Tagungsbericht 5/1995

Applied and Computational Convexity

January 29 – February 4, 1995

The conference, organized by P. Gritzmann (Trier), V. Klee (Seattle) and P. Kleinschmidt (Passau), was attended by 38 participants, who gave a total of 37 lectures ranging from 25 to 45 minutes.

The meeting reflected exciting new developments in the area of Applied and Computational Convexity. The roots of this field lie jointly in geometry, in mathematical programming and in computer science. Typically, the problems are algorithmic in nature, the underlying structures are geometric with special emphasis on convexity, and the questions are usually motivated by practical applications in mathematical programming, computer science, and other less obviously mathematical areas of science.

According to the concept of this conference, the participants belonged to four different fields: classical convexity theory, mathematical programming, computational geometry and computer science.

The talks dealt with various topics of the wide spectrum of subjects covered by Applied and Computational Convexity. Some lectures were devoted to integer programming and polyhedral combinatorics. A new approach based on Gröbner bases and Newton-polytopes was presented, and new results, some general, some related to particular applications were given which utilize polyhedral approaches for solving large-scale combinatorial optimization problems. Various lattice point problems were studied, partly from the point of view of integer programming.

Other talks dealt with linear, semi-definite and convex optimization problems. New algorithms, partly motivated by results from classical convexity theory were presented, new insight was gained in known methods, and some special purpose approaches were reported on, which are tailored to particular practical applications.

Geometric aspects of nonlinear (smooth and nonsmooth) optimization were scrutinized in some other lectures, geometric partitioning and covering problems turned out to be particularly relevant for global optimization.

Another group of talks focussed on the computation and optimization of certain geometric functionals, one of which was motivated by the Hadamard determinant problem, and on algorithmic reconstruction problems that are related to problems in computer vision or computer tomography. In this context the algorithmic theory of convex bodies played an important role.

Also presented were new results in geometric graph theory, in the theory of polytopes, tilings, and related combinatorial objects, partly theoretical, partly algorithmic. In particular, one of the lectures surveyed the outstanding new results of Richter-Gebert on the realization space of convex polytopes, which solve a large variety of long-standing open problems in polyhedral theory.

The conference showed that even though the participants belonged to different fields that have quite different tool-boxes, approaches and ideas for solving their problems, there is a deep and close connection which is centered around the basic concept of convexity.

Vortragsauszüge

Farid Alizadeh (New Brunswick):

Complementary relations between pairs of convex polar cones with applications to primal-dual algorithms

It has been observed that some interior point algorithms for linear programming can be extended, in a sense word by word, to optimization problems over more general domains, such as the cone of positive semidefinite matrices and the so-called "ice cream" cone. Consider the following pair of primal and dual problems:

$$\min\{\mathbf{c}^T\mathbf{x} : A\mathbf{x} = \mathbf{b}, \mathbf{x} \in \mathcal{K}\} \qquad \text{and} \qquad \max\{\mathbf{b}^T\mathbf{y} : A^T\mathbf{y} + \mathbf{z} = \mathbf{c}, \mathbf{z} \in \mathcal{K}^*\}$$

where $\mathcal{K}^*$ is the polar of the cone $\mathcal{K}$, $\mathbf{x}^T\mathbf{z} = 0$ at the optimum (with appropriate constraint qualifications). For $\mathbf{x}, \mathbf{z} \in \mathbb{R}^n$ the three relations 1) $\mathbf{x} \in \mathcal{K}$, 2) $\mathbf{z} \in \mathcal{K}^*$ and 3) $\mathbf{x}^T\mathbf{z} = 0$, together, actually impose n equality constraints. However, it is not clear what these n constraints are explicitly in general. For the special case of the positive orthant (linear programming), positive semidefinite cone (semidefinite programming) and "ice cream" cone (convex quadratic constrained quadratic programming) the n complementarity relations are actually bilinear forms, and therefore one can extend LP-interior point algorithms to these problems rather naturally. In contrast, for p-cones, complementarity conditions are not bilinear and direct extensions of LP interior point methods seem more difficult. We discuss these extensions and issues involving the complexity of such generalizations.

Imre Bárány (Budapest):

Maximal lattice–point free convex bodies

For a general $m \times n$ matrix A consider the convex bodies $K_b = \{x \in \mathbb{R}^n : Ax \leq b\}$. K_b is lattice–point free if $\operatorname{int} K_b \cap \mathbb{Z}^n = \emptyset$ and maximal if $\operatorname{int} C \cap \mathbb{Z}^n = \emptyset$ for every convex body C properly containing K_b. Motivated by problems in integer programming, we study maximal lattice–point free convex bodies. Such a K_b has one integer point z^i on each of its facets. There is a natural way to identify K_b with the set of all integers $\{z^1, \ldots, z^m\}$ from the facets. This gives rise to an abstract simplicial complex $\mathcal{K}(A)$ associated with A. It turns out that $|\mathcal{K}(A)|$ is homeomorphic to $\mathbb{R}^{m-1}$. Moreover, $\mathcal{K}(A)$ is invariant under translation by integers. The factored out complex, $|\mathcal{K}(A)/\mathbb{Z}^n|$ is homeomorphic to the n–torus $\otimes\mathbb{R}^{m-n-1}$.
(Joint work with R. Howe, H. E. Scarf, D. Shallcross.)

