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Willi Jäger, Heidelberg

Methods of asymptotic analysis are basic in many areas of mathematics and its applications: e.g. in mathematical modelling, reducing complex systems to simpler ones, analysing systems with respect to characteristic parameters, studying systems with different scales, characterising behaviour of solutions, in numerical methods. Theory of singular perturbation covers problems depending on parameters not in a regular way and leading for critical parameters to significant changes e.g. in the character of the relevant equations and their solutions. Up to now there does not exist a unified theory of singular perturbation. Recent developments in the theory of dynamical systems, of bifurcation, of limiting and homogenization were the main impulses for this Oberwolfach meeting. It was the second of its kind organized by the Mathematische Forschungsinstitut. 28 lectures covered mainly two areas: singular perturbation in dynamical systems (mainly nonlinear ordinary and partial differential equations) and homogenization of partial differential equations and of problems of calculus of variation.

To bring together specialists from these different directions and to combine their expertise was an important aim of this conference. The exchange of results and methods, the ongoing discussions have proven to be very important for the participants who usually are split up in different groups. The following topics are examples for what was discussed in detail:

Systems reduction in case of singular perturbed partial differential equations, systems showing metastable behaviour, developing layers in the interior and at the boundary, with interfaces, problems with rapidly oscillatory geometry or other datas, systems with multiple scales. The underlying functional analytic concepts and the proper tools for the different special problems were discussed. The relation of asymptotic methods and numerical algorithms attracted strong interest. The perspectives of applying theoretical results of asymptotic analysis to design more effective numerical algorithms were discussed. They are very promising.

The impulses stirred by the interaction of different mathematical areas were the main result of the conference. An active cooperation in research developed during the meeting. The very special conditions and the unique atmosphere of the Forschungsinstitut were essential for the success obtained. All participants were very grateful to the institute and its staff who helped to make the stay not only pleseant but also scientificly most successful.

Vortragsauszüge

Peter W. Bates:

Asymptotic dynamics of the Cahn-Hilliard equation are the Mullins-Sekerka flow

Writing the Cahn-Hilliard equation as a sytem:

$$u_t^\varepsilon = -\Delta v^\varepsilon$$
$$v^\varepsilon = \varepsilon\Delta u^\varepsilon - \frac{1}{\varepsilon} f(u^\varepsilon) \qquad \text{in} \quad \Omega \subset \mathbf{R}^N,$$
$$\frac{\partial u^\varepsilon}{\partial n} = \frac{\partial v^\varepsilon}{\partial n} = 0 \qquad \text{on } \partial\Omega,$$
$$u^\varepsilon(x, 0) = u_0^\varepsilon(x),$$

we show that there exists a family of inititial functions $\{u_0^\varepsilon\}_{0<\varepsilon\leq 1}$ such that the zero level set of the solution u^ε converges as $\varepsilon \to 0$ to the solution to the Mullins-Sekerka (or Hele-Shaw) problem Γ_t:

$\Gamma_0 \subset \Omega$ a given closed (N - 1)-dimensional submanifold.

$\Delta v = 0$ in $\Omega\backslash\Gamma_t$, $t > 0$

$v = \sigma K$ on Γ_t, K = mean curvature, σ a constant

$\frac{\partial v}{\partial n} = 0$ on $\partial\Omega$

Γ_t envolves with normal velocity $= \frac{1}{2}$(jump in $\frac{\partial v}{\partial n}$ across Γ_t).

Giovanni Bellettini

Approximation by Γ-convergence and discretization of some geometric problems in calculus of variations

We have considered the variational approximation of the functionals

$$J(E) = P_\Omega(E) + \cos\theta\, H^{n-1}(\partial E \cap \partial\Omega) - \int_E k\,dx \qquad E \subseteq \Omega \subseteq \mathbf{R}^n,$$

$$J(u) = \int_\Omega |\nabla u|^2 dx + H^{n-1}(S_n) + \int |u - g|^2 dx \qquad u \in SBV(\Omega).$$

By means of sequences of more regular energies defined on finite dimensional spaces, we have discussed how the relaxation and the discretization parameters must be related in order to have the convergence result.

Finally we discuss semicontinuity problems for the funtional

$$\int_{\partial E} [1 + \psi(K)]dH^1 + \int_E g\,dx, \qquad E \subseteq \mathbf{R}^2,\ E \in C^2,$$

where K is the curvature of ∂E and ψ is a convex function.

Pavol Brunovsky

Tracking invariant manifolds without differential forms

An alternative of a result of Jones, Kaper and Koppell concerning the inclination of invariant manifolds of singularly perturbed differential equations at exit points from neighbourhoods of "slow manifolds" is given. Unlike the original proof, it does not make use of differential forms.

G. A. Chechkin

Homogenization problems with random alternating boundary conditions

Let (Ω, B, μ) be a probability space , $T_\xi : \Omega \to \Omega$ is a mapping semigroup with respect to ξ which leaves the measure invariant.

