MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 22 / 1994

Diskrete Geometrie

22. 5. bis 28. 5. 1994

Unter der Leitung von L. Danzer (Dortmund) und G.C. Shephard (Norwich) trafen sich 45 Teilnehmer aus 11 Ländern zu fruchtbarer Diskussion. Wegen des hohen Maßes an Internationalität – nur ein Drittel der Teilnehmer kam aus Deutschland – wurden die 39 Vorträge ausnahmslos in Englisch gehalten.

Polytope und ihre Kombinatorik bildeten einen ersten thematischen Schwerpunkt der Tagung. Eine ganze Reihe von Vorträgen war solchen Einzelthemen gewidmet, die zeigten, daß es auch in der gewöhnlichen euklidischen Ebene noch viele offene Fragen gibt. In etlichen Vorträgen wurden Verbindungen zu anderen Gebieten der Mathematik — wie Gruppentheorie, algebraische Geometrie, hyperbolische Geometrie, Graphentheorie — hergestellt.

Eine weitere Gruppe von Vorträgen war dem breiten Problemkreis der Überdeckungen, Packungen und Pflasterungen zuzuordnen, wobei vor allem Untersuchungen über Dichten raumfüllender Packungen und Dichten endlicher Packungen (die von M. Henk und J. M. Wills vorgestellten Methoden haben kürzlich zum Beweis der bekannten "Wurstvermutung" von L. Fejes Tóth ab Dimension 45 geführt) sowie quasiperiodische Pflasterungen zu nennen sind; letztere haben für die für die Festkörperphysik Bedeutung. Der computergestützte Vortrag von Daniel Huson zum letztgenannten Problemkreis hob sich dabei in der Art der Präsentation von allen anderen Vorträgen der Tagung ab.

Ferner gab es mehrere Beiträge zu kombinatorisch-geometrischen Fragen (u.a. von H. Harborth, D. Larman, E. Makai, M.Perles, H. Tverberg).

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Ein Höhepunkt war ohne Zweifel der Meinungsaustausch über die Ende letzten Jahres erschienene Arbeit von Wu-Yi Hsiang zur Lösung des Keplerschen Problems über die dichteste Packung kongruenter Kugeln in der Dimension drei. Der anwesende Autor stellte sich (neben dem Vortragsprogramm) am Mittwochabend einer mehrstündigen, sehr ins Detail gehenden Diskussion mit den Tagungsteilnehmern, von denen sich einige intensiv mit diesem oder mit einschlägigen Problemen befaßt haben. Die mit großer Offenheit geführte Diskussion konnte natürlich keine letzte Klarheit schaffen, hat aber doch zu der gegenseitigen Einsicht geführt, daß zwar einerseits die vorliegende Publikation wegen erheblicher Lücken in den Details nicht als vollgültiger Beweis angesehen werden kann, daß aber andererseits die Lücken möglicherweise geschlossen werden können.

Neben dem offiziellen Programm der Vorträge von 20 bis 40 Minuten Dauer hielt Peter McMullen in einer der Mittagspausen einen längeren Vortrag "Informal Lecture on Weighted Polytopes", in welchem die von ihm aufgebaute Theorie der Polytop-Algebra weiterentwickelt und damit erneut die Fruchtbarkeit dieser Theorie unter Beweis gestellt wurde. Schließlich wurden auf zwei abendlichen Problemsitzungen am Dienstag und am Donnerstag Anstöße zu weiterer Forschung ausgetauscht.

Erneut erwies sich auf dieser Tagung das Aufeinandertreffen sehr verschiedener Methoden und Ideen als äußerst anregend, was insbesondere auch wieder der Anwesenheit einiger Teilnehmer zu danken ist, die der Diskreten Geometrie benachbarten Gebieten zuzurechnen sind.

Vortragsauszüge

A. ALTSHULER:
Polyhedra without diagonals

By a polyhedron without diagonals – or neighborly polyhedron – we mean a 3-dimensional polyhedron P in which every two vertices are joined by an edge of P. Two such polyhedra are known: The tetrahedron and the Császár torus. In both cases the boundary is a 2-manifold. The number of vertices is 4 and 7, respectively. The next such polyhedron – if it exists – would have at least 12 vertices. We show that if we allow the boundary to be a pseudomanifold, then there exist neighbourly polyhedra with 9 and 10 vertices. (Essentially, a pseudomanifold is obtained from a 2-manifold by pinching some vertices.) Furthermore: if we allow the boundary to be a generalized pseudomanifold (obtained from a 2-manifold by pinching edges), then for every $n \geq 3$ there exists a neighborly polyhedron with $2n$ vertices and with connected interior, whose boundary is a generalized pseudomanifold.

A. BEZDEK:
A solution of Conway's fried potato problem

In order to fry it as expeditiously as possible, Conway wishes to slice a given convex potato into n pieces by $n-1$ succesive plane cuts (just one piece being divided by each cut), so as to minimize the greatest inradius of the pieces. We show that one has to employ equally spaced parallel cuts, choosing the direction of the cuts parallel to the planes of support which determine the minimum width of a well defined, rounded potato. We also show some examples and discuss two variations of the original problem.

G. BLIND:
On packings with incongruent circles

Let $\mathcal{P}$ be a packing of circles K_i with radius r_i. The homogeneity h of $\mathcal{P}$ is usually defined by $h := \inf \frac{r_i}{r_j}$. An often investigated problem is to find good bounds for the packing density D of $\mathcal{P}$, and the bounds will depend on h.

We define a local homogeneity l by $l := \inf \frac{r_i}{r_j}$, where the inf is only over all neighboring circles of $\mathcal{P}$, and we are interested in bounds for D depending on l. We show in particular that $0{,}797... \leq l \leq 1$ implies $D \leq \frac{\pi}{\sqrt{12}}$.

