MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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# The Navier-Stokes Equations: Theory and Numerical Methods

5.6. bis 11.6.1994

The conference was organized by J. G. Heywood (Vancouver), K. Masuda (Tokyo), R. Rautmann (Paderborn), and V. A. Solonnikov (St. Petersburg). It was participated in by 48 scientists from 10 countries. The lectures and discussions centered on new mathematical results for the hydrodynamical equations, particularly concerning the following types of physical problems: 1) flow in exterior domains and in aperture domains, 2) water waves and other free boundary problems, 3) compressible flow, 4) turbulence, 5) thermal convective flow, and 6) flow control. The mathematical methods under discussion were mainly those of: 1) the theory of partial differential equations via functional analysis, 2) stability and bifurcation theory, 3) the theory of attractors within the dynamical systems context, 4) the theory of conservation laws, and 5) the theory of numerical simulation by various schemes including the nonlinear Galerkin method, a new product formula approach, and a new spectral method. Physical experiments from the engineering community and numerical simulations were also presented in movies.

## Abstracts:

J. T. BEALE

### Analysis of Time-Dependent Water Waves away from Equilibrium

We consider the time-dependent motion of a two-dimensional, inviscid, incompressible, irrotational fluid, below a moving interface, with gravity and possibly surface tension. In joint work with T. Y. Hou and J. S. Lowengrub, we have shown that under certain conditions the equations of motion, linearized about an arbitrary time-dependent solution, are well-posed. An interesting qualitative structure is found for the linear equations. A Lagrangian approach is used, and the formulation is similar to that in numerical work using boundary integral methods, in which the interface is tracked explicitly. We prove that certain versions of the boundary integral methods are numerically stable and converge to the exact solution. It is important that the discrete equations have a structure analogous to that of the continuous case. Examples of breaking waves have been computed.

W. BORCHERS

**A new convergence result for the pressure correction method**

It is shown that the pressure correction method for solving the Navier-Stokes initial boundary value problem of incompressible fluids is of first order accuracy under realistic assumptions on the data. In particular, this convergence takes place without assuming the 'virtuell uncheckable' nonlocal compatibility condition of the initial value. Moreover, we present a new splitted version of this scheme (so called characteristic's pressure correction method) for which similar results are proved.

P. DEURING

**Starke Lösungen zum Navier-Stokes-System auf Lipschitz-berandeten Gebieten** (mit Prof. von Wahl)

Wir betrachten das Navier-Stokes-System

$$u_t - \nu \cdot \triangle_x u + u \cdot \nabla_x u + \nabla_x \pi = f, div_x u = 0 \text{ in } \Omega \times (O,T), \qquad (1)$$

wobei $\Omega$ ein beschränktes, Lipschitz-berandetes Gebiet im $\mathbf{R}^3$ bezeichnet. Zusätzlich stellen wir die Anfangsbedingung

$$u(x,0) = u_0(x) \text{ für } x \in \Omega, \qquad (2)$$

sowie die Randbedingung

$$u(x,t) = 0 \text{ für } x \in \partial\Omega, t \in (0,T). \qquad (3)$$

Unter geeigneten Voraussetzungen an $f$ und $u_0$ können wir zeigen: Es gibt eine Zahl $T > 0$ und eine Lösung $(u, \pi)$ von (1) – (3) auf $\Omega \times (O,T)$, so daß

$u \in C^\circ([0,T], W^{1/2,2}(\Omega)^3)$,

$u(\cdot,t) \in W^{2,2}_{\text{loc}}(\Omega)^3 \cap W^{1/2}(\Omega)^3 \cap \bigcap_{\delta>0} W^{3/2-\delta,2}(\Omega)^3$,

$\pi(\cdot,t) \in W^{1,2}_{\text{loc}}(\Omega)^3 \cap \bigcap_{\delta>0} W^{1/2-\delta,2}(\Omega)^3$ für $t \in (0,T)$.

Falls $u_\circ$ und $f$ in geeignetem Sinne klein sind, kan man $T = \infty$ wählen.

R. FARWIG

**Weighted Decay Estimates of Stationary $D$-Solutions of the Navier-Stokes Equations in Exterior Domains.**

Consider a solution $u$ with bounded Dirichlet integral, a so-called $D$-solution, of the stationary Navier-Stokes equations in an exterior domain of $\mathbf{R}^3$ such that $u$ has a nonzero limit $u_\infty$ at infinity in the sense $\| u - u_\infty \|_{L^6} < \infty$. The classical problem to investigate the pointwise decay of $|u(x) - u_\infty|$ for $|x| \to \infty$ was solved by K. I. Babenko and more recently by G. P. Galdi: $u$ is a $PR$-solution in the sense of $R$. Finn and $u(x) - u_\infty$ has the same decay as the fundamental solution of the Oseen system.

In a joint work with H. Sohr (University of Paderborn) we present a new and short proof which is based on the $L^q$-theory of the Oseen equations in $\mathbf{R}^3$ and a perturbation argument. It avoids the use of explicit integral representations, allows for external forces with noncompact support and yields weighted $L^q$-estimates as well as pointwise decay estimates of $u(x) - u_\infty$, $\nabla u(x)$ and of the pressure.

A. V. FURSIKOV

**Certain boundary exact controllability problems for Navier-Stokes equations.**

In a bounded domain $\Omega \subset \mathbf{R}^n (n = 2, 3)$ a solenoidal vector field $v_0(x) \in H^3(\Omega)$ is given. We construct the vector field $z(t,x)$ defined on lateral surface $[0,T] \times \partial\Omega$ of cylinder $[0,T] \times \Omega$ which possesses the properties: a) $z(t,x)$ is tangential to $\partial\Omega$ for any $t \in [0,T]$, b) the solution $v(t,x)$ of the boundary value problem for Navier-Stokes equations with initial value $v_0(x)$ and boundary Dirichlet condition $z(t,x)$ satisfies the equality $v(T,x) \equiv 0$ at sufficiently large given instant T. Moreover

$$\| v(t,\cdot) \|_{H^3(\Omega)} \leq c \exp(-K(T-t)^2) \text{ as } t \to T$$

where $c > 0, K > 0$ are certain constants.

