Tagungsbericht 4/1995

Numerical Methods for Singular Perturbations

22.–28.1.1995

The meeting was organized by P. Hemker (Amsterdam), H.-G. Roos (Dresden) and M. Stynes (Cork). Its topic, "Numerical methods for singular perturbations", had not previously been the subject of an Oberwolfach meeting. Yet many differential equations in the applied sciences – for example in fluid dynamics – are singularly perturbed and standard numerical methods often fail to provide satisfactory solutions to such problems. In the last 25 years, many different techniques have been developed to overcome the shortcomings of the standard methods. Despite the efforts made, our current knowledge is not at the level needed for effective application to practical problems such as nonlinear models in several space dimensions. Recent progress in the use of anisotropic and piecewise uniform grids, a deeper understanding of when fitted operator methods can be applied, and new results on adaptive approaches for singularly perturbed problems galvanized the organizers into bringing together scientists from these areas.

The aims of the meeting were:

1. The dissemination of information about current research and open questions in the numerical and asymptotic analysis of singular perturbation problems;

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2. The evaluation of "optimal" methods for the solution of such problems;

3. An examination of promising future research directions in this area.

In all, 30 lectures were given. They covered the key aspects in the discretization and numerical analysis of singular perturbation problems. Most of the research tools that are effective for these problems were addressed by participants at the meeting; the variety of techniques represented meant that aim (1) above was quite successfully achieved. For aims (2) and (3), it was very important that the schedule left ample time for discussion both for long periods during the day and after lectures. This enabled individuals with different points of view and different approaches to meet and profit from each other's experience. The warm and stimulating atmosphere of the Forschungsinstitut is also a stimulating factor in fostering such mixing. The week's workshop laid the foundation for several future collaborations between different groups and induviduals, which testifies to the attainment of aims (2) and (3).
All participants agreed enthusiastically that the workshop was extremely useful, that contact between different groups should be maintained, and that when sufficient further development of the area has taken place (in, say, three years) a meeting with the same topic should if possible be organized.
The alphabetical list of contributions below can be divided under the following headings: Operator fitted uniformly convergent methods; anisotropic and piecewise uniform meshes; defect correction; stabilized finite element, finite volume and finite difference methods; adaptive methods; methods for shock layers; numerical methods for the Navier-Stokes equations at high Reynolds numbers.

Vortragsauszüge

D. ADAM:

Nonconforming Uniformly Convergent Finite Element Methods for Singularly Perturbed Elliptic Problems in Two Dimensions

A new analysis of a noncnforming Galerkin finite element method for solving linear elliptic singularly perturbed boundary value problems on rectangular domains is given. In the case of ordinary boundary layers the method is shown to be convergent uniformly with respect to the perturbation parameter of order $h^{1/2}$ in the energy norm. The trial functions are exponentials fitted to the differential operator.
Numerical tests confirm that the nonconforming method is numerically stable and the order of convergence obtained is optimal.
Besides this, the method yields satisfactory numerical results for the same kind

of problems on L-shaped domains as well.

L. ANGERMANN:

A Posteriori Error Estimates for Singularly Perturbed Elliptic Problems

The lecture presents a formal extension of Babuška&Rheinboldt's approach to the problem of a posteriori error estimation for singularly perturbed elliptic problems which are discretized by means of an upwinded FEM. It is shown that, for such problems, there can be obtained uniform (w.r.t. the perturbation parameter) error estimates in a suitable chosen norm. Moreover, a new method to simplify the practical computation of the more complicated error indicators is proposed.

T. APEL:

Local Inequalities for Anisotropic Finite Elements

The classical local interpolation error estimates (see e.g. [Ciarlet '78]) were derived under an assumption which is in two dimensions known as Zlámal's minimal angle condition. This condition was weakened by different authors ([Jamet '78], [Babuška/Azis '76], [Křížek '89]) to a maximal angle condition. But the possible advantage of using mesh sizes with different asymptotics in different directions which leads to small angles, was not extracted.
In this presentation, anisotropic interpolation error estimates in two and three dimensions are given. Here, one derives benefit from the different asymptotics of the mesh sizes. Moreover, an anisotropic version of the inverse inequality is presented.
Anisotropic meshes are already successfully applied to Poisson-like problems in domains with edges and to convection-diffusion problems where boundary layers occur.