K. BÖRÖCZKY:

Circle packings

It is well-known that the density of a packing of circles is at most $\frac{\pi}{\sqrt{12}}$ if the radii are chosen from the interval $[q, 1]$ where $q = 0{,}742990\ldots$. On the other hand, J. Molnár constructed an example with $q = 0{,}645707\ldots$ such that the density is $\frac{\pi}{\sqrt{12}}$. L. Danzer has another example where $q = 0{,}654316\ldots$. The union of two touching and non-overlapping circles is called a molecule (the radii of the circles are q and 1). A. Heppes proved that certain circle packings are solid. Two of them provide the densest packing of molecules for $q = \sqrt{2} - 1$ and $q = 0{,}637555\ldots$. In our result the possible values of q constitute a whole interval. The main theorem is the following:

Let $\frac{2}{\sqrt{3}} - 1 \leq q \leq 0{,}189673\ldots$. Then the density of any packing of the corresponding molecules is at most $\dfrac{\pi(1+q^2)}{\sqrt{3} + \dfrac{4\sqrt{q^2+2q}}{(1+q)^2}}$.

K. BÖRÖCZKY jun.:

Intrinsic volumes of and fat packings

Let C be a convex body in $\mathbf{E}^d$ and Q_n be the convex hull of n non-overlapping translates of C. Our goal is to minimize the intrinsic i-volume $V_i(Q_n)$ of Q_n for $i = 1, \ldots, d$. In the case of the volume $(i = d)$ there are various possibilities. For example, the minimal arrangement can be a "sausage" (if C is a cylinder) or Q_n can have large inradius for large n (if C is a spacefiller which is not a cylinder). On the other hand, for $i \leq d-1$ we proved that if $V_i(Q_n)$ is minimal then

$$\frac{r(Q_n)}{R(Q_n)} \geq 1 - \frac{\omega}{n^{\frac{2}{d(d+3)}}} \qquad \text{where} \qquad \omega = 4000\sqrt{d}\left(\frac{R(C)}{r(C)}\right)^{\frac{2}{d+3}}.$$

J. BOKOWSKI:

Dürer's polyhedron in his engraving MELENCOLIA I – Combinatorial tilings on the 3-sphere

The open Steinitz problem "characterize, among all combinatorial $(d-1)$-spheres, the d-polytopal ones", lead to the investigation of equifacetted simple 3-spheres which cannot be obtained as boundaries of 4-polytopes (joint work with Peter Schuchert). Apart from the known example of the Altshuler classification of neighborly 3-spheres, the search for examples within the class up to 10 facets lead to precisely one new example, the combinatorial facet-type of which equals that of Dürer's polyhedron in his famous copperplate engraving.

U. BREHM:

Triangulation of lens spaces with few simplices

(joint work with J. Świątkowski)

For the lens space $L(p,q)$ ($p > q$, p,q relatively prime) a triangulation with $24N(p/q) + 33$ tetrahedra and $3N(p/q) + 9$ vertices is constructed, where $N(p/q)$ denotes the sum of the coefficients of the representation of p/q as a continued fraction. Using Reidemeister's topological classification of lens spaces, this implies that there are at least 2^n topologically distinct triangulated lens spaces with at most $24(n+4)$ tetrahedra and $3(n+4)$ vertices. The core of the proof is the construction of a "reparametrizing complex" for a simplicial torus. Finally, it is indicated how to use these ideas to triangulate arbitrary 3-manifolds with few simplices using a Heegard diagram and Dehn twist.

F. BUEKENHOUT:

Unfolding combinatorial polytopes

This is a report on joint work with S. Bouzette, E. Dony and A. Gottcheiner. The literature on unfoldings of polyhedra seems rather scarce to us. We were developing a theory of unfoldings of a combinatorial polytope P in terms of morphisms from prepolytopes to P and other categorical ideas. It leads to a characterization based on the spanning trees of the 1-skeleton for the dual polytope P^*. Metrical polytopes in d-space and their metrical unfoldings in $(d-1)$-space are studied likewise. The enumeration of all unfoldings of the convex regular polytopes up to symmetry in dim ≤ 4 is close to completion in joint work with M. Parker. It was shown by Bonzette and Vandamme that the dodecahedron and the icosahedron unfold in 43 380 ways.

R. CONNELLY:

Equilateral, equiangular, planar polygons

The problem of calculating the number of components of the space of points in a Euclidean space subject to certain distance constraints is considered. It is possible to use and apply some of the ideas from the theory of rigid structures.

L. DANZER:
SCD, a 3-dimensional "Einstein"

A prototile SCD ($= T(^m/_n, s, h)$, $m, n \in \mathbb{N}$, g.c.d(m,n) = 1, $m, 2 < n$, $s, h \in \mathbf{R}$, $h > 0$) is presented, which

- is a weakly convex polyhedron in $\mathbb{E}^3$,
- permits $2^{\aleph_0}$ face-to-face tilings of $\mathbb{E}^3$,
- none of which is invariant under any nontrivial translation (mirror images excluded).

If and only if n is odd, the species S_0^+ of all face-to-face tilings by congruent copies of SCD (mirror images not permitted) is repetitive.

When SCD is furnished with a finite number of sites for various atoms, every tiling of the latter type yields an (r,R)-set (i.e. a DELONE-set)

- with only finitely many VORONOI-cells (up to rigid motions)
- and without any BRAGG-peak in its X-ray diffraction pattern (i.e. no DIRAC-delta in its FOURIER-transform), except a 1-lattice on the z-axis.

Only a very few of these patterns possess any global symmetry. There may be a reflection in a line. If there are more symmetries, there is a screw with an angle incommensurate to π.

If any of these (r,R)-sets are physically realizable, they will obey a strict long-range orientational order, but not by translations and far away from being quasicrystalline.