G. P. GALDI

**On the asymptotic structure of steady plane viscous flows in exterior domains**

In this talk I show that every solution $v$ to the $2-D$ exterior, steady Navier-Stokes equations having a finite Dirichlet integral and tending pointwise to a non-zero velocity $v_\infty$ at large distances, is "Physically Reasonable" in the sense of Finn & Smith, provided the component of $v$ orthogonal to $v_\infty$ belongs to some Lebesgue space $L^s$ in a neighbourhood of infinity. Such a result have been jointly obtained with H. Sohr.

Y. GIGA

**Two-Phase Stokes flow**

A global-in-time weak solution of the nonstationary two-phase Stokes flow is constructed for arbitrary given initial phase configuration (under periodic boundary condition) where two viscosities are close. The solution presented here tracks the evolution of the interface after it develops singularities. The theory of viscosity solutions is adapted to solve the interface equation. Surface tension effects are ignored here. (joint with S. Talkahashi, to appear in SIAM J. Math. Analysis (1994))

V. GIRAULT

**Weighted $L^p$ spaces for the Laplace operator in $\mathbf{R}^N$ or exterior domains.**

Let $\Omega$ be an exterior domain in $\mathbf{R}^N$ (possibly the whole of $\mathbf{R}^N$) and let $\rho(r) = \sqrt{1+r^2}$ and $lg\ r = ln(2+r^2)$, where $r$ is the distance to the origin, be two basic weights. Let $m \geq 0$ be an integer, $\alpha, \beta, p$ three real numbers with $1 < p < \infty$. We define the integer $k$ by

$$k = \begin{cases} -1 & \text{if } \frac{N}{p} + \alpha \notin \{1, \ldots, m\} \\ m - \left(\frac{N}{p} + \alpha\right) & \text{if } \frac{N}{p} + \alpha \in \{1, \ldots, m\} \end{cases}$$

and then we define the general family of weighted spaces

$$\begin{aligned} W^{m,p}_{\alpha,\beta}(\Omega) &= \{v \in D'(\Omega); 0 \leq |\lambda| \leq k, \rho^{\alpha-m+|\lambda|}(\lg r)^{\beta} D^{\lambda} v \in L^p(\Omega), \\ &\quad k+1 \leq |\lambda| \leq m, \rho^{\alpha-m+|\lambda|}(\lg r)^{\beta-1} D^{\lambda} v \in L^p(\Omega)\} \end{aligned}$$

The weights arise from a generalized Hardy's inequality and are chosen so that on one hand $D(\overline{\Omega})$ is dense in $W^{m,p}_{\alpha,\beta}(\Omega)$ and on the other hand it satisfies some Poincaré inequality.
With these spaces, we can prove that the Laplace operator satisfies the following isomorphisms, for any integer $l \geq 0$ and if $\frac{N}{p'} \notin \{1, \ldots, l\}$ :

$$\Delta : W^{1,p}_{l}(\mathbf{R}^N)/\mathbf{P}_{[1-l-\frac{N}{p}]} \to W^{-1,p}_{l}(\mathbf{R}^N) \perp \mathbf{P}^{\Delta}_{[l+1-\frac{N}{p'}]}$$

$$\Delta : W^{2,p}_{l}(\mathbf{R}^N)/\mathbf{P}_{[2-l-\frac{N}{p}]} \to W^{0,p}_{l}(\mathbf{R}^N) \perp \mathbf{P}^{\Delta}_{[l-\frac{N}{p'}]}$$

$$\Delta : W^{2,p'}_{-l}(\mathbf{R}^N)/\mathbf{P}^{\Delta}_{[2+l-\frac{N}{p'}]} \to W^{0,p'}_{-l}(\mathbf{R}^N).$$

G. GRUBB

**Nonhomogeneous initial-boundary value problems for the Navier-Stokes equations in anisotropic $L_p$ Sobolev spaces**

We consider the Navier-Stokes equations with nonhomogeneous data:

$$
\begin{aligned}
\partial_t u - \Delta u + (u\cdot\nabla)u + \nabla q &= f \text{ in } \Omega \times I = Q_I \\
\operatorname{div} u &= 0 \text{ in } Q_I \\
T_j\{u,q\} &= \phi \text{ on } \partial\Omega \times T = S_I \\
u|_{t=0} &= u_0 \text{ on } \Omega
\end{aligned}
$$

with Dirichlet condition ($T_0\{u,q\} = \gamma_0 u$) or Neumann condition ($T_1\{u,q\} = \chi_1 u - \gamma_0 q\vec{n}, \chi_1 u = \gamma_0(\sum_i(\partial_i u_j + \partial_j u_i)n_i)_{j=1,\ldots,n}$). For bounded smooth domains $\Omega \subset \mathbf{R}^n (n \geq 2)$ we show, in the anisotropic spaces $H_p^{s,s/2}(Q_I)$ (generalizing $H_p^{2l,l}(Q_I) = \{u | \partial_x^\alpha \partial_t^j u \in L_p \text{ for } |\alpha| + 2j \leq 2l\}$ by complex interpolation and duality) and $B_p^{s,s/2}(Q_I)$ (obtained by real interpolation), the unique solvability of the linearized problem with $u \in H_p^{s+2,s/2+1}(Q_I)$ for

$$
\{f, \varphi, u_0\} \in H_p^{s,s/2}(Q_I) \times B_p^{s+1+j-1/p,(s+1+j-1/p)/2}(S_I) \times B_p^{s+2-2/p}(\Omega)
$$

satisfying natural compatibility conditions and $\operatorname{div} u_0 = 0, s > \frac{1}{p} - 1$. This gives solvability of the nonlinear problem (for small data or a small interval) when furthermore $s \geq (n+2)/p - 3$, with $I = \mathbf{R}_+$ in the Dirichlet case. Note that $p > (n+2)/3$ allows $s < 0$, distributional $f$.

