Tagungsbericht 6/1995

Algebraic and Geometric Combinatorics

5. – 12. 2. 1995

The topic for this workshop (organized by Anders Björner, Stockholm, Gil Kalai, Jerusalem, and Günter M. Ziegler, Berlin) was the increasingly active area of contact between algebraic combinatorics and geometry/topology. The fertile interaction in recent years has led to much progress and new points of view, and an increasing number of mathematicians from both sides are now working in this border territory. The meeting brought together a young and enthusiastic group of participants. The overall feeling expressed to the organizers was that the chosen topic is relevant and well-focused (despite the diverse mathematical backgrounds of the participants) and inspiring for future work. The level of interaction was high.

The program consisted of two one-hour lectures each morning (some of them invited in advance) and four half-hour talks mainly in the late afternoon. There were spontaneously organized informal sessions on the following topics: Integer programming, Triangulations, Algebraic shifting, Arrangements of hyperplanes, and Ornaments (Vassiliev-type invariants). Informal discussions were intensive during the afternoons and evenings, and much progress on several projects was made during the meeting.

The excitement and value of the workshop was heightened by the presentation of several recent major breakthroughs (presented here for the first time), such as J. Richter-Gebert's work on 4-dimensional polytopes settling in a unified way a whole host of classical problems in that area, the construction of nonshellable fans of convex cones by P. Mani and N. Mnëv as a byproduct of work on smoothing of manifolds, the work of T. Braden and R. MacPherson that proves the long-standing monotonicity conjecture for Kazhdan-Lusztig polynomials of Weyl groups and makes progress on a conjecture of Kalai on g-polynomials of polytopes, G. Rybnikov's counterexample to Orlik's conjecture on complex hyperplane arrangements, and J. Rambau's counterexample to the generalized Baues conjecture.

The organizers are grateful to the Oberwolfach Institute and its Förderverein for presenting the opportunity and the resources to arrange this successful meeting.

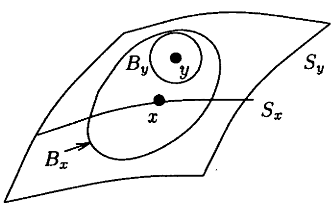
VORTRAGSAUSZÜGE

HÉLÈNE BARCELO:
Lattice of parabolic subgroups associated with Coxeter arrangements

Let L be the lattice consisting of all intersections of hyperplanes in the arrangement associated with a finite real reflection group W. We show that L is isomorphic to the lattice L' consisting of all parabolic subgroups of the reflection group. This isomorphism is used to determine all W for which L is supersolvable. Also when W is irreducible and neither of type A_n nor B_n we used it to show that the only modular elements are $\hat{0}$, $\hat{1}$, and the atoms of L. Let $p_W(t)$ be the characteristic polynomial of L. To every element X of L there corresponds a parabolic subgroup of W denoted $Gal(X)$. As a third application of our isomorphism between L and L' we show that if W is irreducible then an element X of L is modular if and only if $p_{Gal(X)}(t)$ divides $p_W(t)$.
There is a well known combinatorial procedure for the generation of all non-broken circuit bases (NBC bases) of a supersolvable lattice. If the NBC bases of a geometric lattice can be obtained by this procedure, we say that the NBC bases are "obtainable by hands." We show that L is supersolvable if and only if all the NBC bases of L are obtainable by hands.
(This is joint work with E. Ihrig.)

TOM BRADEN:
Intersection homology and polytopes – recent progress

There are (at least) two instances of polynomials of combinatorial interest whose coefficients are Betti numbers of even-dimensional local intersection homology groups $IH_{2i}(B_x)$ for a small ball around a point x in a stratified singular algebraic variety X. These are the g-polynomials of rational polytopes, where X is a toric variety, and Kazhdan-Lusztig polynomials $P_{x,y}$ in Weyl groups, where X is a Schubert variety.
Moving the point x from a larger to a smaller stratum produces maps between these groups: if x, y lie in strata S_x, S_y with $S_x \subset \overline{S_y}$, then the picture



gives induced maps $IH_*(B_y) \to IH_*(B_x)$.
Theorem. If X satisfies the following property, then these maps are always injective: each stratum S has a topological tubular neighborhood $N_S \approx S \times Y$ *which is an open algebraic subvariety* of X.

Both toric varieties and Schubert varieties satisfy this condition, so we get the corollaries:

1. If $g(P)$ is the g-polynomial of a rational polytope P, and F is a face, then $g(P) \geq g(F)$ coefficient by coefficient. (This is a weaker version of a conjecture of G. Kalai, that $g(P) \geq g(F)g(lk_P F)$.)

2. If x, y, z are in a Weyl group W, and satisfy $x < y < z$ in the Bruhat order, then $P_{x,z} \leq P_{y,z}$ coefficient by coefficient.

