Tagungsbericht 1/1995

Symmetrien
01.01. bis 07.01.1995

Organisatoren der Tagung waren Jose M. Montesinos (Madrid), Reinhold Remmert (Münster) und Peter Slodowy (Hamburg).

Die Idee der Tagung war es, Experten aus verschiedenen Gebieten Gelegenheit zu geben, untereinander über die Rolle der Symmetrie in ihrem Forschungsbereich zu berichten. Dabei sollte das Konzept der Symmetrie bzw. ihre Ästhetik im Vordergrund stehen.
Siebzehn Vorträge beschäftigten sich mit Übersichten, Ergebnissen und aktuellen Entwicklungen der folgenden Teilgebiete:

- Geschichte und allgemeine Gruppentheorie
- Hyperbolische Geometrie
- Symmetrie in Kristallographie und Physik
- Symmetrie in der Zahlentheorie
- Symmetrie in algebraischer bzw. analytischer Geometrie

Zum Gelingen der Tagung trugen die offene Atmosphäre, die reichlich vorhandene Gelegenheit zu informeller Diskussion und die sehr gute Ausstattung des Instituts (i.e. Bibliothek) wesentlich bei.
Nicht unerwähnt bleiben sollte auch der abendliche Lichtbildvortrag "Symmetries in the Alhambra" von Jose M. Montesinos, der die in der Innenarchitektur und in den Kachelmustern verborgenen kristallographischen Raumgruppen aufzeigte.
Alle Vorträge wurden in englischer Sprache gehalten.

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Vortragsauszüge

S. Endraß Symmetric Algebraic Surfaces with finitely many Singularities

An old and easy formulated question is the following: What is the maximal number $\mu(n)$ of double points of a surface of degree n in $\mathbb{P}_3(\mathbb{C})$ (with no further degeneracies)? The exact answer is known only for $n \leq 5$ and there exist examples of those surfaces, most of them admitting some symmetry. A family of surfaces of arbitrary degree is introduced which includes all so far known best examples for surfaces with finitely many double points from degree two up to degree six. All these surfaces admit (at least) a rotational symmetry of order n (or $n-1$). The key idea is to restrict the surface to a symmetry plane in such a manner that every singularity of the restricted surface, which is a plane algebraic curve of degree n, induces an orbit of n (or $n-1$) singularities of the surface. So the problem is restricted to finding an algebraic curve (within a given family) with many singularities. For $n \leq 5$ this curve splits off a line, for $n = 6$ this curve is irreducible and admits a symmetry group of order 4 (V_4). The examples of degree five and six have been calculated with the massive use of computer algebra systems; calculations for degrees seven and eight involve huge computations with seventh roots of unity and therefore overstress sometimes the possibilities of such systems.

W.D. Geyer Reciprocity Laws

The symmetries of a number field are given by its Galois group G_K. The study of these groups G_K is closely connected with reciprocity laws. This first talk about this topic is concerned with the 1-dimensional representations of G_K, i.e. with the structure of the abelianised group $G_K^{ab} = G_K/G_K'$, not from a purely group theoretical view but together with its arithmetic structure which corresponds to the decomposition laws of primes in field extensions. The talk started with Fermat's theorem about primes being sum of 2 squares, the interpretation of Gauß as decomposition law in $\mathbb{Q}(\sqrt{-1})$, Euler's way to the quadratic reciprocity law with a side view to the local global principle of quadratic forms over fields resp. rings, the decomposition laws in quadratic extensions and cyclotomic extensions until Artin's reciprocity law.

