MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 20/1994

Variationsrechnung

8.5.-14.5.1994

Die Tagung fand unter der Leitung von Herrn L. Modica (Pisa), Herrn K. Steffen (Düsseldorf) und Herrn E. Zeidler (Leipzig) statt. In Vorträgen und Gesprächen haben die Teilnehmer aus verschiedenen Ländern (z.B. Deutschland, Italien, USA, Schweiz, Australien,...) neue Resultate und neue Entwicklungen in den Gebieten Minimalflächen, harmonische Abbildungen, Elastizitätstheorie, optimale Steuerung, Evolutionsgleichungen und allgemeine elliptische partielle Differentialgleichungen dargestellt und diskutiert.

Vortragsauszüge

P. Rabinowitz *Multibump solutions of differential equations*

Variational methods have been developed recently to obtain multibump solutions of differential equations. These methods involve finding a basic "one bump" solution and then using it to find multibump solutions. We illustrate with an example. Consider

$$-\Delta u + u = f(x, u) \qquad x \in \mathbb{R}^n$$

where we seek solutions $u \in W^{1,2}(\mathbb{R}^n)$. f satisfies

(f_1) $f \in C^1(\mathbb{R}^n \times \mathbb{R})$ and f is 1-periodic in x_i, $1 \le i \le n$

(f_2) f satisfies the usual Sobolev growth condition

(f_3) $f(x, 0) = 0 = f_u(x, 0)$

(f_4) $\exists \mu > 2 : 0 < \mu F(x, u) \le u f(x, u)$. $\quad (u \neq 0)$; $F(x, u) := \int_0^u f(x, t)\, dt$)

Solutions of the PDE are obtained as critical points of $I(u) = \int_{\mathbb{R}^n} \frac{1}{2}(|\nabla u|^2 + u^2) - F(x, u)$.

Let $\mathfrak{K}$ denote the set of critical points of I and $\mathfrak{K}^c = \{u \in W^{1,2} | u \in \mathfrak{K}, I(u) \le c\}$.

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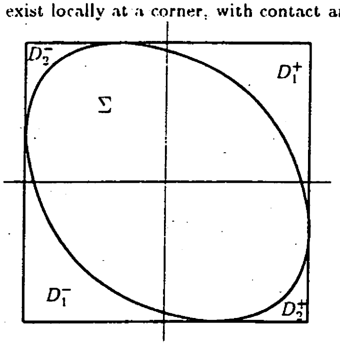
Theorem: Suppose $(f_1)-(f_4)$ hold and there is an $\alpha > 0$ such that $\mathfrak{K}^{c+\alpha}/\mathbb{Z}^n$ is finite (c being the Mountain Pass minimal value). Then for each $k \in \mathbb{N}$ there is an $r_0(k)$ s.t. for $r \in (0, r_0)$: $v_1, \dots, v_k \in \mathfrak{K} \cap I^{-1}(c)$ and $l_1, \dots, l_k \in \mathbb{Z}^n$ with $|l_i - l_j| \geq l_0(r)$, $(i \neq j)$ the set $B_r(\sum \tau_{l_i} v_i) \cap \mathfrak{K}$ is not empty.
(Here $B_r(.)$ denotes an open ball of radius r in $W^{1,2}$ and $\tau_j\, u(x) := u(x-j)$.).

R. Finn *Capillary Wedges*

Capillary surfaces $u(x, y)$ are sought in domains containing a sharp corner. We ask when solutions of the capillarity equation

$$\operatorname{div} Tu = 2H\ , \quad Tu := \frac{\nabla u}{\sqrt{1+|\nabla u|^2}}\ , \quad H \equiv \text{const.}$$

exist locally at a corner, with contact angles γ_1, γ_2 on the sides.



Setting $B_1 = \cos\gamma_1$, $B_2 = \cos\gamma_2$, we find the necessary condition for existence with continuous unit normal up to the corner is that (B_1, B_2) be interior to the closed ellipse $\Sigma\ :\ \ B_1^2 + B_2^2 + 2B_1B_2\cos 2\alpha \leq \sin^2 2\alpha$. It is shown that no solution can exist in D_1^+ and D_1^-, regardless of growth condition at the corner. However, solutions do exist in D_2^+ and D_2^-. The unit normals to these solution surfaces are necessarily discontinuous at the corner.

J. Jost *Generalized harmonic maps between metric spaces*

Starting with the work of Gromov–Schoen on harmonic maps from Riemannian manifolds into locally compact Euclidean Tits buildings harmonic maps between possibly highly singular spaces are attracting geometric interest. The author has developed a general theory of such maps between not necessarily locally compact metric spaces (some more special results were independently obtained by Korevaar–Schoen). The existence result requires that the target space has nonpositive curvature in the generalized sense of Alexandrov. The existence result is reduced to a theorem on the existence of minimizers of convex functionals on spaces of nonpositive curvature; the latter space here arises as an L^2-mapping space. For that existence proof an appropriate version of Moreau–Yosida approximation is used.

U. Dierkes *Curvature Estimates for Hypersurfaces of Variable Mean Curvature*

We generalize the classical pointwise curvature estimates for minimal surfaces due to Heinz ($n > 2$) and Schoen–Simon–Yau ($n \leq 5$) to stable solutions of the Euler equation for the integral $E_\alpha(u) = \int u^\alpha \sqrt{1+|Du|^2}\,dx$ provided suitable dimension restrictions hold. The result is as follows.

Theorem: Let $u \in C^2(B_r(x_0))$, $B_r(x_0) \subset \mathbb{R}^n$, be a positive, stable solution of

$$\operatorname{div} \frac{Du}{\sqrt{1+|Du|^2}} = \frac{\alpha}{u\sqrt{1+|Du|^2}} \qquad (*)$$

and suppose that $\alpha + n < 4 + 2\sqrt{\frac{2}{n+\alpha}}$ (i.e. $n + \alpha < 5.23..$). Then we have the estimate

$$(H^2 + \alpha|A|^2)(x_0) \leq C r^{-2+\frac{2+q}{2+q}} \qquad \forall q \in [0, \sqrt{\tfrac{2}{n+\alpha}})$$

where H denotes mean curvature and $|A|$ is the length of the second fundamental form of $M = \text{graph } u$.