CLARA CHAN:
Cubical polytopes, cubated spheres

It is well known that the f-vectors of simplicial polytopes are completely characterized in terms of their h-vectors, by McMullen's g-theorem.
Much less is known about the f-vectors of cubical polytopes (= polytopes whose proper faces are all combinatorial cubes). We present work toward finding the convex hull of this set of f-vectors, which is an analog of McMullen-Walkup's Generalized Lower Bound Conjecture for cubical polytopes. Using a mirroring construction on cyclic polytopes, we show that the convex hull of f-vectors of PL cubated spheres contains "Adin's g-cone," i.e., the cone of vectors satisfying $g^c \geq 0$ where g^c corresponds to Ron Adin's "cubical h-vector."
(Joint work with Eric Babson and Louis J. Billera, Cornell Univ.)

JESÚS ANTONIO DE LOERA:
A triangulation with few neighbors

Let $\mathcal{A} = \{a_1, a_2 \ldots, a_n\} \subseteq \mathbb{R}^d$ be a configuration of points such that $\dim(conv(\mathcal{A})) = d$. It is well known that any coherent triangulation of $\mathcal{A}$ (in the sense of Gel'fand-Kapranov-Zelevinsky) has at least $n - d - 1$ neighbors under bistellar moves. Here we presented the following contrasting result:
"There exists a non-coherent triangulation of a configuration in $\mathbb{R}^3$ with 13 points with only 6 bistellar neighbors."
We showed a model of this.

ART DUVAL:
Iterated homology of simplicial complexes

We use the exterior face ring of a simplicial complex to develop an iterated homology theory for simplicial complexes. Let Δ be a simplicial complex of dimension $d-1$. For each $r = 0, \ldots, d$, we define r^{th} *iterated homology groups of* Δ, the $r = 0$ case corresponding to ordinary homology, and the $r = 1$ case corresponding to the homology of Δ' if Δ is a cone over Δ'. If a simplicial complex is shellable (in the generalized nonpure sense of Björner and Wachs), then its iterated Betti numbers (vector-space dimensions of the iterated homology groups over a field) give the restriction numbers h_{ij} of the shelling. Iterated

Betti numbers are preserved by Kalai's algebraic shifting, and may be interpreted combinatorially in terms of the algebraically shifted complex in several ways.
(This is joint work with Lauren Rose.)

JÜRGEN ECKHOFF:
f-vectors of colored complexes and clique complexes

The clique vector of a finite graph G is the sequence $C(G) = (C_1, \dots, C_r)$, where $C_k = C_k(G)$ is the number of k-cliques of G and $r = r(G)$ is the clique number of G. It seems to be very difficult to describe the set of integer vectors which occur as clique vectors of graphs. We propose two conjectures:

Conj. A: $C_{k+1} \leq \partial^r_{k/k+1}(C_k)$, $k = 1, \dots, r-1$.
Conj. B: For each graph G having clique number r, there exists an r-partite graph H with $c(H) = c(G)$.

The "pseudopowers" $\partial^r_{k/k+1}$ in Conj. A were introduced in a paper by Frankl, Füredi & Kalai [Math. Scand. **63** (1988), 169-178] where an analogous statement was proved for the f-vectors of r-colorable complexes (providing a complete characterization of such vectors). Notice that the C-vector of a graph is the f-vector of its clique complex (shifted by one). While Conj. B is, so far, supported by numerical evidence only, Conj. A has been established in a number of special cases.

MICHAEL FALK:
An application of shellability to generalized hypergeometric functions

By a theorem of Esnault-Schechtman-Varchenko, the set of germs of generalized hypergeometric functions at an arrangement $\mathcal{A}$ can be identified in part using the Orlik-Solomon algebra of $\mathcal{A}$. Yuzvinsky used sheaf theory on finite topological spaces to establish an isomorphism of this space with the cohomology of the order complex of $\mathcal{A}$, at which stage powerful combinatorial tools, like shelling, can be used. In this talk we will give some background for these ideas, some fundamental results, and an outline of the procedure used to construct bases for the local system cohomology referred to above. In addition, we hope to state precisely the main theorem of a recent joint paper with H. Terao.

SERGEY FOMIN:
Piecewise-linear maps, total positivity, and pseudoline arrangements

The talk is devoted to two problems that turn out to be closely related to each other:

1. study the piecewise-linear maps related to Lusztig's canonical basis in the quantized enveloping algebra of the Lie algebra of the group of unipotent upper-triangular matrices (u.u.m.);

2. study the variety of totally positive u.u.m.

The relation between these two problems was recently discovered by G. Lusztig. (At the moment, we only consider the A_n case.)
Main results (general description):

1. explicit closed formulas for these piecewise-linear maps that avoid the iteration process from their original definition;

2. formulas for decompositions of a unipotent upper-triangular matrix into a product of elementary Jacobi matrices.