The proof goes by a reduction (of G.-Solonnikov) to pseudodifferential parabolic boundary value problems, using a new $L_p$ theory there. For the sake of exterior domains the method has been reworked so that it boils down to treating the exterior Dirichlet and Neumann problems for the Laplace operator and the heat operator, plus a parameter dependent pseudo-differential operator in $\Gamma$.

H. Ch. GRUNAU

## Regularity and Decay Properties of Perturbations of R. Finn's Stationary PR-Solution

In order to investigate instationary Navier-Stokes flows with prescribed constant velocity $v_\infty$ at infinity, we consider perturbations $u$ of $R$. Finn's stationary PR-solution $\overset{(0)}{v}$. The perturbations $u$ have to satisfy the equations $u_t - \Delta u + (\overset{(0)}{v} \cdot\nabla)u + (u\cdot\nabla)\overset{(0)}{v} + (u\cdot\nabla)u + \nabla\pi = f$, $\operatorname{div} u = 0$ in $(0,\infty) \times \Omega$. Beside some regularity results, decay rates for $u$ are presented. Due to the enormous difficulties with respect to the Oseen-semigroup in exterior domains $\Omega$, we calculate decay rates for $u$ only in the model case $\Omega = \mathbf{R}^3$. We assume $(\overset{(0)}{v} - v_\infty) \in L^p$ for some $p \in (2,3)$. With regard to Finn's work, this assumption is realistic, if $v_\infty \neq 0$.

For $(\overset{(0)}{v} - v_\infty)$ small in an appropriate sense, there holds

- for any weak solution $u$ with generalized energy inequality:

$$\| u(t) \|_2^2 \leq C(1+t)^{-\alpha}, \alpha = \min\{\alpha_0, \frac{5}{2}, 1 + \frac{3}{p}\},$$

- if $u$ is also a strong solution:

$$\| u(t) \|_q \leq C(1+t)^{-\{\alpha/2+3/2(1/2-1/q)\}}.$$

Here $\alpha$ is the decay rate for the semigroup solution of the corresponding heat system.

J. G. HEYWOOD

**Spectral computations for the spatially periodic Navier-Stokes equations.**

I think I have a fundamentally new method, that will allow one to penatrate much further into the spectrum.

Y. KAGEI

**Equations of thermal convection in the presence of the dissipation function**

We consider the following two-dimensional equations of thermal convection in $\mathbf{R} \times (0,1)$ :

$$(E)_\eta \quad \begin{cases} \frac{\partial u}{\partial t} - \nu \Delta u + u \cdot \nabla u + \nabla p = f(\Theta) e_2, (t > 0, x = (x_1, x_2) \in \mathbf{R} \times (0,1)) \\ \Delta \cdot u = 0, (t \geq 0, x \in \mathbf{R} \times (0,1)) \\ \frac{\partial \Theta}{\partial t} - \kappa \Delta \Theta + u \cdot \Delta\Theta - u \cdot e_2 = \eta \frac{\nu}{2} \sum_{i,j=1}^{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)^2, (t > 0, x \in \mathbf{R} \times (0,1)), \end{cases}$$

under the boundary conditions:

$u = 0, \Theta = 0 (x_2 = 0, 1); u, p, \Theta$ are periodic in $x_1$.

Here $\nu, \kappa, \eta > 0$ are non-dimensional parameter; $e_2 = (0,1)$.

Under appropriate assumptions on $f$ and $\eta$, we prove the existence of global weak solutions of $(E)_\eta$ and the existence of the global attractor for $(E)_\eta$. We also discuss the convergence of the weak solutions of $(E)_\eta$ as $\eta \to 0$.

H. KOZONO

## Navier-Stokes equations in exterior domains

Consider the stationary Navier-Stokes equations in an exterior domain $\Omega$:

$$(N-S)\begin{cases} -\Delta w + w\cdot\nabla w + \nabla\Pi = \operatorname{div}F, \operatorname{div}w = 0 \text{ in } \Omega \\ w|_{\partial\Omega} = 0, w(x) \to 0 \text{ as } |x| \to \infty \end{cases}.$$

In this talk, we shall discuss the existence, uniqueness and stability of solutions to $(N-S)$. The class of solutions which we consider is not $D$-solutions but the one with finite $L^r$-gradient. It is known that we have to restrict ourselves to the class $\nabla w \in L^{n/2}(\Omega)$, which causes a difficulty because of the lack of unique solvability for the linearized equations. To overcome such an obstraction, we shall introduce the class $\nabla w \in L_{p,q}(\Omega)$, where $L_{p,q}(\Omega)$ denote the Lorenz space. In fact, we can show that the Stokes equations have only one solution $w$ with $\nabla w \in L_{n/2,\infty}(\Omega)$. Based on this fact, we shall prove the following: if $F$ is small enough in $L_{n/2,\infty}(\Omega)$, then there exists a unique small solution $w$ of $(N-S)$ with $\nabla w \in L_{n/2,\infty}(\Omega)$.

D. KRÖNER

## Measure valued solutions of conservation laws

The theory for the Euler equations of gas dynamics in $n-D$ is not very well developed but there are lot of "good" numerical algorithms for solving them in several space dimensions. Also the proof for convergence to a measure valued solution of the numerical solution $u_n$ is not clear, as well as for the solution of the compressible Navier-Stokes equations if the viscosity tends to zero. Therefore we (in a joint paper with Zajaczkowski) considered a higher order regularization to the Euler equations and could show the convergence to a measure valued solution. For scalar conservation laws one can even show that the measure (constructed as the limit of the numerical solutions $u_n$) is equal to a Dirac measure, which means that $u_n$ approximates the weak solution. This step is still open for systems. We also present some results of numerical experiments for solving the Euler equations in complex geometries in 3-D.