A. Huckleberry Reduction of symmetries

Let (M,ω) be a symplectic manifold equipped with a Hamiltonian action of a Lie group G and a moment map $\Phi : M \longrightarrow Lie(G)^*$. If $M_0 := \Phi^{-1}(0)$, then we consider the Marsden-Weinstein reduction $\rho_{MW} : M_0 \longrightarrow M_0/G =: M_{red}$. This quotient may not in general be Hausdorff. So at first we consider the case were G acts properly.
The goal is to push down the symplectic structure to obtain a (singular) symplectic structure on M_{red}. If G is acting freely, then this is no problem. In fact, even if G has finite isotropy, M_0 is smooth and the quotient structure is relatively easy to

understand. By slice arguments the case of constant orbit-dimension can be reduced to that of finite isotropy. Finally, arguing the orbit-type stratification, Sjamaar and Lermann (Ann.of. Math.'91) produced a stratified symplectic structure on M_{red}. Our idea (H. with P.Heinzner and F.Loose, Crelle '94, [HHL]) is to obtain a globally defined quotient structure using methods of complex analysis.
For this we first recall the Hilbert quotient $\rho_H : X \longrightarrow X//G$ of an affine variety with respect to an algebraic action of a reductive group $G = K^{\mathbb{C}}$. This is defined by the invariant algebraic functions $\mathcal{O}_{alg}(X)^G =: R$, i.e. $X//G = Spec(R)$. It is also informative to regard this as being defined by the equivalence relation $x \sim y :\Leftrightarrow \overline{Gx} \cap \overline{Gy} \neq \emptyset$, i.e. the reduction $\rho_H : X \longrightarrow X//G$ is the maximal Hausdorff quotient.
In the complex analytic setting these results have been proved by geometric \ analytic methods, e.g. for K compact and X Stein $\rho_H : X \longrightarrow X//K$ exists with the desired properties. In fact there is an open Runge embedding $X \subset X^{\mathbb{C}}$ in a Stein space $X^{\mathbb{C}}$ where $G = K^{\mathbb{C}}$ acts and $X//K = X^{\mathbb{C}}//K^{\mathbb{C}}$ (Heinzner, Math.Ann.'91).
Let $\varphi : X \longrightarrow \mathbb{R}^{\geq 0}$ be a strictly pluri-subharmonic exhaustion and $\omega := dd^c\varphi$, which even makes sense for singular spaces. It can be shown that each fiber of the reduction $\rho_H : X \longrightarrow X//K$ intersects $\Phi^{-1}(0) = X_0$ in a K-orbit and the associated $K^{\mathbb{C}}$-orbit in the fiber $X^{\mathbb{C}}$ is the unique closed one. In particular, the inclusion $X \supset X_0$ induces a homeomorphism $X//K \cong X_{red} = X_0/K$. Furthermore φ can be pushed down to a strictly pluri-subharmonic function φ_{red} on the complex space X_{red}. Thus, in the complex analytic setting we have the quotient structure $\omega_{red} = dd^c\phi_{red}$.
To handle the original problem of the symplectic structure on M_{red}, given a symplectic manifold (M, τ), we construct a Stein-Kähler manifold (X, ω) with a totally real embedding $\iota : M \hookrightarrow X$ so that $\iota^*\omega = \tau$. For compact groups and in certain cases of proper actions this can be done in an equivariant fashion so that $\Phi_X|M = \Phi_M$. The natural embedding $\iota_{red} : M_{red} \longrightarrow X_{red} = X//K$ yields the the desired singular symplectic structure, i.e. $\tau_{red} := \iota_{red}^*\omega_{red}$.

K. Hulek On the symmetries of the Horrocks-Mumford bundle

The aim of this talk was to explain, how the (finite) Heisenberg group and related groups can be used to construct some interesting objects in algebraic geometry.
For $p \geq 3$ we consider $V = \mathbb{C}^p$ with its standard basis $\{e_i\}_{i \in \mathbb{Z}_p}$. The automorphisms

$$\sigma : e_i \mapsto e_{i-1}, \quad \tau : e_i \mapsto \varepsilon^i e_i \ (\varepsilon = e^{2\pi i/p})$$

have order p. They generate a subgroup H_p of $SL(p, \mathbb{C})$ of order p^3 which is a central extension

$$\begin{array}{ccccc} 1 \longrightarrow \mu_p = \{\varepsilon^i; i \in \mathbb{Z}_p\} & \longrightarrow & H_p \longrightarrow \mathbb{Z}_p \times \mathbb{Z}_p \longrightarrow 0 \\ \varepsilon & \mapsto & \varepsilon \cdot id_V = [\sigma, \tau] \end{array}$$

