definition is equivalent to the usual one of a topological n-dimensional projective space. Topological buildings of type C_2 are topological quadrangles, and topological buildings of type $I_2^{(m)}$ are topological m-gons in the sense of Theo Grundhöfer and Hendrik van Maldeghem.

In the case of topological buildings of type C_n, the above definition implies that shortest chains between almost opposite vertices depend continuously on these vertices. (A pair of vertices is called almost opposite if it is embeddable in a pair of opposite chambers.) This property suggests a definition of a topological polar space which generalizes the definition of a topological quadrangle. In general, this definition of a topological building leads to interesting questions concerning almost opposite flags, shortest chains and projections. The axioms (φ) and (Vop) carry over to residues of topological buildings. Residues of type A_{n-1} of topological buildings of type C_n are topological $(n-1)$-dimensional projective spaces.

Higher rank subspaces in manifolds of non-positive curvature

Christoph Hummel, Zürich

Let M be a compact Riemannian manifold of non-positive curvature. If M is real analytic and dim $M \leq 4$, then all maximal higher rank subspaces of M are closed and there are at most finitely many of them by a result due to Schroeder. We outlined partial results to generalize this for $\dim M = 5$ by considering flats at infinity.

Pasting in 2–dimensional Laguerre planes and 3–dimensional generalized quadrangles

Günter F. Steinke, Christchurch

Semiclassical 2-dimensional Laguerre planes are topological Laguerre planes that are composed of two halves of the classical real Laguerre plane. This can be done in two ways by pasting along two parallel classes or along a circle in the point space. In a joint paper with B. Polster a different kind of pasting process for 2-dimensional circle planes is investigated. Here one glues together two 2-dimensional circle planes of the same type along a 2-dimensional separating set in the circle space. In the case of Laguerre planes we came up with three different methods. The talk presents these constructions and looks at their interpretation in the associated

3–dimensional generalized quadrangles, i.e. the Lie geometries of these 2 dimensional Laguerre planes.

Compact antiregular quadrangles

Andreas E. Schroth, Braunschweig

A generalized quadrangle is called antiregular, if for any three pairwise noncollinear points x, y and z the number of points collinear with x, y and z is either two or zero. In the compact case this geometric property is closely linked to topological properties. More precisely, if a compact quadrangle with parameters (s,t), where $0 < s, t < \infty$, is antiregular, then $s = t \in \{1, 2\}$. Conversely, up to duality, every compact quadrangle with parameters (s,s), with $s \in \{1, 2\}$, is antiregular. This implies that, up to duality, every compact quadrangle with parameters (s,s) where $s \in \{1, 2\}$ is the Lie geometry of a $2s$–dimensional Laguerreplane.

The results also imply a complete solution to the problem of Apollonius in topological circle planes. This problem asks for the number of circles touching three given objects, each of which might be a circle or a point.

Injectivity radius estimates and sphere theorems

Uwe Abresch, Münster

The injectivity radius estimates for manifolds of positive sectional curvature that have been established by W. Klingenberg around 1960 play a fundamental role in the proof of the classical topological sphere theorem and in the proof of the Berger Rigidity Theorem. As a first application of the Gromov Compactness Theorem, M. Berger extended his rigidity theorem in 1983 to a Pinching Below $\frac{1}{4}$ Theorem. This result, however, is only valid for even-dimensional manifolds. The problem is that Klingenberg needed to assume weak quarter pinching and simply connectedness in order to obtain a lower bound for the injectivity radius of odd-dimensional manifolds of positive curvature. This pinching condition has recently been relaxed in some joint work with W. T. Meyer.

Theorem 1. (—, Meyer)

$$\exists \varepsilon > 0 \ : \ \forall (M^n, g) \text{ complete} : \frac{1}{4}(1+\varepsilon)^{-2} \leq K_M \leq 1, \ \pi_1(M^n) = 0$$
$$\Rightarrow \quad \text{inj}(M^n) = \text{conj}(M^n) \geq \pi.$$

In fact, the constant ε ist not only independent of the dimension n but also explicit. The current proof shows that $\varepsilon = 10^{-6}$ works. Since the other arguments in the proof of Berger's Pinching Below $\frac{1}{4}$ Theorem can be applied in the odd-dimensional case as well (c.f. Durumeric ~ 1986), there is the following immediate application of Theorem 1:

Theorem 2. (—, Meyer)

$$\forall n \equiv 1(2) : \exists \varepsilon_n > 0 : \forall (M^n, g) \text{ complete} : \begin{pmatrix} \frac{1}{4}(1+\varepsilon_n)^{-2} \leq K_M \leq 1, \pi_1(M^n) = 0 \\ \Rightarrow M^n \approx_{\text{homeo}} S^n \end{pmatrix}.$$

The constant ε_n in this Theorem comes from a compactness argument and is therefore inexplicit. The proof of Theorem 1 on the other hand is by direct comparison estimates, which involve a new type of Jacobi field estimates and a new lifting construction.