Jürgen Bokowski (Darmstadt):

On the automorphism group of the matroid of the d–cube

In computational synthetic geometry the problem of determining all oriented matroids for a given (questionable underlying) matroid forms a decisive intermediate step. A conjecture by Las Vergnas fits into this framework. It leads to investigating the automorphism group G^d of the matroid of the d–cube. We can prove:

Theorem. *$G^d \cong Q^{d+1}/Z$, where Q^{d+1} denotes the Coxeter group of the $(d+1)$–cube and Z denotes its center.*

Application of this theorem leads to simple proofs of the uniqueness of the reorientation class of the matroid of the d–cube in small dimension thus extending former results of LasVergnas, Roudneff and Salaün in dimension 4.
(Joint work with A. Guerdes de Olivera, U. Thiemann, A. Veloso da Costa.)

Thomas Burger (Trier):

Optimal projections of finite metric spaces

We consider the following optimization problems.

- Given a polytope $P \subset \mathbb{R}^n$ and an integer k, find a k-dimensional subspace U such that the orthogonal projection of P on U has maximum/minimum volume.
- Given a finite metric space Z and an integer k, find a k-dimensional subspace U such that the orthogonal projection of Z on U has minimum "distortion".

We present some applications and related NP–completeness results. We discuss some algorithms for the second problem.
(Joint work with P. Gritzmann.)

James V. Burke (Seattle):

Duality theory and numerical methods for trust–region subproblems

The trust–region approach to the numerical solution of a nonlinear program has provided a range of very effective numerical techniques for unconstrained optimization. Recently, several proposals have been suggested for extending this approach to the constrained case. From a theoretical perspective, these proposals are comparable. Each has a robust global convergence theory, and, under the usual hypotheses, the local theory indicates good convergence rates. However, the practical success of each proposal ultimately depends on an efficient numerical method for solving the associated trust–region subproblems. As a side benefit to our approach to the duality theory, new insights are also obtained for trust–region subproblems arising in unconstrained problems.

Dietmar Cieslik (Greifswald):

Steiner minimal trees in Banach–Minkowski spaces

I consider Banach–Minkowski spaces $M_d(B)$, that means a d-dimensional Banach space with unit ball B and norm $|v|_B = \inf\{t > 0 : v \in tB\}$. Let N be a finite set in $M_d(B)$. Then Steiner's problem is to find a shortest tree interconnecting the points of N. There are a lot of results for solutions of Steiners's problem, for instance (i) combinatorial restrictions for such trees, (ii) results about the local version: Find one point which minimizes the sum of distances to the given points in N, (iii) methods to construct such trees in special spaces, (iv) investigations about the Steiner ratio, that means the relative defect of the length of a Steiner problem solution and the length of a tree interconnecting the points of N only by segments between the points. I will give an idea how we can construct a shortest tree in arbitrary Banach-Minkowski planes.

Ludwig Danzer (Dortmund):

A single prototile, which tiles space, but neither periodically nor quasiperiodically

A prototile SCD $(= T(\frac{m}{n}, s, h), m, n \in \mathbb{N}, \gcd(m, n) = 1, m, 2 < n, s, h \in \mathbb{R}, h > 0)$ is presented, which

- is a weakly convex polyhedron in $\mathbb{E}^3$,
- permits $2^{\aleph_0}$ face–to–face tilings of $\mathbb{E}^3$,
- none of which is invariant under any nontrivial translation (mirror images excluded.

If and only if n is odd, the species S_0^+ of all face–to–face tilings by congruent copies of SCD (mirror images not permitted) is repetitive. When SCD is furnished with a finite number of sites for various atoms, every tiling of the latter type yields an (r, R)–set.

- with only finitely many Voronoi–cells (up to rigid motions)
- and without any Bragg–peak in its X–ray diffraction pattern (i.e. no Dirac–delta in its Fourier–transform) except a 1-lattice on the z–axis.

Only a very few of these patterns possess any global symmetry. There may be a reflection in a line. If there are more symmetries, there is a screw with an angle incommensurate to π. If any of these (r, R)–sets are physically realizable, they will obey a strict long–range orientational order, but not by translations and far away from being quasicrystalline.

Robert M. Freund (Cambridge):

Ill–posedness and the efficiency of solving linear inequalities

Given a data instance (A, b), a linear inequality system $Ax \leq b$ is well–posed to the extent that changes in the data (A, b) do not alter the solvability of the linear inequality system, i.e., the system either remains solvable or remains unsolvable (via a theorem of the alternative) for small changes in the data. We study the dependency of complexity bounds on the ill–posedness of a given linear inequality system for four different algorithms: von Neumann's algorithm, Rosenblatt's algorithm, the ellipsoid algorithm, and a generic interior–point method. We show that the complexity of these algorithms depends on either the reciprocal of the ill–posedness measure or on the logarithm of the reciprocal of the ill–posedness measure, depending on the algorithm.

