MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 21/1994

# Critical Phenomena in Spatial Stochastic Models

15. Mai bis 21. Mai 1994

Organisers: H.-O. Georgii (München) and G.R. Grimmett (Cambridge).

An important characteristic of the study of random spatial processes is the interplay of probabilistic and physical intuition. At this meeting, mathematicians and mathematical physicists from many parts of the world discussed a variety of topics involving critical phenomena in Probability Theory and Statistical Mechanics. The main themes included lattice spin systems and their time evolutions, percolation theory, interacting diffusions and population models, stochastic processes in random media, spin glass models, and properties of random graphs. These were accompanied by talks on a number of closely related topics including, for example, random self-similar sets, and the ergodic properties of billiards. There were 41 participants at the meeting, and 36 talks were presented.

## Abstracts

*Ken A. Alexander (Los Angeles):*

### Simultaneous Uniqueness of Infinite Clusters

In processe such as invasion percolation and certain models of continuum percolation, in which a (possibly) random label $f(b)$ is attached to each bond $b$ of a (possibly) random infinite graph, percolation models for various values of the order parameter $r$ are naturally coupled: one defines a bond $b$ to be occupied at level $r$ if $f(b) \leq r$. If the labeled graph is stationary, then assuming positive finite energy, it is known that for each fixed $r$ there is a.s. at most one infinite cluster, at least for several standard types of models. We give a unifying framework for such fixed-$r$ results, and prove that if the site density is finite and the labeled graph has positive finite energy (which is a sort of "sufficient independence"), then with probability one, uniqueness holds simultaneously for all $r$. Without positive finite energy, one can have fixed-$r$ uniqueness a.s. for each $r$, yet not have simultaneous uniqueness.

*Béla Bollobás (Cambridge), joint work with Graham Brightwell:*

### Random Partial Orders

The random $k$-dimensional partial order $P_k(n)$ is defined by selecting $n$ points at random from $[0,1]^k$, say $x, y, \ldots$, and declaring $x < y$ if $x_i < y_i \; \forall i, 1 \leq i \leq k$. Equivalently, $P_k(n)$ is defined on the set $[n] = \{1, 2, \ldots, n\}$ by taking $k$ random permutations $\pi_1, \ldots, \pi_k$ of $[n]$ and setting $x < y$ if $\pi_i(x) < \pi_i(y) \; \forall i, 1 \leq i \leq k$. The aim is to study the main parameters of posets, like height, width, dimension, maximal degree, largest connected subgraph etc. Perhaps the most frequently studied parameter is the height $H_k(n) = H(P_k(n))$: the maximal number of points in a chain in $P_k(n)$.

The study of $H_2(n)$ goes back to Ulam (1961). Following work by Hammersley and Kingman, it was shown by Vershik & Kerov (1977) and Logan & Shepp (1977) that $H_2(n)/n^{1/2} \longrightarrow 2$ in probability. In 1988 Winkler and I proved that $\frac{H_k(n)}{n^{1/k}} \longrightarrow c_k$ for every fixed $k$, where $2 = c_2 \leq c_3 \leq \cdots \leq e$ and $c_k \underset{k \to \infty}{\longrightarrow} e$.

Following work of Friere (1991), Brightwell and I (1992) proved that $H_k(n)$ is exponentially concentrated; for $k = 2$ this was improved by Talagrand in a recent work. Janson and I have given a lower bound for the variance of $H_2(n)$: $\text{Var}\, H_2(n) \geq cn^{1/8}$ for some $c > 0$.

The talk touched on some recent results obtained jointly with Graham Brightwell concerning the random poset in the case $k(n) \longrightarrow \infty$. Among many other assertions, we proved that if $t = t(n) \geq 2$, $k = k(n) \geq 6 \log n / \log\log n$ and $k \log(t!) - t \log n \longrightarrow c > 0$ then the number of $t$-chains in $P_k(n)$ tends to $\text{Po}(e^{-c})$ in distribution. In particular, in this range of $k = k(n)$ the height is almost completely determined: with probability tending to 1, it takes one of at most $n$ values. The proof uses $\varepsilon$-well spaced chains and is completed by the Stein Chen method, following Arratia, Goldstein and Gordon.

Here is one of the many conjectures left open. Let $P$ be a fixed non-trivial poset and let

$$k_P(n) = \max\left\{k : \mathbb{P}\left(P_k(n) \text{ contains } P\right) \geq \tfrac{1}{2}\right\}.$$

Then $k_P(n)$ is a sharp threshold function:

$$\lim_{n\to\infty} \mathbb{P}\left(P_k(n) \text{ contains } P\right) = \begin{cases} 0 & \text{if } k(n) - k_P(n) \longrightarrow \infty \\ 1 & \text{if } k(n) - k_P(n) \longrightarrow -\infty \end{cases}$$

*Christian Borgs (Los Angeles), joint work with J.T.Chayes:*

**Random Clusters and the Covariance Matrix of the Potts Model**

We consider the truncated two point function of the $q$-state Potts model in $d \geq 2$, expressed in terms of the random cluster model. In the ordered low temperature phase, the two point function

$$G^{mn}(x,y) = \langle q\delta(\sigma_x, m) q\delta(\sigma_y, n)\rangle - \langle q\delta(\sigma_x, m)\rangle \langle q\delta(\sigma_y, n)\rangle$$

is a linear combination of two terms, with coefficients depending on $n$ and $m$. The two terms are the probability $\tau^{fin}(x,y)$ that $x$ ynd $y$ are connected by a finite cluster, and the covariance $C(x,y)$ of the two events that $x$ and $y$ lie in an infinite cluster. Diagonalizing the matrix $G^{mn}$ we find two eigenvalues: an eigenvalue $G_{ord}(x,y)$ corresponding to the trivial representation of the invariance group $S_{q-1}$ in the ordered state, and a $q-2$ times degenerate eigenvalue $\tilde{G}_{ord}(x,y)$ corresponding to the remaining irreducible representations. While $\tilde{G}_{ord}(x,y)$ is proportional to $\tau^{fin}(x,y)$, $G_{ord}(x,y)$ is a linear combination of both $\tau^{fin}(x,y)$ and $C(x,y)$.

We prove the existence of the masses $m_{ord}$ and $\tilde{m}_{ord}$ corresponding to $G_{ord}$ and $\tilde{G}_{ord}$, the inequality $m_{ord} \leq \tilde{m}_{ord}$, and (for $d = 2$, $\beta = \beta_t$ and $q$ large enough) the equality

$$\tilde{m}_{ord}(\beta_t) = 2m_{dis}(\beta_t)$$

where $m_{dis}$ is the standard mass of the disordered high temperature phase.