Refined Jacobi field estimates are also a key ingredient to the next result.

Theorem 3. (—, Meyer)

$$\exists \varepsilon > 0 : \forall n \equiv 1(2) : \forall (M^n, g) \text{ complete} : \left(\begin{array}{c} \frac{1}{4}(1+\varepsilon)^{-2} \leq K_M \leq 1, \pi_1(M^n) = 0 \\ \Rightarrow M^n \approx_{\text{homeo}} \mathbb{S}^n \end{array} \right).$$

Again ε is independent of the dimension and explicit. The geometric arguments are not limited to the odd-dimensional case. They show that either (i) $\operatorname{diam}(M^n) \geq \pi(1+\varepsilon)$ and thus $M^n \approx \mathbb{S}^n$ by the Diameter Sphere Theorem due to Grove and Shiohama or (ii) that there exists a continuous map $f : \mathbb{RP}^n \to M^n$ of degree 1. The mere existence of such a map into a simply connected, odd-dimensional manifold M^n implies that M^n is a homology sphere, and therefore we can refer to Smale's solution of the Poincaré Conjecture in dimension $n \geq 5$. The 3-dimensional case requires a special argument based on R. Hamilton's result on the Ricci flow.

Twin trees: generalized ∞–gons

Mark A. Ronan

This talk concerned joint work with J. Tits. Twin trees are a special case of twin buildings, and these are a generalization of spherical buildings which first arose in the study of Kac–Moody groups. In the 1-dimensional case a twin building is either a generalized m-gon, or a twin tree (the case when $m = \infty$).

A *twin tree* is a pair of trees, together with a codistance between any pair of vertices not in the same tree. This codistance is a natural number $0, 1, 2, \ldots$ and satisfies the property that if two vertices are adjacent in one tree, then their codistance from any vertex in the other tree differs by 1. Moreover given two vertices at codistance at least 1, then among the neighbours of one of these vertices there is a unique one for which the codistance increases. This property of codistances in a twin tree is very similar to that of distances in a single tree, where if two vertices are at distance at least 1, then each one has a unique neighbour for which the distance to the other vertex decreases. One thinks of codistance 0 as meaning that the two vertices are as far apart as possible; increasing the codistance is like decreasing the distance.

A first example arises from the group $GL_2(k[t, t^{-1}])$ in which a suitable pair of vertices at codistance 0 are stabilized by the subgroups $GL_2(k[t])$ and $GL_2(k[t^{-1}])$ respectively. This is an example of a Kac–Moody group, for the generalized Cartan matrix $\begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}$. Other generalized Cartan matrices provide other examples, but there are further examples admitting large automorphism groups (Moufang examples) which do not arise from Kac–Moody data.

In the talk we adumbrated several basic properties of twin trees. For example any two vertices at codistance 0 must have the same valency, from which one shows that when all vertices have valency at least 3, both trees must be biregular (vertices of the same type have the same valency), and isomorphic. We also showed how a twinning of two trees yields a special set of ends. Certain pairs of these ends span what we call twin apartments, and the set of twin apartments, though not the set of ends alone, uniquely determines the twinning. All these results are contained in the recent paper, Twin Trees I, Inventiones **116** (1994).

We also discussed a construction of all twin trees, starting with a single tree and using horoballs centred at the eventual ends of the twinning. This construction will appear in a forthcoming paper.