Komei Fukuda (Tokyo):

Vertex and face enumeration algorithms

In this talk, we present some recent results on the vertex and face enumeration problems for a convex polyhedron P given as the solution set to a system of m linear inequalities in d variables.
From the view point of computational complexity, we show that there is an output-sensitive algorithm to enumerate all v vertices of P in time $O(mdv)$ and in space $O(md)$, under the nondegeneracy assumption. The algorithm is based on the reverse search technique by D. Avis and K. Fukuda. No output–sensitive algorithm is known for the general case. In contrast, we show that the faces of a general polyhedron can be enumerated in time polynomial in the input size and linear in the output size, and in space polynomial in the input size. The idea is simply to use backtracking and linear programming. The same technique cannot be used efficiently for the vertex enumeration because the associated decision problem, called the restricted vertex problem, is $\mathbb{NP}$–complete. These results on the analysis of backtrack algorithms are due to K. Fukuda, Th.M. Liebling and F. Margot.
We also give some empirical results on the double description method of Motzkin *et al.* and present some ideas for its practical implementation to enumerate all vertices of a highly degenerate convex polyhedron.

Peter Gritzmann (Trier):

Determination of finite sets by X–rays

High resolution transmission electron microscopy can efficiently measure the number of atoms of a molecular object on each line in certain directions. The aim is to determine a crystal from a number of different such X–ray images. The talk studies the underlying mathematical problem, gives various theoretical and algorithmic results and outlines generalizations. A combination of methods from convexity and algebraic number theory leads, in particular, to the solution of a problem posed by L. Shepp (at a DIMACS conference on Discrete Tomography).
(Joint work with R. J. Gardner.)

Pierre Hansen (Montreal):

D–C programming, column generation and location theory

Many nonconvex optimization problems may be expressed as D–C programs in which the objective function and constraints' left–hand sides are differences of convex functions. This leads to solution methods for various extensions of Weber's problem in continuous location theory. Moreover, combination of D–C programming with column generation methods of linear and integer programming gives an efficient way of solving the difficult multisource Weber problem.

Andreas Hefner (Passau):

Solving constrained matching problems by polyhedral combinatorics

The following problem (called the Master/Slave–Matching Problem) arises in the area of manpower scheduling: Given an undirected bipartite graph $G = (V, E)$ with bipartition $V = W \cup U$ and a digraph $D = (U, A)$. A Master/Slave–matching in G with respect to D is a matching in G such that for every arc $(u, v) \in A$ the node v is matched whenever the node u is matched. The problem is, to find a Master/Slave–matching of maximum cardinality.

Let k be the maximum size of a (weakly) connected component of D. First we show that the problem is $\mathbb{NP}$–hard and remains $\mathbb{NP}$–hard even if $k = 3$. Second we focus on the case $k = 2$: We show how the Master/Slave–Matching Problem can be transformed to the (non–bipartite) Matching Problem (even if nonnegative edge weights are present). Finally, we use polyhedral combinatorics to establish a min–max equation which well–characterizes the cardinality of a maximum Master/Slave–matching. This equation can be viewed as a generalization of König's min–max theorem.

(Joint work with Peter Kleinschmidt.)

Martin Henk (Berlin):

Notes on shortest and nearest lattice vectors

Let $\Lambda \subset \mathbb{Q}^n$ be a lattice and let K be a centrally symmetric convex body such that K contains the n–dimensional unit ball, and K is contained in the n–unit cube. We show that with respect to the norm given by the distance function of K, the shortest lattice vector problem is polynomial–time Turing–reducible to the nearest lattice vector problem.

Dorit Hochbaum (Berkeley):

The complexity of convex separable optimization problems over linear constraints

We demonstrate the polynomiality of nonlinear separable convex (concave) optimization problems, on linear constraints with a matrix with "small" subdeterminants, and the polynomiality of such integer problems provided the integer linear version of such problems is polynomial. We present a general purpose algorithm for converting procedures that solve linear programming problems with or without integer variables, to procedures for solving the respective nonlinear separable problems. The conversion is polynomial for constraint matrices with polynomially bounded subdeterminants.

Among the important corollaries of the algorithm is the extension of the polynomial solvability of integer linear programming problems with totally unimodular constraint matrix, to such integer separable convex programming problems. In particular, it follows that convex network flow problems in integers or continuous variables are solvable in polynomial time.

It is proved that strongly polynomial algorithms are impossible for convex (nonquadratic) network flow and this entire class of nonlinear problems. We present few special cases when strongly polynomial algorithms exist for quadratic optimization problems. An open question remains regarding the strong polynomiality of the quadratic cost network flow problems, yet we delineate certain promising research directions for its resolution.

Reiner Horst (Trier):

Indefinite quadratic programming, concave minimization and some extensions

Indefinite quadratic optimization and concave minimization over polytopes are considered. After a brief introduction into these problems, their applications and basic solution techniques, a decomposition approach is proposed which can also be used for solving the wider class of biquasiconcave optimization problems and some multiplicative programs. The underlying theory is briefly outlined and numerical results are reported which demonstrate the practical value of this approach.