J. MALEK

## On the Fractal Dimension of Attractors

The asymptotic behaviour of the three-dimensional flows for a system of the Navier-Stokes type is investigated. In the considered model, the viscous part of the stress tensor is generally a nonlinear function of the symmetric part of the velocity gradient. Provided that the function describing this dependence satisfies the polynomial $(p-1)$ growth condition, the unique weak solution exists if either $p \geq \frac{2+n}{2}$ and $u_0 \in H$ or $p \geq 1 + \frac{2n}{n+2}$ and $u_0 \in W^{1,2}(C)^n \cap H$.

In the first case, the existence of a global attractor in $H$ has been proved in [1]. In order to indicate the finite dimensional behaviour of the flow at infinity the fractal dimension of the new invariant set, composed from all short $\delta$-trajectories with initial value in the attractor, is estimated in the $L^2(0,\delta;H)$ topology.

The possibility to estimate the fractal dimension in $H$ is discussed, too.

Having uniqueness only for more regular data in the second case, many trajectories can start from the initial value $u_0 \in H$. This does not allow to define a semigroup on the space $H$. Therefore, the set of short trajectories $X_\delta^s$, closed in $L^2(0,\delta;H)$ along with a semigroup working on this set is introduced. The existence of a global attractor with a finite fractal dimension is then demonstrated.

[1] J. Malek, J.Necas, A finitedimensional attractor for Threedimensional Flow of Incompressible Fluids, Preprint SFB 256, No. 326, Bonn, 1993 (sent to J. Differential Equations).

P. MAREMONTI

## On Decay Properties of Solutions to Stokes System in Exterior Domains

The initial boundary value problem for Stokes system in $\Omega \times (0,T)(\Omega \subseteq \mathbf{R}^n, n \geq 2)$ is considered. Particular interest is given to the properties concerning the time asymptotic behaviour of the solutions, either pointwise estimates or integral properties are obtained. The aim of the talk is to show the improvements of results already known and some their consequences for weak solutions to the Navier-Stokes equations. Finally, it is possible to show that some estimates, quite different from the analogous estimates for Cauchy problem, are sharp.

All the results communicated are extracted from the forthcoming paper:

P. Maremonti, V. A. Solonnikov, On the nonstationary Stokes problem in exterior domains.

M. MARION

## On the time discretization of numerical schemes based on multi-level spatial splittings

We investigate the discretization in time of numerical schemes based on multi level spatial splittings for the two-dimensional periodic Navier-Stokes equations. The approximate solution is computed as the sum of a low frequency component and a high frequency one. These two terms are advanced in time using different time steps. We present error estimates that indicate that the high frequency term can be integrated with a larger time step. We adress implementation issues and show that the method should yield a significant gain in computing time.

W. NAGATA

**Effects of sidewalls on binary fluid convection.**

We consider the onset of oscillatory convection in a 2-dimensional layer of binary fluid, with sidewalls. The problem is reduced using centre manifold theory to that of a double Hopf bifurcation with reflection symmetry, near 1 : 1 resonance. By studying the associated normal form, one determines the existence and stability of various types of oscillatory wave-like solutions, and also the nonlinear interactions between them.

S. A. NAZAROV

**Asymptotics of solutions to Stokes problem in the exterior of paraboloidal and cylindrical domains**

The Stokes equations in the domain $\Omega$, which, outside the ball $\{x \in \mathbf{R}^3 : |x| < R\}$, coincides with the exterior of the paraboloid $M = \{x : (x_3^{\gamma-1}x_1, x_3^{\gamma-1}x_2) \in \omega\}$ where $\gamma > 0$ and $\omega$ is a plane domain with a smooth boundary, is treated. The asymptotics (as $|x| \to +\infty$) of a solution is investigated. For instance, if the solution $u$ possesses finite Dirichlet integral and the right-hand sides of the equations have compact supports, then

$$u(x) = \sum_{i=1}^{3} c_i \phi^i(x) |\log|x||^{-1/2\gamma}[1 + O(|\log|x||^{-1})] + 0(|x|^{-2+\varepsilon})$$

where $c_i$ are constants, $\phi^{(i)}$ denote the columns of the fundamental matrix of the Stokes operator in $\mathbf{R}^3$ ($\phi^i_j(x) = O(|x|^{-1})$) and $\varepsilon$ is an arbitrary positive number.

A. NOVOTNÝ

**Steady compressible Navier-Stokes equations in domains with singular boundaries**

The motion of incompressible Navier-Stokes fluids near the singularities of the boundary is a problem, which is well studied in the mathematical literature. This is not the case for compressible fluids. In the present contribution we investigate a steady motion of compressible fluids near the singularities of the boundary, especially near the conical points. On an example of steady compressible isothermal Navier-Stokes equations in bounded plane domains with one corner point, we show, how to treat the problem. Using the method of decomposition of the compressible flow into its compressible and incompressible parts introduced in Novotny, Padula [Arch. Rat. Mech. Anal., in press], we prove (for small external data) existence of solutions in appropriate weighted Sobolev spaces (Kondrat'ev spaces).

M. PADULA

## Mathematical problems concerning the equations of compressible fluids

Recently, in [NP] it was proved the existence of steady flows of a viscous compressible fluid in exterior domains in Lebesgue spaces. Such result was rendered possible thanks to a coupling of the existence results for the steady Navier-Stokes system with a new decomposition method for the kinetic field of a compressible fluid.

One interesting question left open there, was the analysis of the decay rate for the velocity and the density fields to constant fields at infinity. As well known, for the more explored Navier-Stokes system it was proved the velocity has the same decay rate as that classical one of the linearized Stokes or Oseen systems. In this note, we show that the steady velocity field of a viscous isothermal gas has, at infinity, the same asymptotic structure as that one known for a viscous incompressible fluid.

