Tagungsbericht 3/1995

Enumerative Combinatorics and the Symmetric Groups

15.01. bis 21.01.1995

The conference was organized by George E. Andrews (University Park), Christine Bessenrodt (Magdeburg) and Jørn B. Olsson (København). It was attended by 43 participants, coming from Austria, Belorussia, Canada, Denmark, France, Germany, Great Britain, Italy, Japan, Russia, Sweden and the USA. In 30 talks, a wide spectrum on the interface of algebra and combinatorics was covered; in a special evening session 3 reports were given on algorithmic aspects of related computational problems.

The main aspect of the meeting was to bring together mathematicians from algebra and combinatorics for a fruitful interaction on the overlap of these areas. This was achieved in focussing on topics in the theory of partitions and q-series, symmetric functions, the theory of Coxeter groups (in particular symmetric groups) and Hecke algebras and their representations, and combinatorial aspects of posets. Very recently, some long standing problems of major significance in these areas have been solved; reports on these achievements constituted some of the highlights of this week.
As computer algebra methods have become increasingly important for each of the areas represented at the conference, in a special session algorithms and newly available software packages were demonstrated.

There was also a special session dedicated to Dominique Foata followed by a festive evening. Perhaps the stimulating atmosphere of the institute and our affection for our honoured colleague was best described by the final speaker of the meeting, Richard Stanley, whose only overhead-slide read:

Friendly
Oberwolfach
Atmosphere
Towards
Algebraic combinatorics

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VORTRAGSAUSZÜGE

K. Alladi: Refinements and generalizations of partition conjectures of Capparelli arising from Lie algebras

Motivated by a study of vertex operators in the theory of Lie algebras, S. Capparelli made two similar Rogers-Ramanujan type partition conjectures in 1988 involving partitions with difference conditions on the one hand and partitions with congruence conditions on the other. In 1992 Andrews proved the first conjecture by the use of generating functions; the proof of the second conjecture is implicit in his method. Subsequently, in collaboration with Andrews and Gordon, I obtained generalizations as well as refinements of these partition theorems involving four free parameters by means of a new technique called "the method of weighted words". This approach yields combinatorial (bijective) as well as generating function proofs of the Capparelli conjectures, their generalizations and refinements. In addition, the method also yields several companion partition theorems. Recently, Lie theoretic proofs of the original Capparelli conjectures have been found but the refinements and generalizations we have obtained have not yet been realized through Lie algebras.

G. E. Andrews: Ramanujan, Partitions and Binary Quadratic Forms

Recently I have found two apparently independent projects of mine merging into a combined study. The first concerns four identities of Ramanujan; a typical example is

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1} q^{\binom{n+1}{2}}(1-q^n)}{(1+q^n)^2} = \left(\sum_{-\infty}^{\infty} (-q)^{n^2}\right)^2 \sum_{n=1}^{\infty} \frac{nq^{\binom{n+1}{2}}}{1-q^n}.$$

The second concerns q-series arising in the study of the transitive closure of acyclic digraphs (joint work with D. Crippa and K. Simon). The merger is due to the fact that each is related to questions concerning the class numbers of binary quadratic forms. This leads nicely to new relations between partitions and class numbers.

Ch. Barop: Projective matrix representations of S_n over $\mathbb{C}$

The foundation of the theory of projective representations - especially of S_n - was laid by I. Schur in his 1904/07/11 papers. Among others he constructed one irreducible projective matrix representation of S_n (Hauptdarstellung 2. Art). In the 60's A. Morris began to publish papers about this subject, e.g. he clarified why Clifford algebras are involved. In 1988 M. Nazarov announced a construction of a full system of irreducible projective matrix representations of S_n. In my M.Sc.-dissertation I tried to find a more direct approach to explain this construction. There is also a SYMMETRICA-routine calculating Nazarov's matrices. It is also possible to calculate them via fast fourier transformation in analogy to Clausen's algorithm for S_n. Another - theoretical - approach is via eigenvectors on a maximal commutative subalgebra (Gelfand-Zetlin-bases) using the CSCO-Method.