H. EDELSBRUNNER:
Volume of union of balls

Let $\mathcal{B}$ be a finite set of spherical balls in $\mathbf{R}^d$ and $\bigcup\mathcal{B}$ their union. By the inclusion-exclusion principle,

$$\text{vol} \bigcup\mathcal{B} = \sum_{\sigma \in 2^{\mathcal{B}} - \{\emptyset\}} (-1)^{\text{card}\,\sigma - 1} \text{vol} \bigcap\sigma .$$

Most of the terms in this formula are redundant. We show there is a d-dimensional abstract simplicial complex $K \subseteq 2^{\mathcal{B}}$ so if K is substituted for $2^{\mathcal{B}}$ the formula is still exact. Every simplex $\sigma \in K$ is independent; that is, $\bigcap\tau - \bigcup(\sigma - \tau) \neq \emptyset$ for all $\tau \subseteq \sigma$. The formula can be further reduced to terms of at most d (rather than $d+1$) balls each, using simplex angles as weights for the terms $\text{vol} \bigcap\sigma$.

A. FLORIAN:

On the intersection of a pyramid with a ball

Let Q be an n-sided pyramid contained in the unit ball K and having its apex at the centre O of K. Let U be the radial projection of the base of Q onto the boundary S^2 of K. We denote by $\bar{Q}$ the corresponding n-sided pyramid, the base of which is a regular n-gon with its vertices on S^2 and which satisfies

$$a(\bar{U}) = a(U) \qquad (a = \text{area}).$$

Let $K(\rho)$ be the ball with the same center O and radius ρ, where $0 < \rho \leq 1$.

Theorem: $V(Q \cap K(\rho)) \leq V(\bar{Q} \cap K(\rho))$, where V denotes volume.

When $\rho \geq$ height of $\bar{Q}$, then equality holds only for $Q = \bar{Q}$. Some applications are discussed.

P. GRITZMANN:

Largest simplices in polytopes

(joint work with V. Klee and D. Larman)

The talk gives various results on the computational complexity of problems related to the task of finding a largest j-simplex in an n-dimensional v- or x-polytope. Applications to the Hadamard determinant problem, to the question of bounding the growth rate of pivots in Gaussian elimination with complete pivoting and to weighing designs are indicated; but the main emphasis is placed on the discription of a polynomial-time transformation from the problem of finding a Hamilton cycle in a directed graph which is based on earlier work of Papadimitriou & Yannahahis on the recognition of integer polyhedra.

R. J. HANS-GILL:

The view-obstruction problems

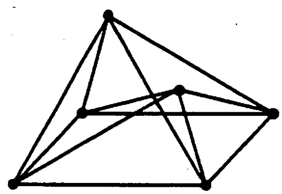
The view-obstruction problem for an n-dimensional closed convex body C containing the origin in its interior was formulated by Cusick in 1973. For $k > 0$, the convex body kC is translated to all points of the set $(-\frac{1}{2}, -\frac{1}{2}, \dots, -\frac{1}{2}) + \mathbb{N}^n$, where $\mathbb{N}$ is the set of natural numbers. When k is large these bodies block all the rays from the origin into the open positive cone. The central problem is to determine the minimal blocking size. Wills (1968) had already considered this problem for boxes while studying certain arithmetical functions. For spheres and boxes centred at 0 for $n = 2, 3, 4$ the problem has been solved by various authors (Betke and Wills, Chen, Cusick, Pomerance, Dumir, Hans-Gill and Wilker).

Several new results regarding this problem and its generalisations have been obtained by Dumir, Hans-Gill and Wilker. In particular the constant for sphere centred at 0 in $\mathbb{R}^5$ has been determined.

H. HARBORTH:
Few distances in small point sets

For n points in the plane the known values for the minimum number $f(n)$ of different distances are $f(3) = 1$, $f(4) = f(5) = 2$, $f(6) = f(7) = 3$, $f(8) = f(9) = 4$, $f(10) = f(11) = f(12) = 5$. The known numbers $r(n)$ of different configurations with $f(n)$ different distances are $r(3) = 1$, $r(4) = 6$, $r(5) = 1$, $r(6) = 9$, $r(7) = 2$, $r(8) = ?$, $r(9) = 3$.
For the platonic solid graphs $G = T, H, O, D, I$ (tetrah., cube, octah., dodecah., icosah.) straight line drawings $D(G)$ (two edges have at most one point in common) in the plane are considered. The minimum numbers A of different edge length are determined in the cases of real $(A(G))$, of integer $(A(G,r))$, of plane $D(G)$'s with real $(A_p(G))$, and of plane $D(G)$'s with integer edge lengths.

	$A(G)$	$A(G,r)$	$A_p(G)$	$A_p(G,r)$
T	2	3	2	3
H	1	1	2	2
O	2	3	3	3
D	1	1	2	2
I	3	3	4	≤ 7



For O all 26 different $D(W_5)$'s and 6 classes of $D(W_5)$'s with one parameter are constructed for the wheel graph W_5 with four spokes. These can be fitted together to get 9 octahedra drawings with two different edge lengths (see one in the figure). For I there are 274 $D(W_6)$'s, and no combinations are possible to get a $D(I)$ with only two different edge lengths.