O. AXELSON:

Uniformly Convergent Difference Methods of Arbitrary High Order for Singularly Perturbed Convection Diffusion Problems

A general framework to construct difference methods for singularly perturbed convection diffusion problems with discretization error estimates of arbitrary high order, which hold uniformly in the singular perturbation parameter, is presented. The method is based on the use of a defect-correction method and special, adaptively graded and patched meshes, with meshsizes varying between h and $\epsilon^{3/2}h$, where h is the meshsize, used in the part of the domain where the solution is

smooth, ϵ is the singular perturbation parameter and $\epsilon^{3/2}h$ is the final meshsize in the boundary layer.
Similar constructions hold for the interior layers. The correction operator is a monotone operator enabling the estimate of errors of optimal order in maximum norm.

C. CANUTO:

Wavelet-Based Adaptive Methods for Advection-Diffusion Boundary Value Problems

Functions which describe phenomena in science and engineering often exhibit a structure, well localized both in physical space and in frequency space (or in scale). Boundary layer problems and turbulence offer examples of such a behaviour. Classical bases are well localized in space (e.g. finite element bases) or in frequency (e.g. Fourier bases), but not in both spaces at the same time. Recently developed bases, like wavelets and hierarchical f.e. bases combine both aspects. They allow an adaptive approximation of functions, in which negligable components are discarded. This is the rationale for stating an adaptive discretization of pde's.
We review the construction of (biorthogonal) wavelets, starting from the classical Haar basis. Attention is paid to those aspects (Bernstein and Jackson inequalities, decay of wavelet coefficients, local and global characterization of functional spaces) which are not relevant to numerical analysis. Next, we consider a model advection-diffusion equation, and we show how the upper-level portion of the approximate solution (in a hierarchical decomposition) can be used as an a-posteriory error indicator. This justifies the wavelet analysis of the approximate solution, in order to change adaptively the upper-level complement to optimize the error distribution. Two examples (1D and 2D) are given; in the 2D example, the tensor product diadic grid used for wavelet analysis is simply superimposed to the (Delaunay) grid used to represent the f.e. solution.

C. CLAVERO:

Uniform Convergence for One Dimensional Problems Using Shishkin'Mesh

In this communication we prove that classical schemes are uniformly convergent where they are defined on special meshes of Shishkin type. We examine the convection-diffusion and the reaction-diffusion problem. In each case we define in appropriate form a piecewise uniform mesh which condensing the points of the mesh in the boundary layers.
Also, we show some numerical results for the numerical integration for parabolic

singularly pertubed problems. The method uses the alternating directions technique and the previously finite difference schemes constructed on Shishkin'meshes for 1D problems. These examples show that the method is uniformly convergent for this type of problems.

A. CRAIG:

A Class of Petrov-Galerkin Methods for the Stationary Convection-Diffusion Equation

An upwinded Petrov-Galerkin method is proposed which is applicable to the n-dimensional convection-diffusion equation. The essential property of the method is that it sets the L_2-projection of the error on element boundaries, into a particular class of functions, to zero. This has the consequence at producing nonoscillating approximations. The method is described and both asymptotic and nonasymptotic analysis are presented. The standard Galerkin method and the cell vertex finite valume method are seen to be limiting cases in the pure diffusion and pure convection limits respectively.

J. DALÍK:

An Explicit Modified Method of Characteristics for the Two-Dimensional Convection-Diffusion Problem with Dominating Convection

I want to motivate and describe a basic mechanism of a certain combination of the characteristics with the finite difference method for a numerical solution of non-stationary twodimensional convection-diffusion problems with dominating convection.
To each triangulation without obtuse angles, this method relates a stable approximate solution. Numerical experiments illustrate that this solution is disturbed by minimal amount of the artificial diffusion. I wish to point out some problems which one has to solve on the road to an error-estimate.

P. A. FARRELL:

Uniformly Convergent Difference Schemes for Semi and Quasilinear Singularly Perturbed ode's

In this lecture results were presented which showed that standard difference schemes, with a fitting factor frozen in the neighbourhood of the boundary layer, on a uniform mesh cannot convergence ϵ-uniformly in the maximum norm to the solution of the differential equation. Uniformly convergent schemes consisting of monotone finite difference schemes on piecewise uniform meshes, condensed in the neighbourhood of the boundary layer(s), were presented for both semi and quasilinear singularly perturbed ode's. Theoretical rates of uniform convergence were given in both cases, and numerical results were presented, which showed that these were in practice conservative estimates.