Definition. A random set $\Gamma \subset \mathbf{R}^{n-1}$ is called nondegenerate if $\exists$ a positive function h on Ω: $h(T_\xi\omega)$

such that for almost all $\omega \in \Omega$, $\forall\ \varphi \in C_0^\infty(\mathbf{R}^n \setminus (\Gamma \times \{\zeta = 0\}))$

$$\int_{\mathbf{R}^{n-1}} h(T_\xi\omega)\varphi^2(\xi, 0)d\xi \le \int_{-\infty}^{0} \int_{\mathbf{R}^{n-1}} |\nabla\varphi(\xi, \zeta)|^2 d\xi d\zeta; \tag{1}$$

$$\text{in addition } \langle h^{-1} \rangle \equiv \int_\Omega \frac{d\mu(\omega)}{h} < \infty. \tag{2}$$

Let Γ_q be random nondegenerate closed sets in $\mathbf{R}^{n-1}$, $q = 1, \ldots, N$. The domain $D \subset \mathbf{R}^n$ has a smooth boundary ∂D. The boundary ∂D consists of finite number of maps $\{V_q\}$, $q = 1, \ldots, N$, with local coordinates $x_q = (x_q^1, \ldots, x_q^{n-1})$. Let $\partial D = \Gamma_\varepsilon \cup \gamma_\varepsilon$, in each map V_q $\quad \Gamma_\varepsilon = \{x_q | \, x_q \in V_q, \frac{x_q}{\varepsilon} = (\xi_1, \ldots, \xi_{n-1}) \in \Gamma_q\}$.

We consider the problem $\Delta u_\varepsilon = 0$ in D, $u_\varepsilon|_{\Gamma_\varepsilon} = 0$, $\varepsilon \frac{\partial u_\varepsilon}{\partial n}\Big|_{\gamma_\varepsilon} = g(x)$. (3)

We will look for asymptotics of the generalized solution $u_\varepsilon(x)$ in the form of $u_\varepsilon(x) \sim u_0(x)W(\frac{x}{\varepsilon})$, where $\Delta u_0 = 0$ in D, $u_0|_{\partial D} = \langle W \rangle g$. (4)

The function $W(\xi, \zeta) = \widetilde{W}(T_\xi\omega, \zeta)$ satisfy the following problem

$$A^{ij}\partial_i\partial_j\widetilde{W} + \frac{\partial^2\widetilde{W}}{\partial\zeta^2} = 0,\ \zeta < 0;\ \widetilde{W}|_{\Omega_0, \zeta=0} = 0,\ \frac{\partial\widetilde{W}}{\partial\zeta}\Big|_{\Omega\setminus\Omega_0, \zeta=0} = 1, \tag{5}$$

where $\Omega_0 = \{\omega\!: T_0\omega \in \Gamma_q\}$, $\partial_i\widetilde{W} := \frac{\partial}{\partial\xi_i}\widetilde{W}(T_\xi w, \zeta)\big|_{\xi=0}$, $i = 1, \ldots, n-1$.

Theorem. The sequence u_ε converges to u_0 as $\varepsilon \to 0$ strongly in $L_{1+x}(D)$, weakly in $L_{1+x}(\partial D)$, $0 < x \ll 1$.

These results are the joint work with A. Ju. Belyaev.

Jack Carr

Metastability in singular problems

Metastable systems have solutions that spend a long time in a state which changes extremely slowly, but are far from equilibrium. Such systems can be described as nightmares to analyse since the equations describing them usually give no clear warning of the pathological behaviour of their solutions and numerical experiemtation can produce confusing results leading to the wrong conclusions. The Becker-Döring equations are an infinite set of ordinary differential equations describing congulation and fragmentation of particles. Two applications of the Becker-Döring equation are given:

(a) to the kinetics of phase transformations in binary alloys,

(b) to micelle formation.

Theoretical and numerical results on metastability for these cases are given.

Francine Diener

Complex rivers as sums of diverging series

We study the asymptotic behaviour of the solutions of the ODE

$$\frac{dy}{dx} = \frac{Q(x, y)}{P(x, y)} := F(x, y)$$

for $(x, y) \in \mathbb{C}^2$ and P, Q polynomials. We shall indicate how to find all possible asymptotic expansions (in rational powers of $1/x$), and how to prove the existence of solutions having these series as asymptotic expansions on sectors at infinity. We use a macroscope technique to reduce the problem to a singular perturbation problem. These solutions are called rivers and their expansions are usually diverging. Most "special functions" of physics, as the Airy function or the Weber function, are examples of rivers.

Marc Diener

Invariant manifolds at infinity for 3-dimensional polynomial vector-fields

We shall consider, on a typical example, a problem of invariant manifolds for a 3-dimensional polynomial differential equation (or equivalently, a solution of $(y + 3v)\, \partial v/\partial x + \partial v/\partial y = -(v^2 + x)$.) The considered manifold exists above a curved-shaped sector at infinity, and is polynomial growth. We shall give a macroscope technique that permits to show the existence of such an invariant manifold. We shall indicate how the expansion in powers of the small parameter $\varepsilon > 0$ introduced by the macroscope gives a two-variables asymptotic expansion of the manifold.