M. HENK:
Finite and infinite lattice packings

For a convex body K in the d-dimensional Euclidean space E^d let $\mathcal{P}_L(K)$ be the family of all packing lattices with respect to K. Further for an integer n let $\mathcal{P}_L(K,n) = \{ C \subset E^d \mid \#(C) = n$ and there exists a $\Lambda \in \mathcal{P}_L(K)$ with $C \subset \Lambda \}$.
We call $\mathcal{P}_L(K,n)$ the set of all finite lattice packings of K (of cardinality n).
With respect to the parameterized densities

$$\delta_L(K,n,\rho) = \max\left\{ \frac{nV(K)}{V(\mathrm{conv}(C) + \rho K)} : C \in \mathcal{P}_L(K,n) \right\},\ \rho > 0,$$

we show

$$\limsup_{n\to\infty} \delta_L(K,n,\rho) = \delta_L(K) \quad \text{for} \quad \rho \geq \rho(K) = \begin{cases} \frac{1}{2}\sqrt{21}\ , & K = B^d \\ 3\ , & K = -K \\ \frac{3}{2}(d+1)\ , & \text{else} \end{cases}$$

where $\delta_L(K)$ is the density of a densest infinite lattice packing.

W.-Y. HSIANG:

The proof of Kepler's conjecture on the sphere packing problem

The sphere packing problem: If one packs a large number of identical spheres into a container, the density of such a packing is defined to be the percentage of the volume of the container occupied by spheres. The l.u.b. of such packings will, of course, depend on the shape and the relative size of the container. If one let the relative size between the container and the spheres tend to infinity, then the above l.u.b. will tend to a common limit value independent of the shapes of the containers. This common limit is essentially the optimal density of sphere packings with the whole space as the container! The sphere packing problem seeks to determine the exact value of the above optimal density.

The Kepler conjecture on sphere packing: In a latin booklet of 1611 entitled: "A new year's gift — on six cornered snowflake" He discussed the above sphere packing problem, compared the densities of a few examples which are, nowadays, called the pattice type packing. Among them, he found the face centered cubic lattice packing is the densest, and then, he boldly conjectured that the f.c.c. packing already achieved the optimal density of all possible sphere packings, namely, in his own words: "The (f.c.c.) packing will be the tightest possible, so that in no other arrangements could more pellets be stuffed into the same container."

The proof of Kepler's conjecture: The sphere packing problem and the Kepler conjecture is, by definition, a problem of infinitely many spheres. The first step is to introduce suitable local invariants so as to reduce the above problem to some problems of finite spheres, if such a reduction is at all possible. The following two local invariants play this crucial role in the proof of Kepler's conjecture, namely (i) local density and (i) a locally averaged density defined as follows.

Definitions: (i) To each given sphere in a packing $\mathcal{P}$, one associates a surrounding polyhedron which consists of those points that are as close to its center as to the centers of the others, and call it the local cell of the sphere in the given packing $\mathcal{P}$. The local density of a sphere S_i in $\mathcal{P}$ is defined to be the ration between the volumes of the sphere and its local cell, namely

$$\rho(S_i,\mathcal{P}) = \frac{\mathrm{vol}(S_i)}{\mathrm{vol}\, C(S_i,\mathcal{P})}.$$

(ii) Two spheres are defined to be close neighbors if their center distance is at most 2.18 times the radii (two spheres are defined to be neighbors id their local cells have a common face). The cluster of spheres consisting of a central sphere S_i and all its close neighbors is called the core packing of S_i and denoted by $\hat{\mathcal{L}}(S_i)$. Set

$$\hat{\mathcal{L}}(S_i) = \{ S_{ij},\ 1 \le j \le \#\hat{\mathcal{L}}(S_i) \}$$

$$\omega_{ij} = \frac{13}{\#\hat{\mathcal{L}}(S_i)} \cdot \mathrm{vol}\, C(S_{ij},\mathcal{P})$$

(iii) The locally averaged density of S_i in $\mathcal{P}$ is defined to be the following weighted average of the local densities of S_{ij} in $\mathcal{P}$, namely

$$\bar{\rho}(S_i,\mathcal{P}) = \frac{\sum_j \omega_{ij}\ \rho(S_{ij},\mathcal{P})}{\sum_j \omega_{ij}}.$$

In a paper published in the october issue (1993) of International Journal of Mathematics, the author proves the following theorems on the optimal upper bound of the local invariants,namely

Theorem 1: The optimal local density is equal to

$$\frac{4\pi}{3} : 20(1-2\cos\frac{2\pi}{5})\tan\frac{\pi}{5} = 0{,}75469\ldots$$

and it can be achieved when and only when the local cell is a circumscribing regular dodecahedron.

Theorem 2: The optimal locally averaged density is equal to $\pi/\sqrt{18}$ and it can be achieved when and only when the double layer local packing (consisting of the central sphere, its close neighbors and all their neighbors) is a sub-cluster of the close packing of Barlow-type.
Since the global density of a packing $\mathcal{P}$ is the following weighted average density,namely

$$\rho(\mathcal{P}) = \limsup \left\{ \frac{\sum_i \bar{\omega}_i\ \bar{\rho}(S_i,\mathcal{P})}{\sum_i \bar{\omega}_i} \right\}$$

where $\bar{\omega}_i = \sum_j \omega_{ij}$, the Kepler conjecture follows directly from Theorem 2. This proves the Kepler conjecture that $\pi/\sqrt{18}$ is indeed the optimal density of all possible sphere packings in the whole E^3.

D. HUSON:

Periodic Delone tilings

Delone complexes (and Voronoi Diagrams) are useful in a large number of fields, from archeology to zoology. Many different algorithms have been devised to compute them. For simplicity and to avoid computational difficulties, the input data, a set of points, is assumed to be in general position and the arising Delone complexes are always triangulations. In the application of Delone complexes to crystal structures, however, it is important it is important that the arising cells or tiles reflect the symmetry of the given points (atoms) and hence special arrangements should not be neglected. In this talk a simple geometric procedure is discussed and demonstrated that computes the Delone complex (i.e. tiling) of a periodic 3D point set, taking special arrangements into account.

H.-C. IM HOF:

Complex volume of orthoschemes

The aim is to define a volume functional for polytopes in projective space P^n with respect to a quadric Q inducing hyperbolic geometry.