H_p is the Heisenberg group of level p (in its Schrödinger representation). Let N_p be the normaliser of H_p in $SL(p, \mathbb{C})$. Then

$$N_p = H_p \rtimes \mathrm{SL}(2, \mathbb{Z}_p)$$

and hence $|N_p| = p^4(p^2-1)$. The role of this group was explained in the following examples:

(1) Given an elliptic curve E one can construct an embedding of E into $\mathbb{P}(V) = \mathbb{P}^{p-1}$ as a normal curve of degree p which is H_p-invariant. H_5 acts on E by translation with p-torsion points. Using representation theory of H_p the (quadratic) equations of E can be determined. This was done explicitly for $p = 5$.

(2) Using H_5-equivariant maps $f^{\pm} : V \longrightarrow \Lambda^2 V$ one can construct the Horrocks-Mumford bundle F on $\mathbb{P}^4$ which is - up to obvious operations - still the only known indecomposable rank 2 bundle on $\mathbb{P}^4$. The bundle F has N_5 as its symmetry group. Its sections are closely related to moduli of abelian surfaces (partly joint work with Kahn and Weintraub).

(3) In case $p = 7$ the H_7-invariant abelian surfaces in $\mathbb{P}^6$ of degree 14 give rise to a Fano 3-fold X of index 1 and genus 12. Manolache and Schreyer have identified X with the variety of polar hexagons to the Klein quartic K.

(4) The quadrics through an elliptic curve of degree five in $\mathbb{P}^4$ define a Cremona transformation on $\mathbb{P}^4$. This can be used to construct (non-minimal) bielliptic surfaces of degree 15 in $\mathbb{P}^4$ (joint work with Aure, Decker, Popescu, Ranestad).

H. Karcher **Klein's (2,3,7)-surface: From the hyperbolic to its algebraic description**

The talk starts from Thurston's description of the surface model in front of the MSRI. My aim is to define two meromorphic functions in the hyperbolic picture and deduce the equation $w^7 = z(1-z)^2$ between them.

First, simpler examples of genus 2 and 3 platonic tessalations (automorphisms transitive on directed edges) were given and functions obtained by dividing through symmetries, with an obvious equation between them. The simplest ones are coverings of platonic tessalations of the sphere.

Klein's surface is tessalated by twentyfour 120°-heptagons. On any platonic surface the connection of any two midpoints of edges extends to a closed geodesic, on Klein's surface one has to extend the segment between nearest neighbours 8-fold. With these closed geodesics one has pair-of-pants-decompositions (not with right angles) in terms of which all the symmetry subgroups can be described. Division by an order 7 cyclic subgroup gives a projection (with three order 7 fixed points) to the sphere, which maps the geodesic $\frac{\pi}{7}$-triangle, with vertices at the fixed points, to a hemisphere. Normalize this map to send the three fixed points to $0, 1, \infty$ and call this function z. Another function w is obtained by mapping the same triangle to a spherical triangle with angles $\frac{\pi}{7}, \frac{2\pi}{7}, \frac{3\pi}{7}$. Fourteen of these cover the sphere three times and analytic continuation by reflection is compatible with the identifications, thus giving w. Clearly $w^7 = z(1-z)^2$.