A. FELGENHAUER:

Mixed Formulation Analysis of L-Spline Galerkin-Petrov F.E.M.

We consider a GALERKIN - PETROV finite element discretization of a singularly perturbed twopoint boundary value problem on the real interval [0,1] proposed by O'RIORDAN and STYNES. The method is determined by piecewise exponential trial and test functions fitted to the convection-diffusion part of the differential operator. In contrast to the standard finite element analysis we do not describe the discretized problem by reduction of the variational equation to a finite dimensional subspace but by variationally formulated side constraints. It will be demonstrated, that abstract results from the theory of mixed formulations are applicable. This new analysis simplifies the proof of convergency, even in the case of piecewise linear f.e.m., and enables a new approach of constructing exponential fitted finite element methods adapted to singularly perturbed differential operators. The application of this method to the two-dimensional case is discussed.

J. E. FLAHERTY:

High-Order Finite Element Methods for Singularly-Perturbed Elliptic and Parabolic Problems

We develop a framework for applying high-order finite element methods to singularly-perturbed elliptic and parabolic differential systems that utilizes special quadrature rules to confine spurious effects, such as excess diffusion and non-physical oscillations, to boundary and interior layers. This approach is more suited for use with adaptive mesh-refinement and order-variation techniques than other problem-dependent methods. Quadrature rules, developed for two-point convection-diffusion and reaction-diffusion problems, are used with finite element software to solve examples involving ordinary and partial differential equations. Numerical artifacts are confined to layers for all combinations of meshes, orders, and singular perturbation parameters that were tested. Radau or Lobatto quadrature used with the finite element method to solve, respectively, convection- and reaction-diffusion problems provide the benefits of the specialized quadrature formulas and are simpler to implement.

A. HEGARTY:

Central Differencing for Shishkin Meshes

It is well-known that upwind finite difference operators on Shishkin meshes yield ϵ-uniformly convergent numerical solutions of linear elliptic singular perturbation problems, where ϵ is the small parameter; however, heretofore, it had been considered that a non-monotone difference operator, such as central differencing, which does not satisfy a maximum principle, would not be uniformly convergent. In earlier paper, we had shown that, for a model parabolic layer problem, central differencing produced a numerical solution which appeared to be uniformly convergent of order greater than one; in this case, the oscillation could be eliminated by an apppropriate choice of the location of the transition from fine to coarse meshes. In the regular layer case, it was shown that, for a model problem which satisfied some compatibility conditions, the numerical solution again indicated uniform convergence of order greater then one. Similar behaviour was observed for a second problem, without such compatibility; nevertheless, in this situation, ϵ-dependence of the convergence of the linear solver was observed. Numerical solutions for small values of ϵ could only be obtained by using powerful iterative solvers, BiCG-Stab(L), which are expansive in their use of 2L matrix products per iteration. Thus, the practicability of this approach needs further investigation.

W. HEINRICHS:

Defect Correction for Singular Perturbation Problems

A defect correction procedure with first order upwind preconditioning is applied to high order finite difference (β-schemes, narrow stencils) and spectral discretizations of singular perturbation problems. By Fourier analysis the preconditioning properties and smoothing effects are studied. For variable coefficient problems flow directed point iterations are proposed. The defect correction is used in a multigrid frame for relaxation. This procedure is applied to the Boussinesq flow problem in vorticity-streamfunction formulation. Numerical results for increasing Rayleigh numbers are presented.

D. HIETEL:

Cell-Orientated Semidiscretizations for the Numerical Solution of Convection-Dominated Problems

The method of lines applied to nonstationary partial differential equations usually leads to stiff systems of ordinary differential equations. The cell-orientated semi-discretization is based on finite-volume-type approximation of conservation laws by using cell and edge averages. The time dependent problem is then a system of differential-algebraic equations. We present the derivation of this method, the stability and convergence properties. Numerical results for the convection-diffusion problem and the linearized Shallow-Water equations show that this method is well suited for convection dominated situations. Finally the method can be interpreted as a PETROV-GALERKIN method which is conforming in the one-dimensional and nonconforming in the two-dimensional case. This motivates a modification of the trial and test functions which can be applied to triangulations which should be studied by future work.