Alexander Hufnagel (Trier):

On the algorithmic complexity of Minkowski's reconstruction theorem

In 1903 Minkowski showed that, given pairwise different unit vectors $u_1, \ldots, u_m$ in Euclidean n-space $\mathbb{R}^n$ which span $\mathbb{R}^n$, and positive reals $\mu_1, \ldots, \mu_m$ such that $\sum_{i=1}^{m} \mu_i u_i = 0$, there exists a polytope P in $\mathbb{R}^n$, unique up to translation, with outer unit facet normals $u_1, \ldots, u_m$ and correponding facet volumes $\mu_1, \ldots, \mu_m$. We consider the computational complexity of the underlying reconstruction problem, to determine a presentation of P as the intersection of its facet halfspaces. After a natural reformulation that reflects the fact that we employ the binary Turing machine model of computation, we show that this reconstruction problem can be solved in polynomial time when the dimension is fixed but is $\#\mathbb{P}$–hard when the dimension is part of the input. This result has application in computer vision.
(Joint work with Peter Gritzmann.)

Marek Karpinski (Bonn):

Lower bounds on randomized decision trees recognizing convex polyhedra

We give an overview of recent lower bound results on randomized algebraic decision trees recognizing convex polyhedra. The underlying proof ideas of counting the number of singularities along the paths of a decision tree are also being presented.

Petar Kenderov (Sofia):

Polygonal approximation of plane convex compacta

A connection is described between the polygonal approximation of a compact convex set in $\mathbb{R}^2$ and some dynamical systems on the unit circumference in $\mathbb{R}^2$. Based on this a numerical procedure is proposed for finding a best approximating n–gone for an arbitrary compact convex set in $\mathbb{R}^2$ (w.r.t. Hausdorff metric). The algorithm provides a solution to a specific global optimization problem where the function to be minimized has more than one local minimum. In one of its equivalent formulations the above approximation problem can be considered as a specific spline approximation problem. From this point of view our algorithm provides also a solution to a specific variable knots spline approximation problem.

Victor Klee (Seattle):

Largest j-simplices in n-polytopes

Relative to a given convex body C, a j-simplex S in C is *largest* if it has maximum volume (j-measure) among all j-simplices contained in C; and S is *stable* (resp. *rigid*) if $\text{vol}(S) \geq \text{vol}(S')$ (resp. $\text{vol}(S) > \text{vol}(S')$) for each j-simplex S' that is obtained from S by moving a single vertex of S to a new position in C. The talk presents a variety of qualitative results that are related to the problems of finding a largest, a stable, or a rigid j-simplex in a given n-dimensional convex body or convex polytope. In particular, the computational complexity of these problems is studied both for $\mathcal{V}$-polytopes (presented as the convex hull of a finite set of points) and $\mathcal{H}$-polytopes (presented as the intersection of finitely many halfspaces). (Joint work with Peter Gritzmann and David Larman.)

Jeffrey C. Lagarias (Murray Hill):

Convexity and the average curvature of plane curves

Given a parametrized closed curve $\gamma : [a,b] \mapsto \mathbb{R}^2, \gamma(a) = \gamma(b)$, its average curvature $M(\gamma)$ is its total (absolute) curvature $K(\gamma)$ devided by its length $L(\gamma)$. Here $K(\gamma) = \int |K| ds$ where γ is a C^2–immersion, and is designed for rectifiable curves γ as the supremum of the sum of exterior angles of an inscribed polygonal approximation.

Theorem. *If D is a closed convex body in $\mathbb{R}^2$ with boundary curve ∂D parametrized by arc length, and γ is any closed curve immersed in ∂D, then $M(\gamma) \geq M(\partial D)$.*

This was proved for $D = disk$ by Fary [1950]. Possible generalizations to $\mathbb{R}^n (n \geq 3)$ and to some non–convex D in $\mathbb{R}^2$ are mentioned.
(Joint work with Thomas J. Richardson, AT&T Bell Labaratories.)

David G. Larman (London):

The 180° art gallery problem

An art gallery with n sides is defined as a simple polygon in the plane with n sides. A guard G can see a point P in the gallery if the line segment G–P lies in the gallery. In general, $\frac{n}{3}$ guards are needed, and are sufficient, to see every point of the gallery. However, even if the vision of a guard is restricted to 180°, I conjecture that the number of guards which are sufficient is still $\frac{n}{3}$. Here I show that $\frac{4n}{9}$ guards are sufficient. With more detailed analysis of small cases, Csizmadia & Toth have reduced this to $\frac{2n}{3}$.
(Joint work with D. H. Bunting.)

Horst Martini (Chemnitz):

Combinatorial geometry of belt bodies

We shall consider the class of *belt bodies*, which was recently introduced by V. Boltyanskí. This class is a natural generalization of the class of zonoids. But whereas the set of zonoids is not dense in the set of centrally symmetric convex bodies, the class of belt bodies is dense in the set of *all* convex bodies.
In particular, we shall present complete solutions of known problems from combinatorial geometry for the class of belt bodies. More precisely, we shall give results of the Hadwiger–Gohberg covering problem, of the Helly dimension problem and of the minimal fixing system problem.

JiříMatoušek (Prag):

A Helly-type theorem for unions of convex sets

We prove that for any $d, k \geq 1$ there exist numbers $q = q(d,k)$ and $h = h(d,k)$ such that the following holds: Let $\mathcal{K}$ be a family of subsets of the d–dimensional space, such that the intersection of any at most q sets of $\mathcal{K}$ can be expressed as a union of at most k convex sets. Then the Helly–number of $\mathcal{K}$ is at most h. We also obtain topological generalizations of some cases of this result.