Some geometric, analytic, and algebraic properties of euclidean Tits buildings

Jürgen Jost, Bochum

The arithmeticity and rigidity theorems of Margulis put strong restrictions on homomorphisms ρ from some lattice Γ acting on an irreducible symmetric space G/K of non-compact type and rank ≥ 2 into linear algebraic groups over $\mathbb{R}, \mathbb{C}$, or $\mathbb{Q}_p$. The harmonic map approach to these questions associates to any such ρ a harmonic map from G/K onto some space on which the image group acts, a symmetric space in case of $\mathbb{R}$ or $\mathbb{C}$, and a Tits building in case of $\mathbb{Q}_p$. One then proceeds by deriving restrictions on the harmonic map that imply the desired restrictions on ρ. A proof of the Margulis theorems along these lines has e.g. been obtained by Jost-Yan using an existence result by Gromov-Schoen for the harmonic map. In fact, Euclidean Tits buildings are in several respects quite analogous to symmetric spaces of non-compact type as is explored in ongoing joint work with K. Zuo.

In other joint work with K. Zuo, this approach is used to obtain restrictions on the possible representations of fundamental groups of projective or quasiprojective manifolds into linear algebraic groups ("nonabelian Hodge theory"). For that purpose, one needs rather general existence results for harmonic maps taking their values in possibly nonlocally compact Tits buildings. Such an existence theory has been developed by J. Jost and in similar, although somewhat more special form by Korevaas–Schoen. In fact, one may show the existence of generalized harmonic maps between metric spaces in case the image has nonpositive curvature in the sense of Alexandrov. This uses among other things mean value constructions in such spaces of nonpositive curvature and the theory of variational or Γ-convergence in the sense of de Giorgi.

In the quasiprojective case, one needs additional constructions, because of the possibility of infinite energy. These constructions, again due to joint work with K. Zuo, need to use the algebraic structure of the domain in a stronger manner.

Harmonic maps into Tits buildings and rigidity theorems

K. Zuo, Kaiserslautern

This is a joint work with J. Jost. Let X be an algebraic variety and $\rho : \pi_1(X) \to G$ be a representation of the fundamental group of X into a simple algebraic group G over K, where K is a p–adic number field or completed function field. Using equivariant pluriharmonic maps into Tits building of G, we proved

Theorem 1. *Suppose that ρ is Zariski dense and unbounded. Then ρ factors through an algebraic map $f : X \to Y$ s.t. $\dim Y \leq \mathrm{rk}_K G$, i.e. there exists an $\tau : \pi_1(Y) \to G$ s.t.*

$$\begin{array}{ccc} \pi_1(X) & \xrightarrow{\ \rho\ } & G \\ {\scriptstyle f_*}\big\downarrow & \circlearrowleft & \nearrow{\scriptstyle \tau} \\ \pi_1(Y) & & \end{array}$$

Remark. $\dim Y = \mathrm{rk}_{K'} G$ is sharp in general. For example we take a p-adic ball quotient $X = \Omega^n/_\Gamma$. There is a natural representation $\rho : \pi_1(x) \to \Gamma \to \mathrm{PSL}_{n+1}(\mathbb{Q}_p)$, which does not factor through any $f : X \to Y$ with $\dim Y < n = \mathrm{rk}_{\mathbb{Q}_p} \mathrm{PSL}_{n+1}(\mathbb{Q}_p)$.

Corollary (p–adic rigidity). *Suppose that ρ is as above in Theorem 1. Let N be the symmetric space of $G(\mathbb{C})$, and $U : \tilde{X} \to N$ the equivariant pluriharmonic map of ρ with $\mathrm{rk}_{\mathbb{R}}\, u > 2\, \mathrm{rk}_{K'} G$. Then ρ is bounded.*

Interpolation and topological incidence geometry

Burkard Polster, Cristchurch

Many of the objects investigated by topological incidence geometers can be interpreted as special kinds of unisolvent sets that are of interest to people dealing with interpolation theory. An n-unisolvent set over an interval $I \subseteq \mathbb{R}$ is a set F of continuous functions $I \to \mathbb{R}$ such that for any choice of n points $(x_i, y_i) \in I \times \mathbb{R}$, $i = 1, 2, \ldots, n$, $x_1 < x_2 < \ldots < x_n$ there exists exactly one $f \in F$ such that $f(x_i) = y_i$. A (half-)periodic n-unisolvent set F is a set of (half-)periodic continuous functions $[0, 2\pi] \to \mathbb{R}$ such that the set of all restrictions of functions in F to $[0, 2\pi)$ is n-unisolvent. Hence the set of non-vertical lines in a 2-dimensional affine plane gives rise to a 2-unisolvent set over $\mathbb{R}$ and the circle set of a 2-dimensional Laguerre plane gives rise to a periodic 3-unisolvent set. Furthermore the set of circles of the projective completion of a 2-dimensional dually affine plane corresponds to a half-periodic 2-unisolvent set .