P. Kramer   Concepts of symmetry for quasicrystals

The symmetry of crystals arises from orbits on $\mathbb{R}^3$ under translation and point groups. Quasicrystals lack translational symmetry but display quasiperiodicity together with non-crystallographic point symmetry. We survey and illustrate symmetry principles for quasicrystals.
(1) $\mathbb{Z}$-modules of rank $N > 3$ allow to extend point symmetry to quasicrystals and their Fourier transform, but are not sufficient to determine their spatial order.
(2) Quasiperiodic tilings generalise the spatial order associated with cells in crystals. New composition rules for these tilings encompass so-called local inflation and matching rules.
(3) Crystallography in dimension $N > 3$ provides lattices whose projections yields $\mathbb{Z}$-modules. Canonical tilings can be projected from faces of Voronoi and dual Delaunay domains of (root) lattices. Inflation rules for tiles are related to Non-Euclidean scaling symmetries of the lattice.
(4) In approaches going beyond N-dimensional crystallography, the commutative translation group is replaced by a free monoid or a free group. Substitution rules for monoids can generate quasiperiodic words. Finite and infinite order automorphisms of free groups generalise the notions of point and scaling symmetry. Notions of formal grammars and automata can handle the spatial realisation of these algebraic structures.

C. Maclachlan   Arithmetic Fuchsian and Kleinian groups

Arithmetic Fuchsian and Kleinian groups form a tractable and attractive class within the class of all Fuchsian and Kleinian groups and can readily be described in terms of quaternion algebras. Each such group Γ comes equipped with number theoretic data which gives additional information on Γ and on the geometry and topology of the quotient orbifold $H^n/\Gamma, n = 2, 3$. For example, the compactness of H^n/Γ, the volume of H^n/Γ, the existence of spherical subgroups and the number of conjugacy classes of elements of finite order can all be decided or determined from the arithmetical data. The existence of non-elementary Fuchsian subgroups in an arithmetic Kleinian group, or equivalently, the existence of immersed totally geodesic surfaces in an arithmetic manifold H^3/Γ, occurs precisely when the defining quaternion algebra A defined over k satisfies $[k : k \cap \mathbb{R}] = 2$ and $A \cong B \otimes_{k\cap\mathbb{R}} k$ where B is a quaternion algebra defining a Fuchsian (sub)group.
Given a Kleinian group Γ of finite covolume, the trace field is a number field $k(\Gamma)$ and one can obtain a quaternion algebra $A(\Gamma)$, in $M_2(\mathbb{C})$, as finite sums of the group elements over k. Arithmeticity can then be detected if the field $k(\Gamma^{(2)})$ and the quaternion algebra $A(\Gamma^{(2)})$ satisfy some "finite determinable" conditions. In general, this quaternion algebra is an invariant of the commensurability class and it may be that quaternion algebras have a wider role to play in the general theory of Kleinian groups of finite covolume and hyperbolic 3-manifolds.

H. Nicolai **Kac Moody Algebras and String Theory**

This contribution summarises recent attempts to understand the root spaces of Kac Moody algebras of hyperbolic type, and in particular the "maximally extended" Kac Moody algebra E_{10}, in terms of a DDF construction appropriate to a subcritical compactified bosanic string. While the level-one root spaces can be completely charactetised in terms of transversal states (the level-zero elements just span the affine subalgebra), longitudinal DDF states are shown to appear beyond level one. The utility of the method is demonstrated by constructing an explicit and comparatively simple representation for certain level-two root spaces.
The embedding of the Kac Moody algebra into a larger Lie algebra of physical states (which is simpler to handle, at least from a physicist's point of view) is also explained. Finally it is pointed out that the structure constants of such Kac Moody algebras can be viewed as S-matrix elements connecting physical string states.