References

[*NP*] Novotný, A., Padula, M.: $L^p$ approach to steady flows of viscous compressible fluids in exterior domains, Arch. for Rational Mech. and Anal., to appear.

A. PASSERINI

## On existence and uniqueness of solutions to the steady Navier-Stokes equations in a 2-dimensional aperture domain

We prove the existence of solutions to the problem

$$\begin{aligned} -\Delta v + v \cdot \nabla v + \nabla p &= 0 \ in \Omega \\ \nabla \cdot v &= 0 \ in \Omega \\ v|_{\partial\Omega} &= 0 \end{aligned}$$

when $n = 2$, and $\Omega$ is an aperture domain with a smooth boundary. The existence is proved in the class of solutions having finite Dirichlet integral and for arbitrary value of the flux through the aperture.

Moreover, we prove that

$$\lim_{|x|\to\infty} |v(x)| = 0$$

and that the solution is unique in the class of solutions goint as $|x|^{-1}$ at infinity.

R. PEYRET

**Chebyshev spectral multidomain method for the Navier-Stokes equations in vorticity-streamfunction formulation.**

For two-dimensional flows (plane or axisymmetric) the vorticity-streamfunction formulation of the Navier-Stokes equations are efficiently solved by using the Chebyshev-collocation approximation associated with the technique of influence matrix for handling the boundary conditions. The rank of this matrix will be discussed according to the type of boundary conditions.

The use of the domain decomposition technique presents several advantages, among them the possibility to deal with singular solutions of the Navier-Stokes equations while avoiding Gibbs phenomenon. The method for constructing the multidomain solution is based on the influence matrix technique for prescribing the matching condition at the interface. The algorithm is direct and efficient in CPU time. Some numerical results will be given for illustration.

K. PILECKAS

**Stationary Stokes and Navier-Stokes systems in domains with layer-like outlets to infinity**

We consider the stationary Stokes problem in a domain $\Omega$ having a form of an infinite layer, i. e. $\Omega = \{x \in \mathbf{R}^3, (x_1, x_2) \in \mathbf{R}^2, 0 < x_3 < 1\}$. The main results reads as follows. Let the external force $\vec{f}$ has a compact support and let $(\vec{u}, p)$ be a solution to the Stokes problem with

$$\vec{u} = O(r^{k+1+\varepsilon}), p = O(r^{k+\varepsilon}), r \to \infty,$$

$k$ is an integer, $\varepsilon \in (0,1)$. Then $(\vec{u}, p)$ have an asymptotic expansion

$$\begin{aligned} p(x) &= P(x_1, x_2) + O(r^{-N}), \\ u_j(x) &= \frac{1}{2\nu} z(z-1) \frac{\partial P}{\partial x_j}(x_1, x_2) + O(r^{-N}), r \to \infty, \end{aligned}$$

$$u_3(x) = O(r^{-N}),$$

where

$$P(x_1, x_2) = \sum_{j=-k}^{N} a_0 \delta_{j0} \ln r + r^{-j}(b_j \cos(j\varphi) + c_j \sin(j\varphi)),$$

$(r, \varphi)$ are polar coordinates.

The results are obtained jointly with S. A. Nazarov.

V. V. PUKHNACHOV

**Microconvection in fluids**

To describe gravitational convection, the system of the Oberbeck-Boussinesq equations is usually used as a traditional mathematical model. An analysis of assumption made in deriving the mentioned system from the exact equation of continuum mechanics shows that the classical model is not applicable to studying convection if the parameter $gl^3/\nu\chi$ is small where $g$ is the gravity acceleration, $l$ is the characteristic linear scale, $\nu$ is the kinematic viscosity coefficient and $\chi$ is the fluid thermodiffusion coefficient. In this case, under the additional assumption that liquid density is a function of temperature only, the asymptotically exact system of equations of motion and heat transfer has been derived. The caracteristic property of the resulting system is the nonsolenoidality of the velocity field. For this system the initial boundary value problem is posed for bounded region with the no-slip condition and the Neumann temperature one. This problem is established to be solved under small variation of the boundary heat flux distribution. It is shown that with linear dependence of the specific volume on temperature, the system can be reduced to the form where the modified velocity vector is solenoidal. This allows an analogue of the stream function to be introduced to the plane and axisymetrical problem and the method of their efficient numerical solution to be found. The comparison of the results of numerical simulation on the basis of the classical model and the new one is presented for two cases: convective flows in an annular cavity at time-periodic microgravity and in a vertical layer with time-periodic heat flux on its sides.

M. PULVIRENTI

**A Statistical Approach to 2-D Turbulence**

Consider the following problem:

$$\begin{cases} -\Delta\psi = \frac{e^{-\beta\psi}}{\int_\Lambda e^{-\beta\psi}dx} & \beta \in \mathbb{R} \\ \psi = 0 \text{ in } \partial\Lambda \end{cases}$$

where $\Lambda$ is an open connected set. The solutions of the above equation (for $\beta \in (-8\pi, +\infty)$) are stationary states for the 2-$D$ Euler equation ($\psi$ is the stream function) which are connected with various entropy-energy variational principles from one side and with the Statistical Mechanics of point vortices according to the Onsager theory (1949). It seems that such states (with negative temperatures) are really observed in large scale simulation of the Navier-Stokes flows. The analysis of the above equation from a mathematical point of view and its physical mathematical derivation has been performed in a joint research with E. Caglioti, P. L. Lions, C. Marchioro.

Ref.s CLMP Comm. Math Phys (1992), CLMP to appear.
See also:
C. Marchioro, M. Pulvirenti, Mathematical: Theory of Non-Viscous Incompressible Flows. Springer Verlag Appl. Math. Sciences 96 (1994).