The main method is a combinatorial Ansatz based on a representation of reduced words by pseudoline arrangements.
(This is joint work with A. Zelevinsky and A. Berenstein.)

JACOB E. GOODMAN:
Some combinatorial questions for convex sets on affine Grassmanians

We discuss three unsolved problems growing out of the convexity structure of the affine Grassmanian $G'_{k,d}$ of k-flats at finite distance in $\mathbb{R}^d$ introduced in [1].

1. How many convex point sets are needed to give a "minimal, irredundant presentation" of a set of n k-flats in general position in $\mathbb{R}^d$ as the set of all of their commom k-transversals? (In [1], it is shown that $2(d-1)(n-d)+2^d$ suffice in the special case $k = d-1$; for example, any n lines in the plane, no two parallel, can be minimally and irredundantly presented by $2n$ line segments.)

2. It is proved in [1] that $G'_{1,3}$, the space of lines in $\mathbb{R}^3$, can be partitioned into three non-empty convex sets, but not into two. More generally, the smallest $n > 1$ for which $G'_{k,d}$ has a partition into n convex sets is no more than $\binom{d-1}{k}+1$ and — if the sets are closed under parallels — at least $d-k+1$. What is the correct value of n as a function of k and d?

3. In [2] we establish the first known polynomial upper bound for the number of "geometric permutations" induced on k-flat transversals by a suitably separated family of n compact convex sets in $\mathbb{R}^d$: $O(n^{k(k+1)(d-k)})$, if k and d are fixed. Much better bounds are known in the special case $k = d-1$: $O(n^{d-1})$. Can our upper bound be reduced in the general case, perhaps to $O(n^{(k+1)(d-k)-1})$?

[1] J. E. Goodman and R. Pollack, Foundations of a theory of convexity on affine Grassmann manifolds, Mathematika, to appear.

[2] J. E. Goodan, R. Pollack, and R. Wenger, Bounding the number of geometric permutations induced by k-transversals, preprint.

TAKAYUKI HIBI:

Ehrhart polynomials of convex polytopes

Let $P \subset \mathbb{R}^N$ be an integral convex polytope of dimension d. Given an integer $n \geq 1$, set $nP := \{n\alpha : \alpha \in P\}$ and define $i(P,n) := \#(nP \cap \mathbb{Z}^n)$. Ehrhart proved that $i(P,n)$ is a polynomial in n of degree d with $i(P,0) = 1$. We define the sequence $\delta_0, \delta_1, \delta_2, \ldots$ by

$$(1-\lambda)^{d+1}\left[1 + \sum_{n=1}^{\infty} i(p,n)\lambda^n\right] = \sum_{i=0}^{\infty} \delta_i \lambda^i.$$

Then $\delta_i = 0$ for every $i > d$. We say that $\delta(P) := (\delta_0, \delta_1, \ldots, \delta_d)$ is the δ-vector of P. We study what can be said about the δ-vector of an integral convex polytope P by using algebraic techniques for Cohen-Macaulay rings.

MONIQUE LAURENT:

The geometry of the set of positive semidefinite matrices with diagonal entries

We consider the convex set $\mathcal{E}_n$ consisting of the positive semidefinite symmetric $n \times n$ matrices whose diagonal entries are all equal to one. $\mathcal{E}_n$ is called an *elliptope* and its elements are known as the correlation matrices. One motivation for the study of $\mathcal{E}_n$ comes from combinatorial optimization. Indeed, the matrices xx^T for $x \in \{\pm 1\}^n$ clearly belong to $\mathcal{E}_n$; they correspond to the cuts of the complete graph K_n and for this reason are called the cut matrices. Hence by optimizing a linear objective function on $\mathcal{E}_n$ one obtains an upper bound for the max-cut problem. This upper bound can be computed in polynomial time and a nice recent result of Goemans and Williamson shows that the upper bound is within 13% of the optimum cut.
Another motivation for the study of $\mathcal{E}_n$ comes from the following problem in linear algebra: Given a partial symmetric matrix (i.e., whose entries are specified only on e subset E) decide whether the unspecified entries can be chosen so as to obtain a positive semidefinite matrix. Let G denote the graph on n nodes with edge set E. An obvious necessary condition for a partial matrix to be completable to a psd matrix is that every principal subdeterminant consisting of specified entries be nonnegative. This condition is also sufficient if and only if the graph G is chordal (result by Grone et al.). Other necessary conditions were given by Barrett et al.; they are sufficient for the graphs with no K_4 minor. The convex set $\mathcal{E}_n$ is, in fact, closely related to the convex body $TH(G)$ which was introduced by Grötschel, Lovász and Schrijver as a positive semidefinite relaxation for the stable set problem.
The following facts are known about the facial structure of $\mathcal{E}_n$: It has vertices (namely, the cut matrices). The possible dimension of its faces are known; they form a lacunary interval. The highest possible dimension for a polyhedral face is the largest integer k such that $\binom{k+1}{2} \leq n$.
(Joint work with S. Poljak)