Shmuel Onn (Haifa):

Partitionable and shellable complexes and posets

The property of *Shellability* is of fundamental importance in the combinatorial and algorithmic theory of Convex Polytopes and Simplicial Complexes. Pictorially, a shelling is a sewing order of the polytope from its facets, where each facet is attached in turn to the previous ones along a single seam.
Related is the broader property of *Partitionability.* Tutte considered it for matroids and Stanley raised the question whether all simplicial spheres are partitionable. We discuss the computational complexity of Shellability and Partitionability: for example, the later is not seen to be in $\mathbb{NP}$ or co–$\mathbb{NP}$. We extend Partitionability to Partially Ordered Sets, show how to read off the *flag h–vector* of a partitionable poset, and give an algorithm for partitioning the barycentric subdivision.
We discuss two classes of (likely nonshellable) spheres, each strictly containing convex polytopes: *Polyhedral Cone Fans*, which we show to be partitionable, and *Oriented Matroid Polytopes*, which we conjecture to be. We show that both classes satisfy McMullen's upper bound theorem in the simplicial case. Various open questions on the subject are raised.
(Partly joint work with Peter Kleinschmidt.)

János Pach (Budapest):

Some geometric Ramsey theorems

Theorem. *Let $f(n)$ denote the largest number such that any family of n plane convex sets has either $f(n)$ pairwise disjoint members or $f(n)$ pairwise intersecting members. Then*

$$n^{1/5} \leq f(n) \leq n^{\log 4/\log 27}.$$

Theorem. *Any complete graph drawn in the plane by straight–line segments, whose edges are coloured with two colours has a monochromatic non–selfintersecting spanning tree.*

Theorem. *Any complete graph of $3n-1$ vertices which is drawn in the plane by straight–lines segments, and whose edges are coloured with two colours, contains n pairwise disjoint edges of the same colour.*

(Joint work with Gyula Károly and Géza Tóth.)

Diethard Pallaschke (Karlsruhe):

Quasidifferentiable functions and minimal pairs of compact convex sets

According to V. Demyanov and A. Rubinov a directionally differentiable function defined on an open subset of a real normed vector space X is called *quasidifferentiable* if the directional derivative (as a function of the direction) can be expressed as a difference of two continuous sublinear functions. Since every continuous sublinear function is uniquely determined by its subdifferential at the origin, there is a natural correspondence

between the directional derivatives of quasidifferentiable functions and pairs of nonempty compact convex sets in the topological dual X' endowed with the weak—*-topology.
This observation leads to the problem of characterizing inclusion minimal representants for elements of the Radström-Hörmander lattice of equivalence classes of pairs of nonempty compact convex sets in a real topological vector space.
Different type of sufficient criteria for inclusion minimal representants of pairs of nonempty compact convex sets in the Radström–Hörmander lattice as well as a cutting plane algorithm for reducing pairs of compact convex sets are presented.
Furthermore some properties of equivalent minimal pairs of nonempty compact convex sets are stated.

Panos M. Pardalos (Gainesville):

Continuous approaches to discrete optimization problems

A large class of discrete optimization problems can be formulated as continuous nonconvex optimization problems. New properties and efficient algorithms have resulted from these continuous formulations. We discuss continuous approaches to several discrete problems, including the satisfiability problem, the maximum clique problem, the traveling salesman problem and the Steiner problem on graphs.

Svatopluk Poljak (Passau):

The facial structure of the set of correlation matrices

A semidefinite matrix X with all diagonal entries $x_{ii} = 1$ is called a *correlation matrix*. The set of all correlation matrices of size $n \times n$ is denoted by $\mathcal{E}^{n\times n}$. $\mathcal{E}^{n\times n}$ is convex but not polyhedral.

Theorem. *$\mathcal{E}^{n\times n}$ has precisely 2^{n-1} vertices, each being of the form xx^T with $x \in \{-1,1\}^n$.*

Theorem. *$\mathcal{E}^{n\times n}$ has some polyhedral k-faces for all k satisfying $\binom{k+1}{2} \leq n-1$.*

We also decribe normal cones and minimal face containing a given matrix. Results are motivated by discrete optimization.
(Joint work with M. Laurent.)

Ricky Pollack (New York):

Bounding the number of geometric permutations induced by k–traversals

Theorem. *A $k-1$ separated family of n compact convex sets in $\mathbb{R}^d$ can be met by a k–traversal in at most $O\left(d^{d^2}\left(\binom{2^{k+1}-2}{k}\binom{n}{k+1}\right)^{k(d-k)}\right)$ different order types, which for d, k fixed is $O\left(n^{k(k+1)(d-k)}\right)$.*

Definition. *A family is k–*separated *if it has no $k-1$ traversal.*

The proof of the theorem depends on the following proposition and theorem.

Proposition. *The orientation of a $(d+1)$–tuple, P, of points in general position in $\mathbb{R}^d$ is determined by the order type of the normal vectors to any set of oriented hyperplanes separating each nonempty subset of P from its complement.*

Theorem. *Let $\mathcal{V}$ be a variety of real dimension k' which is the zero set of $P \in R[x_1, \ldots, x_k]$ and* $\deg P \leq d$. *Given polynomials $\mathcal{P} = \{P_1, \ldots, P_s\} \subset R[x_1, \ldots, x_k]$, each of degree $\leq d$ the number of* cells *of $\mathcal{P}$ over $\mathcal{V}$ is* $\binom{s}{k'}(O(d))^k$.