As a corollary one obtains the following Bernstein type result which has been already been established earlier.

Corollary: There are no entire stable solutions $u \in C^2(\mathbb{R}^n)$ of equation (*).

(Note that the stability condition is essential here.) The proof is based on a generalized Simons inequality (i.e. an estimate for the Laplacian of H and $|A|$) an integral curvature estimate and a Moser type iteration argument on the hypersurface. A further basic ingredient is a new Sobolev inequality for stationary surfaces.

M. Flucher *Harmonic radius and concentration, hyperbolic radius and Liouville's equations $\Delta u = e^u$ and $\Delta u = u^{\frac{n+2}{n-2}}$*

The conformal radius of a simply connected planar domain Ω is defined as $r(x) = 1/|f'(x)|$ where $f : \Omega \longrightarrow B_0^1$ is a conformal equivalence with $f(x) = 0$. Liouville found that Robin's function $t(x) = -\log r(x)$ is the maximal solution of $\Delta t = 4e^{2t}$. We discuss two possible extensions of the conformal radius to multiply connected domains:

1) the harmonic radius $r(x) := \exp(-H_x(x))$, defined in terms of the regular part H_x of the Green's function

2) the hyperbolic radius $R(x) := \exp(-U(x))$, defined in terms of the maximal solution of Liouville's equation $\Delta U = 4e^{2U}$.

Similar definitions are given in higher dimensions. It turns out that the harmonic and the hyperbolic radius share a striking number of common properties. Moreover, their numerical values are close to each other. The hyperbolic radius mainly applies in geometry. $ds = |dx|/R$ is the hyperbolic metric on Ω. The harmonic radius, in particular its maximum points, play a central role in elliptic boundary value problems involving concentration of energy. Moreover, its maximal value appears in various isoperimetric inequalities. Therefore it is important to have an efficient method for the numerical approximation of the harmonic centers.

(joint work with C. Bandle)

Th. Nehring *Embedded minimal surfaces of annulus type solving an exterior problem*

Given a compact, strictly convex body X in $\mathbb{R}^3$ and a closed Jordan curve $\Gamma \subset \mathbb{R}^3 \setminus X$ satisfying several additional assumptions. The existence of a parametric annulus type minimal surface is proved which parametrizes Γ along one boundary component, has free boundary on ∂X along the other boundary component and which stays in $\mathbb{R}^3 \setminus X$. As a consequence of this and a reasoning developed by W.H.Meeks and S.-T.Yau we find an embedded minimal surface with these properties. Another application is the existence of an embedded minimal surface with a flat end, free boundary on ∂X and controlled topology.

J. Bemelmans *On a minimal surface supporting a heavy ball*

We study an elastic membrane that is bounded by a planar curve Σ and on which a heavy ball is placed. The ball is allowed to move on the surface and we are interested in a configuration of minimal total energy. If the elastic energy of the membrane is given by Dirichlet's integral the energy is $E(u) = \frac{1}{2}\int_\Omega |Du|^2\,dx - Gh$, if the elastic energy equals the area of the membrane we consider $F(u) = \int_\Omega \sqrt{1+|Du|^2}\,dx - Gh$. Here G is the weight of the ball and h is the x_3–component of the center of the ball. The gravitational force points into the x_3–direction. If the position of the ball is given we are faced with a standard obstacle problem. We analyze the energy as a function of the coordinates of the center of the ball and show for convex domains Ω that there is at least one minimizer for E. The same conclusion holds for F, provided the weight G is not too large.
(joint work with M. Chipot)

S. Luckhaus *A pointwise convergence result for a singular perturbation of the area functional*

We look at the functionals $F_\epsilon(u_\epsilon) = \int[\frac{\epsilon}{2}|\nabla u|^2 + F(u)]$. It is known that $F_\epsilon \xrightarrow{\Gamma} \alpha\int|\nabla\cdot| + \begin{cases} \infty & \text{if } (|u|-1)^2 = 0 \\ 0 & \text{otherwise} \end{cases}$. Now if u_ϵ are local minima of $F_\epsilon(u) + \int gu$, i.e. $F_\epsilon(u_\epsilon) + \int gu \le F_\epsilon(v) + \int gv$ for all $v - u\,|_{\partial B_\rho} \equiv 0$ and $u_\epsilon \longrightarrow u$ in L^1, then under mild conditions on F (which allow $F(u) = (u^2-1)^2$ or $F(u) = \begin{cases} 1-u^2 & u \le 1 \\ \infty & \text{otherwise} \end{cases}$) we prove
<u>Theorem:</u> $\operatorname{dist}(\Gamma(u_\epsilon), \Gamma(u)) \longrightarrow 0$ where $\Gamma(u_\epsilon) = \{(x, u_\epsilon(x)) \mid x \in \hat\Omega\}$ and $\Gamma(u) = \{(x, u(x)) \mid x \in \hat\Omega \backslash \operatorname{supp}(\nabla\chi) \times [0,1]\}$.
The main tool is the following
Lemma: Suppose $F'\,|_{(-1+\delta,-1+\delta_0)} > 0$, $|u| \le M$, $\inf_{(-1+\delta,-1+\delta_0)} F' \ge \epsilon|g|_\infty$, $F(-1+\delta)\rho \ge \epsilon$, $\rho > 2\epsilon$, then there exists ϵ_0 independent of ϵ or δ such that for k arbitrary $|B_\rho \cap \{u_\epsilon > -1+\delta_0\}| < \epsilon_0|B_\rho|$, $|B_\rho \cap \{u < -1+\delta\}| > \frac{1}{2}$ implies $|B_{\rho/2} \cap \{u_\epsilon > -1+\delta_0\}| < c_k\epsilon^k$.