Paul C. Fife

Phase-field models for diffusion induced grain boundary motion

(joint work with J. W. Lahn and D. Penrose)

Grains are crystals of the same phase differing only in the orientation of their crystalline axes. The migration of boundaries between grains is an important pheneomenon in materials science. Without additional complications the motion is thought to obey a motion - by curvature law. Diffusion induced grain boundary motion (DIGM) involves the enhancement of the motion by the addition of atoms of a solute species; that material diffuses to (on from) the grains from the outside via the boundary (interface). The mechanism for this enhancement is unknown. Phase-field models are employed to model the phenomenon, with a view to discovering possible physical mechanisms. This involves the asymptotic study of traveling waves for a system of partial differential equations - the gradient system for a variety of model "free energy" functionals for another parameter and solute concentration. Several possible mechanisms are indeed found and elucidated.

Martin Flucher

Asymptotics for eigenvalue problems on singularly perturbed domains

We study the influence of a small hole A^r to the solutions of the eigenvalue problem for the vibrating membrane

$$\begin{aligned} -\Delta\phi &= \lambda\phi && \text{in } \Omega, \\ \phi &= 0 && \text{on } \partial\Omega, \\ \int_\Omega |\phi|^2 dx &= 1 \end{aligned}$$

A necessary and sufficient condition for the convergence of the eigenvalues

$$\lambda_k^r := \lambda_k(\Omega \setminus A^r) \to \lambda_k$$

is that the 2-capacity of the holes tends to 0. We obtain the following convergence rates:

$$\begin{aligned} \lambda_k^r &= \lambda_k + O(\mathrm{cap}(A^r)), \\ \|\phi_k^r - \phi_k\|_{L^2(\Omega)} &= o\left(\sqrt{\mathrm{cap}(A^r)}\right), \\ \|\phi_k^r - \phi_k\|_{H_0^1(\Omega)} &= O\left(\sqrt{\mathrm{cap}(A^r)}\right). \end{aligned}$$

For the case of concentration at a single point $A^r \subset B_x^r$ the precise asymptotics are:

$$\lambda_k^r = \lambda_k + \phi_k^2(x)\,\mathrm{cap}(A^r) + o(\mathrm{cap}(A^r)),$$

$$\phi_k^r = \phi_k - \phi(x)\, u^r + O(\mathrm{cap}(A^r))$$

in a space depending on the dimension, where u^r denotes the capacity potential of A^r. A multiple eigenvalue $\lambda_k = , ..., = \lambda_l$ splits according to the above formula with $\phi_k^2(x)$, ..., $\phi_l^2(x)$ replaced by 0, ..., 0, $\sum_{j=k}^{l} \phi_j^2(x)$.

The proof is of variational nature. The projections of the unperturbed eigenfunctions onto the space of the perturbed ones serve as comparison functions for the Rayleigh quotient. A continuous variant of the inverse power method is used for the estimation of the eigenfunctions. Details are given in [1] and [2]. For the planar case the accuracy of the eigenvalue approximation formula is confirmed numerically [3]. For this purpose a finite element approximation in conformal coordinates is used. The resulting discrete eigenvalue problem is solved by the inverse vector iteration scheme accelerated to third order by adapted spectral shifts. For the initial spectral shift we use the above eigenvalue approximation formula. This trick also permits to comptue higher eigenvalues by simple vector iteration although in this case the conjugate gradient method has to deal with a non-positive stiffness matrix.

References:

[1] FLUCHER, M. Apporximation of Dirichlet eigenvalues on domains with small holes, to appear J. Math. Anal. Appl.

[2] FLUCHER, M. Eigenfunction estimation methods for singularly perturbed domains, in preparation

[3] FLUCHER, M., KOOP, A. Numerical solution of a singularly perturbed eigenvalue problem, Preprint series SFB 256 No. 309 (1993), Universität Bonn

Giorgio Fusco

Some aspects of slow dynamics for the Cahn-Hilliard equation

We discuss the following theorem concerning the dynmacics of the Cahn-Hilliard Equation for $\varepsilon << 1$.

Theorem: There exist $\bar{\nu}, \bar{\varepsilon} > 0$ such that given $\nu \in (0, \bar{\nu})$, $\xi_0 \in \partial\Omega$, for each $\varepsilon \in (0, \bar{\varepsilon})$ there is a solution $u^\varepsilon: [0, \infty) \to H^3$ of

$$(CH)\begin{cases} \varepsilon^\nu u_t = \Delta(-\varepsilon^2\Delta u + F'(u)), & x \in \Omega \\ \dfrac{\partial u}{\partial \nu} = \dfrac{\partial \Delta}{\partial \nu} = 0, & x \in \partial\Omega \end{cases}$$

such that

$$\begin{cases} \lim\limits_{\varepsilon \to 0} u^\varepsilon(x, t) = 1, & x \in \overline{\Omega}\backslash\{\xi(t)\}, \\ \lim\limits_{\varepsilon \to 0} u^\varepsilon(\xi(t),t) = -1, & x = \xi(t), \end{cases}$$

where $\xi(\cdot) : [0, \infty) \to \partial\Omega$ is the solution of the o.d.e.

$$\begin{cases} \dot{\xi} = \chi \text{ grad } K(\xi), \\ \xi(0) = \xi_0. \end{cases}$$

Here $K(\xi)$ is the mean curvature of $\partial\Omega$ at $\xi \in \partial\Omega$ and χ is a positive constant.

Rustem R. Gadyl'shin

Asymptotics of eigenvalues and scattering frequencies for Laplacian in singularly perturbed domains

We present some results obtained by using the method of matching asymptotic expansions.