We identify 2-dimesional affine planes and Laguerre planes as the second and third steps of an infinite sequence of circle geometries. Step n geometries correspond to a special (half-) periodic n-unisolvent sets if n is (even) odd. The point sets of the corresponding geometries are homeomorphic to the (Möbius strip) cylinder. Furthermore, deriving in any point of such a step n geometry yields a step $(n-1)$ geometry.

We also describe how affine parts of step n geometries can be integrated to form step $(n+1)$ geometries. In particular, 2-dimensional affine planes can be integrated to form 2-dimensional Laguerre planes and foliations of $\mathbb{R}^2$ can be integrated to form affine planes.

3D–Reconstruction

Thomas Buchanan, Darmstadt

This talk was motivated by problems in computer vision or more specifically in photogrammetry. Camera projections are considered as central projections $\pi : \mathbb{P}_3 \to \mathbb{P}_2$. In an attempt to understand π better, we just look at central projection in lower dimensions $\mathbb{P}_2 \to \mathbb{P}_1$. The problem of 3D–reconstruction (Hauptaufgabe der Photogrammetrie) was stated. This talk applies ideas from the book A. COBLE: "Algebraic Geometry and Theta Functions" to 3D-reconstruction problems.

A "scene" $S = (P_1, \cdots, P_6)$ consisting of an ordered set of six points is assumed. Algebraic invariants of S and $\pi(S)$ are described briefly. Transcendental invariants are illustrated in the special cases where in the $\mathbb{P}_3 \to \mathbb{P}_2$ case $\pi(S)$ lies on a conic (or in the $\mathbb{P}_2 \to \mathbb{P}_1$ case where S lies on a conic). We consider the Weddle surface $W \subseteq \mathbb{P}_3$ and an algebraic curve $H \subseteq \mathbb{P}_2$ "interpolating" the scene $P_1, \cdots, P_6$ and the center of projection.

Theorem. *SCHOTTKY's parametrization of W using the jacobian $J(H)$ relates the centers of projection for the cases $\mathbb{P}_2 \to \mathbb{P}_1$ and $\mathbb{P}_3 \to \mathbb{P}_2$.*

Rigidity of quasi–isometries and a generalization of Mostrow rigidity

Bruce Kleiner, Berkeley

If X and Y are metric spaces and $\varphi : X \to Y$ is a map, then φ is a *quasi–isometry* if for some pair (C, t),

i) $\forall x_1, x_2 \in X : C^{-1}d(x_1, x_2) - t \leq d(\varphi(x_1), \varphi(x_2)) \leq Cd(x_1, x_2) + t$

ii) $\forall y \in Y : d(y, \varphi(X)) < t$.

X is *quasi–isometric* to Y if there is a quasi–isometry from X to Y; it follows easily from i) and ii) above that quasi–isometry is an equivalence relation.

Quasi–isometries occur naturally in several geometric contexts, but they play a particularly important role in geometric group theory, where one studies finitely generated groups via the geometry of their word metrics. One application of the theorems below is a characterization of the finitely generated groups which are quasi–isometric to certain symmetric spaces of noncompact type (see theorem 5).

Theorem 1 (joint with M. Kapovich and B. Leeb). *For $1 \leq i \leq m$, $1 \leq j \leq n$, let M_i, N_j be Hadamard manifolds with sectional curvature ≤ -1, and let Nil, Nil' be two simply connected nilpotent Lie groups with left invariant Riemannian metrics. Let $M = Nil \times \prod_{i=1}^{m} M_i, N = Nil' \times \prod_{j=1}^{n} N_j$ be the Riemannian products. If $\varphi : M \to N$ is a quasi-isometry, then*

i) $m = n$

ii) After reordering the factors, there are quasi-isometries $\varphi_i : M_i \to N_i$ such that the following diagram is commutative up to bounded error (i.e. $\sup_{x \in M} d((\pi' \circ \varphi)(x), ((\pi \varphi_i) \circ \pi)(x)) < \infty$):

$$\begin{array}{ccc} M & \xrightarrow{\varphi} & N \\ \pi \downarrow & & \downarrow \pi' \\ \prod_{i=1}^{m} M_i & \xrightarrow[\prod \varphi_i]{} & \prod_{j=1}^{m} N_j \end{array}$$

Here π, π' are the projections coming from the product structure. In particular, the fibers of π are carried by φ to sets within finite Hausdorff distance of fibers of π', so Nil is quasi–isometric to Nil'.