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The variety we use is $G_{k,d}$, the k subspaces of $\mathbb{R}^d$. A cell of $\mathcal{P}$ over $\mathcal{V}$ is a connected component of $\{x \in \mathcal{V} | P_1(x)\delta_1, P_2(x)\delta_2, \ldots, P_s(x)\delta_s\}$ where $\delta_i \in \{< 0, = 0, > 0\}$.
(Joint work with E. Goodman and R. Wenger.)

Nagabhushane Prabhu (Weat Lafayette):

Sections of polytopes

Consider the problem of intersecting the relative interiors of all the j-faces of a d–polytope by a flat of dimension k. We show that there exist d–polytopes with arbitrarily large numbers of vertices the relative interior of all of whose j-faces can be intersected by a flat of dimension $2(d-j)$. Further we show that if a k–flat intersects the relative interiors of all the j–faces of a d–polytope then $k = \min\{d, 2(d-j)\}$ thereby proving a tight lower bound for the problem.
In connection with a different problem concerning sections of polytopes we show that it is possible to position two d–simplices $\Delta_1, \Delta_2 \subset \mathbb{R}^d$ such that $0 \in \text{int}(\Delta_1) \cap \text{int}(\Delta_2)$ and every hyperplane through the origin contains no more than one of the $2d+2$ facets of Δ_1 and Δ_2. The above configuration of simplices leads to a counter-example to a certain conjecture.

William R. Pulleyblank (Yorktown Heights):

On the strength of cuts in integer programming

Let $\tilde{x}$ be a fractional vertex of a polyhedron $P = \{x : Ax \le b\}$. Let $A^= x = b^=$ be the set of inequalities tight for $\tilde{x}$ and let C be the cone generated by the rows of $A^=$. The valid cuts are inequalities $a^T x \le \lfloor a\tilde{x} \rfloor$ for integral $a \in C$. The *strength* of the cut $ax \le \lfloor a\tilde{x} \rfloor$ is defined to be $\langle a\tilde{x} \rangle / ||a||$, where $\langle \cdot \rangle$ denotes the fractional part. We describe a pseudo polynomial method for finding a strongest cut and show how this can be used to produce a polynomial cutting plane algorithm for integer programs of dimension 2.

Peter Recht (Dortmund):

Partial derivatives in non–smooth–optimization

We will construct a "generalized gradient" for the investigation of a broad class of nondifferentiable functions. Starting from this tool, we deduce in a natural way "generalized partial derivatives" as uniquely determined real numbers. Using this instrument, we can provide a variety of local information of non–smooth functions, e.g. necessary and sufficient conditions in the nondifferentiable situation.

Uriel G. Rothblum (Haifa):

Linear problems and linear algorithms

Using predicate logic, the concept of a linear problem is formalized. The class of linear problems is huge, diverse, complex, and important. *Linear and randomized linear algorithms* are formalized. For each linear problem, a randomized linear algorithm is constructed that *completely solves* it, that is, for any data of the problem, the output set of the randomized linear algorithm is identical to the solution set of the problem. A single machine, called a Universal Randomized Linear Machine, completely solves every linear problem. Also, for every randomized linear algorithm a linear problem is constructed that the algorithm completely solves. These constructions establish a one–to–one and onto correspondence from equivalence classes of linear problems to equivalence classes of randomized linear algorithms.

Joseph Stoer (Würzburg):

Infeasible interior point algorithms for linear complementary problems

There exist many interior point algorithms for linear, convex quadratic programs and for the linear complementary problem (LCP). These problems are all of the same general form when defined geometrically. Based on some properties of such geometric (monotone) LCP's, we propose and analyze a simple infeasible–interior–point algorithm for solving them. The algorithm is a path–following method of predictor–corrector type for pursuing a central path. It features global convergence, polynomial time convergence if there exist a solution that is "smaller" than the initial point, and quadratic convergence if there exists a strictly complementary solution.
(Joint work with S. Mizuno and F. Jarre.)

Rekha R. Thomas (Berlin):

Variation of cost functions in integer programming

Let $IP_{A,c}(b) = \max\{c^T x : Ax = b, x \in \mathbb{N}^n\}$. Two cost functions c_1 and c_2 are equivalent (w.r.t. A) if $IP_{A,c_1}(b)$ and $IP_{A,c_2}(b)$ have the same set of optimal solutions for all $b \in \mathbb{Z}^n$.
We show that each such equivalence class is an open polyhedral cone in $\mathbb{R}^n$, the collection of which form a polyhedral fan called the *Gröbner fan* of A. This is the outer normal fan of an $(n-d)$–polytope $St(A)$, called the state polytope of A. The set of edge directions of $St(A)$ is shown to be the universal Gröbner basis of A and the family of polytopes $\operatorname{conv}\{x \in \mathbb{N}^n : Ax = b\}$ as b varies over $\mathbb{Z}^m$.
(J work with Bernd Sturmfels.)