A. Björner: Affine permutations of "type A"

Denote by $\tilde{S}_n$ the group (under composition) of all bijections $a : \mathbb{Z} \to \mathbb{Z}$ such that $a(x) + n = a(x+n)$, $\forall x \in \mathbb{Z}$, and $a(1) + \ldots + a(n) = \binom{n+1}{2}$. With respect to the adjacent transpositions (mod n) $(i, i+1)$, $i = 1, \ldots, n$, this gives a realization of the affine Coxeter group $\tilde{A}_{n-1}$ (Lusztig, 1983). It has appeared also in the work of Shi (1986) and H. Eriksson (1994).
In this work we establish three combinatorial facts about $\tilde{S}_n$:

1. A bijection between $\tilde{S}_n^J$ (the minimal coset representatives modulo $S_n = \langle s_1, \ldots, s_{n-1} \rangle$) and the set of all partitions with less than $n-1$ parts. This gives an elementary bijective proof for Bott's (1956) formula $\prod_{i=1}^{n-1} \frac{1+q+\ldots+q^i}{1-q^i}$ for the length generating function of $\tilde{A}_{n-1}$.

2. A rule for comparing $a, b \in \tilde{S}_n$ in Bruhat order. For $a \in \tilde{S}_n$, $a_i = a(i)$, and $j \in \mathbb{Z}$, let $\varphi_j(a) = \sum_{i:a_i<j} \lceil \frac{i-a_i}{n} \rceil$. Then:

   $$a \le b \Leftrightarrow \varphi_j(a_1+n, \ldots, a_k+n, a_{k+1}, \ldots, a_n) \le \varphi_j(b_1+n, \ldots, b_k+n, b_{k+1}, \ldots, b_n)$$

   for all $0 \le k \le n-1$ and $\min\{a_i\} < j \le \max\{a_i\}$.
   For $a, b \in \tilde{S}_n^J$ it suffices to take $k = 0$ on the right hand side.

3. A rule for comparing $a, b \in \tilde{S}_n$ in left weak order. Define the inversion graph $I(a)$ of $a \in \tilde{S}_n$ as the directed multigraph on vertices $1, \ldots, n$ with an edge of multiplicity $| \lfloor \frac{a_j - a_i}{n} \rfloor |$ between i and j, for $i < j$, and directed from i to j if $a_i < a_j$ and from j to i otherwise. Then: $a \le b \Leftrightarrow I(a) \subseteq I(b)$.

Partial work on affine permutations of type $\tilde{C}_n$ was also mentioned.
(Joint work with F. Brenti.)

F. Brenti: Combinatorial properties of Kazhdan-Lusztig polynomials

We introduce a new family of polynomials, easily computable by simple recursions, into which any Kazhdan-Lusztig polynomial (of any Coxter group) can be expanded linearly, and we give a combinatorial interpretation to the coefficients in this expansion. This gives a combinatorial rule for computing the Kazhdan-Lusztig polynomials in terms of the enumeration of paths in a certain directed graph, and a completely combinatorial reformulation of the nonnegativity conjecture.

D. M. Bressoud: Some observations on the Borwein conjecture

Let

$$\prod_{i=0}^{n-1}(1-q^{3i+1})(1-q^{3i+2}) = A_n(q^3) - qB_n(q^3) - q^2C_n(q^3).$$

The Borwein conjecture states that for all n, A_n, B_n, and C_n have non-negative coefficients. We discuss the relationship between these polynomials and the generating functions for partitions with prescribed hook differences. In particular, if $\alpha_n(q)$ is the generating function for partitions λ with $\lambda_1, \lambda_1' \leq n$ and for every i such that $\lambda_i \geq i$ we have either $\lambda_i = \lambda_{i+1}$, or $\lambda_i \geq \lambda_i' > \lambda_{i+1}$, then $\alpha_n(1) = A_n(1) = 2 \cdot 3^{n-1}$ and $A_n(q) - \alpha_n(q)$ is a polynomial with "small" coefficients.

K. Erdmann: Dimensions of simple modules for the symmetric groups

Let K be an algebraically closed field of characteristic $p > 0$ and let D^λ be the simple module of the group algebra KS_r of the symmetric group, where λ is a p-regular partition of r. The dimensions of D^λ for λ with at most n parts are the same as the multiplicities of indecomposable direct summands of $E^{\otimes r}$ where E is the natural n-dimensional module for the group $GL_n(K)$. We determine generating functions for $\dim D^\lambda$, for all partitions λ with two parts, by applying some new results from the representation theory of $GL_n(K)$. The results are explicit rational functions.