R. RANNACHER

**The Role of Hydrodynamic Stability for A-posteriori Error Control in Flow Computations**

Recently, in joint work with C. Johnson, a new approach towards quantitative error control in computational fluid mechanics has been developed. Combining the concepts of so-called "strong stability" and "Galerkin orthogonality" sharp a-priori as well as a-posteriori error estimates can be obtained with error constants depending on the Reynold number Re. In turn, the Re-dependence is directly related to the stability of the flow considered and appears crucial for its computability. The theoretical or computational determination of the corresponding stability constants is connected to an interesting new aspect in hydrodynamic stability theory. In predicting the stability or instability of a base flow, one has to examine the maximum growth of small pertubations. Due to the non-normal character of the linearized operator this pertubation growth can be rather large leading to instability even if (e.g., in the stationary case) all relevant eigenvalues are safely positive but small.

R. RAUTMANN

**Some New Convergence Results to Navier-Stokes Approximations**

In this talk we will consider 3 different approximations to solutions of the Navier-Stokes initial boundary value problem: (i) The solutions of a sequence of locally-in-time linearized problems, (ii) a semi-discrete approximation scheme ("Rothe's scheme"), and (iii) a product formula approach for solving the sequence of locally linearized problems. For all 3 methods we will present error bounds, the combination of which leads to explicit convergence rates in $L^2$, $H^1$ and $H^2$ of the product formula approach to nonstationary Navier-Stokes equations. The details of proofs are given in a joint paper with K. Masuda.

K. G. ROESNER

**Kohärente Strukturen im laminar-turbulenten Übergangsbereich rotierender Fluide, numerische Behandlung der kompressiblen Navier-Stokesschen Gleichungen**

Die Stabilität einer durch rotierende Scheiben induzierten Wirbelströmung in einem würfelförmigen Volumen wird numerisch untersucht. Durch die Vorgabe periodischer Randbedingungen wird das Wirbelfeld im Raum fortgesetzt.

In einer ersten Anordnung wird Periodizität nur in $x$- und $y$-Richtung angenommen. Die Strömung im kubischen Volumen wird durch Rotation von Deckel- und Bodenflächen angetrieben. Die rotierenden Flächen stellen Kreisschreiben dar, deren Durchmesser gleich der Kantenlänge des Kubus sind.

Der zweite Fall stellt die periodische Fortsetzung des Wirbelfeldes in allen drei Raumrichtungen dar.

Für Machzahlen nahe bei 1 werden die Strömungsfelder mittels eines pseudospektralen finiten Volumenverfahrens berechnet. Es handelt sich um ein Tschebyscheff-Kollokationsverfahren. Damit werden die kompressiblen Navier-Stokesschen Gleichungen auf die Lösung eines gekoppelten Systems gewöhnlicher Gleichungen zurückgeführt. Das Anfangs-Randwertproblem wird mittels eines Adams-Bashforth Verfahrens numerisch gelöst.

M. RUMPF

**The Equilibrium State of an Elastic Solid in an Incompressible Fluid Flow**

The interaction of an elastic solid and an incompressible flow at a common boundary is analysed. We will in detail examine the configuration of a liquid flowing around an elastic obstacle and a nonfixed elastic body falling in a fluid container. The elastic body is influenced by the boundary stresses of the liquid and conversely the body influences the fluid by changing its domain. Existence of a velocity, a pressure, and a deformation are proved, assuming the data is small. Smallness is expressed in the case of first application in terms of the prescribed velocity on the outer boundary or for the free falling solid in terms of the density of the elastic solid.

M. RUZICKA

**Regularity for Steady Solutions of the Navier-Stokes Equations**

One of the main unsolved problems in the mathematical theory of the Navier-Stokes equations is the question of regularity and uniqueness of weak solutions of the three-dimensional unsteady Navier-Stokes equations. ¿From the scaling invariance of the equations follows that the time variable scales like a two-dimensional space variable (see e. g. [1]) and thus there is some similarity between the unsteady three-dimensional and the steady five-dimensional case. Based on the observation (see [2]), that there exist weak solutions of

$$\begin{aligned} -\Delta u + u \cdot \nabla u + \nabla p &= f \\ \operatorname{div} u &= 0 \end{aligned} \qquad \text{in } \Omega \subseteq \mathbf{R}^5$$

which satisfy for $\Omega_0 \subset\subset \Omega$

$$\sup_{x \in \Omega_0} \left( \frac{u^2}{2} + p \right) \leq c_{\Omega_0},$$

one can show certain apriori estimates for $u, \nabla u, p$ in Morrey spaces. Using this information it is possible to prove the existance of regular solutions of (1) in several situations, see [3], [4], [5].

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M. SCHONBEK


**Decay of solutions to the Boussinesq system of equations.**

We show that solutions to the Boussinesg system of equations decay in $L^2$ at an algebraic rate.

The main ideas used are Fourier splitting methods combined with bounds coming from the special entropy corresponding to the underlying hyperbolic equations.

A. SEQUEIRA

**On a Vector Transport Equation with Applications to Non-Newtonian Fluids**

When one attempts to show existence of regular solution for the equations of non-newtonian fluids of grade $n$, one finds it most natural to split the problem into two auxiliary problems, namely a Stokes-like problem and a vector transport equation of the form

$$(P)\begin{cases} w + v\cdot\nabla w &= F + \nabla p \\ \nabla\cdot w &= 0 \\ w\cdot n|_{\partial\Omega} &= 0 \end{cases} \qquad \text{in } \Omega\subset \mathbf{R}^3$$

where $\nabla\cdot v = 0, v\cdot n|_{\partial\Omega} = 0$

The aim of this work is to show that, for $\Omega$ bounded domain, exterior, half-space or whole space, problem $(P)$ admits one and only one solution in the space $W^{m,q}(\Omega)$, for $m\geq 0, 1 < q < \infty$.