First result: Let $\Omega \subset \mathbf{R}^3$ be a bounded simply connected domain with a smooth boundary Γ_0, λ_0 be a simple eigenvalue of the Neumann boundary value problem for Laplacian in Ω and φ_0 be the corresponding eigenfunction normalized in $L_2(\Omega)$. Assume that Ω coincides with the half-space $x_3 > 0$ at some neighborhood of the origin and $\omega_\varepsilon = \{x : x\varepsilon^{-1} \in \omega\}$, where ω is a bounded domain in the plane $x_3 = 0$, $0 < \varepsilon << 1$. Then the eigenvalue λ_ε of the boundary value problem

$$-\Delta\varphi_\varepsilon = \lambda_\varepsilon\varphi_\varepsilon,\ x \in \Omega,$$
$$\frac{\partial\varphi_\varepsilon}{\partial\nu} = 0,\ \ x \in \Gamma_\varepsilon = \Gamma_0 \setminus \overline{\omega_\varepsilon},\ \ \varphi_\varepsilon = 0,\ \ x \in \omega_\varepsilon,$$

converging to λ_0, has the following asymptotic expansion:

$$\lambda_\varepsilon = \sum_{j=0}^{\infty} \varepsilon^j\lambda_j,\ \ \lambda_1 = 2\pi\varphi_0^2(0)c_\omega,$$

where $c_\omega > 0$ is the capacity of the plate ω.

Second result: The asymptotic expansion of the scattering frequency τ_ε of Helmholtz resonator $\Omega_\varepsilon = \mathbf{R}^3 \setminus \overline{\Gamma_\varepsilon}$, converging to $k_0 = \sqrt{\lambda_0} \neq 0$, has form

$$\tau_\varepsilon = k_0 + \sum_{j=1}^{\infty} \varepsilon^j\tau_j,\ \tau_1 = \pi\varphi_0^2(0)c_\omega/(2k_0),$$
$$\mathrm{Im}\varepsilon_2 = -\sigma(\pi\varphi_0(0)c_\omega)^2/2,$$
$$\sigma = \lim_{R\to\infty} \int_{|x|=R} |G^{ex}(x, 0, k_0)|^2 ds,$$

where $G^{ex}(x, y, k)$ is the Green function of the exterior limit problem outside $\overline{\Omega}$.

Third result: Let Ω' be bounded simply connected domain, $\overline{\Omega} \subset \Omega'$, Ω' coincide with half-space $x_3 > -h$ at some neighborhood of $x_0 = (0, 0, -h)$, $h > 0$, $\Omega^{ex} = \mathbf{R}^3 \setminus \overline{\Omega'}$. Then asymptotic expansion of the scattering frequency $\tau_\varepsilon \to k_0 \neq \frac{\pi m}{h}$ of the acoustic resonator $\Omega^\varepsilon = \Omega \cup \Omega^{ex} \cup \{\omega_\varepsilon \times [-h, 0]\}$ has form

$$\tau_\varepsilon = k_0 + \sum_{i=2}^{\infty} \varepsilon^i\tau_i,$$
$$\tau_2 = \frac{1}{2}\mathrm{ctn}\,(k_0h)\varphi_0^2(0)|\omega|,\ \mathrm{Im}\tau_3 = 0,\ \mathrm{Im}\tau_4 = -\frac{1}{2}(\varphi_0(0)|\omega|\sin^{-1}(k_0h))^2\sigma',$$
$$\sigma' = \lim_{R\to\infty} \int_{|x|=R} |G^{ex}(x, x_0, k_0)|^2 ds.$$

If $k_* = \frac{\pi m}{h}$, $m \neq 0$, and k_*^2 is not the eigenvalue of the limit interior problem in Ω, then

$$\tau_\varepsilon = k_* + \sum_{i=1}^{\infty} \varepsilon^i \tau_i,$$

$\tau_1 = 2q_\omega k_* h^{-1}$, $\mathrm{Im}\tau_2 = -k_*^2 h^{-1}|\omega|\sigma'/2$, where the negative constant is an analog of c_ω.

Fourth result: If Ω^{ex} is a bounded domain, then the second eigenvalue of the boundary value problem

$$-\Delta\varphi_\varepsilon = \lambda_\varepsilon \varphi_\varepsilon,\ x \in \Omega^\varepsilon,\ \frac{\partial \varphi_\varepsilon}{\partial \nu} = 0,\ x \in \partial\Omega^\varepsilon$$

has the asymptotic expansion

$$\lambda_\varepsilon^{(2)} = \varepsilon^2 h^{-1}|\omega|(|\Omega|^{-1} + |\Omega^{ex}|^{-1}) + O(\varepsilon^3).$$

Steffen Heinze

Homogenization of Flame Fronts

It is proven, that travelling waves in periodically diffusive and convective media with combustion nonlinearity can be homogenized. An equation for the homogenized limit is derived. A crucial step is the derivation of uniform lower and upper bounds for the wave velocity from which uniform gradient bounds for the wave profile follow. These estimates allow to pass to the homogenized limit.