An orthoscheme is a simplex described by a linear Coxeter diagram. The volume of a reducible orthoscheme is inductively defined to be the product of the volumes of its components.

An irreducible orthoscheme induces a configuration of $n+3$ hyperplanes, which in turn define a set of $2n+7$ polytopes. One of these lies inside Q, it therefore has a positive real volume. It differs from the given orthoscheme by an algebraic sum of reducible orthoschemes. This allows us to assign a complex volume to any orthoscheme in P^n.

The Schläfli differential form continues to hold in our more general context.

A. IVIĆ WEISS:

Polytopes related to the Picard group

For each positive integer m we show that there is a finite 4-polytope with simplicial facets whose vertex-figures are regular abstract polyhedra of type $\{3,m\}$. In the construction we employ certain quotients of the Picard group. Distinct polytopes of the same type can then arise, some of which may be chiral; but in each instance, facets and vertex-figures are regular.

R. KELLERHALS:

Sphere packings and volumes of hyperbolic manifolds

We improve estimates from below for volumes $\mathrm{vol}_n(M^n)$ $(n \geq 3)$ of compact hyperbolic space forms $M^n = H^n / \Gamma$, $\Gamma < \mathrm{Iso}(H^n)$ discrete and torsionfree, by making use of sphere packings and density estimates (following ideas of R. Meyerhoff and Martin).

Let $r(x,M) := \sup\{ r \mid \exp_x$ injective on a ball of radius $r \}$ and $r(M) := \sup\{ r(x,M) \mid x \in M \}$. Then, by Martin's result, $r(M) > \frac{1}{2 \cdot 9^{2+[\frac{n}{2}]}}$. The Buser-Karcher estimate says that

$$\mathrm{vol}_n(M^n) \geq \frac{\Omega_{n-1}}{n} \cdot \left(\frac{1}{4^{n+3}}\right)^n, \quad \Omega_{n-1} = \mathrm{vol}_{n-1}(S^{n-1}).$$

By Böröczky's theorem on the upper bound of a sphere packing (here with balls of radius $r(M)$), we derive the lower bound

$$\mathrm{vol}_n(M^n) \geq \mathrm{vol}_n(R(\alpha)), \text{ where}$$

$$\Sigma(R(\alpha)) : \bullet\overset{\alpha}{—}\bullet—\bullet \cdots\cdots \bullet—\bullet, \quad \frac{1}{n} < \cos(2\alpha) = \frac{\cosh(2\,r(M))}{1+(n-1)\cosh(2\,r(M))} < \frac{1}{n-1}$$

denotes the characteristic hyperbolic n-orthoscheme associated to the regular simplex $S_{\mathrm{reg}}(2\alpha)$ by $S_{\mathrm{reg}}(2\alpha) = (n+1)!\,R(\alpha)$.

W. KUPERBERG:

Mutually contiguous translates of a plane region
(joint work with András Bezdek and Krystyna Kuperberg)

Two sets are *contiguous* if they have disjoint interiors and a common boundary point. Members in a family of sets are *concurrent* if all of the sets in the family have a common point. We consider a family of mutually non-overlapping translates of a plane region (bounded by a simple closed curve), and we prove that if the translates are concurrent, then the family concists of at most four members. Generalizing this result, we prove that if a family of mutually contiguous translates of a region has at least four members, then the translates are concurrent, and therefore the family consists of exactly four members. This solves a problem of J. Mitchell, posed in 1989. Also, we characterize those regions which admit four concurrent, mutually non-overlapping translates.

D. G. LARMAN:

The art gallery problem with 180° guards
(joint work with Ellen Bunting)

An art gallery room with n sides is a simple plane polygon with n sides. The original art gallery problem was to place a minimal number of guards in the gallery so that every point of the gallery can be "seen" by at least one guard. An ingenious argument of Chvatal shows that in general $\frac{n}{3}$ guards will suffice and that this is the best possible result. Urrutia asked for the analogous number when the guards vision is restricted to 180o. Here we show that $\left\lfloor \frac{4n+1}{9} \right\rfloor$ guards will do but perhaps again the correct answer is $\left\lfloor \frac{n}{3} \right\rfloor$. If it could be shown that an art gallery with eleven sides requires only 3 guards then the result could be improved to $\frac{2}{5}n$.

C. LEE:
Combinatorial volume

Notation: For $(r_1, \dots, r_n) \in \mathbb{Z}_+^n$ and indeterminates $x_1, \dots, x_n$, $|r| = r_1 + \dots + r_n$, $r! = r_1! \cdot \dots \cdot r_n!$, $x_r = x_1^{r_1} \cdot \dots \cdot x_n^{r_n}$, and $\operatorname{supp}(x^r) = \{ i \mid r_i > 0 \}$. Let Δ be an abstract simplicial $(d-1)$-complex with vertex set $\{1, \dots, n\}$ having an underlying topological space which is an orientable manifold, $v_1, \dots, v_n$ be generic points in $\mathbb{R}^d$, and M be the $d \times n$ matrix $[v_1, \dots, v_n]$. Then there exists a polynomial $V(x) = \sum_{|r|=d} b_r \frac{x^r}{r!}$ satisfying $b_r = 0$ when $\operatorname{supp}(x^r) \notin \Delta$ and $M \nabla b = 0$.