Theorem 2 (joint with B. Leeb). *The conclusion of Theorem 1 also holds if some of the M_i, N_j are irreducible symmetric spaces of noncompact type (of dimension > 1).*

Theorem 3 (with B. Leeb). *Let M be an irreducible symmetric space of noncompact type with* $\operatorname{rank}(M) \geq 2$. *If N is a symmetric space of noncompact type, and $\varphi : M \to N$ is a quasi-isometry, then $d(\varphi, \varphi_0) < \infty$ for some homothety $\varphi_0 : M \to N$.*

Theorem 3 confirms a conjecture of Margulis. Theorem 2 and 3 together with results of Mostow imply:

Corollary 4. *If M and N are quasi-isometric symmetric spaces, then M and N are affinely equivalent. Equivalently, after renormalizing the metrics on the irreducible factors of N. M is isometric to N.*

We also obtain the following charaterization of finitely generated groups which are quasi-isometric to certain symmetric spaces:

Theorem 5. *Let M be a symmetric space of noncompact type which contains no Euclidean, hyperbolic plane, or Complex hyperbolic factors in its irreducible decomposition. Then any finitely generated group Γ which is quasi-isometric to M (with respect to some word metric on Γ) is a finite extension of a uniform lattice in the isometry group of M.*

Differentiable projective planes

Joachim Otte, Kiel

An affine or projective plane is called smooth if both the point space and the line space are smooth manifolds such that the geometrical operations are smooth. Smooth planes only occur in the dimensions $2, 4, 8, 16$. Examples are the planes over $\mathbb{R}, \mathbb{C}, \mathbb{H}, \mathbb{O}$. Every plane isomorphic to one of these examples is called classical.

Theorem 1. *In each possible dimension, there exist nonclassical smooth affine translation planes.*

Theorem 2. *Every smooth projective translation plane is classical.*

Theorem 3. *In each possible dimension, there exist nonclassical smooth projective planes.*

The nonclassical examples proving Theorem 1 and Theorem 3 are constructed by distorting the multiplication of the classical algebras.

In the situation of Theorem 2, the projectivity group of a point row consists of diffeomorphisms of the classical sphere and hence is comparatively small. This property characterizes the classical planes.

On the geometry of twin buildings

Peter Abramenko, Frankfurt

Let $\mathcal{G}$ be a simple (and simply connected) algebraic group, defined and isotropic over a field k. Denote by Δ_ε ($\varepsilon \in \{+,-\}$) the Bruhat-Tits building of $\mathcal{G}(k((t^{-\varepsilon})))$ and set $\Gamma := \mathcal{G}(k[t])$, $G := \mathcal{G}(k[t,t^{-1}])$. An explicit geometric description of the quotients $\Gamma \backslash \Delta_+$ and $G \backslash \Delta_+ \times \Delta_-$ is derived by using the following

Facts: *1) G possesses a twin BN-pair (G, B_+, B_-, N, S).*
2) The components Δ_+ and Δ_- of the twin building $\Delta(G, B_+, B_-, N, S)$ associated to the twin BN pair coincide with the Bruhat-Tits buildings introduced above

as well as the following

Proposition: *Let $\Delta = (\Delta_+, \Delta_-, \delta^*)$ be the twin building of a twin BN pair (G, B_+, B_-, N, S), $\Sigma = (\Sigma_+, \Sigma_-)$ a twin apartment of Δ stabilized by N, c_- a chamber of Σ_- and set $F := \{(a_+, a_-) | a_+ \in \Sigma_+ , a_- \subseteq c_-\}$. Then it holds:*

i) $G \cdot F = (\Delta_+, \Delta_-)$.
ii) *Assume $a = (a_+, a_-)$, $a' = (a'_+, a'_-) \in F$, $g \in G$ and $g \cdot a = a'$. Then $a_- = a'_-$, and there exists an $n \in N \cap G_{a_-}$, such that $n \cdot a_+ = a'_+$.*

The description of $\Gamma \backslash \Delta_+$ follows from the proposition by interpreting $\mathcal{G}(k[t])$ as the stabilizer in $\mathcal{G}(k[t, t^{-1}])$ of a vertex of Δ_-. In this way one obtains a new proof for and a generalization of a theorem of Soulé.