S. Müller *Microstructures with finite surface energy*

Certain alloys undergo solid–solid phase transformations that lead to a complicated arrangement of different phases on a microscopic scale. Mathematical models for this phenomenon lead to ill–posed variational problems and the existence of minimizers is in general open. Here we study the problem of characterizing stress–freee states (or pointwise minimizers of the integrand). Let $K = SO(2) \cup SO(2)$; $\begin{pmatrix} \lambda & 0 \\ 0 & \mu \end{pmatrix}$, $0 < \lambda < 1 < \mu$; let $\Omega \subset \mathbb{R}^2$, $u : \Omega \longrightarrow \mathbb{R}^2$, $U \in W^{1,\infty}$ and $E = \{x \in \Omega : Du(x) \in SO(2)\}$.
<u>Theorem:</u> If $Du \in K$ a.e. and $Per_\Omega E < \infty$ then u is locally a function of one variable and ∂E consists of straight lines which can only intersect at $\partial\Omega$.
Remark: The conclusion does not hold if the assumption $Per E < \infty$ is dropped.

R. Klötzler *A modification of L.C. Young's flow problems*

This paper deals with the metamorphoses of problems of the calculus of variations into transportation flow problems. This approach allows not only a simplification of existence theorems but also the inclusion of extended problems with several sources and sinks and branching out solutions. These investigations are imbedded in a duality between transportation flow problems and deposit problems which generates sufficient and necessary criterions for optimality.

S. Pickenhain *Second order sufficient optimality conditions in optimal control and an application to a class of problems in calculus of variations*

For an optimal control problem of Dieudonné–Rashevský–type second order sufficient optimality conditions for strong local minimality are developed by using methods of duality in optimal control. These second order condition generalizes the known sufficient conditions aldso in the calculus of variations. The obtained theorem is applied to the variational problem

$$\int_\Omega x^k(t)\sqrt{1+|Dx|^2}\,dx \longrightarrow \text{min!} \qquad \Omega \subset \mathbb{R}^n,\ k \geq 1$$

L. Ambrosio *On lower semicontinuity of quasi–convex integrals in SBV*

In variational problems involving integral functionals depending on vector valued functions

$$F(u) = \int_\Omega f(\nabla u)\,dx \qquad u : \Omega \subset \mathbb{R}^n \longrightarrow \mathbb{R}^k$$

a natural question is the research of necessary and sufficient conditions which ensure the lower semicontinuity in Sobolev spaces. The natural condition, introduced by Morrey, is the quasi–convexity:

$$f(A)meas(\Omega) \leq \int_\Omega f(A+\nabla\varphi)\,dx \qquad \forall A \in \mathbb{M}^{n\times k},\ \varphi \in C_0^1(\Omega,\mathbb{R}^k).$$

Quasi–convex functions include convex functions of the determinants of the minors of the matrix, the so–called polyconvex functions. We extended the lower semicontinuity results of Morrey, Acerbi–Fusco and others to functionals defined in $SBV(\Omega,\mathbb{R}^k)$, the simplest one being $F(u) = \int_\Omega f[\nabla u)\,dx + \mathcal{H}^{n-1}(S_u)$. The main feature of these functionals is that discontinuities of u are allowed and are penalized by their $\mathcal{H}^{n-1}$-dimensional measure. The space $SBV(\Omega,\mathbb{R}^k)$ consists of those functions $u \in BV(\Omega,\mathbb{R}^k)$ such that the (distributional) Jacobian of u is the sum of an absolutely continuous measure and a measure supported in the jump set S_u.

G. Alberti *On the structure of the singular set of a convex function*

Let f be a real convex function on $\mathbb{R}^n$. We are concerned with the sets

$$\sum{}^k f := \{x \in \mathbb{R}^n\ .\ \dim \partial f(x) \geq k\ \}$$

where $k = 1,2,\ldots,n$. ∂f is the subdifferential of f, then $\partial f(x)$ is always a non-empty closed convex set in $\mathbb{R}^n$. We prove that for every k the set $\sum^k f$ is $(n-k)$ dimensional.

More precisely, we show that it can be covered by countably many manifolds of class C^2 and dimension $n-k$ up to an $\mathcal{H}^{n-k}$-negligible set. (We call this property $(n-k)$-rectifiability of class C^2.) This result allows to give some weak (but still pointwise) definition of the second fundamental form of the singular sets $\sum^k$. Of course it may be extended to convex surfaces of arbitrary dimension and in this case we generalize an old result by Besicovitch (and others).

S. Hildebrandt *On freely stable minimal surfaces*

Consider a boundary configuration $\langle\Gamma, \mathcal{S}\rangle$ consisting of a "support surface" $\mathcal{S}$ and a Jordan arc Γ that meets $\mathcal{S}$ exactly in its endpoints P_1 and P_2. Let $\mathcal{C}(\Gamma,\mathcal{S})$ be the class of surfaces $X \in H^{1,2}(B,\mathbb{R}^3)$, $B = \{w = u + iv : |w| < 1, v > 0\}$, which map $C = \{|w| = 1, \operatorname{Im} w \geq 0\}$ monotonically onto Γ and $I = (-1,1) = \partial B - C$ a.e. into $\mathcal{S}$. Let $N : B \longrightarrow \mathbb{R}^3$ be the Gauss map associated with X, $\mathfrak{N}(x)$ be the surface normal of $\mathcal{S}$ defined on a tubular neighbourhood of $\mathcal{S}$, and $S = -\nabla\mathfrak{N}$ be the Weingarten map of $\mathcal{S}$. A variation

$$\tilde{X}(.,\epsilon) = X + \epsilon\lambda N + \frac{\epsilon^2}{2}Z + o(\epsilon^2) \quad , \quad Z = \mu\mathfrak{N} \text{ on } I,$$