S. Fomin: Noncommutative Schur functions

We develop (jointly with Curtis Greene) a theory of Schur functions in noncommuting variables, assuming certain commutation relations that are satisfied in many well-known examples, such as the plactic, nilplactic and nilCoxeter algebras and the degenerate Hecke algebra $H_n(0)$. As an application, we prove Schur-positivity and obtain generalized Littlewood-Richardson and Murnaghan-Nakayama rules for a large class of (ordinary) symmetric functions, including stable Schubert and Grothendieck polynomials.

F. Garvan: Cranks, t-cores and the combinatorics of partition congruences

We survey the known combinatorial interpretations (or cranks) of Ramanujan's partition congruences. In an earlier paper with Kim and Stanton we found cranks which combinatorially proved Ramanujan's partition congruences modulo 5, 7, 11 and 25. We extend these methods to find a crank which combinatorially explains and proves Ramanujan's partition congruence $p(49n + 47) \equiv 0 (\mathrm{mod}\ 49)$.

I. Goulden: The combinatorial relationship between nonseparable rooted planar maps and two stack sortable permutations

West conjectured and Zeilberger proved that the number of permutations of $1, ..., n$ that can be sorted with two passes through a stack (TSS permutations) is $2(3n)!/(n + 1)!(2n + 1)!$. This is precisely Tutte's formula for the number of nonseparable rooted planar (NS) maps with n+1 edges, but the combinatorial relationship between these two sets is not at all clear from Zeilberger's proof. Dulucq et al. have found a direct bijection between the NS maps and another class of permutations, which together with a sequence of nine further bijections between sets of permutations gives a bijection between TSS permutations and NS maps. In addition, they prove that their bijection identifies the numbers of vertices and degree of the root face of the map with the number of descents +2 and the number of right to left maxima +1, respectively, of the corresponding permutation, but no direct description of their bijection is apparent.

In this talk, a new bijection is given that preserves the above statistics in a straightforward manner. The TSS permutations are characterized by an associated lattice path, called the Raney path of the permutation since these paths were used by Raney in his combinatorial proof of Lagrange's implicit function theorem. Simple path bijections then lead directly to TSS bijections that are exactly analogous to Tutte's NS bijections, giving the required combinatorial relationship between TSS permutations and NS maps. This is joint work with Julian West.

D. M. Jackson: The genus series for maps

The genus series for maps is the counting series for the number of (rooted) maps on orientable and nonorientable surfaces with respect to the degrees of vertices, faces and number of edges. The series for orientable surfaces and locally orientable surfaces have expressions in terms of Schur functions and Zonal polynomials, and there is then a connexion with Jack symmetric functions. Both series have representations in terms of integrals and it is significant that these can be used, with some difficulty to determine counting series for subclasses of map, such as monopoles. For example, the result of Harer and Zagier is recoverable. If there is time I will mention the use of Pfaffians in connexion with monopoles on locally orientable surfaces. The genus series is related to the partition function in random matrix models in physics.

G. D. James: Some representations of Hecke algebras

We discuss some recent work, in collaboration with Andrew Mathas, on the decomposition matrices of Hecke algebras H which are associated with the symmetric groups S_n. The algebra H which we consider is defined over a field of characteristic zero, and the parameter q in its definition is set equal to -1. As a consequence, the decomposition matrix is a "first approximation" to the 2-modular decomposition matrix of S_n. The starting point of our investigation is a theorem which says that the first colums of the decomposition matrix has 1 opposite every hook partition whose 2-core is not $(2,1)$ and 0 opposite every other partition. We combine this result with a theorem which shows that all the columns can be calculated, using the Littlewood-Richardson Rule, when the 2-core is large. Our conclusions include a determination of all the rows of the decomposition matrix which are indexed by partitions into at most four parts.

W. Johnson: Polynomials of q-binomial type

A polynomial sequence $\{p_n(x)\}$ is said to be of binomial type if it satisfies

$$p_n(x+y) = \sum_{k=0}^{n} \binom{n}{k} p_k(x) p_{n-k}(y).$$

In the same way that the ordinary binomial theorem is a model for this definition, we take Schützenberger's noncommutative q-binomial theorem

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k}_q x^k y^{n-k},$$

where $yx = qxy$, as a model for a definition of q-binomial type. Thus we study polynomial sequences satisfying

$$p_n(x+y) = \sum_{k=0}^{n} \binom{n}{k}_q p_n(x) p_{n-k}(y) \quad \text{where} \quad yx = qxy$$

We can give a few general properties of such sequences (e.g. a characterization in terms of generating functions) and a few examples that have some combinatorial significance.