V. JOHN:

A Parallel and Adaptive Algorithm for the Stokes- and Navier-Stokes-Equations

We present a parallel and adaptive algorithm for solving the Stokes and Navier-Stokes equations. The linear, respectively linearized, equations are discretized using the non-conforming P_1/P_0 element of Crouzeix/Raviart. A residual a posteriori error estimator by Verfürth is used for the adaptive mesh design. We obtain expected results. But there is still the open question in which norm the error should be estimated. We demonstrate on some numerical tests with scalar convection diffusion equations that the meshes and quality of solution using different a posteriori error estimators (for $||\cdot||_{H^1}$ and $||\cdot||_{L^2}$) are sometimes rather different.

J. LORENZ:

Boundary Conditions for Low - Mach - Number Flows

We derive the model system

$$w_t + \begin{pmatrix} U & 1/\epsilon \\ 1/\epsilon & U \end{pmatrix} w_x = v \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} w_{xx}$$

with $w = \begin{bmatrix} u(x,t) \\ q(x,t) \end{bmatrix}$, where u and q describe velocity and pressure fluctuations, respectively. Here ϵ is the Mach number and $U > 0$ is the velocity of a base flow. Simple boundary conditions to be posed at artificial boundaries are $u + q = u_x - \epsilon u_t = 0$ at $x = -L$ (inflow) and $u - q = 0$ at $x = L$ (outflow).
The same principle can be applied to the simple advection-diffusion problem $U_t + aU_x = vu_{xx}$, $a > 0$. If one uses these boundary conditions for Burger's equation $u_t + uu_x = vu_{xx}$ by freezing u at outflow, one encounters difficulties if $u = 0$ initially at outflow. A possibility is to switch from the outflow condition

$u_x = 0$ to $u_t + uu_x = 0$ if u exceeds a threshold at $x = L$.

G. LUBE:

Resolution of Boundary Layers with Anisotropic Finite Elements

Standard Galerkin finite element solutions of the diffusion-advection-reaction problem may suffer from numerical instabilities which are generated by dominant convection and / or reaction terms unless the mesh is sufficiently refined. As a remedy, we consider a stabilized Galerkin method (Galerkin/Least-squares FEM) which is in contrast to standard upwind type methods consistent with the weak formulation.
We extend the a-priori analysis for isotropic meshes to anisotropically refined meshes at least in boundary layers. As a result we find (nearly) uniformly valid error estimates. In detail we discuss the construction of the layer mesh and the design of the numerical dumping parameters.
We present some numerical results and consider the application of domain decomposition method for such problems.

J. J. H. MILLER, E. O'RIORDAN, G. I. SHISHKIN:

On the Non-Existence of ε-Uniform Fitted Operator Methods on uniform Meshes for the Singularly Perturbed Heat Equation

It has been shown that it is not possible to an ϵ-uniform fitted finite difference method on a uniform rectangular mesh for singularly perturbed equations having solutions containing a parabolic boundary layer. This negative result is true even in the simplest case of the heat equation with a small coefficient of conductivity. On the other hand it is easy to construct a simple fitted piecewise uniform mesh, on which standard finite difference operators yield numerical methods that are ϵ-uniform for such problems.
The key steps in the proof of the non-existence result are discussed.

B. O'MALLEY, J. LAFORGUE:

Shock Layer Solution for Viscous Shock Equations

An explicit solution to the steady Burgers' problem

$$\epsilon u_{xx} = uu_x, \quad u(-1) = 1 \mp e^{a/\epsilon},\ 0 < a < 1,\ u(1) = -1$$

was provided, giving a shock at $a - 1$ or $1 - a$.
An explicit solution to Burgers' problem

$u_t + uu_x = \epsilon u_{xx},\ u(\pm 1, \epsilon) = \mp 1,\ t > 0, \quad u(x, 0)$ decreasing, $u(\pm 1, 0) = \mp 1$ was given via the Cole-Hopf transformation and the shock was shown to follow the slowly moving asymptotic profile

$$\tanh\left(\frac{X - X_\epsilon(te^{-1/t})}{2\epsilon}\right) \quad \text{with } X_\epsilon[\tau) \sim \frac{\epsilon}{2}\log\left(\frac{1 + \tanh(-\frac{1}{q\epsilon}\int_{-1}^{1} u(s,0)ds)}{1 - \tanh(-\frac{1}{q\epsilon}\int_{-1}^{1} u(s,0)ds)}\right).$$