Further geometric concepts associated with twin buildings, namely coprojections and coconvex hulls, are introduced briefly.

Projective planes and isoparametric hypersurfaces

Norbert Knarr, Braunschweig

A compact hypersurface in the sphere is called *isoparametric* if it has constant principal curvatures. It was proved by E. Cartan in 1939 that there are precisely 4 examples if the number of different principal curvatures is equal to 3. A new proof for this result was given by associating a compact connected Moufang plane with each such isoparametric hypersurface. The hypersurface can be canonically identified with the flag space of this Moufang plane, and this eventually proves Cartan's result. The proof is in the spirit of Thorbergson's classification of isoparametric submanifolds of euclidean space whose codimension is at least 3.

This is joint work with Linus Kramer.

Compact groups on topological projective planes

Barbara Priwitzer, Tübingen

Let $\mathcal{P} = (P, \mathcal{L})$ be a topological projective plane with locally compact, connected point space. If $\dim P < \infty$, then $\dim P \in \{2, 4, 8, 16\}$, (Löwen 1983).

Classical examples: $P_2\mathbb{R}, P_2\mathbb{C}, P_2\mathbb{H}, P_2\mathbb{O}$.

Let Φ be a compact, connected subgroup of the group $\mathrm{Aut}(\mathcal{P})$.

Theorem (Stroppel 1994; Salzmann, Löwen). *Then one of the following is true:*

a) $\Phi \cong \mathcal{E}$ = *elliptic motion group of the classical plane, and* $\mathcal{P} \cong P_2\mathbb{F}, \mathbb{F} \in \{\mathbb{R}, \mathbb{C}, \mathbb{H}, \mathbb{O}\}$
b) $\dim \Phi \leq \dim \mathcal{E} - \dim \mathcal{P}$

From now on: $\dim \Phi = \dim \mathcal{E} - \dim \mathcal{P}$.

Then can easily be shown: $\Phi \cong \mathcal{E}_0$ = point stabilizer in the elliptic group.

There exist 2- and 4-dimensional, non-classical planes which admit $\Phi \cong \mathcal{E}_0$ as a group of automorphisms, Salzmann 1963, Schellhammer, Sperner 1990.

Theorem. *Let*

$$\Phi \cong \mathcal{E}_0 = \begin{cases} U_2\mathbb{C} \\ U_2\mathbb{H} \times U_1\mathbb{H} \\ \mathrm{Spin}_9\mathbb{R} \end{cases}$$

act on a topological projective plane $\mathcal{P}$ *with dimension* $\begin{cases} 4 \\ 8 \\ 16 \end{cases}$. *Then:*

i) The action of Φ *on the point space* P *is equivalent to the usual action of* Φ *on the point space* $P_2\mathbb{F}, \mathbb{F} \in \{\mathbb{C}, \mathbb{H}, \mathbb{O}\}$*, of the classical plane.*
ii) The lines through the origin are the classical lines.
iii) For $\dim P \in \{8, 16\}$ *the plane* $\mathcal{P}$ *is uniquely determined by a 2-dimensional subplane* $\mathcal{E}$*, which admits the torus* $\mathbb{T}$ *as a group of automorphisms.*

Periodic flats in $A_1 \times A_1$ complexes

Sergei Buyalo, St. Petersburg (joint work with V. Kobelskii)

Let X be a compact metric space of nonpositive curvature, whose universal cover $\tilde{X}$ contains a flat (i.e. a totally geodesic subspace which is isometric to $\mathbb{R}^2$). The main question is: is it true that X contains also a periodic flat (what is equivalent, $\pi_1(X)$ contains $\mathbb{Z} \oplus \mathbb{Z}$).

We are focused on the case when X is a 2-dimensional chamber complex whose chambers are standard unit squares. Our result shows that, roughly speaking, the set of all such X for which the answer is "yes" is "open and everywhere dense" on the one hand, and if the counterexamples do exist, they have to constitute a sufficiently ample set.

Moreover, we show that periodic flat problem for $A_1 \times A_1$-complexes can be reduced to the same questions about irreducible lattices in the product of two trees.