Ulrich Hornung

Weighted Two-Scale Convergence

Let $\Omega \subset \mathbf{R}^n$ be a bounded domain and $Z = [0, 1]^n \subset \mathbf{R}^n$ the unit cube. According to Nguetseng (1989) and Allaire (1992) a family of functions $u^\varepsilon \in L^2(\Omega)$ is said to *two-scale converge* to a function $u \in L^2(\Omega; L^2(Z))$ iff

$$\int_\Omega u^\varepsilon(x)\varphi(x, \frac{x}{\varepsilon})\, dx \to \int_\Omega \int_Z u(x, y)\varphi(x, y)\, dy\, dx$$

holds for any test-function $\varphi \in C_0^\infty(\Omega; C_\#^\infty(Z))$, where # denotes Z-periodicity. This notion has shown to be very useful in proving convergence of homogenization processes for diffusion and flow in media with periodic structures including problems in perforated domains. The topic of this talk is to generalize this notion to diffusion problems taking place on surfaces and/or curves. Applications to adsorption, fractured media, and macropore problems are discussed.

Xiao-Biao Lin

Singularly perturbed system of reaction-diffusion equations

I shall construct matched asymptotic expansions for formal solutions to any desired order in ε for a singularly perturbed system of *n* reaction-diffusion equations, assuming that the 0th order solutions in regular and singular regions are all stable. The formal solution shows that there is an invariant manifold of wave-front-like solutions that attracts other nearby solutions. With an additional assumptions on the sign of the wave speed, the wave-front-like solutions converge slowly to stable stationary solutions on that manifold. If time permits, I shall indicate how to prove the existence of an exact solution near the formal solution.

Stefan Luckhaus

Homogenization for two phase flow

The subject is to derive the so-called double porosity model for oil water flow through porous rocks by homogenization starting from the two phase Darcy flow with strongly varying permeability. The equations:

$$\Theta\, \partial_t s = \operatorname{div}\left([\chi(\varepsilon x) + \varepsilon^2(1 - \chi(\varepsilon x))]\, k(s)\nabla(p_c(s) + (\rho_2 - \rho_1)z) + \tilde{k}(s)v\right)$$
$$\operatorname{div} v = 0,$$
$$v = (\chi(\varepsilon x) + \varepsilon^2(1 - \chi(\varepsilon x))\, [K(s)\nabla(p(s) + \tilde{K}(s)\nabla z]$$

with a periodic [0, 1] function χ and their formal limit

$$\Theta\, \partial_t \tilde{s} = \operatorname{div}\left(A\, k(\tilde{s})\nabla(p_c(\tilde{s}) + (\rho_2 - \rho_1)\, z) + \tilde{k}(\tilde{s})v\right) + f,$$
$$\operatorname{div} v = 0,$$
$$v = A(K(\tilde{s})\nabla p + \tilde{K}\nabla z),$$
$$f = \frac{1}{|Q - Y|}\int_Y \partial_t \sigma\,,$$

$Q = \{0 \le x_i \le 1\}$, $Y = \{x \mid \chi(x) = 0\} \cap Q$ where σ satisfies $\partial_t\sigma - \operatorname{div}(k(\sigma)\nabla p_c(\sigma)) = 0$ in Y, $\sigma = s(x)$ in ∂Y were derived by Arbogast, Douglas, Hornung. Here we [A. Bourgeat, A. Mikelic, S. Luckhaus] show this result rigorously. Using methods of C. Vogt and an extension for the doubly degenerating s-equation.

Andro Mikelic

On the boundary conditions at contact interface between a porous medium and a free fluid

We consider a slow viscous fluid flow in 2D domain consisting of a porous medium and a free fluid domain. At the boundaries of the solid part of porous medium we suppose the no-slip boundary condition supposing a periodic porous medium with period proportional to the characteristic pore size and a fixed porosity and using the homogenization, we find conditions on the interface linking pressures and velocities. Furthermore, we estimate the L^2-norm of the difference between the solution for some pore size ε and the combination of solutions of the limit problems and show the convergnce as $\varepsilon \to 0$. We distinguish two important cases:

a) The balanced flow in free fluid domain and in porous medium. We obtain continuity of the normal velocities at the interface. Finally, the value of Darcy's pressure at the interface is zero.

b) The small flow in porous medium caused by the flow in free domain. At the interface the Darcy pressure is equal to the pressue of the free fluid minus a constant coming from an auxiliary problem multiplied by the value of tangential computation of normal stress at the interface.

Stefan Müller

Singular perturbations of nonconvex minimization problems

Consider the problem

$$I_\varepsilon(u) = \int_\Omega W(Du) + \varepsilon^2 |Du|^2 \to \min$$

subject to $u = u_0$ on $\partial\Omega$, where $u : \Omega \subset \mathbf{R}^n \to \mathbf{R}^m$. In view of applications to solid-solid phase transitions we are interested in the case where W is not convex (and not even strongly elliptic). Then, for $\varepsilon = 0$ the infimum may not be attained and minimizers of I_ε form particular minimizing sequences. As not much is known in the general case three model problems are studied.