is admissible if $\tilde{X}(.,\epsilon) \in \mathcal{C}(\Gamma,\mathcal{S})$ for $|\epsilon| \ll 1$. Taking into account that for a stationary X in $\mathcal{C}(\Gamma,\mathcal{S})$ we have

$$\frac{1}{2}\int_B |\nabla N|^2\, dudv = \int_B E|K|\, dudv < \infty ,$$

which follows from Gauß–Bonnet's formula and the free boundary condition of X on I, we compute that

$$\overline{\delta}^2 A(X,\lambda) := \frac{d^2}{d\epsilon^2} A(\tilde{X}(.,\epsilon)) = \delta^2 A(X,\lambda) + \int_B (X_u Z)_u + (X_v Z)_v\, dudv$$

where $\delta^2 A(X,\lambda) = \int_B (|\nabla\lambda|^2 + 2EK\lambda^2)dudv$. Suppose that $\lambda \restriction_C = 0$. The assumption $\tilde{X}(.,\epsilon) \in \mathcal{C}(\Gamma,\mathcal{S})$ for $|\epsilon| \ll 1$ uniquely determines Z on I, namely $Z = \lambda^2\langle N, S(X)N\rangle\mathfrak{N}(X)$ on I. A stationary surface X in $\mathcal{C}(\Gamma,\mathcal{S})$ is said to be "freely" stable if $\overline{\delta}^2 A(X,\lambda) \geq 0$ for all λ with $\lambda \restriction_C = 0$. The authors describe conditions on $\langle\Gamma,\mathcal{S}\rangle$ guaranteeing that $\langle\Gamma,\mathcal{S}\rangle$ bounds exactly one freely stable stationary minimal surface. This result is used to investigate the existence and behaviour of stationary minimal surfaces in a wedge.
References: S.Hildebrandt, F.Sauvigny: Uniqueness of stable minimal surfaces with partially free boundaries. Bonn, SFB 256, Preprint Nr. 290. To appear in: J.Math.Soc.Japan
(joint work with F. Sauvigny)

F. Tomi *Hypersurfaces with prescribed Gauss curvature and boundary*

In my talk I report on the following result which was obtained in collaboration with Nina Ivochkina (St.Petersburg). Let $B \subset \mathbb{R}^{n+1}$ be a strictly convex compact body of class $C^{4,\alpha}$, $(0 < \alpha < 1)$ and $\varphi : S^{n-1} \longrightarrow \partial B$ an embedding of class $C^{4,\alpha}$. Furthermore, let $c \in C^{2,\alpha}(B)$ be a positive function such that $\max c < \min K_{\partial B}$ where K_F denotes the Gauss curvature of a hypersurface F. Then there exists a strictly convex hypersurface diffeomorphic to an n-dimensional closed ball such that $\partial M = \varphi(S^{n-1})$ and $K_M(x) = c(x)$ for all $x \in M$.

H.C. Wente: *Constant Mean Curvature Bubbletons of Finite and Infinite Type*

The classical Bäcklund Transformation produces a new constant Gauss curvature (CGC) surface with $K = -1$ from a given one by an appropriate tangent transformation. L. Bianchi observed that a similar but more complicated procedure applies to the case $K = +1$ and by extension to the $H = 1/2$ parallel surface. Such surfaces correspond to solutions of the elliptic sinh-Gordon equations. $w(u,v)$. Starting with the round $H = 1/2$ cylinder with corresponding $w_0(u,v) \equiv 0$ one obtains a sequence $w_n(u,v)$ of solutions with corresponding $H = 1/2$-surfaces. These are the bubbletons. The n^{th} bubbleton resembles an infinite round cylinder to which n-clusters of spheres have been attached. They move freely with respect to each other. These bubbletons are of finite type in the sense of soliton theory. We show that any cmc immersion of a cylinder with flat ends must be of finite type. Finally, one can take limits of the n- bubbleton sequence to create a new cmc surfaces of infinite type

F. Helein *Ginzburg–Landau équation and renormalized energy*

We are interested in the asymptotic behaviour of maps u from a two-dimensional domain Ω with values into the complex numbers which solve the so-called Ginzburg-Landau equation

$$\Delta u + \frac{1}{\epsilon^2} u(1 - |u|^2) = 0$$

as the parameter ϵ goes to zero. F. Bethuel. H. Brezis and myself proved that under some hypotheses such solutions converge to a harmonic map with values into the circle S^1. with isolated singularities. The position of these singularities is gouverned by some variational principle: they are critical points of a renormalized energy. which may be computed "explicitely". Various extensions or improvements where found by M. Struwe. F. Bethuel/T. Rivière and S. Baraket.

G. Buttazzo *Nonconvex Functionals on Various Function Spaces*

It is known that on $L^p_\mu(\Omega)$ a "local" functional F is weakly lower semicontinuous if and only if it is convex and in this case it takes the form $F(u) = \int_\Omega f(x,u)\,d\mu$. This is not true if $L^p_\mu(\Omega)$ is substituted by the space $M(\Omega)$ of all measures with finite total variation. In this case a local weakly-*-l.s.c. functional is characterized by the form

$$F(\lambda) = \int_\Omega f(x,\lambda^a)\,d\mu + \int_{\Omega\setminus A_\lambda} \varphi(x,\lambda^s) + \int_{A_\lambda} g(x,\lambda(x))d\#$$

for suitable integrands f, φ, g. In the case of functionals on $BV(\Omega, \mathbb{R}^m)$ a complete characterization of all local l.s.c. functionals is not known. We give some partial results on the lower semicontinuity and relaxation of functionals of the form

$$F(u) = \int_\Omega f(\nabla u)\,dx + \int_{\Omega\setminus S_u} \varphi(D^s u) + \int_{S_u} g([u] \otimes \nu)\,d\mathcal{H}^{n-1}$$

which can be seen as models of energies for elastic bodies with fractures.