*Francis Comets (Paris), joint work with J.Neveu:*

**High Temperature Sherrington–Kirkpatrick Spin-Glass Model**

Let $B_{ij}(.)$ be a collection of linear Brownian motions and

$$Z_N(t) = E_\sigma \exp\left\{\frac{1}{\sqrt{N}} \sum_{1\leq i<j\leq N} B_{ij}(t)\sigma(i)\sigma(j) - \frac{N-1}{4}t\right\}.$$

with $E_\sigma$ denoting the average over $\sigma \in \{-1,1\}^N$. Then $Z_N(t)$ has (up to some constant factor) the same distribution as the partition function of the SK model

Gefördert durch DFG Deutsche Forschungsgemeinschaft

at inverse temperature $\beta = t^{1/2}$. It has the advantage of stochastic calculus; as a positive martingale it can be represented as

$$Z_N(t) = \exp\left\{M_N(t) - \frac{1}{2}\langle M_N\rangle(t)\right\}$$

for some continuous martingale $M_N$. We prove

1) $\langle M_N\rangle \xrightarrow[N\to\infty]{P}$ some deterministic $\Phi$ for $t \in [0,1[$.

2) $M_N \xrightarrow[N\to\infty]{\mathcal{D}}$ some Gaussian process $M$ on $[0,1[$.

and then a fluctuation result for $Z_N(t)$ as well as some thermodynamical quantities.

*Donald Dawson (Ottawa):*

**Critical Phenomena in some Spatially Distributed Population Models**

We consider a class of interacting particle systems which invoke critical branching, catalytic branching and mutually catalytic branching. The catalytic branching process involves two types of particles. The first one undergoes a critical branching random walk (or Brownian motion). The second type undergoes random walk and in addition branching catalyzed by the first type, that is, the second type branches at a rate proportional to the mass of the first type at the same location. The mutually catalytic system is similar to the catalytic system except that each of the two types catalyzes the other type. We introduce the system at the particle level and then consider the process obtained in the diffusion limit both on the lattice $\mathbb{Z}^d$ and $\mathbb{R}^d$. The catalytic process has been studied in joint work with K. Fleischmann via the Loy-Laplace equation which in this case is a nonlinear evolution equation in a random medium where the random medium is provided by the first type. The mutually catalytic system has a dual process which can be used to compute moments. However the resulting moments problems are not well-posed. The consequence of this is that the study of the mutually catalytic system is much more difficult than that of the catalytic system. Some preliminary results obtained jointly with E.A.Perkins on this system are presented.

*Jean Dominique Deuschel (Berlin), joint work with Ofer Zeitouni (Haifa):*

**Limiting Curves for i.i.d. Random Records**

Let $Z_1 = (X_1, Y_1), Z_2 = (X_2, Y_2), \ldots, Z_N = (X_N, Y_N)$ be i.i.d. random variables on $[0,1]^2$ with distribution $\rho(dx, dy) = \rho(x,y)dxdy$. We say that $\underline{Z} = (Z_{\alpha_1}, Z_{\alpha_2}, \ldots Z_{\alpha_l})$ forms a record of length $l$ if $Z_{\alpha_i} < Z_{\alpha_{i+1}}$, $i = 1, 2, \ldots, l-1$. Let $\underline{Z}_{max}$ be a record of maximal length $l_{max}$. We show that under the condition $l_{max} = N$, $\underline{Z}_{max}$ concentrates around curves $\{\varphi_1, \varphi_2, \ldots, \varphi_l\}$ which solve the variational problem

$$\bar{J} = J(\varphi) = \max_{\varphi} \int_0^1 \sqrt{\rho(x, \varphi(x))\dot{\varphi}(x)}dx$$

or equivalently which solve the Euler equation

$$\ddot{\varphi} = \frac{\rho_x}{\rho}\dot{\varphi} - \frac{\rho_y}{\rho}\dot{\varphi}^2, \qquad \varphi(0) = 0, \varphi(1) = 1.$$

Also $l_{max}/N^{1/2} \longrightarrow 2J$ in probability and $\underline{Z}_{max}$ concentrates around the curves $\{\varphi_1, \varphi_2, \ldots, \varphi_l\}$.

*Jiangfeng Feng (Peking), joint work with D.Chen, M.Qian:*

**The Metastable Behaviour of the *d*-Dimensional Ising Model**

Let $\Lambda_N = \{1, 2, \ldots, N\}^2$ with periodic boundary condition and $\overline{X} = \{-1, 1\}^{\Lambda_N}$ the state space. $\xi_n$ is the Markov process of Glauber dynamics and $\sigma(A)$ the first hitting time of a set $A$. We have

(i) If $l_1 \wedge l_2 < L = \left[\frac{2}{h}\right] + 1$

$$\lim_{\beta \to \infty} P_\xi^\beta (\sigma(-1) < \sigma(1)) = 1;$$

(ii) If $l_1 \wedge l_2 \geq L = \left[\frac{2}{h}\right] + 1$

$$\lim_{\beta \to \infty} P_\xi^\beta (\sigma(-1) > \sigma(1)) = 1;$$

(iii) $\lim_{\beta \to \infty} \frac{1}{\beta} \log E_{-1}^\beta \sigma(+1) = P(L);$

(iv) $\lim_{\beta \to \infty} \frac{E_{+1}^\beta \sigma(-1)}{E_{-1}^\beta \sigma(+1)} = \infty;$

(v) a characterization of the set of critical droplets.

Here $h$ is the external field, $\beta$ the inverse temperature, and $\xi$ starts at a configuration for which the $+1$ spins form a rectangle of size $l_1 \times l_2$ in the sea of $-1$ spins. We generalize the results above to the case of higher dimensions.

*Klaus Fleischmann (Berlin), joint work with A. Greven:*

**Diffusive Clustering of Interacting Diffusions**

Infinte systems of interacting diffusions of the Fisher-Wright type are studied. The migration term is chosen in such a way that we are in the critical regime of diffusive clustering, a notion introduced by Cox and Griffeath (1986) in the case of the 2-dimensional voter model. That is, clusters of components with states close to the boundary grow on different random scales. The main results are universal in that they do not depend on the shape of the diffusion coefficient in the resampling term, and also not on the concrete ergodic initial law, except its intensity. Main methods are duality, coupling and moment comparison.