<u>Theorem</u>: $V(x)$ is unique up to scalar multiple, and $V(1) \neq 0$. In the case that $\Delta \equiv \partial \operatorname{conv}\{v_1, \dots, v_n\}$ then $V(x) = \operatorname{vol}\{ y \mid y \cdot v_i \leq x_i, i = 1, \dots, n \}$ when $x_i = 1$, and so $V(1)$ is obviously nonzero. Let $A_\Delta = \mathbb{R}[x_1, \dots, x_n]/I_\Delta$ be the Stanley-Reisner ring of Δ, $\theta_j = \sum_{i=1}^{n} v_{ij} x_j$, $j = 1, \dots, d$, and $B = A_\Delta/(\theta_1, \dots, \theta_d) = B_0 \oplus \dots \oplus B_d$. Then $\dim_{\mathbb{R}} B_0 = \dim_{\mathbb{R}} B_d = 1$. As a corollary to the theorem, $(x_1 + \dots + x_n)^d \neq 0$ in B_d and so multiplication by $(x_1 + \dots + x_n)^d$ induces a bijection between B_0 and B_d.
(This is joint work with Sue Foege at the University of Kentucky.)

E. MAKAI:
Lower bounds on the numbers of shadow-boundaries and illuminated regions

Let $K \subset \mathbb{R}^n$, $n \geq 2$, be a closed convex set, with $\emptyset \neq \operatorname{int} K$, $K \neq \mathbb{R}^n$. We denote by $\operatorname{shb}_{pt}(K)$, $\operatorname{shshb}_{pt}(K)$, $\operatorname{illr}_{pt}(K)$ the number of different non-empty shadow-boundaries, non-empty sharp shadow-boundaries, and illuminated regions (in the strict sense) of K, different from bd K, w.r.t. arbitrary point light sources outside K. (A shadow boundary is sharp if each light ray that supports K has at most one common point with K.) If we consider parallel illumination, from directions whose opposite direction does not belong to the characteristic cone of K, we denote the analogous quantities by $\mathrm{shb}_{\mathrm{par}}(K)$, $\mathrm{shshb}_{\mathrm{par}}(K)$, $\mathrm{illr}_{\mathrm{par}}(K)$. Any of these six quantities is finite if and only if K is polyhedral. If K is a both-way infinite cylinder over an $(n-1)$-dimensional convex set L, then, apart from some simple cases, each of these quantities has the same value for K and L. Let therefore K be line-free. Let j denote the dimension of the characteristic cone of K. Then, if K varies by fixed j, the minimum of $\mathrm{shb}_{\mathrm{pt}}(K)$, $\mathrm{shshb}_{\mathrm{pt}}(K)$, $\mathrm{illr}_{\mathrm{pt}}(K)$ (resp. $\mathrm{shb}_{\mathrm{par}}(K)$, $\mathrm{shshb}_{\mathrm{par}}(K)$) is attained if and only if K is a j-fold one-way infinite cylinder over an $(n-j)$-simplex (resp. $(n-j)$-parallelotope).

P. MANI-LEVITSKA:

On Nikolai Mnëvs universality theorem

We, that is Daniel Lehner and I, are presenting a slightly modified and elaborated proof of N. Mnëv's universality theorem (Springer Lecture Notes 1346): Given any semialgebraic set M, there exists a finite sequence c of points in the real projective plane $\mathbf{P}_2$, whose reduced configuration space $[c]_0$ is stably homeomorphic to M. Consider a resolution $\sigma = (\sigma_1, \dots, \sigma_r)$ of the defining polynomials $f_1, \dots, f_r$ for M. σ_i is a rooted labeled binary tree, which describes how f_i is composed from constants and variables by successive application of the operations $+, -, \cdot, :$. There is a map ε "forgetting the logical structure", from the vertex set $V(\sigma)$, the disjoint union of the vertex sets $V(\sigma_i)$, into the field $\mathbb{Q}(x_1 — x_k)$ of rational functions over $\mathbb{Q}$. ε maps the root of σ_i into f_i. Let $\mathrm{reg}(\sigma)$ be the set of those $p \in \mathbf{R}^k$ where $\varepsilon(v)$ is regular, for every $v \in V(\sigma)$. Call two points p, q in $\mathrm{reg}(\sigma)$ equivalent, if $\varepsilon(v) - \varepsilon(w)$ has the same sign, $+$, $-$ or 0, at p and at q, for every $v \neq w$ in $V(\sigma)$. Denote by $\Pi(\sigma)$ the set of equivalence classes.

Proposition 1. For every $K \in \Pi(\sigma)$ there exists a sequence c in $\mathbf{P}_2$ such that K is stably homeomorphic to $[c]_0$.

Proposition 2. For an appropriate choice of σ, there is some $K \in \Pi(\sigma)$, stably homeomorphic to M.

We have found that, after a few modifications, N. Mnëv's proofs of these statements are correct.

I have learned from Günter Ziegler that Harald Günzel in Aachen has come to the same conclusion, so that a clean text should be available in the near future.

P. MCMULLEN:

"Bare hands" constructions of regular polytopes

Because of the close connexion between regular polytopes (as presently understood) and string C-groups, the most popular method used these days to construct new regular polytopes is by means of their automorphism groups. However, when Grünbaum relaunched the subject in 1975, he frequently used direct methods, even in some instances constructing regular polytopes by fitting together their facets one by one. In this talk, we wish to show that there is still some milage to be gained from such "bare hands" techniques.

B. NOSTRAND:

Ring extensions and chiral polytopes

Abstract polytopes are partially ordered structures which generalize the notion of polyhedra in a combinatorial sense. There are abstract polytopes which correspond to all of the classical regular polytopes and many other well-known structures. Chiral polytopes are repetitive structures with maximal rotational symmetry which lack reflexive symmetry. While much is known about regular polytopes, little is yet known about chiral polytopes. The simplest chiral polytopes are all twisted tessellations of torii. While these chiral torii can be used to construct locally toroidal chiral polytopes of rank 4, we can also construct locally spherical polytopes. We use hyperbolic honeycombs to construct the symmetry groups of abstract polytopes over finite rings. The corresponding polytopes belong to families with related local symmetry. Ring extensions allow us to construct additional members of these families and to alter global structure.