M. Chipot *Analysis of microstructures*

Let $\varphi : \mathbb{R}^{n\times n} \longrightarrow \mathbb{R}^+$ be continuous and such that $\varphi(w_i) = 0 \quad \forall i = 1,\ldots,k$. $(\mathbb{R}^{n\times n}$

denotes the set of $m \times n$-matrices.) Let $A \in C_0(w_i)$, the convex envelope of the w_i's. One of the model result that we address is:
Theorem: Assume $Rk(w_i - w_j) = 1 \quad \forall j \neq i$. Then there exists a constant C independent of h such that

$$\inf_{V_A^h} \int_\Omega \varphi(\nabla v(x))\, dx \leq C h^{1/2}$$

where $V_A^h = \{v : \Omega \longrightarrow \mathbb{R},\, v \text{continuous}\,,\, v\restriction_K \in P, \forall K \in \mathfrak{C}_h, v = 0 \text{ on } \partial\Omega\}$. h is the mesh size of the triangulation $\mathfrak{C}_h$. More generally we look at the question of producing sequences $u_h \in V_A^h$ such that $\int_\Omega \varphi(\nabla u_k) \longrightarrow 0$. Such a sequence presents usually oscillations that indicate the appearance of microstructures in applications.

M. Struwe *The asymptotic behaviour of minimizers of the Ginzburg–Landau model in 2 dimensions*

Let $\Omega \subset \mathbb{R}^2$ be a smooth possibly multiply connected domain with boundary $\partial\Omega = \Gamma_1 \cup \ldots \cup \Gamma_K$, and let $g : \partial\Omega \longrightarrow S^1$ be a smooth map of total degree $d = \sum_k d_k$, where d_k is the degree of $g\restriction_{\Gamma_k} : \Gamma_k \cong S^1 \longrightarrow S^1$, $1 \leq k \leq K$.
Define $H_g^1 = \{u \in H^{1,2}(\Omega, \mathbb{R}^2) : u = g \text{ on } \partial\Omega\}$, and for $\epsilon > 0$ consider minimizers $u_\epsilon \in H_g^1$ of the Ginzburg–Landau energy

$$E_\epsilon(u) = \frac{1}{2}\int_\Omega |\nabla u|^2 + \frac{1}{2\epsilon^2}(1 - |u|^2)^2\, dx\,, \quad u \in H_g^1\,.$$

Then there holds:
Theorem: For $\epsilon_n \downarrow 0$ let $u_n = u_{\epsilon_n}$ minimize E_{ϵ_n}. Then a subsequence $u_{n'} \rightharpoonup u$ weakly in $H^{1,2}_{loc}(\Omega \backslash \{x_1, \ldots x_J\}, \mathbb{R}^2)$ and weakly in $H^{1,p}(\Omega, \mathbb{R}^2)$ for any $p < 2$ as $n' \longrightarrow \infty$, where $u : \Omega \backslash \{x_1, \ldots, x_J\}' \longrightarrow S^1$ is harmonic.
Remark: For a star shaped domain (necessarily simply connected) this result is due to Bethuel–Brezis–Helein (BBH). From their work it then also follows that in the theorem above $J = |d|$ and $x_j \in \Omega_0$, $1 \leq j \leq |d|$. Moreover, the position of the singular points $x_1, .., x_{|d|}$ can be characterized by means of the renormalized energy introduced by BBH.

K. Beyer *On the initial-value problem for capillary surface waves*

The Cauchy–Poisson problem is considered here which asks for the solution of the initial value problem for the potential flow of an ideal incompressible fluid having a free boundary. The approach applies nonlinear Fourier analysis to the corresponding Hamiltonian system of evolution equations. Assuming the initial data to be sufficiently regular existence of a unique local in time solution is proved.

M. Fuchs *Approximations for some model problems in nonlinear twodimensional elasticity*

Let Ω denote a bounded region in the plane representing the undeformed state of the elastic material under consideration and consider a stored energy density of the form $\frac{1}{2}|F|^2 + h(\det F)$ where $h \in C^2(0,\infty)$ is convex, nonnegative and satisfies the natural growth condition $\lim_{t\downarrow 0} h(t) = +\infty$. For $u_0 \in H^{1,2}(\Omega, \mathbb{R}^2)$ such that $\lambda \leq \det\nabla u_0 \leq \Lambda$ (a.e.) for positive constants λ, Λ it is easy to prove that the variational problem

$$E(u) := \int_\Omega \frac{1}{2}|\nabla u|^2 + h(\det\nabla u) \longrightarrow \min! \qquad (*)$$

in $\mathfrak{C} := u_0 + \overset{\circ}{H}{}^{1,2}$ admits a solution but nothing is known about the regularity of minimizers. We therefore replace h by a sequence h_δ of suitable approximations and show that the corresponding minimizers u_δ converge strongly to a solution u of (*). Moreover, we have the following results:

- there exists an open subset Ω_δ of Ω such that $u_\delta \in C^{1,\gamma}(\Omega_\delta)$ for any $0 < \gamma < 1$ and $|\Omega - \Omega_\delta| \xrightarrow{\delta \downarrow 0} 0$.

  x_0 belongs to Ω_δ iff x_0 is a Lebesgue point for ∇u_δ such that $det\, \nabla u_\delta(x_0) > 0$ and $⨍_{B_r(x_0)} |\nabla u_\delta - (\nabla u_\delta)|^2 \xrightarrow{r \downarrow 0} 0$.

The same theorems hold in case $\Omega \subset \mathbb{R}^n$, $n \geq 3$, if $|\nabla u|^2$ is replaced by $|\nabla u|^m$ for some $m \geq n$ in the stored energy functional. Motivated by the above results we conjecture partial C^1-regularity (up to a set of vanishing Lebesgue measure) for solutions u of problem (*).