For the viscous shock problem

$u_t + (f(u))_x = \epsilon u_{xx}$ with $f(u) < f(-1) = f(1)$ for $|u| < 1$ and $f'(1) < 0 < f'(-1)$,

boundary conditions $u(\pm 1, t, \epsilon) = \mp 1$, and decreasing initial value, the steady state is implicitly defined via

$$\eta = \int_0^{\varphi} \frac{dv}{f(v) - f(\pm 1)}$$

where $\varphi(\eta) \sim \mp 1 \pm L^{\pm} e^{-A_{\pm}\eta} \infty \eta \pm \infty$, where $A_{\pm} = \pm f^1(\pm 1) > 0$ and $L^{\pm} > 0$ are known. After shock forms, the limiting solution is given by

$$\varphi\left(\frac{X - X_\epsilon(\tau)}{\epsilon}\right) \quad \text{for} \quad \tau = te^{-A/\epsilon} \quad \text{and} \quad A = 2\left(\frac{1}{A_+} + \frac{1}{A_-}\right)^{-1}.$$

Here, X_ϵ satisfies the initial value problem for

$$\frac{dx_\epsilon}{d\tau} = c\left\{e^{-(x_\epsilon(\tau) - x_\epsilon(\infty))A_-/\epsilon} - e^{(x_\epsilon(\tau) - x_\epsilon(\infty))A_+/\epsilon}\right\}$$

for some $c > 0$ and the stable steady-state $X_\epsilon(\infty) \sim \frac{A_+ - A_-}{A_+ + A_-}$. Numerical aspects are being studied with Marc Garkey (Lyon 1).

E. O'RIORDAN, J. J. H. MILLER, G. I. SHISHKIN :

Central Ideas for the Proof of an ϵ-uniform Convergence of a Shishkin Mesh

In a singularly perturbed problem in two-dimensions on a rectangle where the solution contains only regular layers, it is known that a monotone finite difference operator and a piecewise-uniform Shishkin mesh yield an ϵ-uniformly convergent finite difference method. The proof of this result and other more general results has been given by Shishkin several years ago. Here, we outline the key ideas that might be helpful to a reader of the full proofs.

U. RISCH:

Global and Local Error Estimates for an Upwind Finite Volume Discretization of Singularly Perturbed Equations

We consider the equation $-\epsilon\Delta u + b(x)\nabla u + c(x)u = f(x)$ in $\Omega \subset R^2$ with $0 < \epsilon \ll 1$. The finite volume discretization is based on cells constructed as dual polygons for an (in general non-uniform) triangular mesh. For a class of upwind methods the error in an energetic (or maximum, resp.) norm is estimated by $c\sqrt{h}||u||_{2,2,\Omega}$ (or $c\sqrt{h}||u||_{2,\infty,\Omega}$, resp.) with C independent of ϵ.
Furthermore, in subdomains outside boundary layers and additional numerical layers, the factors $||u||_{...}$ can be omitted. The thickness of numerical boundary layers is asymptotically $O(h)$ at ordinary layers and $O(\sqrt{h})$ at parabolic ones.

R. SACCO:

Divergence Free Exponentially Fitted Finite Elements for Convection-Diffusion Problems

We deal with a new nonconforming finite element method for the numerical solution of convection-diffusion equations with a dominating convective term. In the applications that we keep in mind, attention is paid to the study of current continuity equations arising in the Drift-diffusion model for semiconductor devices. The proposed finite element approach extends to the $2D$ problem in the case of triangular decompositions the well-known Scharfetter-Gummel exponentially fitted scheme by a proper choice of the trial functions for approximating the unknown $u(\underline{x})$.
This latter turns out to be nonconforming over the whole domain Ω and gives rise to divergence-free current fields $\underline{I}(u) = \epsilon\underline{\nabla}u - u\underline{\beta}$ over each element of the triangulation. The test functions are piecewise linear continuous over $\bar{\Omega}$. The resulting method is therefore a nonconforming Petrov-Galerkin finite element scheme. The basic properties of the novel trial functions are illustrated and the performance of the P-G method are tested on several classical model problems of convection-dominated flows.

