MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 25/1994

Nichtlinearitäten vom Hysteresetyp

12.06. bis 18.06.1994

Die Tagung fand unter der Leitung von Prof. Karl-Heinz Hoffmann (München), Prof. Ingo Müller (Berlin) und Prof. Jürgen Sprekels (Berlin) statt. Sie wurde von 23 Teilnehmern aus Europa und den USA besucht, die durch Vorträge und Diskussionen zur Gestaltung beitrugen.

Im Mittelpunkt des Interesses stand die Behandlung von Systemen, die hysteretischen Charakter haben. Beispiele dafür sind das Regelverhalten von Thermostaten und anderen Schaltgliedern sowie das Materialverhalten von Gummi, Ferromagnetika, Formgedächtnislegierungen und plastischen Körpern.

Das Spektrum der Vorträge reichte von den physikalischen Grundlagen über die mathematische Modellierung solcher Vorgänge, deren Behandlung in Hinblick auf Existenz und Eindeutigkeit von Lösungen, bis hin zu resultierenden Anwendungen.

Mehrere Vorträge, die sich mit der Beschreibung der pseudoelastischen Hysterese in Formgedächtnislegierungen beschäftigten, deuteten auf die Notwendigkeit der Berücksichtigung nichtlokaler Wechselwirkungen in den Theorien zur Beschreibung des Phänomens.

Des weiteren wurden verschiedene mathematische Modelle zur Beschreibung von Phasenübergängen untersucht. Insbesondere wurden das Stefan-Problem behandelt und phänomenologische Modelle für das Verhalten von Formgedächtnislegierungen sowie Erweiterungen des klassischen Preisachmodells. Es wurden Existenz und Eindeutigkeit der damit verbundenen Rand-Anfangswert-Probleme für verschiedene Grenzfälle gezeigt.

Anwendungen fanden sich in der Schädigungsbestimmung von Materialien unter zyklischer Belastung mittels Hystereseoperatoren in der Plastizitätstheorie, und es wurde gezeigt, wie durch die Temperaturabhängigkeit der Materialparameter beim Phasenübergang in Formgedächtnislegierungen das Schwingverhalten von Verbundwerkstoffen kontrolliert werden kann.

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In den Diskussionen waren besondere Schwerpunkte die Frage nach der Notwendigkeit nichtkonvexer Potentiale und Phasengrenzenergien zur Beschreibung von Hysteresen, sowie die Möglichkeit der Kopplung hysteretischer Prozesse mit stochastischen Einflüssen, um Aussagen über das Zeitverhalten solcher Systeme zu erhalten.

Ein genauerer Überblick über die behandelten Themen läßt sich aus den folgenden Zusammenfassungen der Vorträge gewinnen:

Hysteresis Operators in Plasticity

Martin Brokate

In the uniaxial case, there is a close relation between rate independent elastoplastic constitutive laws, the notion of a hysteresis operator and the rainflow counting method. The latter forms the basis for solving engineering problems in connection with damage estimation, reconstruction and extrapolation of uniaxial loading sequences acting on parts of a mechanical structure. Our main goal is to contribute to an extension of these techniques to multiaxial situations. We present the continuous version of the constitutive model due to Mróz and describe a method for decomposition of a multiaxial loading sequence. We finally discuss the possibility and relevance of inverting the Mróz model.

Phase Transitions in Liquid Crystals

Gianfranco Capriz

The study of simple problems of phase transition in cells of liquid crystals offers the occasion, on the one hand, to predict the possible occurrence of not yet widely explored phenomena (such as optical biaxiality of certain phases or the "melting" of the crystal along transition boundaries) and, on the other hand, to suggest appropriate tools to deal with roughly similar phenomena in any continuum with microstructure, e.g., the definition of average quantities for multigranular material elements; a different approach to the topological theory of defects, etc..

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On the Frémond Model for Shape Memory Alloys

Pierluigi Colli
Pavia

After a reminder of the basic constitutive assumptions and thermodynamic laws of the Frémond thermo-mechanical model for shape memory alloys, the aim of the talk is to give a look to what is known on the mathematical analysis of corresponding initial-boundary value problems. Some existence, uniqueness and regularity results are reviewed and various open questions are pointed out and discussed.