Eckhard Weidner (Trier):

Back to the future? A missing link in linear programming

We consider the method of center–of–gravity cuts based on Grünbaum's result a predecessor of the ellipsoid method. According to that, we present a predecessor (missing link) of Karmarkar's algorithm which captures the essential features of projective transformations and inscribed and circumscribed ellipsoids. The algorithm has the same complexity as the ellipsoid method, hence it is essentially independent of the number of constraints, and also applies to convex bodies given by separation oracles.
(This is joint work with Peter Gritzmann.)

Günter M. Ziegler(Berlin):

Richter–Gebert's universality theorem for 4–dimensional polytopes

The *realization space* of a d–dimensional polytope P is the space of all polytopes $P' \subset \mathbb{R}^d$ that are combinatorially equivalent to P, modulo affine transformations. We report on very recent work by Jürgen Richter-Gebert, which shows that realization spaces of 4–dimensional polytopes can be "arbitrarily bad": Namely, for every primary semialgebraic set V defined over $\mathbb{Z}$, there is a 4–polytope $P(V)$ whose realization space is "stably equivalent" to V. This implies that the realization space of a 4-polytope can have the homotopy type of an arbitrary finite simplicial complex, that all algebraic numbers are needed to realize all 4–polytopes, that there is no finite set of obstructions for realizability of a 3–sphere, and that the realizability problem is NP–complete. The proof is constructive.
These results sharply contrast the 3–dimensional case, where realization spaces are contractible and all polytopes are realizable with integral coordinates (Steinitz' Theorem). So far no similar universality result was known for any fixed dimension. Thus Richter-Gebert's results represent a substantial break–through in several basic problems of polytope theory — he solved all the basic questions on 4–polytopes that arose in view of Steinitz' work on 3–polytopes more than seventy years ago.

Berichterstatter: T. Burger, A. Hefner, E. Weidner

Tagungsteilnehmer

Prof.Dr. Farid Alizadeh
RUTCOR, Centre of Operations
Research, Mathematical Sciences
Rutgers University
P.O. Box 5062

New Brunswick , NJ 08903-5062
USA

Dr. Dietmar Cieslik
Fachrichtung Mathematik/Informatik
Universität Greifswald
Friedrich-Ludwig-Jahn-Str. 15a

17489 Greifswald

Prof.Dr. Imre Barany
Mathematical Institute of the
Hungarian Academy of Sciences
P.O. Box 127
Realtanoda u. 13-15

H-1364 Budapest

Prof.Dr. Ludwig W. Danzer
Fachbereich Mathematik
Universität Dortmund

44221 Dortmund

Prof.Dr. Jürgen Bokowski
Fachbereich Mathematik
TH Darmstadt
Schloßgartenstr. 7

64289 Darmstadt

Prof.Dr. Robert M. Freund
MIT
Sloan School of Management
50 Memorial Dr.

Cambridge , MA 02142-1347
USA

Thomas Burger
Fachbereich IV
Abteilung Mathematik
Universität Trier

54286 Trier

Prof.Dr. Komei Fukuda
Graduate School of Systems Managem.
University of Tsukuba
3-29-1 Otsuka
Bunkyo-ku

Tokyo 112
JAPAN

Prof.Dr. James V. Burke
Dept. of Mathematics
GN-50
University of Washington
C138 Padelford Hall

Seattle , WA 98195
USA

Prof.Dr. Peter Gritzmann
Fachbereich IV
Abteilung Mathematik
Universität Trier

54286 Trier

Prof.Dr. Pierre Hansen
GERAD, Ecole des Hautes Etudes
Commerciales - Departement des
Methodes Quantitative et Sys. d'Inf
5255 avenue Decelles

Montreal, Quebec H3T 1V6
CANADA

Andreas Hefner
Wirtschaftswissenschaftliche
Fakultät
Universität Passau

94030 Passau

Dr. Martin Henk
Konrad-Zuse Zentrum
Heilbronner Str. 10

10711 Berlin

Prof.Dr. Dorit S. Hochbaum
Department of Industrial
Engineering and Operations Research
University of California
Etcheverry Hall

Berkeley , CA 94720-1777
USA

Prof.Dr. Reiner Horst
Fachbereich IV
Abteilung Mathematik
Universität Trier

54286 Trier

Alexander Hufnagel
Fachbereich IV
Abteilung Mathematik
Universität Trier

54286 Trier

Prof.Dr. Marek Karpinski
Institut für Informatik
Universität Bonn
Römerstraße 164

53117 Bonn

Prof.Dr. Petar S. Kenderov
Institute of Mathematics with
Computer Center
Bulgarian Academy of Sciences
P. O. Box 373

Sofia 1113
BULGARIA

Prof.Dr. Victor L. Klee
Dept. of Mathematics
GN-50
University of Washington
C138 Padelford Hall

Seattle , WA 98195
USA

Prof.Dr. Peter Kleinschmidt
Wirtschaftswissenschaftliche
Fakultät
Universität Passau

94030 Passau

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DFG Deutsche Forschungsgemeinschaft

Prof.Dr. Jeffrey C. Lagarias
AT & T Bell Laboratories
P.O. Box 636
600 Mountain Avenue