NATI LINIAL:
On the geometry of graphs

The basic idea in this study is to gain information about graphs by viewing them as geometric objects. Similar ideas can be found in the literature, where geometry models the graph in one of two ways:
(1) Represent the topological properties of the graphs (e.g. planarity, embedding to other 2-dim. manifolds etc.)
(2) Represent the adjacency/nonadjacency relationship among vertices (e.g. Koebe-Andreev-Thurston Thm., Lovász and then Goemans-Williamson and Karger-Motwani-Sudan embed graphs on the unit sphere so that adjacent vertices get mapped to remote points, Linial-Lovász-Wigderson characterized graph connectivity through "convex embedding" in Euclidean space etc.).
Here we attempt to model correctly the metric of the graph. While isometric embeddings of graphs were already looked at previously, we consider a more relaxed notion where distances are allowed to be distorted to some extent. Specific results are: Characterization of the least distortion with which a given graph can be embedded in ℓ_2 (always $O(\log n)$ by a result of Bourgain), and efficient algorithms to find such embeddings. Similarly for embedding in other ℓ_p spaces. Another set of results has to do with graphs whose embeddings are better than the worst case. Such graphs have small balanced separators, they have good low-diameter decompositions (in the sense of Linial-Saks) and good low-diameter covers (à la Awerbuch-Peleg). Finally the gap between max-flow and min-cut in multicommodity flow can be interpreted within our framework and can be shown to be $O(\log k)$, where k is the number of source-sink pairs.
Many open problems remain in this area.
An early version of this work appeared in FOCS'94. An updated version can be obtained via e-mail from nati@cs.huji.ac.il

PETER MANI-LEVITSKA:
Convex polytopes and smooth manifolds

We know that every (compact) differentiable manifold can be triangulated. In the opposite direction, several efforts have been made to understand the obstructions for imposing a smooth structure on a piecewise linear manifold. We have been thinking about this problem in the framework of J. Munkres' smoothing theory, and came up with the following answer: Let M be a compact topological manifold, and $\tau = (C, f)$ a triangulation of M. M has a smooth atlas if, and only if, for every vertex v of C, there exists a convex polytope P_v and a simplicial isomorphism $\varphi_v : link(v, C) \to \partial P_v$.
Among the corollaries are two negative answers to fairly old problems:

- There exist nonshellable fans.
- There exist simplicial complexes A, B with $\bigcup A = \bigcup B$ such that one cannot find a common multiple stellar subdivision.

Many other questions, however, are still open.
(This is joint work with Nikolai Mnëv.)

JIŘÍ MATOUŠEK:
Combinatorial bounds for discrepancy

Recently, it has been shown that tight or almost tight upper bounds for the discrepancy of many geometrically defined set systems can be derived from simple combinatorial parameters of these set systems. Namely, if the *primal shatter function* of a set system $\mathcal{R}$ on an n-point set X is bounded by $const \cdot m^d$, then $|\mathcal{R}| = O(n^{1/2-1/2d})$ (which is known to be tight), and if the *dual shatter function* is bounded by $const \cdot m^d$, then $|\mathcal{R}| = O(n^{1/2-1/2d}\sqrt{\log n})$. We prove that for $d = 2, 3$, the latter bound also cannot be improved in general. We also show that bounds on the shatter functions alone do not imply the average (L_1) discrepancy to be much smaller than the maximum discrepancy; this contrasts results of Beck and Chen for certain geometric cases. In the proof we give a construction of a certain asymptotically extremal bipartite graph, which may be of independent interest.

PETER MCMULLEN:
Tensor weights and polytope algebras

The universal abelian group for valuations on polytopes is the polytope (or Minkowski) ring Π; Minkowski addition induces a multiplication on Π. It is known (Pukhlikov & Khovanskii) that the quotients of Π by powers of the ideal corresponding to translations are (essentially) graded algebras. They have families of separating functions taking tensor values — basically, volume, moment vector, inertia tensor, and so on, on faces of polytopes.
This suggests developing an independent algebra of tensor valued weights on polytopes. These are governed by the Minkowski connexions — analogous of the Minkowski relations for scalar valued weights — on each face of a polytope. With a multiplication geometrically induced by Minkowski addition, a graded algebra results. If P is a simple polytope, the corresponding algebra $\Omega(P)$ is (almost certainly) isomorphic to the face ring of the dual polytope P^* — the Hilbert functions are the same.