Stefan Problems as Asymptotic Limits of the Penrose-Fife Model,
Part 2

Pierluigi Colli
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Italy

We consider the following Stefan problem

$$(1)\quad \begin{cases} \partial\left(\omega\theta + L\chi\right) + k\Delta\left(\frac{1}{\theta}\right) = g & \text{in } \Omega \times (0,T)\ , \\ \beta\left(\chi\right) \ni L\left(\frac{1}{\theta_c} - \frac{1}{\theta}\right) & \text{in } \Omega \times (0,T)\ , \\ k\frac{\partial}{\partial n}\left(\frac{1}{\theta}\right) = \gamma\left(\frac{1}{\theta_\Gamma} - \frac{1}{\theta}\right) & \text{in } \partial\Omega \times (0,T)\ , \\ \left(\omega\theta + L\chi\right)(.,0) = \varphi & \text{in } \Omega \end{cases}$$

where ω, L, k, θ_c are positive constants and g, γ , θ_Γ are given functions. In particular, θ_c denotes the temperature of phase change, and θ_Γ represents the outside temperature. The maximal monotone graph β coincides with the subdifferential of the indicator function $I_{[0,1]}$. In order to find the absolute temperature $\theta > 0$ and the phase fraction $\chi \in [0,1]$, we approximate (1) by a Penrose-Fife initial -boundary value problem, where the second equation of (1) is replaced by s

$$\mu\chi_t - \epsilon\Delta\chi + \beta\left(\chi\right) \ni L\left(\frac{1}{\theta_c} - \frac{1}{\theta}\right) \qquad \text{in } \Omega \times (0,T)\ .$$

along with the boundary and initial conditions

$$\frac{\partial\chi}{\partial n} = 0 \qquad \text{in } \partial\Omega \times (0,T)\ ,$$
$$\chi(\cdot,0) = \chi_0 \qquad \text{in } \Omega\ .$$

We prove existence and uniqueness for (1). The existence result is obtained by investigating the asymptotic behaviour of the Penrose-Fife problem as $\mu \searrow 0$, and $\epsilon \searrow 0$. We show that the solutions of this problem converge (in some sense) to the solution of (1). We also study the intermediate cases μ fixed, $\epsilon \searrow 0$ and $\mu \searrow 0$, ϵ fixed, thus getting two relaxed Stefan problems. These results have been obtained jointly with J. Sprekels, Berlin.

Elastic Invariants and Uniform Rearrangements in the Theory of Defective Crystals

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In the context of a crystal model proposed by Davini (1986) it is possible to derive a defect notion from a property of invariance of certain integral forms under the class of elastic deformations. Together with the known measures of the classical theory of dislocations, the approach provides a finite set of defect measures that fully describe defectiveness in a suggestive sense. Here I give a general account of the kinematics involved and focus in particular on a type of elasto-plastic decomposition theorem that can be proved within this rather general context.

Collisions of Rigid Bodies

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Consider a point and a rigid body. Because the distance of the point to the rigid body varies, the system formed by the point and the solid is deformable. The strain rate can be defined by $D\left(\vec{U}\right)(t) = \vec{U}_1(t) - \vec{U}_2\left(\vec{x}_1(t), t\right)$, where $\vec{x}_1(t)$ is the position of the point, $\vec{U}_1(t)$ its velocity and $\vec{U}_2\left(\vec{x}_1(t), t\right) = \vec{U}_0(t) + \omega \times \left(\vec{x} - \vec{x}_0\right)$ the velocity field of the rigid body. In a collision the velocities are discontinuous ($\overrightarrow{V^-}$ is the velocity before the collision, $\overrightarrow{V^+}$ the velocity after and $\left[\vec{V}\right] = \overrightarrow{V^+} - \overrightarrow{V^-}$). Because the system is deformable, we can define internal forces. The internal percussion is defined by its virtual work,

$$-\overrightarrow{P^{int}} \frac{D\left(\overrightarrow{U^+}\right) + D\left(\overrightarrow{U^-}\right)}{2}.$$