T. Józefiak: A recursive formula for Hall-Littlewood functions

In a recent article by P. Di Francesco, C. Itzykson and J.-B. Zuber, Polynomial averages in the Kontsevich model, Commun. Math. Phys. 151 (1993), 193-219, the authors proved Kontsevich's formula and Witten's conjecture in the intersection theory of the moduli space of punctured curves using a family of symmetric functions which they introduced in the paper by the following formula:

$$f_\nu(X) := 2^k(-1)^{|\nu|} \sum_I \prod_{\substack{p<q \\ p,q\in I}} \frac{x_p + x_q}{x_p - x_q} \prod_{\substack{i\in I \\ j\in I^c}} \frac{x_i + x_j}{x_i - x_j} \det(x_{i_p}^{\nu_q}),$$

where $X = \{x_1, \ldots, x_n\}$ is a set of variables, ν is a partition of length k, $k \leq n$, with distinct parts, $|\nu| = \sum_{i=1}^{k} \nu_i$, and the summation is over all k-element subsets $I = \{i_1 < \ldots < i_k\}$ of $\{1, \ldots, n\}$ with I^c being the complement of I in $\{1, \ldots, n\}$. It is apparent that the authors were not aware that the functions were, up to sign, so-called Q-functions defined at the beginning of the century by I. Schur in connection with spin representations of symmetric groups.
The aim of the talk was to bring up this relationship in the context of more general Hall-Littlewood (H-L) symmetric functions by using Macdonald's definition of H-L functions and by deriving a recursive formula for H-L functions which is equivalent to the original definition of D. E. Littlewood.

A. Kerber: Enumerative combinatorics and the symmetric groups

It was described how the first 7-designs were found. Here they are:

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0011100010100100110001101000010010000000101101100010111100000000100100101000011011101100101011 1000
1100011101011011001110010111101101111111010010011101000011111110110110101111001000100110101000111

The corresponding Kramer-Mesner matrix can be found in the paper by Magliveras/Leavitt in Computational Group Theory (Atkins ed., Acad. Press 1984). This matrix was evaluated along a subgroup ladder in S_n (implementation A. Betten), and the 0-1-vectors were found using an implementation of an improved LLL-algorithm (A. Wassermann).

S. Kerov: Big Young diagrams and long interlacing sequences

Let ω_n denote any one of Young diagrams (Y. d.) with n boxes, which has largest dimension (= the number of standard Y. tableaux), $\dim \omega_n = \max\limits_{\lambda \in Y_n} \dim \lambda$. It will be convenient to consider Y. d. as a piecewise linear function, $v = \lambda(u)$.
Theorem (K. & Vershik, 1985)

$$\lim_{n\to\infty} \frac{1}{\sqrt{n}} \omega_n(u\sqrt{n}) = \Omega(u) = \begin{cases} \frac{2}{\pi}(\arcsin \frac{u}{2} + \sqrt{4-u^2}) & , |u| \le 2 \\ |u| & , |u| \ge 2. \end{cases}$$

A similar result holds for typical Y. d. with respect to Plancherel measure of S_n given by $M_n(\lambda) = \dim^2 \lambda / n!$; $\lambda \in Y_n$. Consider now a pair of interlacing sequences, $x_1 < y_1 < x_2 < \ldots < x_{d-1} < y_{d-1} < x_d$, say, the roots of orthogonal polynomials $P_{d-1}(z), P_d(z)$. It can be uniquely represented by a Y. d.-looking piecewise linear function $v = \omega_d(u)$ with derivative $\omega_d'(u) = \pm 1$, the minima points at $x_1, \ldots, x_d$ and the maxima at $y_1, \ldots, y_{d-1}$. Surprisingly, the limiting shape for ω_d, under mild assumptions, also exists and coincides with the same function Ω.