C. G. SIMADER

**The Exterior Dirichlet Problem for $\Delta$ and Stokes' System on $L^q$.**

For an exterior domain $G\subset \mathbf{R}^n (n\geq 2)$ we give a sketch of joined work with Prof. H. Sohr (Paderborn). We first consider the weak Dirichlet Problem (=: $DP$) for $\Delta$ in certain $L^q$-spaces $(1 < q < \infty)$. Further we discuss the strong DP and moreover so-called (by us) $(2+k)$-solutions of

$$\text{“} -\Delta u = f \text{ on } G, \quad u|_{\partial G} = 0\text{”}.$$

Last mentioned type of solutions means that the datum $f$ has derivatives of $k$-th order $(k\geq 0)$ on $L^q(G)$ whereas the lower order derivatives have only $L^q_{\text{loc}}(G)$-properties. For the solutions we therefore may expect only that the $(2+k)$-th order derivatives belong to $L^q$ and the lower order derivatives have only $L^q_{\text{loc}}(G)$-properties. Further we sketch how these results can be translated to Stokes' system by means of a suitable extension of Helmholtz's decomposition.

Since we make essentially use of the properties of the completion of $C_0^\infty(\mathbf{R}^n)$ with respect to order-homogeneous-norms (say for simplicity with respect to $\|\nabla\cdot\|_q$-norm), in the case $n\leq q<\infty$, too, we explain how a certain remark on the famous paper by J. Deny and J. L. Lions (Ann. Inst. Fourier 5, 305 - 370, 1954, see p. 319) was completely misleading. It turned out to be an obstacle for the development of at least the French school on this direction - while exactly arguments of that type had been used successfully since decades by our Russian colleagues (as we learned at this conference, too.)

H. SOHR

**The Navier-Stokes equations in $L^q$-spaces with weights**

We consider the nonstationary Navier-Stokes equations for exterior domains $\Omega \subset \mathbf{R}^3$ in $L^q$-spaces with weights of the form $|x|^\alpha$, $\alpha > 0$, $x \in \Omega$. The first result yields the existence of a weak solution $u$ (under some conditions on the data) such that $\| \, |\cdot|^\alpha u \, \|_{2,\infty} +$ $\| \, |\cdot|^\alpha \nabla u \, \|_{2,2} < \infty$ if $0 < \alpha < \frac{1}{4}$. Further, if $\alpha$ is sufficiently small we can prove resolvent and semigroup estimates for the Stokes operator and the existence of a local in time unique strong solution in $L^q$-spaces with weights. This method is applicable to all dimensions $n \geq 2$ and also to the stationary case.

M. SPECOVIUS-NEUGEBAUER

**The Helmholtz decomposition and the resolvent of the Stokes operator in weighted $L^r$-spaces**

Let $\Omega \subset \mathbf{R}^3$ be an exterior domain with smooth boundary $\partial\Omega$. For $1 < r < \infty$, $\delta \in \mathbf{R}$ we define weighted $L^r$-spaces $L^r_\delta(\Omega)$ by

$$L^r_\delta(\Omega) = \{u, \| \, u \, \| = \left( \int_\Omega (1 + |x|^2)^{\delta r/2} |u(x)|^r dx \right)^{1/r} < \infty\}$$

Let $X^r_\delta(\Omega)$ denote the closure of all smooth solenoidal testfunctions in $L^r_\delta(\Omega)^3$ and $G^r_\delta(\Omega) = \{\nabla p, p \in L^r_{\delta-1}(\Omega), \nabla p \in L^r_\delta(\Omega)\}$. Then for $\delta \in (\frac{-3}{r}, \frac{3}{r'}) \setminus \{\frac{-3}{r} + 1, \frac{3}{r'} - 1\} =: I$ we get the "classical" Helmholtz decomposition: $L^r_\delta(\Omega)^3 = X^r_\delta(\Omega) \oplus G^r_\delta(\Omega)$.

Let $P$ be the projection operator onto $X^r_\delta(\Omega)$, then the operator $P\Delta$ defines a closed operator on $X^r_\delta(\Omega)$ with domain $D(P\Delta) = X^r_\delta(\Omega) \cap \{u, D^2 u \in L^r_\delta(\Omega), \gamma u = u|_{\partial\Omega} = 0\}$. It can be shown that for any $\lambda \in \mathbf{C} \setminus (-\infty, 0], f \in X^r_\delta(\Omega), \delta \in I$ there exists a unique solution $u \in D(P\Delta)$ of the resolvent equation

$$\lambda u - P \Delta u = f.$$

Moreover, the Stokes operator $P\Delta$ generates a holomophic semigroup of class $C_0$ on $X^r_\delta(\Omega)$.

A. TANI

**On the solvability of the equations for the motion of a vortex filament with or without axial flow**

We consider the solvability of the localized induction equation

$$\vec{x}_t = \vec{x}_s \times \vec{x}_{ss}$$

without axial flow and

$$\vec{x}_t = \vec{x}_x \times \vec{x}_{ss} - \alpha[\vec{x}_{sss} + \frac{3}{2}\vec{x}_{ss} \times (\vec{x}_s \times \vec{x}_{ss})]$$

with axial flow.

E. S. TITI

**The Efficiency of the Nonlinear Galerkin Methods.**

In this talk I will survey the theory of approximate inertial manifolds and the nonlinear Galerkin methods. I'll present some computational study which supports the theoretical estimates on the rate of convergence of the nonlinear Galerkin methods. In this talk I'll present the conditions under which the nonlinear Galerkin method is superior to the usual Galerkin method and the conditions under which it is "comparable".

L. TOBISKA

**Numerical Solution of the Boussinesq Approximation of the Navier-Stokes Equation**

We consider stable numerical methods for solving the Boussinesq approximation of the Navier-Stokes equation. For an external force $f = \nabla\phi$ the exact velocity field $u$ vanishes identically whereas the discrete velocity field $u_h$ only admits an error estimate of order $O(\mathrm{Reh})$, consequently for higher Reynolds number a very small meshsize is needed to guarantee a certain accuracy. A new technique for getting better velocity approximations $u_h$ is proposed giving estimates of order $O(\mathrm{Reh}^2)$ in the no-flow case. Numerical tests demonstrate the inprovements of the new method.