The equations of the movement of the point result from the principle of virtual work. For a collision we get

$$\overrightarrow{P^{int}} = \overrightarrow{P^{reac}} + \overrightarrow{P^d}\left(D\left(\overrightarrow{U^+}\right) + D\left(\overrightarrow{U^-}\right)\right),$$

where $\overrightarrow{P^{reac}}$ is the reaction which prevents interpenetration of the point and the body. Its value is 0, if the contact does not persist after the collision, it is $P\vec{N}$, with $P \leq 0$, if the contact persists after the collision ($\vec{N}$ is the outward normal to the solid). The dissipative percussion $\overrightarrow{P^d}$ describes friction, interactions, ... occuring in the collision. Examples are given with $\overrightarrow{P^d} = \mathcal{F}\left(D\left(\overrightarrow{U_N^+}\right) + D\left(\overrightarrow{U_N^-}\right)\right)$, where

U_N is the normal velocity, for instance, $k\left(D\left(\overrightarrow{U_N^+}\right)+D\left(\overrightarrow{U_N^-}\right)\right)$ with $k>0$. The theory can describe multiple collisions (i.e. collisions occuring at the same time at different points of two solids (a plane and a "concave" wheel for instance)). It can be extended to collisions of a point with a tip of solid angles, i.e. to collisions with non-smooth rigid bodies.

Vibration of a Composite Plate - Amplitude Control

Y. Huo
TU München

We consider a composite material with NiTi shape memory fibers in the form of a plate. Its vibration frequency and amplitude can be influenced by changing the temperatures of the NiTi fibers. A mathematical model is proposed, which assumes the form of a system of partial differential equations with nonlinear hysteretic terms. Numerical calculations are made for a simplified system of PDEs, which demonstrate that the frequency and amplitude of the vibration can be controlled by controlling the temperature of the fibers.

Multidimensional Models for Plastic Hysteresis

Pavel Krejcí

The classical Prandtl-Reuss one-yield constitutive law for elastoplastic materials can be used in several different ways as a basic building block for multiyield models. A standard application of rheological combination principles generates the Prandtl-Ishlinskii model which preserves good analytical properties (continuity, monotonicity, energy inequalities), but its complicated memory does not allow for efficient practical application. This drawback has been removed in the model of Mróz, which, however, does not exclude in general the violation of the second law of thermodynamics. Modifying the Mróz flow rule one obtains a thermodynamically consistent model with a simple memory structure which , on the other hand, is in general neither monotone nor locally monotone. These properties constitute the limits of applicability of each model. Note that all these models coincide in the one-dimensional (uniaxial) case.

The Stefan Problem with Hysteresis

A. Meirmanov
Universidade da Beira Interior
Portugal

The Stefan problem with phase function as a simple hysteresis functional was considered. For the one-dimensional case, we prove existence and uniqueness of weak solutions, where temperature and phase function satisfy the hysteresis law almost everywhere. The uniqueness is also true for the multi-dimensional case.

Gases and Rubbers

Ingo Müller
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Despite widely different appearances, gases and rubbers are closely related thermodynamically. The elasticity of both is not related to energetic potential wells but rather to the entropic tendency for spreading states evenly over the available configurations. Or, as a joking scientist has once remarked: Rubbers are the ideal gases among the solids. These, however, are fair statements only for gases at low densities and rubbers at small deformations - up to 300%. Outside this range the energy of molecular interaction plays a role. But the similarity between gases and rubbers persists even though the gas condenses to a fluid and the amorphous rubber crystallizes. There is only one significant difference: the phase transition in rubber is hysteretic while in the fluid it is reversible. The reason suggested for this difference is the different size of the energy of the phase interfaces in the two cases. In rubber this energy is appreciable, while in the fluid it is negligible.