*József Fritz (Budapest), joint work with B. Rüdiger:*

**Time Dependent Critical Fluctuations for a One-Dimensional Kac Model**

We consider an Ising ferromagnet with Kac type interaction

$$H_\gamma(\sigma) = -\frac{1}{2}\sum_{k\in\mathbf{Z}} \sigma_k h_k^\gamma(\sigma), \qquad h_k^\gamma(\sigma) = \sum_{j\neq k} \gamma J(\gamma k - \gamma j)\sigma_j,$$

where $J = J(x)$ is a symmetric density and $\gamma > 0$ is a small parameter. Let $\lambda_\gamma^\beta \sim e^{-\beta H_\gamma}$ denote the Gibbs state at inverse temperature $\beta > 0$; $\beta = 1$ is the critical point. One version of the associated Glauber dynamics is defined by jump rates

$$c_k(\sigma) = \frac{1}{2}(1 - \sigma_k \mathrm{th}\beta \mathrm{h}_k^\gamma(\sigma)).$$

Assuming $1 - \beta = \varrho\gamma^{2/3}$ we set

$$\xi_\gamma(\tau, x) = \gamma^{-1/3} h_{[x\gamma^{-4/3}]}^\gamma \left(\sigma\left(\tau\gamma^{-2/3}\right)\right),$$

then $\xi_\gamma \Rightarrow \xi$ in the Skorokhod space of distributions as $\gamma \longrightarrow 0$, and $\xi$ is specified by

$$d\xi = \left(\frac{D}{2}\frac{\partial^2\xi}{\partial x^2} - \varrho\xi - \frac{1}{3}\xi^3\right) d\tau + \sqrt{2}dw,$$

where $w$ is white noise in space and time. If $\varrho > 0$ then the initial distribution can be chosen as $\lambda_\gamma^\beta$, thus a description of critical fluctuations is given. Results of Bertini–Presutti–Rüdiger–Saada are improved here.

*Siegfried Graf (Passau):*

**Statistically Self-Similar Fractals**

A compact random set $K$ in $\mathbb{R}^d$ is called statistically self-similar with respect to a distribution $\mu$ on the $N$-tuples $(S_1, \dots, S_N)$ of contractive similitudes on $\mathbb{R}^d$ if, for i.i.d. copies $K_1, \dots, K_N$ of $K$, the random set $S_1(K_1) \cup \dots \cup S_N(K_N)$ has the same distribution as $K$. The talk surveyed results concerning the Hausdorff dimension and the exact dimension function of statistically self-similar sets. Moreover, fractal analogues of the Lebesgue density theorem were discussed and the tangential structure of self-similar sets was described in the deterministic case.

*Andreas Greven (Berlin), joint work with D. Dawson, J. Vaillancourt:*

**Interacting Fleming–Viot Systems**

A class of infinite systems of interacting measure-valued diffusions each with state space $\mathcal{P}([0,1])$, the set of probability measures on $[0,1]$, is constructed. These systems arise as diffusion limits of population genetics models with infinitely many possible types of individuals (labeled by $[0,1]$), spatially distributed over a countable collection of sites (colonies) and evolving as follows. Individuals can migrate and after an exponential waiting time a colony replaces its population by a new generation where the types are assigned by resampling from the empirical distribution of types at this site.

Next the properties of the equilibrium states, respectively patterns of cluster formation if no nontrivial equilibria exist, are studied following the scheme of the "Multiple Space-Time Scale Analysis". This procedure associates with the system in the "mean-field" limit a nonhomogeneous Markov chain with state space $\mathcal{P}([0,1])$. Qualitative properties in the longtime behaviour of the system are reflected in structural properties of this Markov chain.

Another aspect of this analysis is to study rigorously the problem of renormalization and universality.

*Malte Grunwald (Berlin):*

**Sanov Results for Mean Field Mixtures of Measures**

We define the dependent measures underlying the annealed dynamics for the Sherrington–Kirkpatrick (SK) spin-glass. For these measures we state a Sanov result, i.e. a Large Deviation Principle (LDP) for the distribution of the empirical measure in both the "field" and the "path" variables. In the symmetric case a new order parameter appears.

Our result is based on a method for mixing large deviation systems by measures obeying an independent LDP. First consequences for the SK dynamics using jump processes were stated.

*Frank den Hollander (Utrecht), joint work with J. Naudts, F. Redig and P. Scheunders:*

**Long Time Tails for Diffusions in Random Media**

We give a mini-review of the phenomenon of "long time tails" (LTT's), occuring in autocorrelation functions associated with the diffusion of a particle in a random medium. A LTT is a behaviour of the type

$$\langle \rho(0)\rho(t)\rangle \sim At^{-\alpha} \qquad (t \longrightarrow \infty),$$

where $\rho(t)$ is e.g. the velocity of the diffusing particle at time $t$, $\langle . \rangle$ denotes expectation of the motion, and $A, \alpha$ are the amplitude resp. exponent of the LTT ($A \neq 0$, $\alpha > 0$).

After describing the physical origin of LTT's, we present two results for a random walk $\{X(t) : t \geq 0\}$ on $\mathbb{Z}^d$ in a random field $W = \left\{W(x) : x \in \mathbb{Z}^d\right\}$ which has the effect of locally speeding up or slowing down the motion, viz. $1/W(X(t))$ is the jump rate (="velocity") of the random walk at time $t$. It is shown that for $\rho(t) = 1/W(X(t))$ there is a LTT with:

(1) $\alpha = d/4$, $A$ random: in the quenched case (i.e., $W$ is fixed).

(2) $\alpha = d/2$, $\Lambda$ nonrandom: in the annealed case (i.e., averaged over $W$ afterwards).

*Abel Klein (Irvine), joint work with H. van Dreifus and J.F. Perry:*

**Taming Griffiths Singularities**

A proof is given of the infinite differentiability of the magnetization and of all quenched correlation functions for disordered spin systems at high temperature or strong magnetic field in the presence of Griffiths singularities. Uniqueness of the Gibbs state and exponential decay of truncated correlation functions is also shown.

The proof relies on a new simple modified high temperature / low activity expansion whose convergence can be displayed by elementary means (i.e. elementary probabilistic arguments). The results require no assumptions on the probability distributions or the random parameters, except for the obvious one of no percolation of infinite couplings, and in the strong field regime for the also obvious requirement that zero magnetic fields do not percolate.