Theorem (K., 1993) Denote by c_n^2, b_n the coefficients of recurrence relation for polynomials $P_n(z)$:

$$P_{n+1}(z) + (b_n - z)P_n(z) + c_n^2 P_{n-1}(z) = 0.$$

Assume that $\lim \frac{c_{n-1}}{c_n} = 1$ and $\lim \frac{b_n - b_{n-1}}{c_n} = 0$. Then

$$\lim_{n\to\infty} \frac{1}{c_n} \omega_n(c_n x + b_n) = \Omega(x), x \in \mathbb{R}.$$

The same shape arises in many other contexts, too. The fact is partially explained by recent "free probability theory" by D. Voiculescu.

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A. S. Kleshchev: Branching rules for modular representations of symmetric groups and their applications to representation theory, cohomologies, and problem of Mullineux

Let K be a field of characteristic $p > 0, \Sigma_n$ the symmetric group on n letters, D^λ the irreducible $K\Sigma_n$-module corresponding to a (p-regular) partition λ of n. The main object of our interest is the restriction, $D^\lambda|_{\Sigma_{n-1}}$, of D^λ to the natural subgroup $\Sigma_{n-1} < \Sigma_n$. We obtain various results about $D^\lambda|_{\Sigma_{n-1}}$ which can be considered as characteristic-free versions of the classical Branching Theorem. These results turn out to be useful for many other problems about symmetric groups because they provide a tool for using induction.
As one of the applications we propose a combinatorial algorithm for description of the bijection b on the set of p-regular partitions of n defined from

$$D^\lambda \otimes sgn \cong D^{b(\lambda)},$$

where sgn is the 1-dimensional sign representation of Σ_n.

K. Koike: A Hecke algebra of $(\mathbb{Z}/r\mathbb{Z}) \wr \mathfrak{S}_n$ and construction of its irreducible representations

In this talk, we define a "Hecke algebra" $\mathfrak{H}_{n,r}$ of $G_{n,r} = (\mathbb{Z}/r\mathbb{Z}) \wr \mathfrak{S}_n$ (the wreath product of $(\mathbb{Z}/r\mathbb{Z})$ with $\mathfrak{S}_n$) and show that this "Hecke algebra" has appropriate properties as deformation of the group algebra of $G_{n,r}$. Namely $\mathfrak{H}_{n,r}$ is a free module over $A_0 = \mathbb{Z}[q, q^{-1}, u_1, u_2, \ldots, u_r]$ of rank $n!r^n$ and for suitable values of parameters n and $r, \mathfrak{H}_{n,r}$ are isomorphic to Iwahori Hecke algebras of type A_n and B_n. All the irreducible representations of $\mathfrak{H}_{n,r}$ are naturally parametrized by r-tuples $\alpha = (\alpha^{(1)}, \alpha^{(2)}, \ldots, \alpha^{(r)})$ of Young diagrams of total size n and each irreducible representation space of $\mathfrak{H}_{n,r}$ is realized on a vector space spanned by the standard Young tableaux $\mathbb{T}$'s of shape α. We describe the above irreducible representation by giving a set of representation matrices of the generators of $\mathfrak{H}_{n,r}$, which are the natural generalization of Young's seminormal forms. Also we give explicit description of the center of $\mathfrak{H}_{n,r}$. This work is a collaboration with Ariki and appeared in Adv. in Math. Vol 106 (1994) 216-243.

Ch. Krattenthaler: HYP and HYPQ - Mathematica packages for handling hypergeometric and basic hypergeometric series

Hypergeometric and basic hypergeometric series (q-series) are of great importance in areas like special functions theory, combinatorics, probability theory, representation theory, computer science, physics. For combinatorics, probability theory, and computer science this comes from the fact that (almost) all binomial sums can be written as hypergeometric series, and (almost) all q-binomial sums can be written as basic hypergeometric series. For special functions, representation theory, physics this comes from the fact that many important special functions and orthogonal polynomials are hypergeometric or basic hypergeometric series. For a non-expert the problem about (basic) hypergeometric series is two-fold: Computations with (basic) hypergeometric series tend to be cumbersome, and because of lack of comprehensive tables it is difficult to find identities that one might need. My Mathematica packages HYP and HYPQ make hypergeometric and basic hypergeometric series accessible to the non-expert. They allow the user to: (A) convert (q-)binomial sums into (basic) hypergeometric notation, (B) manipulate (basic) hypergeometric expression, (C) find and apply applicable transformation formulas, (D) find and apply applicable transformation formulas, (E) apply contiguous relations, (F) do formal limits of (basic) hypergeometric expressions, (G) use Gosper's and Zeilberger's algorithms, (H) transform (basic) hypergeometric expressions into TEX-code, and provide the user with the largest list of identities that is currently available in one spot. The packages are available by anonymous ftp at pap.univie.ac.at (type cd math, cd hyp_hypq after having logged in).