Murray Hill , NJ 07974-0636
USA

Prof.Dr. Janos Pach
Mathematical Institute of the
Hungarian Academy of Sciences
P.O. Box 127
Realtanoda u. 13-15

H-1364 Budapest

Prof.Dr. David G. Larman
Department of Mathematics
University College London
Gower Street

GB-London , WC1E 6BT

Prof.Dr. Diethard Pallaschke
Institut für Statistik und
Mathematische Wirtschaftstheorie
Universität Karlsruhe

76128 Karlsruhe

Prof.Dr. Horst Martini
Fakultät für Mathematik
Technische Universität
Chemnitz-Zwickau
Reichenhainer Str. 41

09126 Chemnitz

Prof.Dr. Panos M. Pardalos
Dept. of Industrial & Systems Eng.
Center f. Optimiz. & Combinatorics
University of Florida
303 Weil Hall

Gainesville , FL 32611-2083
USA

Prof.Dr. Jiri Matousek
Department of Applied Mathematics
Charles University
Malostranske nam. 25

118 00 Praha 1
CZECH REPUBLIC

Prof.Dr. Svatopluk Poljak
Universität Passau
Fakultät für Mathematik und
Informatik
Innstr. 33

94032 Passau

Dr. Shmuel Onn
Industrial Engineering - Management
Science - Technion
Israel Institute of Technology

Haifa 32000
ISRAEL

Prof.Dr. Richard Pollack
Courant Institute of
Mathematical Sciences
New York University
251, Mercer Street

New York NY 10012-1110
USA

Dr. Nagabhushana Prabhu
Dept. of Economics
Purdue University

West Lafayette , IN 47907-1395
USA

Prof.Dr. Josef Stoer
Institut für Angewandte Mathematik
und Statistik
Universität Würzburg
Am Hubland

97074 Würzburg

Prof.Dr. William R. Pulleyblank
IBM Corporation
Thomas J. Watson Research Center
P. O. Box 218

Yorktown Heights , NY 10598
USA

Dr. Rekha Thomas
Cowles Foundation
Yale University
Box 2125, Yale Station

New Haven , CT 06520
USA

Prof.Dr. Peter Recht
FB Wirtschaftswissenschaften
Universität Dortmund
Postfach 500500

44221 Dortmund

Eckhard Weidner
Fachbereich IV
Abteilung Mathematik
Universität Trier

54286 Trier

Prof.Dr. Uriel G. Rothblum
Faculty of Industrial Engineering &
Management
Technion
Israel Institute of Technology

Haifa 32000
ISRAEL

Prof.Dr. Günter M. Ziegler
Fachbereich Mathematik
Sekr. 6 - 1
Technische Universität Berlin
Straße des 17. Juni 135

10623 Berlin

e–mail–Adressen

Alizadeh, Farid	alizadeh@rutcor.rutgers.edu
Bárány, Imre	h2923bar@ella.hu
Bokowski, Jürgen	bokowski@mathematik.th-darmstadt.de
Burger, Thomas	burger@dm7.uni-trier.de
Burke, Jim	burke@math.washington.edu
Cieslik, Dietmar	cieslik@math-inf.uni-greifswald.de
Danzer, Ludwig W.	office@steinitz.mathematik.uni-dortmund.de (mail by fax prefered: ++49-231-755-5307)
Freund, Robert M.	rfreund@mit.edu
Fukuda, Komei	fukuda@ifor.math.ethz.ch
Gritzmann, Peter	gritzmann@dm1.uni-trier.de
Hansen, Pierre	pierreh@crt.umontreal.ca
Hefner, Andreas	hefner@winf.uni-passau.de
Henk, Martin	henk@math.tu-berlin.de
Hochbaum, Dorit	dorit@hochbaum.berkeley.edu
Horst, Reiner	horst@orsun3.uni-trier.de
Hufnagel, Alexander	hufnagel@dm4.uni-trier.de
Karpinski, Marek	marek@cs.uni-bonn.de
Kenderov, Petar	pkend@bgcict.bitnet
Klee, Victor L.	klee@math.washington.edu
Kleinschmidt, Peter	kleinsch@winf.uni-passau.de
Lagarias, Jeff	jcl@research.att.com
Larman, David	dgl@ucl.ae.uk
Martini, Horst	horst.martini@mathematik.tu-chemnitz.de
Matoušek, Jiří	matousek@kam.mff.cuni.cz
Onn, Shmuel	onn@ie.technion.ac.il
Pach, János	pach@cims6.nyu.edu
Pallaschke Diethard	lh09@dkuni2.bitnet
Pardalos, Panos	pardalos@ufl.edu
Poljak, Svatopluk	poljak@fmi.uni-passau.de
Pollack, Richard	pollack@geometry.nyu.edu
Prabhu, Nagabtrushana	prabhu@ecn.purdue.edu
Pulleyblank, Bill	wrp@watson.ibm.com
Recht, Peter	or-pere@wiso.wiso.uni-dortmund.de
Rothblum, Uri	rothblum@ie.technion.ac.il
Stoer, Josef	stoer@vax.rz.uni-wuerzburg.d400.de
Thomas, Rekha R.	thomas@zib-berlin.de
Weidner, Eckhard	weidner@dm8.uni-trier.de
Ziegler, Günter M.	ziegler@math.tu-berlin.de