S.J. Patterson **Asymptotics of groups**

The purpose of this talk was to discuss how group structures lead to strong asymptotic results on various counting functions. The primary example is the circle problem, where all of the deeper results rely on the Voronoi summation formulae and these reflect the representation theory of the group of Euclidean motions of the plane. Following this one considers the action of discrete groups of motions of the hyperbolic plane. Here the simplest result is the analogue of the circle problem. It can be used, for example, in connection with the modular group to estimate the asymptotics of

$$\{(a,b,c) \in \mathbb{Z}^3 \mid \mathrm{b}^2 - 4\mathrm{ac} = -\mathrm{D}, 0 < \mathrm{a}+\mathrm{c} \leq \mathrm{x}\}$$

($D > 0$ fixed) as $x \to \infty$. This represents binary quadratic forms of discriminant $-D$. The analogous problem for positive disrciminants is more subtle as the corresponding homogeneous space is not Riemannian. One can estimate the asymptotics of

$$\{(a,b,c) \in \mathbb{Z}^3 \mid \mathrm{b}^4 - 4\mathrm{ac} = \mathrm{D}, |\mathrm{az}^2 + \mathrm{bz} + \mathrm{c}| \leq \mathrm{x}\}$$

where $z \in \mathbb{C} - \mathbb{R}$ and $D > 0$ are fixed and $x \to \infty$. Finally a brief discussion of the corresponding problem for integral binary cubics was given; thus, for example, it is possible to estimate the asymptotics of

$$\{(a,b,c,d) \in \mathbb{Z}^4 \mid -27\mathrm{a}^2\mathrm{d}^2 + 18\mathrm{abcd} + \mathrm{b}^2\mathrm{c}^2 - 4\mathrm{ac}^3 - 4\mathrm{b}^3\mathrm{d} = \mathrm{D}, |\mathrm{a}| + |\mathrm{b}| + |\mathrm{c}| + |\mathrm{d}| \leq \mathrm{x}\}$$

for fixed D as $x \to \infty$. These results are, in their most recent forms, contained in papers by W. Duke, Z. Rudnik and P. Sarnak, and of A. Eskin and C. McMullan (both in Vol 71 of the Duke Mathematical Journal), and of A. Eskin, S. Mozes and N. Shah (unpublished). The methods are by no means limited to arithmetic groups but can be applied to wide classes of discontinous groups.

W. Plesken Finite unimodular groups and Bravais manifolds

k-modular lattices with good properties can be obtained by using finite unimodular groups. The cone $\mathcal{F}_{\mathbb{R}}$ of positive definite forms inside the space of all G-invariant quadratic forms of a finite unimodular group G is called the Bravais manifold of G. The normaliser $N_{\mathbb{Z}}$ of G in the full unimodular group acts properly discontiniously on $\mathcal{F}_{\mathbb{R}}$. The investigation of a fundamental domain leads to nice quadratic forms and also to sub-Bravais manifolds, i.e. finite unimodular groups containing G. In case $\mathcal{F}_{\mathbb{R}}$ is 2-dimensional or the commuting algebra of G in $\mathbb{R}^{n\times n}$ is a 2×2-matrix ring over $\mathbb{R}$ or $\mathbb{C}$, $\mathcal{F}_{\mathbb{R}}$ carries a hyperbolic structure respected by $N_{\mathbb{Z}}$. In this case each element of $\mathcal{F}_{\mathbb{R}} \cap \mathbb{Z}^{n\times n}$ represents a modular lattice, once one of the generic lattices in $\mathcal{F}_{\mathbb{R}} \cap \mathbb{Z}^{n\times n}$ is modular. Examples are discussed where $N_{\mathbb{Z}}/G$ acts as a hyperbolic reflection group on $\mathcal{F}_{\mathbb{R}}$. These examples involve unimodular lattices like E_8 and the Leech lattice, the 2-modular Barnes-Wall lattice of dimension 16 and the 3-modular Coxeter-Todd lattice of dimension 12, but give many other modular lattices at the same time.

W. Schempp Symmetries govern Magnetic Resonance Imaging (MRI)

Magnetic resonance imaging (MRI) was introduced into clinical medicine in 1981, and in the short time since then it has assumed a role of unparalleled importance in diagnostic medicine and cognitive neuroscience. It is the most important imaging advance since the introduction of X-rays by Wilhelm Conrad Röntgen in 1895. It is shown that the MRI modality is governed by the symmetries of the non-split central group extension

$$\mathbf{R} \triangleleft G \longrightarrow \mathbf{R} \oplus \mathbf{R}$$

where the normal subgroup $\mathbf{R}$ is isomorphic to the one-dimensional center of the Heisenberg nilpotent Lie group G. The symmetries find their manifestation in the coadjoint orbit model $Lie(G)^\star/G$ of the unitary dual $\hat{G}$ of the Heisenberg group G.