Shape Memory Hysteresis : a Micromechanical Approach

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The shape memory effect is related to martensitic transformation. Due to the lower symmetry of the martensite, the parent phase may transform into several variants of martensite. These variants are crystallographically equivalent but differ by their orientations. This crystallographical feature is the origin of many peculiar properties of shape memory alloys. We propose to account for the martensitic microstructure by use of a micromechanical approach. The first step is to determine constitutive relations for a representative element of the material. Due to the granular structure of metallic alloys, grains are chosen as such representative elements. Kinematical considerations allow to relate strain occuring in a grain with the transformation strain associated to each individual variant (that is a crystallographical data) and with the volume fraction f_n of variants actually formed inside this grain. The determination of f_n is obtained through the definition of a thermodynamic potential for which these f_n are chosen as internal variables. Hysteresis phenomena are then accounted for by the definition of a pseudo-dissipative potential, which is assumed to be proportional to the cumulative volume fraction of the different variants. The resistive forces are considered to be equal on all the variants. From these elements the constitutive relations are derived and physical constraints exerted on f_n are respected. The second step is to compute the effective response of the granular material. This homogenization problem is solved using a self-consistent method previously developped by M. Berveiller and P. Lipinski to describe the plasticity in polycrystalline materials. Results obtained in such a way , include well-described experimental observations (stress-strain curve, kinetics of the transformation) and the theory has predictive capacity.

Hysteresis, Metastability and Nonlocal Interactions

Robert C. Rogers
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In this talk I discussed a spatially nonlocal energy model. The model deals only with static equilibrium and contains relaxed stored energy functionals. In the introductory section I discussed the relationship between this nonlocal energy model and

1. gradient models,
2. interfacial energy models,
3. coherence energy models and
4. order parameter models.

I then described two types of calculations that showed how the model produces at least an indication of hysteresis by inducing multiple metastable states. The first set of results were numerical calculations on a simple one-dimensional model of deformation of a shape memory alloy. I showed calculations of a deformation-controlled hysteresis loop that showed yielding by motion of two phase transitions that nucleated at the ends of the sample. The second result showed that a very simple calculation on the nonlocal model gives a fairly good fit to relationship between orientation and width of hysteresis in the James/Chu biaxial loading experiment.

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On Nucleation and Growth of Martensitic Layers during the Pseudoelastic Phase Transformation

Stefan Seelecke
TU Berlin

The pseudoelastic hysteresis of certain classes of shape memory alloys has been recognized to be greatly influenced by the large number of interfaces evolving during the phase transition. However, the cause for this behavior had not been understood yet, because it appears that an increase in interfaces should also increase the total energy and therefore be unfavorable. Using Eshelby's inclusion method within the framework of a beam theory approach, we derive a new expression for the free energy of such an alloy. Through its dependence on the position and size of an arbitrary number of transformed layers this energy accounts for the nonlocal interactions within the body. It can be shown that due to these nonlocal interactions the formation of new layers corresponds to states of relative minima of the free energy. Therefore these represent favorable configurations of the system despite the apparent increase in interfacial energy.

Stochastic Differential Equations with Hysteresis

T. I. Seidman

Originally motivated by an optimal control problem with a thermostat hysteresis (elementary hysteron) giving discontinuous behavior, we consider the introduction of a small noise in the hope that expected values of cost functionals will become smooth functions of controllable parameters, i.e., stochastic smoothing. Modeling this as an SDE of Ito type, one must obtain existence of a unique stochastic solution with nonstandard Fokker-Planck equation. We show how to compute, e.g., the expected time between transitions and so get the desired smoothness of the dependence of parameters for this component of a cost functional. [Also mentioned were some asymptotic considerations for time scales appropriate to hysteresis problems.]

Stefan Problems as Asymptotic Limits of the Penrose-Fife Model, Part 1

Jürgen Sprekels
Berlin

We study an initial-boundary value problem for the nonlinear system of PDEs

$$c_0\theta_t - \lambda'(\chi)\,\chi_t + k\Delta\left(\frac{1}{\theta}\right) = g \quad , \tag{0.1}$$

$$\mu\chi_t - \epsilon\Delta\chi + \beta(\chi) \ni s'(\chi) + \frac{\lambda'(\chi)}{\theta} \quad , \tag{0.2}$$

where c_0, k, μ, ϵ are positive constants. Moreover, λ and s denote smooth nonlinearities, and the maximal monotone graph β denotes the subdifferential of the indicator function of the interval $[0,1]$ (i.e. $\beta = \partial I$). The system (0.1), (0.2) can be viewed as a system of phase-field equations modelling the kinetics of a diffusive phase transition. In this connection, θ and χ denote absolute temperature and the order parameter of the transition, respectively. In fact, if the free energy is assumed as