Bounded geodesics in rank-1 locally symmetric spaces

C. S. Aravinda, Bombay

Let M be a rank-1 locally symmetric space of non-compact type with finite Riemannian volume. The geodesic flow on the unit-tangent bundle SM of M is known to be ergodic. Consequently, for almost all $(p,\nu) \in SM$, where $p \in M$ and ν is a unit tangent-vector at p, the geodesic through p in the direction of ν is dense in M. The set C_p of unit-tangent vectors positioned at some point $p \in M$, lying on any non-constant C^1 curve in the unit tangent sphere S_p and determining bounded geodesics from p (namely those with compact closure in M) is shown to be of Hausdorff dimension 1. This has the implication on the dynamics of the geodesic flow that the set C of $(p,\nu) \in SM$ for which the corresponding geodesic is bounded has Hausdorff dimension equal to $2n - 1$.

The proof involves showing that the set C_p is an "α-winning set" of a certain "(α,β)-game" introduced by W. M. Schmidt which is known to have full Hausdorff dimension. This is achieved with the use of trigonometric formulae for rank-1 symmetric spaces.

Loops, groups and foliations

Karl Strambach, Erlangen

The aim of the talk was to show that nice properties of sharply transitive sections $\sigma : G/H \to G$ of a group G as well as regularity conditions of sets of projectivities in 3-webs define natural classes of loops. In particular, loops L for which the set of left translations $\lambda_a = [x \mapsto a \cdot x] : L \to L$ is invariant under the inner automorphisms of the group G generated by all λ_a has been studied. If such L is differentiable then G is a Lie group and contains a normal subgroup N which operates sharply transitively on L; the tangent space of N in 1 coincides with the tangent space of the manifold $\{\lambda_a \,;\, a \in L\}$ in 1. Also the problem for which other classes of differentiable loops the group generated by all left translations is a Lie group has been discussed. We conjecture that a differentiable connected loop for which the group generated by all left and right translations is a Lie group satisfies the Moufang identities.

Recent Results in the theory of generalized polygons

Hendrik van Maldeghem, Gent

I mention 5 recent results.

1. The classical embeddings of the finite Moufang hexagons are characterized by some natural axioms (joint work with J. A. Thas).
2. If a finite generalized n-gon admits a group acting transitively on all ordered $(n+1)$-gons, then it must be a Moufang polygon (but not conversely!). For $n = 4$, this is joint work with J. A. Thas.
3. Recently, J. Tits wrote down a proof of the fact that every Moufang polygon satisfies the commutation relations imposed by the appropriate root system.
4. A generalized polygon is called *regular* if for every point p the set S_q of elements at distance d from p lying on geodesics through any opposite point q only depends on p, d and any two elements of S_q.

   Theorem: *Regular generalized n-gons exist only for $n \in \{3, 4, 6\}$ and for $n = 6$, there is a complete classification by work of M. A. Ronan.*

5. An imaginary line in a polygon is the set of points not opposite all points not opposite two opposite points. It is called *long* if its projection onto any element at codistance 1 from all its elements is either constant or bijective (it is always injective).

   Theorem: *All imaginary lines are long only in the symplectic quadrangles and in the split Cayley hexagons over a field with characteristic 2. (Joint work with J. van Bon and H. Cuypers).*

Extensions of local isomorphisms of twin buildings

Bernhard Mühlherr, Tübingen

This is a joint work with M. Ronan.

Twin buildings have been introduced in order to study groups of Kac–Moody-type from a geometrical point of view. They turn out to be natural generalizations of spherical buildings. It is conjectured that the famous extension theorem for spherical buildings (Theorem 4.1.2. in Tits' Lecture Notes) holds also for twin buildings of locally finite type ($m_{ij} \neq \infty$).

A first step toward a proof of this conjecture has been done by M. Ronan and J. Tits. Their result says that local isomorphisms extend to a "half of the twin".

We have the following result.

Theorem. *Local isomorphisms extend to the whole twin if the chambers opposite to a given chamber are connected.*

Using a result of A. Brouwer one can prove that the assumption made in the theorem above is almost always satisfied.