ALEXANDER B. MERKOV:
Finite-order invariants of ornaments

An ornament is a collection of plane curves no three of which intersect at the same point. The homotopy classification of ornaments is a model example of a wide class of similar problems about submanifolds with restrictions on their singularities and mutual disposition. We investigate the finite-order invariants of ornaments generalizing Vassiliev invariants of knots and links. This problem leads naturally to the homological and combinatorial study of generalized "k-equal manifolds," connected multigraphs, and configuration spaces. On the other hand, the invariants guessed by means of homological calculations allow very classical and elementary descriptions.

RICKY POLLACK:

Complexity and algorithms in real algebraic geometry — recent progress

Theorem 1. (Quantifier Elimination)
Given a real closed field R, a family $\mathcal{P} = \{P_1, \ldots, P_s\}$ of s polynomials in $k+\ell$ variables, $X_1, \ldots, X_k, Y_1, \ldots, Y_\ell$ with coefficients in $D \subset R$ that have degree at most d, and a first-order formula

$$\Phi(Y) \quad = \quad (Q_\omega X^{[\omega]}) \ldots (Q_1 X^{[1]}) F(P_1, \ldots, P_s),$$

where $Q_i \in \{\forall, \exists\}$, $Q_i \neq Q_{i+1}$, $Y = (Y_1, \ldots, Y_\ell)$ is a block of ℓ free variables, $X^{[i]}$ is a block of k_i variables, $\sum_{1 \leq i \leq \omega} k_i = k$, and $F(P_1, \ldots, P_s)$ is a quantifier-free Boolean formula with atomic predicates of the form

$$P_i(Y, X^{[\omega]}, \ldots, X^{[1]}) \; \sigma \; 0, \qquad 1 \leq i \leq s,$$

where $\sigma \in \{>, <, =\}$, there exists an equivalent quantifier-free formula,

$$\Psi(Y) \quad = \quad \bigvee_{i=1}^{I} \bigwedge_{j=1}^{J_i} (P_{ij}(Y) \; \epsilon_{ij} \; 0),$$

where $P_{ij}(Y)$ are polynomials in the variables Y, $\epsilon_{ij} \in \{>, <, =\}$

$$I \leq s^{(\ell+1)\Pi_i(k_i+1)} d^{(\ell+1)\Pi_i O(k_i)},$$

$$J_i \leq s^{\Pi_i(k_i+1)} d^{\Pi_i O(k_i)},$$

and the degrees of the polynomials $P_{ij}(y)$ are bounded by $d^{\Pi_i O(k_i)}$. Moreover, there is an algorithm to compute $\Psi(Y)$ using

$$s^{(\ell+1)\Pi(k_i+1)} d^{(\ell+1)\Pi O(k_i)}$$

arithmetic operations in D.

Theorem 2.
Suppose $Q, P_1, \ldots, P_s \in R[x_i, \ldots, x_k]$ have degrees $\leq d$ and the variety $\mathcal{V} = \{x \in R^k | Q(x) = O\}$ has dimension k', then the number of cells (connected components of realizable (over $\mathcal{V}$) sign conditions of $P_1, \ldots P_s$) is bounded by

$$\binom{s}{k'} O(d)^k.$$

(This is joint work with S. Basu and M.-F. Roy.)

JÖRG RAMBAU:

A counterexample to the "Generalized Baues Conjecture"

Associated with every projection of polytopes $\pi : P \to \pi(P)$ one has a partially ordered set of all "locally coherent strings" in P: the families of proper faces of P that project to valid subdivisions of $\Pi(P)$, partially ordered by the natural inclusion relation.
The "Generalized Baues Conjecture" posed by Billera, Kapranov & Sturmfels asked whether this partially ordered set always has the homotopy type of a sphere of dimension $\dim(P) - \dim(\pi(P)) - 1$.
The special case $\dim(\pi(P)) = 1$ appeared as a conjecture of Baues in the theory of combinatorial models of loop spaces and has been proven by Billera, Kapranov & Sturmfels.
It turns out that the conjecture also holds for $\dim(P) - \dim(\pi(P)) \leq 2$, but fails otherwise.
There is an explicit counterexample $\pi : P \to \pi(P)$, where P is a 5-dimensional simplicial 2-neighborly polytope with 10 vertices and 42 facets and $\pi(P)$ is a hexagon in $\mathbb{R}^2$.
The construction is based on an analysis of the geometric relation between the various normal cones of fibers of the projection.
(This is joint work with G. M. Ziegler, Berlin.)

VICTOR REINER:

Triangulations of cyclic polytopes: the higher Stasheff-Tamari posets

We consider a partial order $S(n,d)$ on the set of all triangulations of a cyclic d-polytope $C(n,d)$ with n vertices. In the case $d = 2$, this is the well-known Tamari poset T_n on triangulations of an n-gon. For $d = 3$ we prove a simple encoding of this partial order which shows that $S(n,d)$ is a lattice for $d \leq 3$ in which open intervals (x,y) have the homotopy type of either a sphere or ball, and that the set of all triangulations of $C(n,d)$ is connected by bistellar operations for $d \leq 5$.
(Joint with P.H. Edelmann, Univ. of Minnesota)

JÜRGEN RICHTER-GEBERT:

Realization spaces of 4-polytopes are universal

Studying the set of all convex polytopes with n vertices in dimension d leads to two major questions:

- what *combinatorial types* occur?
- what does the *"space of all realizations"* of a combinatorial type look like?