W. VARNHORN

**On nearly convergent approximation of potentials**

We present a new approximation procedure for hydrodynamical and other volume - and surface potentials based on ideas of V. Maz'ya. The main point is to use a suitable approximate partition of the unity to represent the potential densities. This leads to explicit expressions for the potentials, containing a 1-D integration only. Numerical results for the Stokes and the Poisson equations are demonstrated and illustrate the accuracy of the method.

W. VELTE

**On the optimal constants in some related inequalities**

Es werden drei verschiedene Ungleichungen betrachtet, deren optimale Konstanten in einem einfachen Zusammenhang stehen. Für ein Gebiet $\Omega \subset \mathbf{R}^2$ (beschränkt, einfach zusammenhängend, glatter Rand) werden in den Sobolevräumen $(H_0^1(\Omega))^2$ bzw. $(H^1(\Omega))^2$ betrachtet:

$$\int_\Omega |\nabla \underline{u}|^2 \, \mathrm{dxdy} \leq C(\Omega) \int_\Omega (\operatorname{div} \underline{u})^2 \, \mathrm{dxdy} \quad \text{für } \underline{u} \in (H_0^1)^2 \ominus_{D(\cdot,\cdot)} \ker(\mathrm{div})$$

$$\int_\Omega |\nabla \underline{u}|^2 \, \mathrm{dxdy} \leq K(\Omega) \frac{1}{4} \int_\Omega (u_{j,k} + u_{k,j})(u_{j,k} + u_{k,j}) \quad \text{für } \underline{u} \in (H^1(\Omega))^2$$

mit der Nebenbedingung $\int_\Omega (u_{j,k} - u_{k,j}) \, \mathrm{dxdy} = 0$ (Kornsche Ungl. in der sogen. "ursprünglichen Form").

In dem von Friedrich (1937) betrachteten Hilbertraum (über $\mathbf{R}$) der analytischen Funktionen $w(z) = u(x,y) + iv(x,y)$ in $\Omega$ mit

$$\| w \|^2 = \int_\Omega (u^2 + v^2) \, \mathrm{dxdy} < \infty$$
$$(w_1, w_2) = \int_\Omega (u_1 u_2 + v_1 v_2) \, \mathrm{dxdy}$$

wird betrachtet:

$$\int_\Omega u^2 \, \mathrm{dxdy} \leq \Gamma(\Omega) \int_\Omega v^2 \, \mathrm{dxdy} \qquad \text{für alle } w = u + iv \in F \text{ mit } \int u = \int u = 0.$$

Für die optimalen Konstanten $C(\Omega), K(\Omega), \Gamma(\Omega)$ ist bekannt:

$$C = P + 1, \quad K = 2C.$$

Es werden die zu den quadratischen Formen gehörigen Eigenwertprobleme (Variationsgleichungen) diskutiert, bekannte Resultate und einige Ergänzungen dargestellt, die einen einfachen Zusammenhang zwischen den Spektren ergeben.

W. von WAHL

**Stability of Steady Flows up to and at Criticality**

We give a necessary and sufficient condition in order that the marginal cases for monotonic energy-stability und linearized stability coincide. Evaluation of this condition singles out some of the rare examples where monotonic energy stability is followed up by instability and thus the stability problem is solved completely. As such an example we consider plane parallel shear flow in an infinite layer heated from below.

In the second part of talk we consider stability behaviour of a steady flow at criticality and between criticality and monotonic energy stability, provided there is the usual gap between these quantities.

O. WALSH

**On nonlinear Galerkin methods for the Navier-Stokes equations using finite elements**

The nonlinearity in the Navier-Stokes equations couples the large and small scales of motion in turbulent flow. The nonlinear Galerkin method (NGM) consists of inserting into the equation for the large scale motion the small scale motion as determined by an "approximate inertial manifold". Despite the conceptual appeal of this idea, its theoretical justification has been recently thrown into question. However, its actual performance as a computational method has remained largely untested. Temam and collaborators have reported a 50 % speed up in their spectral code for spatially periodic flow but their experiments have been recently criticized. In any case, spatially periodic computations are of little practical use. The aim of this thesis has been to test the NGM in the more practical context of the finite element method.

Using finite elements, there is ambiguity and difficulty because the coarse grid has no natural supplementary space. We analyze a family of supplementary spaces and it is found that the quality of the asymptotic error estimates depends on the choice. Choosing the space by the $L^2$-projection, we prove that the resulting approximation is "asymptotically good". These results extend and improve upon recent error estimates of Marion and collaborators. For any other choice, the estimates are weaker and if - as we suspect - they are optimal it seems possible that the NGM may actually decrease the accuracy of calculations. We also analyzed a variant of the NGM that we call "microscale linearization" (MSL). We prove that the MSL is "asymptotically good" for any member of this family of supplementary spaces. Turning to calculations, choosing the supplementary space by the Ritz projection, we implemented the NGM by modifying a 2-D Navier-Stokes code of Turek; it performed very poorly. We implemented a variant of the MSL. It performed better, but still not as well as the original code. We sought a further understanding of these results by considering the 1-D Burgers equation. In conclusion, we find no numerical evidence that these methods are better than the standard finite element method. In fact, unless the coarse mesh is itself very fine, all versions performed poorly.

M. WIEGNER

**Decay of higher order norms of solutions of the Navier-Stokes-equations**

We study solutions of the Cauchy-problem for the Navier-Stokes-equations - either strong ones for $n \leq 5$, or weak ones with generalized energy inequality for $n \leq 4$, which become strong after some finite time.

We show, that an energy-decay $\| u(t) \|_2 = O(t^{-\mu})$ implies a decay $\| D^m u(t) \|_2 = O(t^{\frac{-m}{2}-\mu})$ for all $m \in \mathbb{N}$. The decay of other $L_p$-norms, $p \geq 2$, follows by interpolation, e. g. $\| u(t) \|_\infty = O(t^{-\mu-\frac{n}{4}})$.

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