(joint work with G. Seregin)

J.F. Grotowski *Energy Minimzing harmonic maps with an obstacle at the free boundary*

We consider partial regularity for energy minimizing maps satisfying a partially free boundary condition. This condition takes the form of the requirement that a relatively open subset of the boundary of the domain manifold be mapped into a closed submanifold with non-empty boundary, contained in the target manifold. We obtain an optimal estimate on the Hausdorff dimension of the singular set of such a map, viz we show $\mathcal{H} - \dim(Sing(u) \cap \Sigma) \leq m - 3$, $m \geq 3$ and additionally $Sing(u) \cap \Sigma$ is discrete in $M \cap \Sigma$ if $m = 3$, where the domain manifold has dimension m, Σ is the free boundary, and u is the energy minimizing map under consideration. Our result can be considered to be a regularity result for a vector valued Signorini, or thin obstacle, problem.

(joint work with F. Duzaar)

E. Kuwert *Existence and Compactness for Disk-Type Minimal Surfaces of Least Area with boundary curve in a Given Homotopy Class*

Let S be a closed set in $\mathbb{R}^n$ and let $\alpha \in \pi_1(S)$ be a homotopy class of free loops in S. We study the problem to minimize $\frac{1}{2}\int_B |dX(w)|^2\, dudv$ among maps from the disk into $\mathbb{R}^n$ with boundary value $x = (X\,|_{\partial B}) \in \alpha$. Introducing a modified concept $\tilde{\pi}_1(S)$ (which is natural), we discuss the assignment of homotopy classes to $x \in H^{1/2}(\mathbb{R}/2\pi, S)$ and particularly its compactness properties. We give an example showing that there can be infinitely many different homotopy classes which can be realized by loops with equibounded energy (here S is a smooth genus two surface in $\mathbb{R}^3$). This disproves corresponding statements made in previous papers on the subject by Tolksdprf and R.Ye. Our main result concerning the minimum problem is as follows:

Theorem: Let $S \subset \mathbb{R}^n$ be compact and $\alpha \in \tilde{\pi}_1(S)$ with infimum energy $E_*(\alpha) < \infty$. Then any minimizing sequence has a subsequence which decomposes in a well-defined way both in energy and in homotopy into a finite or countably infinite collection of surfaces $X^i : B^i \longrightarrow \mathbb{R}^n$, each of which being a minimizer with respect to its own nontrivial homotopy class.

If Douglas' sufficient condition is imposed on α then $E_*(\alpha)$ is attained and the set of minimizers is strongly compact in $H^{1/2}(\mathbb{R}/2\pi, S)$ modulo the automorphism group of the disk.

E. Scholz *On quasi-minimal and minimal surfaces with boundary in a collection of affine-linear spaces*

Consider affine–linear spaces α_k $(k = 1, \ldots, N+3)$ in $\mathbb{R}^p$ $(p \geq 2)$ satisfying $\alpha_{k-1} \cap \alpha_k \neq \emptyset$ and further technical conditions. We search for minimal surfaces $X : B \longrightarrow \mathbb{R}^p$ with $X(\gamma_k) \subset \alpha_k$, where the arcs γ_k come from a suitable decomposition of ∂B $(\ \gamma_k = \{ w = e^{i\tau} \mid \tau \in [\tau_k, \tau_{k+1}] \}\ )$. At first we fix a parameter vector $\boldsymbol{\tau} = (\tau_1, \ldots, \tau_{N+3})$ and consider then the variational problem

$$D(X) = \frac{1}{2} \iint_B X_u^2 + X_v^2 \; dudv \longrightarrow \min! \qquad (*)$$

for $X \in C^2(B) \cap C^0(\overline{B})$ with $X(\gamma_k) \subset \alpha_k$. Then minimal surfaces appear by minimization over $\boldsymbol{\tau}$. We used and modified methods which were worked out for the case that all α_k are lines in $\mathbb{R}^p$. The following results have been proved.

- It exists a unique solution $X^{\boldsymbol{\tau}}$ of (*) which analytically depends on $\boldsymbol{\tau}$.
- The function $\Theta(\boldsymbol{\tau}) := D(X^{\boldsymbol{\tau}})$ is analytic in $\boldsymbol{\tau}$.
- The critical points of Θ correspond with minimal surfaces bounded by the given configuration.

J. Souček *Composition of weak diffeomorphisms*

We consider the composition of
(i) a map $f : \hat{\Omega} \subset \hat{\mathbb{R}}^n \longrightarrow \mathbb{R}^N$ with a transformation $u : \Omega \subset \mathbb{R}^n \longrightarrow \hat{\Omega} \subset \hat{\mathbb{R}}^n$
(ii) two weak diffeomorphisms $v \circ u$, $\quad v : \hat{\Omega} \longrightarrow \tilde{\Omega} \subset \tilde{\mathbb{R}}^n$.
Typical theorems are
Theorem 1: If $u \in Dif^{p,q}(\Omega, \hat{\Omega})$, $v \in Dif^{r,l}(\hat{\Omega}, \tilde{\Omega})$ with $1/q + 1/r \leq 1$ then $v \circ u \in Dif^{s,m}(\Omega, \tilde{\Omega})$ with $1/s = 1/p + q'/(rp')$. $1/m = 1/l + r'/(ql')$.
Theorem 2: If $u_k \in Dif^{1,q}(\Omega, \hat{\Omega})$, $\sup_k \|u_k\|_{dif^{1,q}} < \infty$ and $v_k \in Dif^{r,1}(\hat{\Omega}, \tilde{\Omega})$, $\sup_k \|v_k\|_{dif^{r,1}} < \infty$ and $1/q + 1/r < 1$ then $v_k \circ u_k \rightharpoonup v \circ u$ in $dif^{1,1}(\Omega, \tilde{\Omega})$.
Here:
$dif^{p,q}(\Omega, \hat{\Omega}) := \{u \in L^p(\Omega, \hat{\Omega} \ : \ |M(Du)| \in L^p(\Omega)$, supp $\partial G_u \subset \partial\Omega \times \partial\hat{\Omega}$, $\exists \hat{u} \in L^q(\hat{\Omega}, \Omega)$ with $|M(D\hat{u}| \in L^q(\hat{\Omega})$ s.t. $G_u = \hat{G}_{\hat{u}}\}$
where $M(Du) = (1, Du, M^2Du, \ldots, det\, Du)$ minors of Du, G_u the current integration on the graph of u ; $\quad u_k \overset{dif^{p,q}}{\rightharpoonup} u :\Longleftrightarrow u_k \overset{L^1}{\rightharpoonup} u$ and $\sup_k \|u_k\|_{dif^{p,q}} < \infty$.
$\left(\quad \|u\|_{dif^{p,q}} = \int_\Omega |M\, Du|^p + \int_{\hat{\Omega}} |M\, D\hat{u}|^q = \int_\Omega |M\, Du|^p + \frac{|M\, Du|^q}{(det\, Du)^{q-1}} \quad \right)$. $Dif^{p,q}(\Omega, \hat{\Omega})$ is the smallest class containing the C^1–diffeomorphisms and being closed under $\overset{dif^{p,q}}{\rightharpoonup}$.