It is the purpose of the talk to show that one cannot expect "nice" answers to these questions:

- boundary complexes of 4-polytopes cannot be characterized locally

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- realization spaces of 4-polytopes can be "arbitrarily bad" (i.e., stably equivalent to any given semialgebraic set V).

So far no similar result was known for any fixed dimension.
The proof proceeds by modelling elementary additions and multiplications as constraints in realization spaces of "Addition Polytopes" and "Multiplication Polytopes." These polytopes are used to encode the defining equations of a semi-algebraic set into the boundary structure of a 4-polytope.

GRIGORI RYBNIKOV:
Non-matroid topological invariants of complex hyperplane arrangements

We consider arrangements of hyperplanes in a complex projective space. The homotopy type of the complement M of such an arrangement is the main object of our investigation. It is well known that the cohomology ring $H^*(M, \mathbf{Z})$ depends only on the matroid of the arrangement. We prove that the fundamental group of M is not determined by the matroid uniquely.
We use an algebraic invariant of the lower central series of the fundamental group $\pi_1(M)$, which appears to be a triple Massey product on $H^1(M, \mathbf{Z})$. Unlike Massey products on $H^*(M, \mathbf{C})$, the Massey product on the integer cohomology ring can be non-trivial. Using this we show for two combinatorially equivalent arrangements of eight lines on $\mathbf{C}P^2$ that there is no isomorphism of the fundamental groups of their complements which agrees to the canonical isomorphism of first homology groups. This leads to construction of two line arrangements of thirteen lines with *the same underlying matroid* such that the *fundamental groups of their complements are not isomorphic.*

KARANBIR S. SARKARIA:
Combinatorics $\longleftrightarrow$ Topology

In this seminar we have been looking at many diverse combinatorially defined homologies of a simplicial complex K. Let's look at a few of these:
(a) *Subcochain complexes of* $(C^*(K_{assoc}), \delta)$, the usual cochain complex of functions f on vertex sequences, are given by imposing on the f's the requirement $f(v_0v_1\ldots v_q) = (-1)^\pi f(v_{\pi 0}v_{\pi 1}\ldots v_{\pi q})$ as π runs over (i) all permutations (ii) only rotations and reversals (iii) rotations only (iv) reversals only and (v) id's (no other permutation groups will work). Of these homologies perhaps the most interesting are (ii) and (iii) for which (over characteristic zero) the answer is

$$H^*_{dih}(K_{assoc}) \cong \bigoplus_{j\geq 0} H^{*-4j}(K) \quad \text{and} \quad H^*_{cycl}(K_{assoc}) \cong \bigoplus_{j\geq 0} H^{*-2j}(K).$$

Now let us look at **(b)** *some subchaincomplexes of* $(C^*(K_{comm}), \delta)$, the chain complex determined by increasing (with respect to a chosen linear order on vert(K)) vertex sequences: viz. for each $r \geq 1$, we can ask that no vertex repeats more than r times. Then Bier has shown that for r odd one gets the usual homology, but

$$\text{for } r \text{ even} \qquad \widetilde{H}_*(K_{comm,r}) \;\cong\; \bigoplus_{\sigma\in K} H_*(lk_K(\sigma)).$$

This result is close to **(c)** ***homology of some deleted joins***, viz. define

$$K \bullet K \;:=\; \{(\sigma,\theta)\in K\cdot K \text{ the join } |\sigma\cap\theta=\emptyset,\ \sigma\cup\theta\in K\}.$$

Then one has

$$\widetilde{H}_*(K\bullet K) \;\cong\; \bigoplus_{\sigma\in K} \widetilde{H}_*(lk_K(\sigma)).$$