1. $$\int_0^1 \varepsilon^2 u_{xx}^2 + (u_x^2 - 1)^2 + u^2 \to \min.$$

Thm. $\exists\ \varepsilon_0 > 0$, such that for $\varepsilon < \varepsilon_0$ minimizers are periodic with period $L_0\varepsilon^{1/3} + O(\varepsilon^{2/3})$. (joint work with R. V. Kohn)

2. $u: (0, L) \times (0, 1) \to \mathbf{R}$, $u = 0$ at $x = 0$

$$I_\varepsilon(u) = \int_0^L \int_0^1 u_x^2 + (u_y^2 - 1)^2 + \varepsilon^2 u_{yy}^2$$

A standard boundary layer construction suggests that $\min I_\varepsilon \approx C\varepsilon^{1/2}L^{1/2}$ but

Thm. For ε sufficient small one has $c\varepsilon^{2/3}L^{1/3} \le \min I_\varepsilon(u) \le C\varepsilon^{2/3}L^{1/3}$.

For the upper bound one uses a test function that has (almost) self-similar refinement near $x = 0$.

(joint work with G. Dolzmann and V. Sverak)

3. $u: \Omega \subset \mathbf{R}^2 \to \mathbf{R}^2$, $u_0^{(x)} = F_x$, $F \in M^{2\times 2}$, $W : M^{2\times 2} \to \mathbf{R}$, $W \ge 0$,

$W^{-1}(0) = K = SO(2)A \cup SO(2)B$, $\det A, \det B > 0$.

$$I(u) = \int_\Omega W(Du) + \varepsilon |D^2 u| dx.$$

Thm. If $F \in K^{qc}$ (the quasi convex hull), then $\exists\ \alpha > 0$ such that $\min I_\varepsilon \le C\varepsilon^\alpha$.

Furthermore $\lim_{\varepsilon \to 0} \frac{1}{\varepsilon} \min I_\varepsilon = \infty$.

Nikolai N. Nefedov

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PDE's with Interior and Boundary Layers and Differential Inequalities Method

We consider boundary value problems

$$\varepsilon^2 \Delta u = f(u, x, \varepsilon), \qquad x \in D \subset R^2,$$
$$u = g(x), \qquad x \in \partial D,$$

where ε is a small parameter, f, g and the boundary ∂D of the domain D are sufficiently smooth. We assume that the nonlinearity f satisfies the following conditions

(A1) $f(\bar{u}, x, 0) = 0$ has three solutions $\bar{u} = \varphi_1(x)$, $\bar{u} = \varphi_0(x)$, $\bar{u} = \varphi_2(x)$, where $f_u(\varphi_i(x), x, 0) > 0$ for i = 1, 2, and $f_u(\varphi_0(x), x, 0) < 0$, $x \in \bar{D}$.

(A2) $I(x) = \int_{\varphi_1(x)}^{\varphi_2(x)} f(u, x, 0)\, du = 0$ if x belongs to some smooth closed curve.

(A02) $I(x) \equiv 0$, $x \in \bar{D}$.

For both cases we construct asymptotic expansion of any order. The proof of correctness is based on differential inequalities. Similar results are given for singularly perturbed parabolic problems.

Olga Oleinik

Homogenization problems for elliptic equations in partially perforated domains

(joint work with W. Jäger and A. S. Shamaev)

We consider homogenization problems in partially perforated domains for the Dirichlet, Neumann and mixed type boundary conditions on holes and also corresponding eigenvalue problems. We find homogenized boundary value problems and get estimates for the difference between solutions in a perforated domain and a solution of the limit equation. In the case of the Dirichlet problem and the mixed type boundary value problem we construct the second term of the asymptotic expansions for solutions and eigenvalues. The proofs are based on the two-scale asypmtotic method and theorems on the existence of solutions of boundary layer type. For eigenvalue problems we use the Theorem from the book: "Mathematical problems of elasticity and homogenization", North-Holland, 1992.

R. E. O'Malley

Using Exponential Asymptotics to Describe the Motions of Shocks and Transition Layers

(joint work with J. Laforgues)

We consider special initial-boundary value problems for singularly perturbed advection-diffusion and reaction-diffusion equations and the approach of their solutions to steady-states over an exponentially long time-scale. Equations include Burgers', Cahn-Allen and their generalizations, in m-space variable. The shock typically takes place over an

$O(\varepsilon)$ space interval. Using a profile $\varphi(\eta)$ appropriate to the steady-state limit, the evolution follows the Ansatz

$$u(\eta, \tau, \varepsilon) = \varphi(\eta) + e^{-A/\varepsilon}u(\eta, \tau, \varepsilon) + \cdots \quad \text{for} \quad \sigma = te^{-A/\varepsilon},\ A > 0.$$

Determining u, provides A and the shock layer location x_ε as a solution of an equation like

$$e^{A/\varepsilon}\frac{dx_\varepsilon}{dt} = k_- e^{(\beta_0 - x_\varepsilon)A_-/\varepsilon)} - k_+ e^{(x_\varepsilon - \beta_0)A_+/\varepsilon)},$$

where $k_\pm$, $A_\pm$ are known constants and β_0 is the ultimate shock location.