G. Huisken *New estimates for the evolution by mean curvature*

Let $F_0 : \Gamma \longrightarrow \mathbb{R}^2$ be the smooth embedding of a curve in $\mathbb{R}^2$. Then we consider the curve shortening flow

$$\frac{d}{dt} F(p, t) = H\nu(p, t) \, , \qquad F(p, 0) = F_0(p) \, .$$

and compare the extrinsic and intrinsic distance functions $d, l : \Gamma \times \Gamma \times [0, T] \longrightarrow \mathbb{R}$, namely $d(p,q,t) = |F(p,t) - F(q,t)|_{\mathbb{R}^2}$, $l(p,q,t) = \int_p^q ds(t)$. It is shown that for embedded curves the quotient d/l can be bounded from below uniformly in time if this is true on the boundary of the curve. For closed curves the same result is shown for the quotient d/ψ where $\psi = \psi(l) = \frac{L}{\pi} \sin \frac{l}{L} \pi$ and L is the total length of the curve. As an application a new proof of Grayson's theorem concerning the smooth contraction of embedded closed curves is obtained. Furthermore, long-time existence and asymptotic behaviour can be analysed for curves with boundary and complete nonclosed curves.

K. Smoczyk *Symmetric hypersurfaces in Riemannian manifolds contracting to Lie-groups by mean curvature*

Let $\widetilde{M}^m$ be a hypersurface smoothly immersed in a Riemannian manifold $\widetilde{N}^{m+1}$ and let $\widetilde{M}_0 := \widetilde{M}^m$ be given locally by some diffeomorphism $\widetilde{F}_0 : \widetilde{U} \subset \mathbb{R}^m \longrightarrow \widetilde{F}_0(\widetilde{U}) \subset \widetilde{M}_0 \subset \widetilde{N}^{m+1}$. Then we want to find a family $\widetilde{F}(.,t)$ of diffeomorphisms belonging to hypersurfaces $\widetilde{M}_t$ such that they satisfy the evolution equation

$$\frac{d}{dt} \widetilde{F}(\widetilde{x},t) = \vec{H}(\widetilde{x},t) \ , \ \widetilde{x} \in \widetilde{U} \ , \qquad \widetilde{F}(.,0) = \widetilde{F}_0$$

where $\vec{H}(\widetilde{x},t)$ is the inward pointing mean curvature vector on $\widetilde{M}_t$. Let G^k be a Lie-group that acts by isometries on $\widetilde{N}$ and assume further that the action is smooth, proper and free. Then we consider the case where $\widetilde{M}_0$ is invariant under the Lie-group action and show that the cross-sections $M_t := \widetilde{M}_t/G$ contract to a single point in finite time, if $m - k \geq 3$ and M_0 satisfies a strong convexity assumption. As a direct consequence we get that $\widetilde{M}_t$ contracts to a single fiber $[p] := \{q \mid \exists \psi \in G \text{ with } \psi(p) = q\}$ in finite time.

K. Ecker *On Regularity for Mean Curvature Flow of Hypersurfaces*

We give a survey of the current state of the regularity theory for hypersurfaces moving by their mean curvature. The main contribution here is still Brakke's regularity theorem proved in 1978. In the special case of the first singular time it states the following: Let $(M_t)_{t \in (0,T)}$ be a smooth family of hypersurfaces in $\mathbb{R}^{n+1}$ moving by mean curvature such that $\mathcal{H}^n \lfloor M_t \xrightarrow{t \to T} \mathcal{H}^n \lfloor M_T$ in some ball $B_R(x_0) \subset \mathbb{R}^{n+1}$ (here $\mathcal{H}^n$ denotes n-dimensional Hausdorff measure) where M_T is an n-rectifiable subset of $\mathbb{R}^{n+1}$ (this is the so called unit density hypothesis). Then the singular set S_T at time T satisfies $\mathcal{H}^n(S_T \cap B_{R/2}(x_0)) = 0$. In view of certain examples, e.g. a thin torus contracting to a circle, one would expect an improved estimate on the singular set namely that $\dim S_T \leq n - 1$. In the special case where the mean curvature of the hypersurfaces M_t is positive, results in this direction have recently been proved by B.White and T.Ilmanen. One would also like to remove the very restrictive unit density hypothesis in Brakke's theorem. We show how certain techniques involving curvature integrals and also a new mean value inequality can be used to greatly simplify Brakke's original proof which so far has been inaccesible due to a rather difficult expositional style.