(The homology of the full deleted join $K * K = \{(\sigma,\theta)|\sigma\cap\theta=\emptyset\}$ is more interesting and harder. Note also that one has similar definitions $K *_G K$, $K \bullet_G K$ for all groups G and their homologies are all very interesting.)
This result is best explained via **(d)** *some spaces of* K*:* Instead of just looking at $|K|$, i.e., $Conv(K) = \bigcup_{\sigma\in K} conv(\sigma)$ in $\mathbb{R}^{vert(K)}$, it makes equal sense to look e.g. at $Aff(K) = \bigcup_{\sigma\in K} aff(\sigma)$ and $Lin(K) = \bigcup_{\sigma\in K} lin(\sigma)$. The latter is contractible, but with very nice coordinate ring (so enters algebraic geometry...) and a very nice link $Sph(K)$ at the origin. In fact $K\bullet K$ triangulates $Sph(K)$, so above formula follows by Goresky-MacPherson (or can be proved directly). Likewise the simplicial space $K \bullet_{S^1} K$ — or its cyclic counterpart — triangulates $Sph_{\mathbb{C}}(K)$: one can also consider above spaces over p-adics and use ℓ-adic cohomologies. The equivariant homology of $K\bullet K$: this gives that of $Proj(K)$, and of $K\bullet_{S^1} K$ that of $Proj_{\mathbb{C}}(K)$ (the quotients of $Sph(K)$ and $Sph_{\mathbb{C}}(K)$ under $\mathbb{Z}/2$ and S^1, respectively).
(e) ***Non-abelian chains***, i.e. the free non-abelian group $F_*(K_{comm})$ generated by K_{comm}, was used by Moore to define a homology $H_*^{Moore}(K_{comm}) = ker\partial_0/im\partial_0$ where

$$(ker(\partial_1\cap ker\partial_2\cap\ldots) \xrightarrow{\partial_0} (ker(\partial_1\cap ker\partial_2\cap\ldots)$$

(here ∂_i denotes the omission of the ith vertex; one checks that $im\partial_0$ is indeed a normal subgroup of $ker\partial_0$) which was calculated by Milnor:

$$H_*^{Moore}(K_{comm}) \;\cong\; \pi_*(S^0\cdot K).$$

I'll wind up by finally saying that **(f)** ***one can use characters other than*** $(-1)^i$ ***also:*** for example if the coefficients contain p-th roots of unity one can equip $C_*(K_{comm})$ with operator

$$\partial(v_0v_1\ldots) = \sum_r \omega^r(v_0v_1\ldots\widehat{v_r}\ldots), \qquad \omega=\exp(\frac{2\pi i}{p}),$$

which now obeys $\partial^p=0$, and so define, for each pair (r,s) with $r+s=p$, the homology $H_{*;r,s}(K_{comm}) = ker\partial^r/im\partial^s$. Its calculation over cyclotomic integers is involved but over field coefficients the only nonzero cyclotomic homology is $H_{kp+r-1;r,s}(K) \cong H_{kp+s-1;r,s}(K) \cong H_{2k}(K)$ and $H_{kp-1;r,s}(K) \cong H_{kp-1;s,r}(K) \cong H_{2k-1}(K)$.
(Many more simplicial homologies will be given in the Chandigarh Topology Seminar 1994-95 lecture notes.)

ALEXANDER SCHRIJVER:
Colin de Verdière invariants and characterization of planarity

We discuss the new graph invariant introduced by Yves Colin de Verdière. This invariant is monotone under taking minors, and moreover it characterizes planar graphs. We give the short proof due to Hein van der Holst of this fact, and we make some further observations.
(Joint work with H. van der Holst, M. Laurent, and L. Lovász.)

BERND STURMFELS:
Polyhedral methods for sparse polynomial systems

Our object of interest is a system of n equations

$$f_i(X) = \sum_{a \in A_i} c_{i,a} x^a \qquad (*)$$

in n variables $x = (x_i, \dots, x_n)$ with fixed supports $A_i \subset \mathbf{Z}^n$. In this talk we survey results and open problems concerning complex roots and real roots of $(*)$. Our starting point is Bernstein's Theorem, which states that — for generic $c_{i,a}$ — the number of roots of $(*)$ in $(\mathbf{C}^*)^n$ equals the mixed volume $M(Q_1, \dots, Q_n)$ of the Newton polytopes $Q_i = conv(A_i)$. An algorithmic proof is sketched. Concerning real roots, the situation is much less understood. Khovanskii's Fewnomial Theorem states that $(*)$ has at most $2^n(n+1)^{\binom{N}{2}}$ roots in $(\mathbb{R}^*)^n$ where $N = \#(A_1 \cup \ldots \cup A_n)$. This appears to be far from the truth, however. We present evidence that the maximum number of real roots is far less than Khovanskii's bound.

REKHA R. THOMAS:
Using Gröbner bases to solve integer programs (A Comparison between linear and integer programming)

Let $IP_{A,c}(b)$ denote the integer program $\min\{cx : Ax = b, x \in \mathbb{N}^n\}$, where $A \in \mathbb{Z}^{d \times n}$, $b \in \mathbb{Z}^d$, $c \in \mathbb{R}^n$ and $\operatorname{rank}(A) = d$. We show that the reduced Gröbner basis with respect to A and c forms a minimal test set for the family $IP_{A,c}(\cdot)$ as b varies. This leads to the construction of an $(n-d)$-polytope $St(A)$ which is shown to be normally equivalent to $\int_b P_b^I db$, where $P_b^I = conv\{x \in \mathbb{N}^n : Ax = b\}$. The edges directions of $St(A)$ are precisely the edge directions of P_b^I as b varies and this is shown to be the Universal Gröbner basis of A. We compare these results with their analogues for linear programming.
(This is joint work with Bernd Sturmfels.)