B. Leclerc: Kostka-Foulkes polynomials and crystal graphs of type A_n

Kostka-Foulkes polynomials are q-analogues of the weight multiplicities in the irreducible $sl(n,\mathbb{C})$-modules. They are defined by means of the expansion

$$s_\lambda = \sum_\mu K_{\lambda\mu}(q) P_\mu(q)$$

where s_λ and $P_\mu(q)$ are respectively the Schur and Hall-Littlewood functions. Kashiwara has attached to the irreducible $U_q(sl_n)$-module V_λ a crystal basis and a crystal graph G_λ describing the action of certain renormalized lowering operators on the crystal basis.

The aim of the talk is to present a combinatorial description of the $K_{\lambda\mu}(q)$ in terms of the geometry of the graph G_λ. This leads to $n-1$ variables refinements of the q-multiplicities of the "rectangular" weights $\mu = (k^n)$, which are the generating functions of Kostant generalized exponents for the $sl(n,\mathbb{C})$ simple modules. (Joint work with A. Lascoux and J.-Y. Thibon).

S. Martin: The infinitesimal Schur algebras

One approach in studying the representation theory of $G = GL_n$ in characteristic p is to look at representations of the 'thickened' group schemes G_rT associated with the r'th Frobenius subgroup of G. A second approach is to exploit the interplay between polynomial representations of G and rational representations of the monoid M of matrices. This reduces matters to studying the classical Schur algebras $S(n,d)$. In joint work with Doty and Nakano we try to study the scheme M_rD (D the diagonals in M) and set up a polynomial representation theory of so-called infinitesimal Schur algebras, $S(n,d)_r$. These algebras, constructed by truncation of the coordinate ring of M, have very interesting homological properties. Their character theory may be relevant to computing the modular irreducible characters of G.

M. Nazarov: Classical dual pairs and affine Hecke algebras

Let G be one of the classical groups $GL(N)$, $O(N)$, $Sp(N)$ acting on the vector space $U = \mathbb{C}^N$. The question how the n-th tensor power of the representation U decomposes into irreducible summands amounts to studying the centralizer $C(n)$ in End $(U)^{\otimes n}$ of the image of G. By the definition of the algebra $C(n)$ we have the chain of subalgebras $C(1) \subset C(2) \subset \ldots \subset C(n)$. There is a canonical orthogonal basis in every irreducible representation of the algebra $C(n)$ associated to this chain. For the group $GL(N)$ the centralizer $C(n)$ is generated by the action of the symmetric group $S(n)$ in $U^{\otimes n}$. The action of $S(n)$ in the canonical basis was described by Alfred Young in 1931. The aim of the talk is to describe the action of the other two centralizer algebras in the canonical basis. This description implies formulas for the dimensions of irreducible representations of the other two classical groups. An object larger than the group $S(n)$ may be recognized in the construction of Young. This is the affine degenerate Hecke algebra $He(n)$ which originates from the representation theory of the group $GL(n)$ over a p-adic field. It will be explained in the talk what plays the role of $He(n)$ for each of the other two centralizer algebras.

S. Okada: Minor-summation formulas and their applications

In this talk, I will give several applications of the following minor summation formula of Pfaffian. Let n be an even integer. For an arbitrary $n \times p$ matrix

$T = (t_{i,j})_{1\leq i\leq n, 1\leq j\leq p}$ and a $p\times p$ skew symmetric matrix $A = (a_{i,j})_{1\leq i,j\leq p}$, we have

$$\sum_{1\leq i_1<\ldots<i_n\leq p} \mathrm{Pf}\,(a_{i_k,i_l})_{1\leq k,l\leq n} det(t_{k,i_l})_{1\leq k,l\leq n} = \mathrm{Pf}(TA^tT),$$

where Pf B is the Pfaffian of a skew-symmetric matrix B. As an application of this formula, we can prove the Littlewood's formula and their variations. Also we can give the irreducible decompositions of some restrictions and tensor products for "rectangular-shaped representations" of classical groups.