F. SCHIEWECK:

On the Numerical Solution of the Navier-Stokes Equations for High Reynolds Numbers

We study the discretization error for an upwind finite element approximation of the stationary incompressible Navier-Stokes equations in the case of high Reynolds numbers. Since the existing theory does not cover this case we do some "experimental" convergence analysis for numerical test problems to get first answers at all. It turns out that the interpolation error of the pressure may cause an $O(Re)$-error in the velocity. This effect can be removed for a certain range of Reynolds numbers by means of a pressure separation.

V. SHAIDUROW, A. THIELE, L. TOBISKA

Fitted Quadrature Rules in the Finite Element Method for Singularly Perturbed Problems

Standard finite element methods for solving convection dominated convection-diffusion problems results in instabilities unless the mesh is very fine. Therefore, several modifications have been developed in the literature to overcome this difficulty and to stabilize the schemes. Some of the most used modifications are the different upwind techniques, the streamline diffusion or Galerkin least square method, the exponentially fitted techniques and the construction of special grids.

We present a new approach for stabilizing convection-diffusion problems of the form

$$-\varepsilon\Delta u + \operatorname{div}(bu) = f \quad \text{in } \Omega, \quad u = 0 \quad \text{on } \Gamma,$$

where ε is supposed to be a small parameter. The main idea of the new technique consists of using a special integration rule on each finite element with weights depending on the streamline direction and the mesh Péclét number, thus it can be easily implemented in existing computer programs. First, we apply this idea to the one-dimensional case when standard piecewise linear trial and test functions are used on a uniform grid with mesh-size h. It is proved that the solution u^h of the quadrature-modified scheme converges, uniformly with respect to ε, to the exact solution u of first order in the grid nodes. Then the two-dimensional case with an exponential boundary layer is considered. The quadrature-fitted technique results in a five-point scheme with an M-matrix. Finally, we give two numerical illustrations in two-dimensional case which confirm theoretical considerations.

G. I. SHISHKIN:

Grid Approximation of the Solution and Diffusion Flux for Singularly Perturbed Equations with Neumann Boundary Condition

Neumann boundary value problems for singularly perturbed parabolic equations are considered on a segment and on a rectangle. The second order derivatives of differential equations contain a small parameter ε^2 which can take any value in the interval (0,1]. For the zero value of the parameter, the parabolic equation is

reduced to a first order (with respect to the time variable) one. On the boundary the normalized diffusion flux (that is the product of the solution gradient and the parameter ϵ) orthogonal to the boundary is given. It is required to find the solution and the normalized diffusion fluxes.
As it is well known, the solution of classical finite difference schemes on uniform grids does not converge ϵ-uniformly. Moreover, we show that, when the parameter tends to zero, the approximate solution and the error can unboundedly increase for some schemes, or the error can tend even to the exact solution for other schemes.
For the boundary value problems under consideration special finite difference schemes are constructed. These schemes allow to approximate the solution and the normalized diffusion fluxes ϵ-uniformly. The method of special condensing grids is applied to construct the schemes and to study their convergence.

G. STRAUBER:

Numerical Methods for Pollutant Transport in Rivers with dead Zones

We numerically investigated the following dead-zone model describing the transport of pollution in rivers (or in soil):

$$\begin{aligned} \frac{\partial c_1(x,t)}{\partial t} + v\frac{\partial c_1(x,t)}{\partial x} - D\frac{\partial^2 c_1(x,t)}{\partial x^2} &= \frac{1}{\tau_1}(c_2(x,t) - c_1(x,t)) - kc_1(x,t) \\ \frac{\partial c_2(x,t)}{\partial t} &= \frac{1}{\tau_2}(c_1(x,t) - c_2(x,t)) - kc_2(x,t) \end{aligned} \tag{1}$$

$(0 \leq x \leq L\,,\ t \geq 0)$ where $L \gg 1$ (say $L = 10^5$ m) that is $\frac{D}{vL} \ll 1$.

Several difference schemes – among them two new modified box schemes – approximating system (1) were investigated and their accuracy and stability and monotonicity conditions were compared to each other. The modified box schemes proved to be more accurate for practically used steplengths than classical schemes.