SINIŠA VREĆICA:
On (f, β)-pairs of multicomplexes

We define a multicomplex as an order ideal of monomials (not necessarily square-free) over $\{x_1, \ldots, x_n\}$, generalizing the notion of simplicial complex. In joint work with Anders Björner we try to establish the relations among f-vectors and Betti numbers of multicomplexes.
We show in the case of the order homology that Betti numbers depend only on the square-free monomials of M. For another notion of homology theory obtained by generalizing the boundary operator from the simplicial category we construct a topological model for multicomplexes. We also establish some necessary and some sufficient conditions on (f, β)-pairs of multicomplexes. For example, we show that the f-vector and Betti numbers of a multicomplex satisfy the relations $\partial^k(f_{2k+1} + \beta_{2k}) \leq f_{2k-1}$, $k \geq 1$.

MICHELLE WACHS:
On Lie k-algebras and homology of partition posets

We define a Lie k-algera to be a $(k+1)$-ary skew symmetric operation on a bigraded vector space which satisfies a certain relation of degree $2k+1$. The notion of Lie 1-algebra coincides with the notion of Lie superalgebra. An ordinary Lie algebra is precisely a Lie 1-algebra with odd elements. We show first that the boundary map in the Koszul complex squares to zero. We then show that the 1^{nk+1} homogeneous part of the free Lie k-algebra with $nk+1$ even generators is isomorphic as an S_{nk+1}-module to the cohomology of $\Pi^{(1)}_{nk+1}$, the poset of all partitions of $nk+1$ in which every block size is $\equiv 1 \bmod k$. This result is analogous to a classical result relating the free Lie algebra with n generators to the cohomology of the partition lattice. We also construct an explicit basis for the 1^{nk+1} graded piece of the free Lie k-algebra with $nk+1$ even generators. Lastly we compute the Lie k-algebra homology of the free Lie k-algebra.
(This is joint work with Phil Hanlon.)

RADE ŽIVALJEVIĆ:
Combinatorial geometry on vector bundles

A collection of 3 red, 3 white, and 3 blue points in the plane $\mathbb{R}^2$ can be partitioned into 3 vertex-disjoint multicolored triangles with a common point. Something similar is possible in the 3-space $\mathbb{R}^3$ but this time we need 5 points of each color in order to guarantee existance of such a partition. The well known Kuratowski nonplanarity criterion implies that $K_{3,3}$ is not embeddable in $\mathbb{R}^2$ which implies that for any collection of 3 red and 3 blue points in the plane there exist two intersecting vertex disjoint line segments with end points of different color.
These three (topological) statements can be abbreviated as follows.

$$(K_{3,3,3} \to \mathbb{R}^2) \implies 3 \mapsto 1$$

$$(K_{5,5,5} \to \mathbb{R}^3) \implies 3 \mapsto 1$$
$$(K_{3,3} \to \mathbb{R}^2) \implies 2 \mapsto 1$$

For example the last statements says that for every continuous map on the left there exist three points in three disjoint triangles which are mapped to the same point in $\mathbb{R}^3$.

The results above can be extended in a systematic way to include results in which the existence of a common point (common 0-dimensional transversal) is replaced by the existence of a common k-dimensional transversal. An example is the statement

$$(K_{6,6} \to \mathbb{R}^3) \implies (4 \mapsto line)$$

which says that for every collection of 6 red and 6 blue points in $\mathbb{R}^3$ there exist 4 multicolored line segments with a common line transversal.

The motivation for these problems comes actually from a different line of thought, namely from the so called Tverberg-Vrećica problem (Europ. J. Comb. 1993). The reader can find more information about this and related problems in the review paper *H. Edelsbrunner, S. Vrećica, R. Živaljević, Combinatorics and geometry of partitions of masses and point sets in* $\mathbb{R}^n$ (to appear).

A general picture seems to indicate that a natural environment for both the formulations and solutions of these problems is "Combinatorial Geometry on vector bundles", an extension of the usual Combinatorial Geometry of points, lines etc. in $\mathbb{R}^d$ where $\mathbb{R}^d$ is replaced by a vector bundle, points are systematically replaced by continuous cross sections of this bundle etc. Indeed, all results above and many more are formulated and proved in this context (*R. Ž., Combinatorial geometry on vector bundles*, in preparation). The main technical novelty in all proofs of these results is a systematic use of the parametrized version of the ideal-valued cohomological index theory as developed by J. Jaworowski, S. Husseini, E. Fadell, A. Dold etc.

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Gefördert durch
DFG Deutsche Forschungsgemeinschaft

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