K. Ono: Some partition theorems

In this lecture we discuss new results regarding the arithmetic nature of various standard partition functions. First we mention new results regarding the parity of the ordinary partition function $p(n)$. We show that $p(n)$ is even infinitely often in every arithmetic progression and $p(n)$ is odd in an arithmetic progression provided that it is odd once. We also present a proof of the t-core partition conjecture if $c_t(n)$ is the number of t-core partitions of n, then we show for $t \geq 4$ that $c_t(n) > 0$ for all n. This implies that for primes $p \geq 5$ every symmetric group and alternating group possesses at least one defect 0 p-block. We also consider $b_p(n)$, the number of p-regular partitions of n. For every positive integer k and every prime p, we show that $b_p(n) \equiv 0 \bmod p^k$ for almost all n.

P. Paule: Algorithms for q-identities - recent progress

Recent progress concerning the algorithmic treatment of q-hypergeometric sums is discussed. Based on a "discrete" version of square-free-factorization, a natural algebraic approach to a q-analogue of Gosper's algorithm is presented. This analogue led to an efficient Mathematica implementation of a q-analogue of Zeilberger's "fast" algorithm for definite q-hypergeometric summation. The applications include, for instance, a simple "four-line" computer proof of the celebrated Rogers-Ramanujan identities, as well as several results obtained by qWZ dualization.

R. A. Proctor: A New Lie Theoretic Subject Contained Entirely in the Category of Combinatorics?

Last year we combinatorialized some special cases of some representation theoretic basis results of Lakshmibai and Seshadri. (Their general work was recently used by Littelmann to completely solve the century-old tensor product problem for "all" Lie algebras.) The central objects in this new combinatorial treatment are certain posets which we call "d-complete". A poset possesses an LS type basis if and only if it is d-complete. We can classify all possible d-complete posets using Dynkin diagrams. Our proofs of these results are entirely combinatorial. It now appears that this class of posets may provide the answer to two pre-existing purely combinatorial problems, as is described below. If the conjectures below can also be proved using only combinatorial techniques, then it could be said that these results comprise a mathematical subject of the semisimple Lie kind which exists entirely in the category of combinatorics. Consider the decomposition of a poset P into a filter F and an ideal I. Fix a natural numbering of the elements of I. Choose a natural ordering of the elements of F, and successively "slide out" the empty locations of F according to Schützenberger. If the same result is obtained for each ordering of F for all decompositions P into such an F and an I and all extensions of I, we say that P is a jeu de taquin poset. The classification of jeu de taquin posets has been regarded by some as being intractable. Empirical evidence indicates that all d-complete posets should be jeu de tacquin posets. In his thesis Stanley defined a generating function $U(P;x)$ for the p-partitions on a poset P. He found a special family of posets for which $U(P;x)'s$ had a strikingly beautiful form. Gansner (and Sagan) found another family of "hook length" posets around 1979. Empirical evidence indicates that all d-complete posets should be hook length posets.

B. Sagan: Coxeter subspace arrangements and characteristic polynomials

Andrews Blass and I show how the characteristic polynomial of a Coxeter subspace arrangement can be interpreted as an Ehrhart quasi-polynomial of an associated polytope. This method can be used to show that such polynomials factor partially over $\mathbb{Z}^+$ and have nonnegative coefficients when expanded in a suitable basis for the polynomial ring. I will also mention several related topics: a generalization of this idea to symmetric functions, a new way to calculate and combinatorially explain the Möbius function of an arbitrary lattice, and topological considerations.

Th. Scharf: Non-commutative cyclic characters

(joint work with B. Leclerc, J.-Y. Thibon (Paris)).