L. TOBISKA:

Stabilized Finite Element Methods for the Navier-Stokes Equations

A robust Navier-Stokes solver for higher Reynolds numbers seems to require (at least) two ingredients:

1. A stable discretization method allowing one to measure the error in a suitable norm in terms of the approximation error with a multiplicative constant which is (almost) independent of the Reynolds number.

2. A meshrefinement strategy to adapt the mesh to the singular behaviour of the solution in local regions as for instance near internal or boundary layers.

The paper gives an overview of different techniques for stabilizing finite element methods for solving scalar convection-diffusion problems, the linearized and complete Navier-Stokes equations. Thereby the main features of upwind type and streamline-diffusion type methods will be clarified.

Z. UZELAC:

Some Spline Difference Scheme for Solving Singularly Perturbed Problems

For the non-turning point case of problem

$$L_\epsilon y(x) \equiv \epsilon y''(x)+p(x)y'(x)-q(x)y(x)=f(x)\,,\quad x\in(0,1)\,,\quad y(0)=\alpha_0,\ y(1)=\alpha_1$$

ϵ is a small parameter, $p(x)\neq 0$, $g(x)\geq 0$, we derive a family of difference schemes on uniform meshes. As a collocation equation we use: $\tilde{L}u(x)\equiv\epsilon\tilde{u}''+\tilde{p}\tilde{u}'-\tilde{q}\tilde{u}'=\tilde{f}$, where $\tilde{p},\tilde{q}$ and $\tilde{f}$ are piecewise constant approximations of the coefficients $p(x),q(x)$ and $f(x)$. As an application function we use an exponential spline $e(x)\in\operatorname{span}\{1,x,e^{-\rho_i x},e^{\rho_i x}\}$, ρ_i are tension parameters, $e(x)=e_i(x)=\tilde{u}_j+m_jh+\frac{a_j}{\rho_j}(\cosh\mu_j-1)+\frac{b_j}{\rho_j}(\sinh\mu_j-\mu_j)$. Free parameters μ_j, a_j and b_j we find from conditions: $e(x)\in C^1[0,1]$ and $\tilde{L}e_i(x)=\tilde{f}$ for $x=x_i$ and $x=x_{i+1}$. Defining ρ_j in a way that a scheme is exact for solutions of $\tilde{L}u(x)=0$ and choosing $\tilde{p}_j=\frac{1}{2}(p(x_j)+p(x_{j+1}))$, $\tilde{q}_j$ and $\tilde{f}_j$ in the same way, we generate El Mistikawy Werle scheme (EMW) choosing $\tilde{p}_j=p(x_{j+1/2})$, $\tilde{q}_j$ and $\tilde{f}_j$ in the same way we derive a scheme we call it IEMW scheme. Both schemes are second order uniform and have classical accuracy. We derive a new one in the following way:

$$4(1-\beta)\,IEMW-\beta EMW\,,\quad\text{where}\quad \beta=\begin{cases}1/2 & \text{for}\quad h\leq\epsilon\\ \epsilon & \text{for}\quad \epsilon<h\end{cases}.$$

We proved that this combination leads to the scheme which is $O\,(h^4/(\epsilon^2+h^2))$.

R. VULANOVIC:

Exploiting Monotonicity in Numerical Methods for Singular Perturbation Problems

Singular perturbation problems whose solutions are monotone and have interior shocks are solved by interchanging the independent and dependent variables and using equidistant finite-difference schemes. Examples illustrate that this approach can locate the shock accurately, which is not the case with the standard methods.

G. ZHOU:

How Accurate is the Streamline Diffusion Finite Elemnet Method?

Since the streamline diffusion finite element method was proposed, various convergence results have been given. On usual quasi-uniform meshes, the pointwise accuracy was proved by Johnson et al to $O(h^{5/4})$, which was later improved by Niijima to $O(h^{11/8})$. By orienting the mesh in the streamline direction and imposing a uniformity condition on the mesh, this result has been improved again by Zhou and Rannacher to its optimal order of $O(h^2)$. In this paper, we investigate the actual accuracy of the streamline diffusion finite element method. A special structured mesh has been analyzed for showing that the convergence order in the L^2 norm changes from $O(h^{2/3})$ to $O(h^2)$ depending on some mesh parameter. And the pointwise error is bounded to $O(h^{3/2})$. Numerical tests verify the analysis and show that the convergence order of $O(h^{3/2})$ cannot be improved without any mesh condition. Some open questions are raised for discussions.

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