*Roman Kotecký (Praha), joint work with L. Chayes and S. Shlosman:*

**Order Induced by Disorder**

We consider Potts models on a lattice in annealed distribution with the Hamiltonian

$$H(n,\sigma) = -\sum_{\langle ij\rangle} n_i n_j \left(\delta_{\sigma_i\sigma_j} - 1\right) - \mu \sum_i n_i - \varkappa \sum_{\langle ij\rangle} n_i n_j .$$

Here the occupation variables are $n_i = 0, 1$, and the spin variables are $\sigma_i = 1, 2, \ldots, q$. The phase diagram shows a new phase for intermediate values of the chemical potential $\mu$ and inverse temperature $\beta$ characterized by existence of an "antiferromagnetic" order for the occupation variables. The existence of this phase is proven with help of reflection positivity or Pirogov–Sinai theory. It can be understood in terms of an effective repulsion of occupied sites caused by loss of entropy if neighbouring sites are occupied. A phase of similar type occurs in a large class of site diluted models including models with continuous spin like the $x$-$y$ model.

*Flora Koukiou (Cergy and Palaiseau):*

**Mean Field Theory of Directed Polymers in Random Media and Spin-Glass Models**

The thermodynamic functions and the phase diagram of the mean-field models defined on homogeneous graphs are investigated. The main objects of study are some random measures given by iterated multiplications and defined on the configuration space. The case of complex temperature is also studied.

*Volkmar Liebscher (Jena), joint work with K.H. Fichtner and N. Poschadel:*

**Random Permutations of Countable Sets and the Bose Gas**

We consider the position distribution of the equilibrium states of a Bose gas on $\mathbb{R}^d$ with respect to the Hamiltonian $H = -\Delta + U$, where $\Delta$ is the Laplacian with natural boundary conditions and $U$ is a suitable potential.

In the case of a system with finitely many particles we find by using the Feynman–Kac formula a description of this position distribution as the result of applying a clustering mechanism on a Gibbs distribution on a suitable loop space. This suggests the same procedure in the case of an infinite system, which comes up as Gibbs problem on the loop space.

For the ideal gas (no particle interaction) this Gibbs problem has exactly one solution, which is a Poisson process. Thus the position distribution is in this case uniquely determined and turns out to be an infinitely divisible point process on $\mathbb{R}^d$, the clustering representation of which is given by the clustering mechanism mentioned above.

The problem of finding back the clusters in the position distribution is equivalent to the problem of determining a certain random permutation of the realization, which comes from the reprentation on the loop space. For finding this random permutation we suggest another Gibbs problem, which has to be solved realization-wise.

*Christian Maes (Leuven):*

**(Dis)agreement Percolation in the Study of Markov Fields**

We study the dependence of Markov fields on boundary conditions by taking a suitable coupling. This coupling leads to a new uniqueness condition. Our coupling shows certain similarities to the Fortuin–Kasteleyn representation.

We extend some of the known relations between percolation and the dependence of Gibbs states on the boundary conditions for Ising ferromagnets to other systems. Similar results hold e.g. for Widom–Rawlingson type models on the continuum $\mathbb{R}^2$.

*Péter Major (Budapest), joint work with P. Bleher:*

**Renormalization in Dyson's Hierarchical Model**

We discussed one-dimensional equilibrium states in statistical mechanics, called Dyson's hierarchical model. We considered the distribution of the average spin of the equilibrium state in a large volume at all temperatures. We are interested in the question for which Hamiltonians the model has a phase transition at low temperatures. This problem leads to the investigation of the powers of an integral operator when applied to a function which depends on the temperature. There is a high temperature and low temperature approximation of the operator which enabled us to handle the problem at these temperatures. We also discussed the case where none of these approximations work.

*Anna de Masi (L'Aquila):*

**Phase Separation and Fluctuations in a Spin System**

I consider the Glauber spin flip dynamics with $\pm 1$ valued spins interacting in $\mathbb{Z}$ via a Kac potential scaled by $\gamma < 1$. The temperature is fixed below the critical (Lebowitz-Penrose) temperature and the initial measure is Bernoulli with zero average corresponding to the value of the magnetization which is thermodynamically unstable. We (DM, Orlandi, Presutti, Frido) prove that the pure phases $\pm m_\beta$ separate at times which grow logarithmically in the scaled length of the Kac interaction and that the separation has a nontrivial spatial structure. Furthermore the lengths of the clusters are typically of order $\sqrt{\log \gamma^{-1}}$ and they are correlated. I compare this spatial structure with the one typical for the infinite volume Gibbs measure. In equilibrium the lengths of the clusters are typically of order $e^{c\gamma^{-1}}$ and they are uncorrelated. Then I discuss some conjectures and open problems on what should happen after phase separation.

*Roland Meester (Utrecht):*

**Continuity of the Critical Density in Boolean Models**

Take a Poisson process with intensity $\lambda$ in $\mathbb{R}^d$ and place a ball with random radius around each point. All balls are independent of each other and the point process and have the same distribution function $F$. The critical density of this process is denoted by $\lambda_c(F)$, i.e. for $\lambda < \lambda_c(F)$ no unbounded covered components exist a.s. but for $\lambda > \lambda_c(F)$ they do. Next, take a sequence $F_n$ of distribution functions such that $F_n \Rightarrow F$ (weak convergence). If in addition the supports of $F_n$ and $F$ are uniformly bounded, we show that $\lambda_c(F_n) \longrightarrow \lambda_c(F)$. This also implies the continuity of the so called "critical covered volume fraction", the fraction of space covered at criticallity. We also show that this critical covered volume fraction does <u>not</u> depend on $F$, as some physicists argued.

*Mikhail Menshikov (Moscow), joint work with S. Aspandiarov and R. Jasnogorodski:*

**Passage Time Moments for Nonnegative Stochastic Processes and an Application to Random Walks in an $n$-Dimensional Quadrant**

We consider the problem of finding effective criteria for the finiteness or infiniteness of the passage time moments for nonnegative discrete-time stochastic processes. (Here passage-time means time to hit a compact set away the origin.) All criteria are closely connected with the well known criterion of Foster for the ergodicity of Markov chains and are given in terms of semimartingales. We apply this method to reflected random walks in an $n$-dimensional quadrant. Our main method is the construction of Lyapunov functions for some random walks. Closely related ideas were used in papers by S.R.S. Varadhan and R.J. Williams.

*Alain Messager (Marseille), partly in collaboration with S. Miracle-Sole:*

**New Phase Transitions in the Falicov–Kimball Model**

We consider the Falicov-Kimball model on a square lattice

$$H_n = t\sum_{(x,y)}\left\{C_x^*C_y + C_y^*C_x\right\} + 2U\sum_x n_x W(x) + \mu_i \sum_x W(x) + \mu_e \sum_x n_x$$

where $C_x^*$ (respectively $C_x$) is the creation operator (respectively annihilation operator) of the electron, $n_x$ its number operator at site $x$ and $W(x)$ is a random variable $W(x) = 0, 1$, denoting the place of the ions.