J. PACH:

On Conway's thrackle conjecture

A thrackle is a graph $G = (V(G), E(G))$ drawn in the plane by simple arcs such that

(i) any two edges that do not share an endpoint cross exactly once, and

(ii) no two edges sharing an endpoint have any other point in common.

Conway conjectured that $|E(G)| \leq |V(G)|$ for any thrackle G.

Theorem 1. Any bipartite graph that can be drawn as a thrackle is planar.

Theorem 2. $|E(G)| \leq 2\,|V(G)|$ holds for every thrackle G.

Analogous results can be proved when we impose some weaker parity conditions on the pairwise intersections of the arcs. This is joint work with Mario Szegedy.

M. PERLES:

Transversals of polytopes

(joint work with N. Prabhu from Purdue University)

Theorem 1: Let P be a convex d-polytope in $\mathbb{R}^d$. There is no k-flat that meets the relative interiors of all j-faces of P, unless $k \geq \min(d, 2(d-j))$.

The proof uses the fact, due to B. Grünbaum, that the $\lfloor \frac{d}{2} \rfloor$-skeleton of a d-polytope P is not isomorphic to the $\lfloor \frac{d}{2} \rfloor$-skeleton of a polytope Q of dimension $\neq d$.

Theorem 2: For each $d \geq 2$ and $n \geq d+1$ there exist a simple d-polytope $P^* = P^*(d,n)$ in $\mathbb{R}^d$, having exactly n facets, with $0 \in \operatorname{int} P^*$, and a nested sequence $(J^{2\nu})_{\nu=0}^{[d/2]}$ of linear subspaces of $\mathbb{R}^d$, with $\dim J^{2\nu} = 2\nu$, such that $J^{2\nu}$ meets the relative interior of every $(d-\nu)$-face of P^* $(0 \leq \nu \leq [d/2])$.

P^* can be chosen as the polar of a translate of a standard cyclic d-polytope with n vertices (= convex hull of n points on the moment curve $(t, t^2, \ldots, t^d)$), with $J^{2\nu} = \operatorname{span}\{e_1, e_2, \ldots, e_{2n}\}$.

J. RUSH:

On lattice packing densities

Superballs are bodies of the form $\{ x : f(x) < 1 \}$ where f is a superball function. A function $f : \mathbf{R}^k \longrightarrow \mathbf{R}$ is a superball function if it satisfies:

(1) $f(0) = 0, f(x) > 0$ if $x \neq 0$

(2) $f(x) = f(-x)$

(3) $\forall\, t > 0\ \exists\, A \in GL_k(\mathbb{R})$ s.t. $tf(x) = f(Ax)\ \forall\, x \in \mathbb{R}^k$

(4) $f(\theta x + (1-\theta)y) \leq \theta f(x) + (1-\theta)f(y)\ \forall\, x, y \in \mathbb{R}^k, 0 \leq \theta \leq 1.$

We conjecture that the maximum lattice-packing density δ_L of the superball

$$f(x_1, \ldots, x_k) + f(x_{k+1}, \ldots, x_{2k}) + \ldots + f(x_{n-k+1}, \ldots, x_n) < 1 \quad \text{in } \mathbf{R}^n,$$

where k divides n, is asymptotically

$$\delta_L = \left(\frac{1}{2} \sup_{A \in GL_k(\mathbf{R})} \sqrt{\frac{\int_{\mathbf{R}^k} \exp(-f(Ax))\, dV}{\sum_{\mathbf{Z}^k} \exp(-f(Ax))}} \right)^{n(1+o(1))} \quad \text{for large } n.$$

P. SCHMITT:

Space filling knots

In this talk two examples of knotted space fillers are described, i.e. two bodies in Euclidean space which are topologically equivalent to a torus but are embedded like a trefoil knot, and which tile space by congruent copies. These space fillers can be realized as toroidal polyhedra.

Furthermore, the idea used in the construction can be generalized to obtain arbitrarily knotted (or linked) space fillers.

Remark: Different constructions were found independently by W. KUPERBERG and by COLIN DAVIS. An earlier example is due to PETER MCMULLEN (unpublished).

E. SCHULTE:

Toroidal adventures

A central problem in abstract polytope theory is the classification of polytopes by their local or global topological type. On the group level this amounts to the classification of C-groups in terms of generators and relations. The classical theory corresponds to the spherical case. Partial results are discussed for the case when the polytopes are locally spherical or euclidean space-forms. The situation is best understood in the toroidal case.

G. C. SHEPHARD:

Ceva, Menelaos, and friends

The theorems of Ceva and Menelaos are well known. These are just two of a vast number of theorems which state identities of the following kind. Let Y_i be a point defined in some geometrically significant way on each edge or diagonal $[V_i, V_{i+1}]$ of an n-gon $[V_1, V_2, \dots, V_n]$. Then a product $\prod_{i=1}^{n} [V_i Y_i / Y_i V_{i+1}] = +1$ or -1. In this lecture, all these identities were shown to be consequences of a fundamental theorem (which is too long to state here) concerning d-dimensional analogues of polygons in A^d, namely polyacrons. These are defined as sets of points in A^d; in addition to edges there are 2-faces (triangles), 3-faces tetrahedra etc. defined by adjacent sets of 3, 4, ... vertices. The analysis of the results was facilitated by CW-diagrams. These show immediately which products of the above type have the required property, namely the value $+1$ or -1 for all n-acrons. Details will be published shortly.

H. TVERBERG:

Almost-transversals for families of translates satisfying T(4)

We consider a family of disjoint convex sets in the plane. They are translates of one compact set K. Assume that every 4 sets have a transversal, i.e. a straight line meeting all of them. Then it follows from a result of Katchalski and Lewis that there will be a transversal for the whole family, with the exception of at most 603 sets. We state this as

$$T(4) \Rightarrow T-603 .$$

Katchalski and Lewis have a conjecture that $T(3) \Rightarrow T-2$, which of course implies $T(4) \Rightarrow T-2$. We indicated a proof, using a result of Eckhoff, that $T(4) \Rightarrow T-6$. This can probably be strengthened by using another one of Eckhoff's results about $T(4)$.