E. Scholz: Shifts of the concept of symmetry during the 19th century

For a historical understanding of the notion of "symmetry" we ought to distinguish between
– symmetrical practices (e.g. in mathematics pre-1800: Lagrange 1771, theory of algebraic equations; in crystallography: Romé de l'Isle, modification series of crystal forms)
– explicit conceptual use (e.g. ca. 1800 in mathematics: reflection symmetry only; architecture: rule governing relations between parts and the whole)
– metaphorical use (e.g. ca. 1800 architecture: origin of beauty, rejected in other arts)
The dual pair of categories "structure" and "symmetry" was introduced in crystallography during the turn from natural history to a physical science. R.J. Haüy characterised structure by a geometrized atomistic theory (18 classes of convex polyhedra serving as "hypothetical cores" from which crystal figures are derived by building up

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layers of "subtractive molecules", and symmetry by a morphological description of equality with respect to a hypothetical core). A group of crystallographers pursuing a dynamistic programme of matter explanation (C.S. Weiss, J.F.C. Hessel, M.L. Frankenheim, J.G. Grassmann) invented and developed vectorial structures and symmetry studies with their own symbolical expressions. That lead to a complete classification of finite symmetry systems in Euclidean space (Hessel) and a permutation representation of certain symmetry constellations (holoedrical symmetries of 6 crystal systems) by J.G. Grassmann (father of Hermann G.).
About 1850 A. Bravais modified the atomistic paradigm of crystal structure to that of polyhedral molecules distributed over a regular point lattice. In that context he studied the symmetries of polyhedra (finite) of point lattices (infinite) and of molecular lattices. He characterised thus the symmetry of most of the later symmorphic space group types (71 of 73) C. Jordan took this as the starting point for his 1869 "Mémoire sur les groupes de mouvement" with the first explicit use of the group concept in geometry.
The further influence of Bravais-Jordan on the formulation of transformation groups by Lie and Klein as well as on Sohnke-Fedorov-Schoenflies was shortly outlined. A last question was posed, not answered, whether the surprising rise of semantical "resonance" or symbolical "homology" (in the sense of natural history) in different fields of knowledge like algebra/group theory and crystallography might find a historical explanation in considering general rules underlying the discourses which governed the shifts in knowledge production from 18th to 19th century mathematics and natural history/crystallography respectively.

J. Schwermer **Another look at reciprocity laws**

Let E/F be a Galois extension of an algebraic number field; given an irreducible finite dimensional representation $\rho : Gal(E/F) \longrightarrow GL(V)$, $dim_{\mathbb{C}} V = n$, the non-abelian Artin L-functions $L_{E/F}(s, \rho)$ are known to be meromorphic in the whole complex plane and to satisfy a functional equation similar to that of the Zeta function but little is known about their poles. In the abelian case (i.e. $Gal(E/F)$ abelian, thus ρ is one dimensional) it is known that these L-functions are entire (for $\rho \neq \mathbf{1}$); this may be viewed as a reinterpretation of Artin's reciprocity law. It seems to be expected that the Artin L-functions are entire in the general case. One way that has been suggested by R.P. Langlands to show this is to show that the Artin L-functions are equal to L-functions attached to automorphic forms. The partial solution in the case $n = 2$ as given by Langlands (using Hecke's theory of modular forms) was indicated, and some ideas of the general programme were outlined.