We give a new representation of this model in terms of an Ising type model with long range interaction. Then we prove the existence of a sequence of phase transitions between the set of phases of period 3, between the set of phases of period 4 and then between the set of phases of period 5. These results are valid for large values of $U$ and small values of $\mu_i$ and $\mu_e$.

*Robin Pemantle (Madison):*

**Intersections of Random Sets**

The probability of a Markov process intersecting a set may be estimated up to a factor of 2 by the capacity of that set in the Martin kernel. Given such a kernel, a random fractal may be constructed such that the probability of this fractal intersecting a set is estimated by the same capacity up to a constant factor (depending on the dimension). Thus the range of a Markov process, such as Brownian motion, is *intersection-equivalent* to a random fractal, the latter having a simpler dependence structure.

*Charles Pfister (Lausanne), joint work with R. Fernandez:*

**Quasilocality of Projections of Gibbs Measures**

Let $\mu^+$ be the Gibbs measure of the Ising model on $\mathbb{Z}^d$ with + boundary condition. Let $T$ be a subgroup of $\mathbb{Z}^d$ and $\mu_T^+$ the projection of $\mu^+$ on the $\sigma$-algebra $\mathcal{F}_T$ generated by the spin variables $\sigma_i$, $i \in T$. We first construct in a natural way a specification on $T$, i.e. a family of probability kernels $q_\Lambda^+(\mathrm{d}r|\omega)$ indexed by the finite subsets $\Lambda$ of $T$, so that for any $\mathcal{F}_T$-measurable

function $f$

$$\int \mu^+(\mathrm{d}\sigma)f(\sigma) = \int \mu^+(\mathrm{d}\sigma)\int q_\Lambda^+(\mathrm{d}\eta|\sigma)f(\eta).$$

A point $\omega \in \{-1,1\}^T$ is a point of quasilocality of the specification $\left\{q_\Lambda^+\right\}$ if for all bounded local functions $f$, all $\Lambda \subset T$, $|\Lambda| < \infty$, the functions $\omega' \mapsto q_\Lambda^+(f|\omega')$ are continuous at $\omega$.

Let $\Omega_q = \left\{\omega : \omega \text{ is a point of quasilocality for } \left\{q_\Lambda^+\right\}\right\}$. We prove

a) $\mu_T^+(\Omega_q) = 1$ or $\mu_T^+(\Omega_q) = 0$.

b) If in the original model there is a unique Gibbs measure then $\mu_T^+(\Omega_q) = 1$.

*Agoston Pisztora (New York):*

**Surface Order Large Deviation Behaviour of the Ising Model for $d \geq 3$**

We consider the $d$-dimensional Ising model in a large box $\Lambda_n$ with ferromagnetic nearest neighbor interaction at inverse temperature $\beta$. Let $M_{\Lambda_n}$ denote the magnetization inside $\Lambda_n$. Assuming that $\beta$ is above a limit of "slab thresholds" (conjectured to coincide with $\beta_c$) we derive surface order large deviation upper bounds for $m_{\Lambda_n}$ holding uniformly with +, free and periodic boundary conditions. This result is based on corresponding large deviation results for high-density percolation as well as on a renormalization argument in the context of FK-percolation.

*Senja Shlosman (Moscow-Irvine), joint work with R. Schonmann:*

**News about the Ising Model**

We study the following question: How long does it take to relax to the (+) phase for the Ising model in positive magnetic field $h$, if one starts from the configuration $\sigma \equiv -1$. We consider the usual Glauber dynamics. We show that if we take a finite box of the size $B/h$ with $B > B_{cr}(T)$ and wait the time $t = \exp\{\lambda/h\}$ then for the two-dimensional system, as $h \downarrow 0$, we will observe $(-)$ phase if $\lambda < \lambda_{cr}(T)$ and $(+)$ phase if $\lambda > \lambda_{cr}(T)$. The quantity $\lambda_{cr}(T)$ is given by

$$\lambda_{cr}(T) = \frac{w_T^2}{8m^*(T)},$$

where $w_T$ is the surface energy of the unit Wulff droplet at temperature $T$, and $m^*(T)$ is the magnetization. The result is valid for all $T < T_{cr}$.

For the infinite system $\lambda_{cr}(T)$ should be replaced by $3\lambda_{cr}(T)$; the last statement is a plausible hypothesis as of today.

*Herbert Spohn (München):*

**Bulk Diffusitivity of Lattice Gases Close to Criticality**

Lattice gases are specified through a finite range Hamiltonian $H$ and nearest neighbor exchange rates $c(x,y,\eta)$ satisfying the reversibility condition

$$c(x,y,\eta) = \exp\{-\Delta_{xy}H(\eta)\}\, c(y,x,\eta^{xy}).$$

The bulk diffusitivity is defined by $D = \sigma/\chi$, where

$$\chi = \sum_x \left(\langle \eta(x)\eta(0)\rangle - \varrho^2\right)$$

with $\langle . \rangle$ the Gibbsmeasure for $H$ with fixed density $\varrho$ and $\sigma$ is given in terms of the variational formula

$$\sigma = \inf_G \frac{1}{2} \sum_{j=1}^{d} \left\langle c(0,e_j,\eta) \left[(e_1 \cdot e_j)(\eta(0) - \eta(e_j)) + D_{0e_j} \sum_x \tau_x G\right]^2 \right\rangle$$

Jointly with H.T. Yau I prove a lower bound of the form $\rho(1-\rho)\sigma_- \leq \sigma$ with $\sigma_- > 0$ and independent of the density. This result implies that the diffusitivity vanishes exactly as $\chi^{-1}$ close to criticallity (critical slowing down). One small piece of the conventional van Hove theory is thereby verified.

*Jeffrey Steif (Göteborg), joint work with Bob Burton:*

**Failure of the Variational Principle for Gibbs States on Trees**

On $\mathbb{Z}^d$, the variational principle of statistical mechanics states that for all translation invariant measures $\mu$ on $\{\pm 1\}^{\mathbf{Z}^d}$ $H(\mu) - e(\mu) \leq P$ with equality if and only if $\mu$ is a Gibbs state. Here $H(\mu)$ is the entropy, $e(\mu)$ the energy, and $P$ the pressure. If $T$ is a homogeneous 3-ary tree and $\pm 1$ are the spin variables, one can show that entropy, energy and pressure can all be defined although these can depend on boundary conditions since surface/volume $\not\to 0$.