Jeannine Saint Jean Paulin

Reticulated structures with several small parameters

We consider periodic structures (period ε) made of very thin bars as layers - the thickness of the material is $\varepsilon\delta << \varepsilon$, because δ is also a small parameter. The thickness e of the tall structure or of the network is a third small parameter. We study thermal problems on elasticity problems in such structures when the three small parameters tend to zero (in different orders). In all the cases considered, the limit coefficients are explicit functions of the constant of the material. For instance we find, for the tall structure, the limit problem

$$-\left(a_{22} - \frac{a_{21}a_{12}}{a_{11}}\right)\frac{\partial^2 v^*}{\partial z_2^2} = 2f(qz_2) \qquad \text{in } (0, L)$$

$$v^*(0) = 0,$$

$$\frac{\partial v^*}{\partial z_2}(L) = 0$$

and for the deflection in the network

$$\frac{\mu}{12}\frac{3\lambda + 2\mu}{\lambda + \mu}\left(\frac{\partial^4 v_3}{\partial z_1^4} + \frac{\partial^4 v_3}{\partial z_2^4}\right) + \frac{2\mu}{3}\frac{\partial^4 v_3}{\partial^2 z_1^2 \partial^2 z_2^2} = F_3.$$

In several cases, we loose the H^1 coercivity and we work on other functional spaces on which coercivity holds.

Most of these results are joint work with D. Cioranescu (Paris 6). Some extensions are joint woirk with I. Charpentier (who also made numerical computations), S. El Otmani, R. Kauffmann, S. M Sac Epeé.

Björn Sandstede

Bifurcations of homoclinic orbits in the Fitz-Hugh-Nagumo system

(joint work with M. Krupa & P. Szmolyan)

We consider the system

$$\begin{cases} \dot{u} = v \\ \dot{v} = cv - f(u) + w, \qquad f(u) = -u(u-1)(u-a) \\ \dot{w} = \frac{\varepsilon}{c}(u - \gamma w) \end{cases}$$

which decribes travelling waves for the Fitz-Hugh-Nagumo system. We show that the well-known slow and fast waves of this equations emanate from a heteroclinic cycle of the limit system for $\varepsilon = 0$, $c = 0$ and $a = \frac{1}{2}$. Moreover, we prove that, for large $\gamma > 0$, the slow wave undergoes an inclination-flip bifurcation. This gives rise to the existence of N-pulse solutions which do not possess an oscillatory toil.

Klaus R. Schneider

Jumping behavior in reaction rates for bimolecular reactions

We study the behavior of the reaction rates r_1/ε, r_2/ε and r/ε on $[0, T]$ of bimolecular reactions which are described by the differential equations

$$\varepsilon dx/dt = \varepsilon I_a(t) - \varepsilon g_1(x) - r_1(x) - r(x, y),\ x(0, \varepsilon) = x^0,$$
$$\varepsilon dy/dt = \varepsilon I_b(t) - \varepsilon g_2(x) - r_2(x) - r(x, y),\ y(0, \varepsilon) = y^0,$$

where all functions are sufficiently smooth and satisfy $r_1(0) = r_2(0) = r(0, y) = r(x, 0) = 0$. This system is a singular singularly perturbed one whose associated system has two equilibria $\bar{x}_0 \equiv 0$ and $\bar{x}_1(t)$ (depending on the parameter t). We prove the following results: The reaction rates have a transition layer near the zeros of $\bar{x}_1(t)$. The proof is based on asymptotic expansions, differential inequalities techniques and a special coordinate transformation. Open equation: Does the jumping behavior trigger pattern formation when diffusion is included.

Daniel Sevcovic

Smoothness of the singular limit of invariant manifolds

The aim is to study the C^1 singular limit dynamics of invariant manifolds for semidynamical systems generated by the following system of sinularly perturbed evolution equation in Banach spaces

$$u_t + Au = g(u, w)$$
$$\varepsilon w_t + Bw = f(u, w)$$

where A, B are generators of analytic semigroups in Banach spaces X and Y, resp., g and f are nonlinear functions and $0 < \varepsilon << 1$ is a small parameter. The main purpose is to prove, under suitable assumptions on f, g, that for small values of the vanishing parameter $\varepsilon > 0$ the system has a finite dimensional invariant center-unstable manifold M_ε which is in the C^1 topology close to the invariant manifold M_0 for the reduced problem where $\varepsilon = 0$.

Pavel E. Sobolevskii

Asymptotic of viscoelastic model solution when viscous coefficient tends to zero

Initial boundary value problem

$$\partial v/\partial t + \sum_{k=1}^{3} v_k \cdot \partial v/\partial x_k - \varepsilon\Delta v - \int_0^t \Psi(t, s)\Delta v ds - \text{grad } p = f,$$

$0 \le t \le T, x \in \Omega,$

(1) $\operatorname{div} v = 0,$

$v = 0,$ $\quad 0 \le t \le T, x \in \partial\Omega,$

$v(0, x) = u(x),$ $\quad x \in \overline{\Omega},$

$\int_\Omega p(t, x)dx = 0,$ $\quad 0 \le t \le T,$

is considered. Here Ω is an open bounded domain of points $x = (x_1, x_2, x_3) \in R^3$ with boundary $\partial\Omega$, and $\overline{\Omega} = \Omega \cup \partial\Omega$; $v = (v_1, v_2, v_3)$, u and f are vector-functions; p and Ψ are scalar functions; $\varepsilon > 0$, $T > 0$ are constants. Under assumption of smoothness of the problem data $\partial\Omega$, f and u and under condition of nondegenerate elasticity