Berichterstatter: E. Scholz

Tagungsteilnehmer

Dr. Giovanni Alberti
Istituto Matematico "U. Dini"
Universita di Pisa
Via Bonanno 25/B

I-56126 Pisa

Prof.Dr. Reinhold Böhme
Institut f. Mathematik
Ruhr-Universität Bochum
Gebäude NA

D-44780 Bochum

Prof.Dr. Luigi Ambrosio
Istituto Matematico Applicata
Via Bonanno 25/B/S

I-56100 Pisa

Prof.Dr. Giuseppe Buttazzo
Dipartimento di Matematica
Universita di Pisa
Via Buonarroti 2

I-56127 Pisa

Franz Auer
Mathematisches Institut
Universität Freiburg
Hebelstr. 29

D-79104 Freiburg

Prof.Dr. Michel Chipot
Mathématiques
Université de Metz
Faculté des Sciences
Ile du Saulcy

F-57045 Metz Cedex 1

Prof.Dr. Josef Bemelmans
Lehrstuhl und Institut
für Mathematik
RWTH Aachen
Templergraben 55

D-52062 Aachen

Dr. Ulrich Dierkes
Mathematisches Institut
Universität Bonn
Beringstr. 4

D-53115 Bonn

Prof.Dr. Klaus Beyer
Fachbereich Mathematik/Informatik
Universität Leipzig
Augustusplatz 10

D-04109 Leipzig

Dr. Frank Duzaar
Institut für Angewandte Mathematik
Universität Bonn
Wegelerstr. 6

D-53115 Bonn

Dr. Klaus Ecker
Department of Mathematics
University of Melbourne

Parkville , Vic. 3052
AUSTRALIA

Dr. Joseph Grotowski
Fachbereich 9 - Mathematik
Universität des Saarlandes
Postfach 151150

D-66041 Saarbrücken

Prof.Dr. Robert Finn
Department of Mathematics
Stanford University

Stanford , CA 94305-2125
USA

Prof.Dr. Michael Grüter
Fachbereich 9 - Mathematik
Universität des Saarlandes
Postfach 151150

D-66041 Saarbrücken

Dr. Martin Flucher
Forschungsinstitut für Mathematik
ETH-Zentrum, HG G 38.1

CH-8092 Zürich

Prof.Dr. Erhard Heinz
Mathematisches Institut
Universität Göttingen
Bunsenstr. 3-5

D-37073 Göttingen

Prof.Dr. Martin Fuchs
Fachbereich 9 - Mathematik
Universität des Saarlandes
Postfach 151150

D-66041 Saarbrücken

Dr. Frederic Helein
CMLA
ENS Cachan
61, Avenue du President Wilson

F-94235 Cachan Cedex

Prof.Dr. Mariano Giaquinta
Dipart. di Matematica Applicata
Universita degli Studi di Firenze
Via di S. Marta, 3

I-50139 Firenze

Prof.Dr. Stefan Hildebrandt
Mathematisches Institut
Universität Bonn
Wegelerstr. 10

D-53115 Bonn

Prof.Dr. Gerhard Huisken
Mathematisches Institut
Universität Tübingen
Auf der Morgenstelle 10

D-72076 Tübingen

Prof.Dr. Jürgen Jost
Institut f. Mathematik
Ruhr-Universität Bochum
Gebäude NA

D-44780 Bochum

Prof.Dr. Rolf Klötzler
Fachbereich Mathematik/Informatik
Universität Leipzig
Augustusplatz 10

D-04109 Leipzig

Ernst Kuwert
Mathematisches Institut
Universität Bonn
Wegelerstr. 10

D-53115 Bonn

Prof.Dr. Stephan Luckhaus
Institut für Angewandte Mathematik
Abteilung Funktionalanalysis und
Numerische Mathematik
Wegelerstr. 6

D-53115 Bonn

Prof.Dr. Erich Miersemann
Fachbereich Mathematik/Informatik
Universität Leipzig
Augustusplatz 10

D-04109 Leipzig

Prof.Dr. Stefan Müller
Mathematisches Institut
Universität Freiburg
Albertstr. 23b

D-79104 Freiburg

Dr. Thomas Nehring
Mathematisches Institut
Universität Heidelberg
Im Neuenheimer Feld 288

D-69120 Heidelberg

Dr. Sabine Pickenhain
Fachbereich Mathematik/Informatik
Universität Leipzig
Augustusplatz 10

D-04109 Leipzig

Prof.Dr. Paul H. Rabinowitz
Department of Mathematics
University of Wisconsin-Madison
480 Lincoln Drive

Madison WI. 53706-1388
USA

Prof.Dr. Friedrich Sauvigny
Institut für Mathematik
Technische Universität Cottbus
Postfach 101344

D-03013 Cottbus

Prof.Dr. Michael Struwe
Mathematik Department
ETH Zürich
ETH-Zentrum
Rämistr. 101

CH-8092 Zürich

Dr. Erasmus Scholz
Institut für Mathematik
Technische Universität Cottbus
Postfach 101344

D-03013 Cottbus

Prof.Dr. Friedrich Tomi
Mathematisches Institut
Universität Heidelberg
Im Neuenheimer Feld 288

D-69120 Heidelberg

Knut Smoczyk
Institut f. Mathematik
Ruhr-Universität Bochum

44780 Bochum

Prof.Dr. Anthony J. Tromba
Mathematisches Institut
Universität München
Theresienstr. 39

D-80333 München

Prof.Dr. Jiri Soucek
Institute of Mathematics of the
CSAV
Zitna 25

115 67 Praha 1
CZECH REPUBLIC

Prof.Dr. Henry C. Wente
Dept. of Mathematics
University of Toledo
2801 W. Bancroft St.

Toledo, OH 43606
USA

Prof.Dr. Klaus Steffen
Mathematisches Institut
Heinrich-Heine-Universität
Gebäude 25.22
Universitätsstraße 1

D-40225 Düsseldorf

Prof.Dr. Eberhard Zeidler
Fachbereich Mathematik/Informatik
Universität Leipzig
Augustusplatz 10

D-04109 Leipzig

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