D. Singerman **Symmetric Riemann surfaces and real curves**

A Riemann surface X is said to be symmetric if it admits an anticonformal involution $T : X \longrightarrow X$. Whereas a compact Riemann surface represents an irreducible algebraic function $F(z, w) = 0$ (connected complex curve), a symmetric compact Riemann

surface represents a real curve, i.e. the coefficients of f can be chosen to be real. The fixed point set of T consists of $0 \leq k \leq g+1$ disjoint simple closed curves or mirrors as we call them. We give a survey of many recent results, often first proved by S.M. Natanzon, concerning the mirrors of symmetries.
(1) If T_1 and T_2 are symmetries of X (genus ≥ 2) both with $g+1$ mirrors then T_1 and T_2 commute and T_1T_2 is the hyperelliptic involution. Moreover, no compact Riemann surface can admit more than 2 symmetries with $g+1$ mirrors.
(2) If T_1 and T_2 do not commute, so that the order of T_1T_2 is $n > 2$ then the total number of mirrors of T_1 and T_2 is less than or equal to $\frac{4g}{n}+2$, if n is even and less than or equal to $\frac{2g-2}{n}+4$ if n is odd. Both bounds are attained.
(3) We also discussed the maximum number of mirrors of $m > 2$ commuting symmetries.
Another interesting result concerns the number of conjugacy classes of symmetries (conjugacy in $Aut\,X$, the group of holomorphic and antiholomorphic automorphisms of X). This is the number of real models of the complex curve. This number is $\leq 2(\sqrt{g}+1)$, and this bound is sharp. The talk ended with other examples of symmetric surfaces such as Klein's curve of genus 3 and the modular curves $X(n)$.

J. Tits The Monster group as a group of symmetries

The main purpose of the lecture was to construct, from scratch, a 196883-dimensional $\mathbb{Q}$-vector space X and a symmetric bilinear form $\beta : X \times X \longrightarrow \mathbb{Q}$ such that $O(\beta) = Aut(X,\beta)$ is the Fischer-Griess Monster group M. From that readily follows a description of M as the symmetry group of a (pointed) Euclidean lattice or, equivalently, as the symmetry group of a convex polytope in the 196883-dimensional Eucledian space. The proof of the main result heavily relies on:

R.L. Griess, Jr., The friendly giant, Inv. Math. 69 (1982), 1-102,

J. Tits, On R.Griess' "Friendly giant", Inv.Math. 78 (1984), 491-499,

and uses (a small and save part) of the classification of finite simple groups.

H.R. Trebin Topological Defects in Quasiperiodic Tilings

Quasiperiodic tilings model point-atom-structures of quasicrystalline intermetallic compounds, which have been discovered exactly ten years ago. The tilings can be represented as planar cuts through higher-dimensional periodic crystals, whose "atoms" are polyhedra. If the polyhedra are connected by "steps" a topologically complicated hypersurface evolves with many branch-points. Circumventing these points with the cutting plane along closed loops results in permutations of the vertices of the tiling. With suitable choices of the loops vertices can be transported to infinity in a self-diffusive way. Thus quasicrystals are expected to exhibit extraordinary selfdiffusion mechanisms.
The geometrical background and the self-diffusion steps are illuminated for the octagonal Ammann-Beenker- and the icosahedral Ammann-Kramer-Penrose tilling.

J. Wess Quantum groups in physics

The quantum group $SL_q(2)$ and its Lie algebra version was discussed in detail to exhibit the structure of a quantum group. The Hopf algebra structure as well as the Poincaré Birkhoff Witt property was emphasised. The $\hat{R}$ matrix approach was put forward and the RRT relation as well as the Young Baxter equation were explained. As symmetric object the Manin plane as comodule was introduced and the differential calculus of this plane was developed. This opens the way to study physical systems which are based on inhomogeneous groups (Poincarë group) and following Wigner's definition, q-deformed one particle states and their wave equations can be studied. For non relativistic systems the differential calculus leads to a q-deformed Heisenberg algebra. Quantum mechanical systems based on this algebra can be studied. The remarkable consequence is that these models live on lattices.

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