(2) $\Psi(t, t) \ge \Psi_0 > 0,$

there exist $t_0 \in (0, T)$ and $M(N) \in [0, +\infty)$ $(N = 0, 1, ...)$, which do not depend on $\varepsilon > 0$, such that problem (1) has a unique solution $v(t, x;\varepsilon) \in C([0, t_0], W_2^2(\Omega)) \cap C^1([0,t_0], L_2(\Omega))$, $p(t, x; \varepsilon) \in C([0,t_0], W_2^1(\Omega))$, and the following estimates are true:

(3) $\quad \|v(t,x;\varepsilon) - \sum_{m=0}^{N} \frac{\varepsilon^m}{m!} v^m(t, x)\|_{W_2^2(\Omega)} \le M(N) \cdot \varepsilon^{N+1},$

(4) $\quad \|\frac{\partial}{\partial t}[v(t,x;\varepsilon) - \sum_{m=0}^{N} \frac{\varepsilon^m}{m!} v^m(t, x)\|_{L_2(\Omega)} \le M(N) \cdot \varepsilon^{N+1},$

(5) $\quad \|p(t,x) - \sum_{m=0}^{N} \frac{\varepsilon^m}{m!} p^m(t, x)\|_{W_2^1(\Omega)} \le M(N) \cdot \varepsilon^{N+1}.$

The pair $[v^m(t, x), p^m(t, x)]$ is an unique solution of initial-boundary value poblem, which we will obtain, if we will formally differentiate relations (1) m-times with respect to ε and will put in the received relations $\varepsilon = 0$.

Uwe Stroinski

Order theory and oscillation at delay diffential systems

We introduce weakly oscillating C_0-semigroups and present a spectral characterization. The main tool in the proof is a theorem of Landau-Widder on the Laplace transform of positive functions. Furthermore, we apply our result to a delay differntial system and generalize a result of Arino and Györi. With help of the Perron-Frobenius theory of positive matrices we obtain "easy" computable sufficient conditions for oscillation of such equations. This solves a problem posed by Györi and Ladas.

Luc Tartar

A problem in micromagnetics

In a bounded open set Ω of $\mathbf{R}^3$ occupied by a crystal one has a field m corresponding to a macroscopic spin and the constitutive relation B = H + m and as H = grad u one has $-\operatorname{div}(\operatorname{grad} u + m\chi_\Omega) = 0$. One wants to minimize the total energy T(m) sum of the magnetostatic energy $\frac{1}{2}\int |\operatorname{grad} u|^2 dx$, the anisotropc energy $\int_\Omega \varphi(m)dx$, the effect of an applied magnetic $\mathbf{R}^3$ field $-\int_\Omega H_0 \cdot m\, dx$ and the exchange energy $\varepsilon^2 \int_\Omega |Dm|^2 dx$.

In the case $\varepsilon^2 \to 0$ the study of minimizing sequences leads to using Young measures for describing the limit of $\int_\Omega \varphi(m)dx$ and H-measures for describing the limit of $\frac{1}{2}\int |\operatorname{grad} u|^2 dx$.

Some informations on the relation between Young measures and H-measures obtained with Francois Murat slow that minimizing sequences $m_u \to m_\infty$ are such that $\operatorname{div}((m_u \to m_\infty)\chi_\Omega) \to 0$ in H^{-1} strong and that the relaxed problems are obtaining by using $|m| \le 1$ and replacing φ by its convex envelope.

Ping J. Xun

Traveling waves as limits of solutions on bounded domains

In this talk the speaker reported some results about the bistable equation $u_t = \varepsilon^2 u_{xx} - f_a(u)$ with Neumann boundary condition where a typical example of $f_a(u)$ is $f_a(u) = (u + a)(u^2 - 1)$. After the rescaling $x \to \varepsilon x$, the equation is defined in a large domain $(-\frac{1}{\varepsilon}, \frac{1}{\varepsilon})$ and the speaker shows how the solutions of the rescaled equation with some appropriate initial conditions approach the traveling wave solution of the equation $v_t = v_{xx} - f_a(v)$, $|x| < \infty$ and $v(\pm\infty) = \pm 1$.

Wen-An Yong

Existence and Asymptotic Stability of Traveling Wave Solutions of a Model System for Reacting Flow

The talk presents some results about existence and asymptotic stabiltiy of traveling wave solutions to the following model system for reacting flow in Lagrangian coordinates:

$$v_t - u_t = 0, \qquad u_t - p(v, s)_x = 0, \qquad s_t = \frac{s_e(v) - s}{\varepsilon \cdot v}.$$

The traveling wave solutions smooth out steady shock waves for the reduced system, which is derived by letting $\varepsilon \to 0$ from the original system,

$$v_t - u_t = 0, \qquad u_t + p(v, s_e(v))_x = 0.$$

The results are based on the stability condition:

$$-p_v(v, s) > p_v(v, s) \cdot s_e(v) > 0 \qquad \text{for } v > 0, s \in [0, 1],$$

and the genuine nonlinearity condition:

$$\frac{d^2}{dv^2}\, p_v(v,\, s_e(v)) \neq 0 \qquad \text{for } v > 0.$$

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