We also explained how the fact that $T(5) \Rightarrow T-0$ implies that if there is counterexample to $T(4) \Rightarrow T-2$, then there is one with a small number of sets (definitely ≤ 49, and most likely ≤ 15). Note that $T(4) \Rightarrow T-2$ holds for circles as well as squares, and that even $T(4) \Rightarrow T-1$ might be true.

W. WHITELEY:

Constraining plane configurations in CAD

A "constrained diagram" in Computer Aided Design is

(i) a collection of geometric objects (points, lines, circles)

(ii) a set of incidences (point on line, line tangent to plane ...)

(iii) a set of numerical constraints: distances, angles ... .

Some basic questions in CAD ask

(a) when a constrained diagram is

  (i) unique, up to congruence, translation ... ;

  (ii) locally unique, up to congruence, translation ... ;

  (iii) fixed, at first-order, up to congruence, translation ... ?

(b) when are the constraints

  (i) independent? (One can be changed, creating a nearby configuration)

  (ii) minimal, for first-order uniqueness?

  (iii) possible new constraints which are independent?

Most problems of this type are unsolved. Even points, lines and angles alone are not characterized.

This talk summarized two known cases.

(c) points with distance constraints (plane rigidity);

(d) points with directions for pairs (parallel drawings);

and presented new results to characterize the combined pattern of

(e) points with both distance and direction constraints.

In this combined problem the 'special position' diagrams are described by interesting Euclidean properties.

J. B. WILKER:

The Apollonian gasket and periodic isoclinal sequences

One approach to estimating the Hausdorff dimension of the Apollonian gasket led to a sequence of disks in the plane with radii in geometric progression and any four consecutive disks mutually tangent. This motivated the definition of an isoclinal sequence of $(n-1)$-spheres in inversive n-space with any $n+2$ consecutive $(n-1)$-spheres required to have the same inclination γ to one another. It turns out that for such a sequence to exist the inclination γ must satisfy $\gamma < -\frac{1}{n+1}, \gamma \neq -\frac{1}{n}$. When n is odd and $-\frac{n+2}{n^2+3n+1} < \gamma < -\frac{1}{n+1}$, the Möbius transformation which advances the sequence upon itself is conjugate to an $\frac{n+1}{2}$-fold rotation of the n-sphere. The condition for this tranformation to be periodic can be expressed in terms of the existence of concurrent chords in a regular N-gon. This yields a complete enumeration of periodic isoclinal sequences: there are infinitely many such sequences in dimension $n = 1$ and there is exactly one such sequence in dimension $n = 3$. The sequence in dimension $n = 3$ consists of 24 2-spheres and has $\gamma = \frac{1-2\sqrt{2}}{7}$.

J. M. WILLS:

A new approach to packing and covering

Let $K \subset E^d$ be a convex body with volume $V(K) > 0$ and let $K_i = K + c_i, i = 1, \ldots, n$ and $C_n = \{c_1, \ldots, c_n\}$. Then

$$\theta(K, C_n, \rho) = n \frac{V(K)}{V(\mathrm{conv}\, C_n + \rho K)}$$

is called the density function of the arrangement $C_n + K$, where $\rho \in \mathbf{R}$ is a parameter. For $\rho > 0$ $V(\mathrm{conv}\, C_n + \rho K)$ can be estimated via mixed volumes; for $\rho < 0$ by other convexity tools. If $\dim(\mathrm{conv}\, C_n + \rho K) < d$, we set $\theta = \infty$. The idea of the parameter ρ is to control the influence of the boundary region of $C_n + K$. If $\mathrm{int}(K_i \cap K_j) = \emptyset$ for $i \neq j$, then $C_n + K$ is a packing, if $\mathrm{conv}\, C_n \subset C_n + K$, then $C_n + K$ is a covering. So one can define optimal densities for finite packings and coverings:

$$\delta(K, n, \rho) = \sup \{ \theta(K, C_n, \rho) \mid C_n + K \text{ packing} \} \qquad \rho > 0$$

$$\delta(K, n, \rho) = \inf \{ \theta(K, C_n, \rho) \mid C_n + K \text{ covering} \} \qquad \rho \in \mathbf{R}.$$

For suitable ρ these densities tend to classical packing and covering densities (lattice and nonlattice). So one gets a joint theory for finite and infinite packing and covering.

G. ZIEGLER:

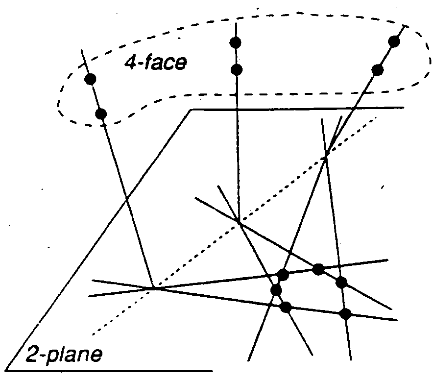
Combinatorics of 4-dimensional polytopes

1. <u>4-Polytopes are not extendably shellable</u>

We explained constructions which show that, while shelling a 4-polytope, one can "get stuck". The basic step is the identification of nonshellable 3-balls (after FRANKL and BING) as subcomplexes of "piles of cubes", which can be lifted to 4-space convexly.

2. <u>For 5-polytopes one cannot prescribe the shape of a 2-face</u>

We sketched the extremely simple, direct construction (due to J. RICHTER-GEBERT) of a 5-polytope for which the shape of a hexagonal 2-face cannot be arbitrarily prescribed – in every realization of the 5-polytope the hexagon vertices lie on a conic.



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