Fix $n \in \mathbb{N}^*$, $\varepsilon := exp(2\pi i/n)$ and $g \in S_n$, an n-cycle. Denote by $X_n^{(k)}$ the irreducible complex character of $< g >$ defined by $X_n^{(k)}(g) := \varepsilon^k$. By a result of Kraskiewicz and Weyman the Frobenius characteristic $l_n^{(k)}$ of $\mathrm{Ind}_{<g>}^{S_n}(X_n^{(k)})$ can be decomposed into ribbon Schur functions:

$$l_n^{(k)} = \sum_{\substack{maj(I)\equiv k \\ \mathrm{mod}\, n}} r_I$$

This suggests an analogue in the algebra of non-commutative symmetric functions (NCSF) in the sense of [Gelfand/Krob/Lascoux/Leclerc/Thibon]:

$$L_n^{(k)} = \sum_{\substack{maj(I)\equiv k \\ mod\, n}} R_I.$$

NCSF can be equipped with an internal product "$*$" which is an analogue of the inner tensor product of representations [GKLLT]. Then

$$L_n^{(k)} * L_n^{(l)} = \sum_{m=0}^{n-1} < l_n^{(k)}, l_n^{(m-l)} > L_n^{(m)}.$$

We note two consequences:
(i) $\mathrm{Ind}_{<g>}^{S_n}(X_n^{(k)}) \otimes \mathrm{Ind}_{<g>}^{S_n}(X_n^{(l)}) = \sum_m < l_n^{(k)}, l_r^{(m-l)} > \mathrm{Ind}_{<g>}^{S_n}(X_n^{(m)})$

(ii) For any $\pi \in S_n$ with $maj(\pi) \equiv m\,(\mathrm{mod}\ n)$, the number of $(\sigma, \tau) \in S_n \times S_n$ s.t. $\pi = \sigma\tau$ and $maj(\sigma) \equiv l\,(\mathrm{mod}\ n)$; $maj(\tau) \equiv k$.

R. Stanley: Graph colorings and symmetric functions

Given a finite graph G, define a symmetric function

$$X_G = \sum_{\kappa: V \to \mathbb{P}} x_{\kappa(v_1)} \cdots x_{\kappa(v_d)},$$

summed over all proper colorings κ of the vertex set $V = \{v_1, \ldots, v_d\}$ of G with positive integers. If we set $x_1 = x_2 = \ldots = x_n = 1$, $x_{n+1} = x_{n+2} = \ldots = 0$, then we obtain $\chi_G(n)$, the chromatic polynomial of G evaluated at n. Hence X_G is a natural generalization of χ_G.

There are many interesting problems involved with expanding χ_G in terms of various bases for the ring of symmetric functions, in particular the bases $m_\lambda, p_\lambda, s_\lambda, e_\lambda$. For instance, if G is the incomparability graph of a poset with no induced subposet isomorphic to  , then it is conjectured that the expansion of χ_G in terms of the elementary symmetric functions e_λ has nonnegative coefficients. Gasharov has shown that X_G is at least Schur-positive. From this one can deduce, for instance, that if c_i is the number of i-element chains in a poset P with not induced  , then all zeros of the polynomial $\sum c_i x^i$ are real.

J. Stembridge: Enriched *P*-partitions

Enriched P-partitions are a generalization to posets of the tableaux for which Schur's Q-functions are the generating functions. They bear the same relationship to Q-functions as "ordinary" P-partitions bear to Schur S-functions. Moreover, nearly every aspect of the theory of ordinary P-partitions has an enriched counterpart. We plant to summarize the highlights of this theory, and discuss some applications to reduced expressions in Coxeter groups, as well as some open problems.

V. Strehl: Transforming Recurrences

There are several reasons (better understanding and circumventing inefficiencies in the multisum Zeilberger algorithm, intriguing examples from combinatorics, special functions and number theory) to consider the following problem: given a holonomic sequence $a = (a_n)_{n \geq 0}$, i. e. a sequence annihilated by a linear difference operator G with polynomial coefficients, and a linear transformation P, what can be said about operators H (order?, degree of coefficients?) annihilating the transformed sequence Pa, the case $H = P \cdot G \cdot P^{-1}$ being of particular interest. It turns out that in the case where P is of "Sheffer type", the transformation $G \mapsto P \cdot G \cdot P^{-1}$ has particularly nice properties which can be profitably used in implementations.

A. Vershik: Vector partitions and limit shapes

The vector partitions appeared in combinatorial problems, geometry, number theory. We will speak about new application of this and about the connexion with statistical physics.

D. Zeilberger FOaTA on one FOOT

Dominique Foata's great contributions to combinatorics, sofar, were described. In particular the Cartier-Foata commutation monoid and the revolution that lead to the combinatorial approach to special functions.

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