4–dimensional projective planes with solvable isomorphism groups

Hauke Klein, Kiel

Let Σ be the group of all continuous collineations of a compact projective plane of topological dimension 4. Then Σ is a Lie–Group and all planes with $\dim \Sigma \geq 7$ are explicitly known. Hence we consider the case $\dim \Sigma = 6$. In this case $\Sigma^1 \simeq \mathbb{R}^2 \rtimes GL_2^+(\mathbb{R})$ or Σ is solvable. In the latter case Σ fixes a flag $v \in W$, i.e. an incident point–line pair. The further classification is based on the orbit structure of Σ acting on $W \backslash \{v\}$ and $\mathcal{L}_v \backslash \{W\}$. In the remaining cases we have: Σ fixes neither a point in $W \backslash \{v\}$ nor a line in $\mathcal{L}_v \backslash \{W\}$ and acts transitively on $W \backslash \{v\}$ or $\mathcal{L}_v \backslash \{W\}$. Further we consider the structure of the maximal connected nilpotent invariant subgroup N of Σ. The planes with $\dim N \geq 5$ or $N \simeq \mathbb{R}^4$ are already classified. First we exclude the possibility $\dim N \leq 3$ and only two cases remain for N. $N \simeq Nil \times \mathbb{R}$ or $N \simeq \mathbb{R}^3 \rtimes \begin{pmatrix} 1 & t & \frac{1}{2}t^2 \\ & 1 & t \\ & & 1 \end{pmatrix}$. The second case for N leads to an unique Lie–Group $\Sigma \simeq N_{6,28}$ and by a detailed analysis of the subgroup–structure of Σ we arrive at a contradiction. Hence the only possible case is: $N \simeq Nil \times \mathbb{R}$.

Actions of Large Groups on Stable Planes

Markus Stroppel, Darmstadt

A linear space is an incidence structure $\mathbf{A} = (A, \mathcal{A})$ with point set A and line set $\mathcal{A}$ such that any two points are contained in a unique line. We assume in addition that there are locally compact topologies on A and on $\mathcal{A}$ such that the geometric operationes $\vee$ (joining points) and $\wedge$ (intersecting lines) are continuous; and that the domain of definition of $\wedge$ is open. If, moreover, the topological dimension of A is positive and finite, we call $\mathbf{A}$ a stable plane. See also the contributions by R. Löwen, H. Löwe, and R. Bödi.

We introduce the notions of actions $\gamma : G \to \operatorname{Aut} \mathbf{A}$ of a topological group G on $\mathbf{A}$, and of morphisms (esp. embeddings) of such actions.

The SALZMANN program for stable planes may be formulated as follows: For "interesting" classes $\mathcal{G}$ of groups, find a set $\mathfrak{A}^{\mathcal{G}}$ of actions, and numbers $b_i^{\mathcal{G}}$ such that the following holds.

If $\gamma : G \to \operatorname{Aut} \mathbf{A}$ is an injective action of a group $G \in \mathcal{G}$ on a stable plane $\mathbf{A}$ of dimension a, then $\dim G > b_a^{\mathcal{G}}$ implies that $\gamma \in \mathfrak{A}^{\mathcal{G}}$ (or, that γ embeds into an element of $\mathfrak{A}^{\mathcal{G}}$).

Apart from the class LCp of locally compact groups, the following subclasses of LCp are important: Cp (compact groups), Ab (abelian), Solv (solvable), AlmS (non abelian and no closed connected non-trivial normal subgroups), SemiS (no closed connected non trivial abelian normal subgroups). The table below indicates results that have been obtained so far.

$\mathcal{G}$	Cp	Ab	Solv	AlmS	SemiS AlmS	LCp
$b_2^{\mathcal{G}}$	1	2	5	0	0	5
$b_4^{\mathcal{G}}$	4	4	10	3	0	11
$b_8^{\mathcal{G}}$	13	8	18	16	18	26
$b_{16}^{\mathcal{G}}$	36	16	40	56	38	61
$\mathfrak{A}^{\mathcal{G}}$	Ell	$\emptyset$	$\emptyset$	S	$\emptyset$	Class

Here Class denotes the usual action of $\operatorname{Aut} P_2\mathbb{F}$ on the projective plane $P_2\mathbb{F}$ over $\mathbb{F} \in \{\mathbb{R}, \mathbb{C}, \mathbb{H}, \mathbb{O}\}$, Ell the restrictions of these actions to maximal compact subgroups (elliptic motion groups), and S is Class plus singular exceptional actions of $PSL_2\mathbb{R}$, $SL_2\mathbb{R}$, $\widetilde{SL_2}\mathbb{R}$ and $SL_2\mathbb{C}$.

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Gefördert durch
DFG Deutsche Forschungsgemeinschaft

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Gefördert durch
DFG Deutsche Forschungsgemeinschaft

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