On $T$, the space of all nearest neighbor interactions invariant under all automorphisms of $T$ is 3-dimensional (after modding out by adding constants to the pair and single spin interactions). Then one can assume that the only interactions are given by $-J_2$ $(-J_1)$ which is the energy for 2 adjacent 1's ($-1$'s) and an external field $h$.

**Theorem** *Let $S$ be the surface in $\mathbb{R}^3 = \{(J_1, J_2, h) : J_1, J_2, h$ real$\}$ given by*

$$e^{J_1+h} - e^{-h} - e^{J_2+h} + e^h = 0.$$

(a) *If $(J_1, J_2, h) \notin S$, there exists a homogeneous measure $\mu$ with $H(\mu) - e(\mu) > H(\nu) - e(\nu)$ for all homogeneous $\nu$. $\mu$ satisfies $H(\mu) - e(\mu) < P$ and is not a Gibbs state.*

(b) *If $(J_1, J_2, h) \in S$, there exists a homogeneous measure $\mu$ with $H(\mu) - e(\mu) > H(\nu) - e(\nu)$ for all homogeneous $\nu$. $\mu$ satisfies $H(\mu) - e(\mu) = P$ and is a Gibbs state.*

*Thomas Strobel (Bonn):*

**Interface Motion in a Planar Spin-Flip Model Derived from One-Dimensional Exclusion on the Line and the Half-Line**

We derive the nonlinear equation

$$\frac{\partial\varphi}{\partial t} = \frac{1}{2}\frac{\frac{\partial^2\varphi}{\partial x^2}}{1+\left|\frac{\partial\varphi}{\partial x}\right|^2} - \gamma\frac{\left|\frac{\partial^2\varphi}{\partial x^2}\right|}{1+\left|\frac{\partial\varphi}{\partial x}\right|}$$

for the interface height of a two-dimensional spin-flip model in the hydrodynamic limit under several initial conditions from results about the one-dimensional asymmetric exclusion process and one on modifications.

The spin-flip model is a limit of Ising dynamics for low temperature and small nonzero external field.

*Domokos Szasz (Budapest), joint work with N. Simányi:*

**Ergodicity of Cylindrical Billiards**

The first result is that the Chernov-Sinai pencase, i.e. a quasi one-dimensional system of $N$ hard balls on an elongated $\nu$-dimensional torus (for geometric reasons $\nu = 2, 3$ or $4$) is a K-system.

The next result concerns a billiard on the 4-torus, $\mathbb{T}^4$ with two cylindric scatterers: $C^1 = A^1 \times B^1$, $C^2 = A^2 \times B^2$, where $A^1, A^2$ are the constituent subspaces and $B^1, B^2$ are two-dimensional balls supported in subspaces $L^1$ and $L^2$ respectively, orthogonal to $A^1$ and $A^2$.

Then, if $\dim(A^1 \cap A^2) = 0$ and $A^1 \perp L^2$, this billiard is K. The proof illustrates two fundamental difficulties arising when one tries to prove the Boltzmann-Sinai ergodic hypothesis. The first one is connected with degeneracies of certain algebraic expressions whereas the second one is connected with the transitivity over $S^{d-1}$ of certain subgroups of $SO(d)$ generated by some of its subgroups.

*Alain-Sol Sznitman (Zürich):*

**Off Diagonal Behaviour for Brownian Motion in a Poissonian Potential**

We consider a Poissonian potential

$$V(.) = \sum_i W(. - x_i)$$

where $W \geq 0$ is bounded compactly supported not a.s. equal to zero. We describe certain critical large deviation properties of quenched and annealed Brownian motion in this potential. These large deviation results involve two families of Lyapunov coefficients.

We discuss some applications to the large time off diagonal asymptotics of the kernel of $e^{t\frac{1}{2}\Delta - V(\cdot)}$, as well as to the case where Brownian motion is replaced by Brownian motion with constant drift. In the latter case, both for the quenched and the annealed situation, there is a transition of regime between small drift and large drift behaviour. The direction dependent critical size of the drift can be expressed in terms of the Lyapunov coefficients.

*Bálint Tóth (Budapest):*

**Generalized Ray–Knight Theory and Scaling Limits for Self-Interacting Random Walks**

Let $w : \mathbb{N} \cup \{0\} \mapsto \mathbb{R}_+$ be a weight function. We consider a nearest neighbour random walk on $\mathbb{Z}$ with

$$\text{Prob(jump along an edge)} \sim w(\#\text{previous visits to that edge}).$$

The concrete cases considered were $w(n) \sim (n + A)^{\alpha}$ with $\alpha \in (0, 1)$ [weakly reinforced], $\alpha = 0$ [finitely reinforced] and $\alpha < 0$ [self repelling]. Let

$$L_{k,t} = \#\{0 \leq j < t | X_j = k + 1, X_{j+1} = k\}$$

be the edge local time process of the random walk and the stopping time

$$T_{k,m} = \inf\{i \mid L_{k,i} > m\}.$$

Define the process $S_{k,m} = L_{k-l,T_{k,m}}$, $l \in \mathbb{Z}$. This is the usual shifted local time process considered in Ray-Knight theory. We proved functional limit theorems for these processes in the following scaling limits.

$$0 < \alpha < 1 : \frac{S_{[Ax],[A^{1/(1-\alpha)}h]}(A\cdot)}{A^{\frac{1}{1-\alpha}}}$$

$$\left.\begin{array}{l}\alpha = 0\\ \alpha = 1\end{array}\right\} : \frac{S_{[Ax],[Ah]}(A\cdot)}{A}$$

The limit processes were given explicitely.

*Maria Eulalia Vares (Rio de Janeiro), joint work with C. Landim and V. Sidoravicius:*

**Some Exponential Estimates for Glauber + Kawasaki Dynamics**

We consider the superposition of a local Glauber dynamics and a speeded-up ($N^2$) symmetric simple exclusion process on $X_N = \{0,1\}^{T_N}$, $T_N = \{0,1,\ldots,N-1\}$ (torus). This model has been studied since several years (also in infinite volume), it is well known to present propagation of chaos, being macroscopically described by a reaction diffusion equation

$$\partial_t \varrho = \frac{1}{2}\partial_r^2 \varrho + G(\varrho).$$

We are now interested in the description of "tunneling times", i.e. if $G(\varrho) = -V'(\varrho)$ with $V$ a double well potential one starts with Bernoulli($m_-$), $m_-$ being one of the stable points and consider the first time the density is "far" from $m_-$ (or close to the other stable point). For this we are combining coupling methods (to get loss of memory) with a good lower bound for such time. For the moment we have an estimate which allows us to see that $e^{N^\delta}$ ($\delta$ suitable) is a lower bound, but also that up to this time the process follows very closely (scale $N^\alpha$, $\alpha < 1$ suitable) the deterministic discretized equation. Being an estimate which allows us to look at smaller scales, it gives us something better than what we could have with our usual large deviation estimates. The basic estimate comes also from an idea presented in a preprint of Yau (metastable behaviour of GL model with conservation law) for a different model. The result (basically) is:

**Theorem** *There exists positive constants $\sigma$ and $C$ depending on the Glauber rates so that*

$$\sup_\eta E_\eta \sup_{s\le t} \exp\left(N^{2/9}\|\eta_s - u_s\|_{-1}^2 e^{\sigma s}\right) \le C e^{Ct}.$$

*where $u$ is the solution of the discretized equation starting at $\eta$, and $\|u\|_{-1}^2 = \langle (1 - N^2\Delta)^{-1} u, u\rangle$, $\Delta$ being the discrete Laplacian.*

We are still working with coupling to prove the exponentiality of tunneling time.

*Wolfgang Woess (Milano):*

**Random Walks on a Hierarchical Tree**

Consider the homogeneous tree $T$ with its boundary $\partial T$ ("ends") and the usual topology that makes $T \cup \partial T$ compact. Select one boundary point $\omega$ and draw a picture of $T$ in horizontal layers (horocycles): $\omega$ is the "mythological ancestor", and the $k$'th horocycle $H_k$ is the $k$'th generation; each member has $q$ sons in the next horocycle $H_{k+1}$ ($q+1$ is the degree of $T$). Every horocycle is infinite. The affine group of $T$, $G = \mathrm{AFF}(T)$, is the group of isometries of $T$ which preserve this hierarchy. Every $g \in G$ maps horocycles to horocycles; hence there is a homomorphism $\Phi : G \longmapsto \mathbb{Z}$ such that $gH_k = H_{k+\Phi(g)}$.

Now let $X_1, X_2, \ldots$ be a sequence of i.i.d. $G$ valued random variables (transformations of $T$). Consider the right and the left random walk

$$R_n = X_1 X_2 \cdots X_n \quad , \quad L_n = X_n \cdots X_1.$$

Their behaviour can be studied in terms of the tree. Under suitable moment conditions on $X_1$ the following results are obtained:

I) Almost sure convergence of $R_n\theta$ to $\partial T$ in the topology of $T \cup \partial T$.

II) A law of large numbers for $d(\theta, R_n\theta)$.

III) A central limit theorem for $d(\theta, R_n\theta)$.

(and others).

A crucial point is the study of the projected random walk $\Phi(R_n)$ on the integers.

*Milos Zahradnik (Praha), joint work with P. Holicky:*

**Stratified Low Temperature Phases of Spin Models**

We develop a new version of the Pirogov-Sinai theory suitable for dealing with these situations. Examples of models to which our method applies include

(1) Phases of Dobrushin's type (1973) with a rigid interface

(2) Various examples of a "rigid wetting" situation appearing either in the whole lattice $\mathbb{Z}^\nu$, or in a half-lattice or a layer.

(3) Ising type models with an additional random field which is "stratified" in the sense that it depends only on the last coordinate $t_\nu$ of $t \in \mathbb{Z}^\nu$.

We consider the model in dimension $\nu \geq 3$. Our formulation of the P.S. theory applies to the study of all stratified phases for a general stratified Hamiltonian. Our main theorem transforms the problem of finding "stable stratified configurations" (giving rise to a Gibbs state whose almost all configurations are "externally equal" to a given configuration) to a problem of finding ground states of some auxiliary one-dimensional Ising type model.

Berichterstatter: Volkmar Liebscher

## Tagungsteilnehmer

Prof.Dr. Ken Alexander
Dept. of Mathematics, DRB 155
University of Southern California
1042 W 36 Place

Los Angeles, CA 90089-1113
USA

Prof.Dr. Donald A. Dawson
Dept. of Mathematics and Statistics
Carleton University

Ottawa, Ont. K1S 5B6
CANADA

Prof.Dr. Béla Bollobás
Dept. of Pure Mathematics and
Mathematical Statistics
University of Cambridge
16, Mill Lane

GB-Cambridge, CB2 1SB

Prof.Dr. Anna De Masi
Department of Pure and Applied
Mathematics
University of L'Aquila
Via Roma, 33

I-67100 L'Aquila

Prof.Dr. Erwin Bolthausen
Institut für Angewandte Mathematik
Universität Zürich
Winterthurerstr. 190

CH-8057 Zürich

Prof.Dr. Jean Dominique Deuschel
Fachbereich Mathematik
Technische Universität Berlin
Straße des 17. Juni 136

D-10623 Berlin

Prof.Dr. Christian Borgs
Dept. of Mathematics
University of California
405 Hilgard Avenue

Los Angeles, CA 90024-1555
USA

Dr. Jianfeng Feng
Mathematisches Institut
Universität Tübingen
Auf der Morgenstelle 10

D-72076 Tübingen

Prof.Dr. Francis M. Comets
U. F. R. de Mathématiques
Case 7012
Université de Paris VII
2, Place Jussieu

F-75251 Paris Cedex 05

Dr. Klaus Fleischmann
Institut für Angewandte Analysis
und Stochastik
im Forschungsverbund Berlin e.V.
Mohrenstr. 39

D-10117 Berlin

Prof.Dr. József Fritz
Mathematical Institute of the
Hungarian Academy of Sciences
P.O. Box 127
Realtanoda u. 13-15

H-1364 Budapest

Prof.Dr. Geoffrey R. Grimmett
Statistical Laboratory
16 Mill Lane

GB-Cambridge CB2 1SB

Prof.Dr. Jürgen Gärtner
Fachbereich Mathematik / FB 3
Sekr. MA 7-5
Technische Universität Berlin
Straße des 17. Juni 136

D-10623 Berlin

Malte Grunwald
Fachbereich Mathematik / FB 3
Sekr. MA 7-4
Technische Universität Berlin
Straße des 17. Juni 136

D-10623 Berlin

Prof.Dr. Hans-Otto Georgii
Mathematisches Institut
Universität München
Theresienstr. 39

D-80333 München

Prof.Dr. Frank den Hollander
Mathematisch Instituut
Rijksuniversiteit te Utrecht
P. O. Box 80.010

NL-3508 TA Utrecht

Prof.Dr. Siegfried Graf
Fakultät für Mathematik
und Informatik
Universität Passau

D-94030 Passau

Prof.Dr. Michael S. Keane
Dept. of Mathematics and
Informatics
Technische Hogeschool Delft