$$\mathcal{F}[\chi,\theta] = \int_\Omega \left(F(\chi,\theta) + \frac{\epsilon}{2}|\nabla\chi|^2\right) dx \ ,$$

with the local free energy density

$$F(\chi,\theta) = -c_0\ln(\theta) + \theta\,(I(\chi) - s(\chi)) - \lambda(\chi) \ ,$$

then (0.1), (0.2) are exactly the phase-field equations resulting from the Penrose-Fife model for diffusive phase transitions. In our contribution, the asymptotic behaviour of solutions to (0.1), (0.2) is studied for the case that $\epsilon \searrow 0$, that is, for vanishing interfacial energy. Global existence of a weak solution to the limiting problem is shown, which is a relaxed Stefan problem with heat flux $\vec{q} = \nabla\left(\frac{1}{\theta}\right)$. The solution is obtained as limit of solutions to (0.1), (0.2) for $\epsilon \searrow 0$. These results have been obtained jointly with P. Colli, Pavia.

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Robust Parameter Identification of Systems with Hysteresis

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The complete-moving-hysteresis (CMH) is an extension of the moving Preisach model, developed to characterize magnetic media. It computes both the reversible and the irreversible components of the magnetization and the relationship between these components. An important question associated with a given model is the corresponding identification strategy which provides a method to experimentally obtain the parameters of the model. In order to be experimentally applicable, such an identification strategy has to be robust in the presence of experimental errors. Non-parametric identification methods use curve fitting techniques but in general behave poorly in the presence of noise. Parametric identification methods use interpolation techniques, require the a-priori knowledge of the type of function, and in general are robust in the presence of noise. This talk presents a parametric identification strategy for the CMH model where the types of functions describing the model are derived based on physical principles. It is found that the robust identification strategy is applicable to a wide range of recording media. It requires the measurement of the major loop, the major remanence loop, the virgin curve and the virgin remanence curve only. It has been experimentally verified for various recording media by comparing the measured and computed higher-order asymmetrical reversal curves. The limitations of the identification method will be discussed. A generalized Preisach function for the CMH model is suggested that may be applicable to soft magnetic materials, as well.

Discontinuous Hysteresis

A. Visintin
Trento

The closure in suitable function spaces of the standard *(delayed) relay operator* is introduced. This can be expressed in terms of a system of two variational inequalities. (Here we denote by $\mathcal{F} : u \longmapsto w$ this closure). A *vectorial* extension of *(delayed) relays* is formulated, and also expressed by means of two coupled variational inequalities. This yields a vectorial extension of the classical Preisach model. Results are then given for problems obtained by coupling the hyteresis conditions $w = \mathcal{F}(u)$ with some PDEs. For instance,

$$\frac{\partial}{\partial t}(u + w) - \Delta u = f \qquad \text{in } \Omega \times]0, T[\ ,$$

or

$$\frac{\partial u}{\partial t} - \Delta u + w = f \qquad \text{in } \Omega \times]0, T[\ ,$$

where Ω is an open subset of $\mathbf{R}^N$ $(N \geq 1)$ and f is a given function.

Statistical Mechanics of Stress-induced Martensitic Transformation

K. Wilmanski
University of Essen

The paper contains the analysis of the statistical properties of the ersatz-model of pseudo-elastic hysteresis. The main aim of the work is to derive the free energy function defined on the space of variables:{stress, volume fraction of martensite, number of interfaces}. This function should describe the scenarios of deformation-controlled processes inside of the stress-strain hysteresis loop. It is shown that such a function follows indeed from the microscopic model of the bar in tension if we account in the Hamiltonian for the shearing deformations in planes perpendicular to the tensile axis. It is also shown that the modulus of the statistical distribution which is inversely proportional to the absolute temperature in the classical statistical mechanics of gases, is proportional to the square root of the number of interfaces. This leads to the definition of "the temperature of mechanical fluctuations" for stress induced martensitic transformations. According to the available experiments this quantity seems to be small which allows to neglect the entropic contribution to the free energy of the macroscopic model. This "low temperature" approximation has been considered earlier and it has produced reasonable results for pseudoelastic materials.

Berichterstatter: Stefan Seelecke

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