NL-2628 CD Delft

Dr. Andreas Greven
Institut für Stochastik
Humboldt-Universität Berlin
Unter den Linden 6

D-10117 Berlin

Prof.Dr. Abel Klein
Dept. of Mathematics
University of California at Irvine

Irvine . CA 92717
USA

Gefördert durch DFG Deutsche Forschungsgemeinschaft

Prof.Dr. Roman Kotecky
Center for Theoretic Study
Ovocny trh. 3

116 36 Praha 1
CZECH REPUBLIC

Prof.Dr. Flora Koukiou
Groupe de Physique Statistique
Université de Cergy-Pontoise
B.P. 8428
47-49 avenue des Genottes

F-95806 Cergy-Pontoise Cedex

Dr. Volkmar Liebscher
Institut für
Angewandte Mathematik
Universität Jena
Leutagraben 1, Univ.-Hochhaus

D-07743 Jena

Prof.Dr. Christian Maes
Institut of Theoretical Physics
Celestijnenlaan 200 D

B-3001 Leuven

Prof.Dr. Péter Major
Mathematical Institute of the
Hungarian Academy of Sciences
P.O. Box 127
Realtanoda u. 13-15

H-1364 Budapest

Dr. Ronald Meester
Mathematisch Instituut
Rijksuniversiteit te Utrecht
P. O. Box 80.010

NL-3508 TA Utrecht

Prof.Dr. Mikhail V. Menshikov
Math. Faculty
Dept. of Probab. Theory
and Stat. Mechanics
Moscow University
(Im Lomonosov)

119899 Moscow
RUSSIA

Prof.Dr. Alain Messager
Centre de Physique Théorique
CNRS
Luminy - Case 907

F-13288 Marseille Cedex 09

Prof.Dr. Robin Pemantle
Department of Mathematics
University of Wisconsin

Madison, WI 53706
USA

Prof.Dr. Charles-Edouard Pfister
Dépt. de Mathématiques
Ecole Polytechnique Fédérale
de Lausanne
Ma-Ecublens

CH-1015 Lausanne

Agoston Pisztora
Courant Institute of
Mathematical Sciences
New York University
251, Mercer Street

New York NY 10012-1110
USA

Prof.Dr. Domokos Szasz
Mathematical Institute of the
Hungarian Academy of Sciences
P.O. Box 127
Realtanoda u. 13-15

H-1364 Budapest

Prof.Dr. Senya B. Shlosman
Institute for Problems of
Information Transmission
Academy of Sciences
ul. Ermolova 19

101447 Moscow GSP-4
RUSSIA

Prof.Dr. Alain-Sol Sznitman
Mathematik
ETH Zentrum
Rämistr.101

CH-8092 Zürich

Prof.Dr. Herbert Spohn
Sektion Physik der
Ludwig-Maximilian-Universität
München
Theresienstr. 37 III

D-80333 München

Prof.Dr. Bálint Tóth
Math. Institute of the Hungarian
Academy of Sciences
Realtanoda u. 13-15

H-1053 Budapest

Prof.Dr. Jeffrey Steif
Dept. of Mathematics
Chalmers University of Technology

S-41296 Gothenburg

Prof.Dr. Maria E. Vares
Instituto de Matematica Pura e
Aplicada - IMPA
Estrada Dona Castorina, 110

22460 Rio de Janeiro , RJ
BRAZIL

Thomas Strobel
Institut für Angewandte Mathematik
Universität Bonn
Wegelerstr. 6

D-53115 Bonn

Prof.Dr. Wolfgang Woess
Dipartimento di Matematica
Universita di Milano
Via C. Saldini, 50

I-20133 Milano

Prof.Dr. Milos Zahradnik
Dept. of Mathematical Analysis
Charles University
Sokolovska 83

18600 Praha 8
CZECH REPUBLIC

# EMAIL Adresses

| | |
|---|---|
| Alexander,K. | alexandr@mtha.usc.edu |
| Bollobás,B. | b.bollobas@pmms.cam.ac.uk |
| Bolthausen,E. | eb@amath.unizh.ch |
| Borgs,C. | borgs@math.ucla.edu |
| Comets,F. | comets@mathp7.jussieu.fr |
| Dawson,D. | ddawson@math.carleton.ca |
| Deuschel,J.D. | deuschel@stoch1.math.tu-berlin.de |
| Feng,J. | jfeng@mailserv.zdv.uni-tuebingen.de |
| Fleischmann,K. | fleischmann@iaas-berlin.d400.de |
| Fritz,F. | jofri@ludens.elte.hu |
| Gärtner,J. | jg@math.tu-berlin.de |
| Georgii,H.-O. | georgii@rz.mathematik.uni-muenchen.de |
| Graf,S. | graf@cantor.fmi.uni-passau.de |
| Greven | greven@namu01.gwdg.de |
| Grimmett,G.R. | g.r.grimmett@statslab.cam.ac.uk |
| Grunwald,M. | grunwald@math.tu-berlin.de |
| den Hollander,F. | denholla@math.ruu.nl |
| Keane,M. | keane@cwi.nl |
| Klein,A. | aklein@math.uci.edu |
| Koukiou,F. | koukiou@u-cergy.fr |
| Kotecky,R. | kotecky@aci.cvut.cz |
| Liebscher,V. | liebscher@minet.uni-jena.de |
| Maes,C. | FGBDA35@BLEKUL11.bitnet |
| Major,P. | major@circle.math-inst.hu |
| De Masi,A. | demasi@vxscaq.aquila.infn.it |
| Meester,R. | meester@math.ruu.nl |
| Menshikov,M. | menshikov@llrs.math.msu.su |
| Messager,A. | messager@cpt.uni-mrs.fr |
| Pemantle,R. | pemantle@math.wisc.edu |
| Pfister,Ch. | cpfister@eldp.epfl.ch |
| Pisztora,A. | pisztora@cims.nyu.edu |
| Shlosman,S. | shlosman@math.uci.edu |
| Spohn,H. | spohn@stat.physik.uni-muenchen.d400.de |
| Steif,J. | steif@math.chalmers.se |
| Strobel,Th. | strobel@uni-bonn.de |
| Szasz,D. | szasz@circle.math-inst.hu |

| | |
|---|---|
| Sznitman,A.-S. | sznitman@math.ethz.ch |
| Toth,B. | balint@circle.math-inst.hu |
| Vares,M.E. | eulalia@impa.br |
| Woess,W. | woess@vmimat.mat.unimi.it |
| Zahradnik,M. | mzahrad@karlin.mff.cuni.cz |