Tagungsbericht 7/1995

Qualitative Aspects of Partial Differential Equations

12. – 18.02.1995

Die Tagung fand unter Leitung von H.Berestycki (Paris), B.Kawohl (Köln) und G.Talenti (Florenz) statt. Leitmotiv war die Gestalt von Lösungen partieller Differentialgleichungen. Inhaltliche und methodische Schwerpunkte lagen auf den drei Themen Rearrangement, neue Anwendungen der Moving Plane Methode und Blow up Phänomene. Die hervorragende Atmosphäre des Instituts und eine Abendsitzung mit 22 offenen Problemen trugen zu einem regen Austausch von Ideen bei.

ABSTRACTS:

James Serrin:
Asymptotic Stability and blow up of solutions of dissipative wave systems

We treat dissipative wave systems subject to the action of strongly nonlinear potential energies. A typical example is

$$u_{tt} - \Delta u + A(t) \mid u_t \mid^{m-2} \pm V(x) \mid u \mid^{p-2} u = 0 \quad (t,x) \epsilon I \times \Omega \qquad (1)$$

$$u(t,x) = 0 \quad (t,x) \epsilon I \times \partial\Omega \qquad (2)$$

where $I = [0,\infty)$ and Ω is a bounded open set in $I\!R^n$. The values of u are taken in $I\!R^N$, $N \geq 1$, while $m,p > 1$ and $A \epsilon C(I \to I\!R_+^{N\times N})$, $V \epsilon C(\bar\Omega \to I\!R_+^1)$. The term $V(x) \mid u \mid^{p-2} u$ represents a **restoring force** if the $+$ sign is used, and an **amplifying force** for the $-$ sign. Let the energy of a solution be defined by $E(u(t)) = \int_\Omega (\mid u(t) \mid^2 + \mid Du \mid^2 \pm \frac{1}{p} V(x) \mid u \mid^p) dx$. Then for the typical case (1)&(2) our results show, that all solutions u satisfy $E(u(t)) \to 0$ as $t \to \infty$, provided that

$$2 \leq m \leq \max(p, \frac{2n}{n-2}), \quad \lim_{t\to\infty} \frac{1}{t^m} \int_0^t \frac{1 + \mid A(s) \mid^m}{h(s)^{m-1}} ds < \infty$$

and the + sign is used; while when the − sign appears all solutions u satisfying $E(u(0)) < 0$ blow up in finite time T provided

$$\max(2,m) < p, \quad \int_0^\infty \min\left(c, \frac{h(t)}{|A(t)|^{\frac{m}{m-1}}}\right)dt = \infty.$$

Here c is any positive constant, and $(A(t)v,v) \geq h(t)\,|v|^2$.

Almut Burchard:
The Riesz Rearrangement Inequality

We determine the cases of equality in the Riesz Rearrangement inequality, which says that for any functions $f,g,h : \mathbb{R}^n \to \mathbb{R}_0^+$ (so that spherically decreasing rearrangements f^*, g^*, h^* can be defined):

$$J(f,g,h) := \int\int f(y)g(x-y)h(x)\,dydx \leq J(f^*,g^*,h^*)$$

This inequality has found applications in functional analysis (such as Young's, the Sobolev, and the Hardy - Littlewood - Sobolev inequalities) and to variational problems of Mathematical Physics (shape of fluid bodies). It is closely related to the Brunn - Minkowski and the isoperimetric inequalities.
The main result states, that for characteristic functions of measurable sets, there are two cases. If the volume satisfies a certain size relation, then equality implies that (essentially) the sets must be already symmetric. If the size relation is violated, then there are many cases of equality. Similarily, if the distribution functions of two of the three functions are continuous, then equality implies that f,g,h are (up to the symmetries of J) already symmetrically decreasing.

Giovanni Alessandri:
On Courant's Nodal Domain Theorem
The Courant nodal domain theorem states, that if u is the $n-th$ eigenfunction in the selfadjoint elliptic eigenvalue problem

$$-\nabla\cdot(A\nabla u) + qu = \lambda\rho u \quad \text{in} \quad \Omega, \quad u=0 \quad \text{on} \quad \partial\Omega \tag{1}$$

and the leading coefficients are Lipschitz continuous then:

$$\sharp \text{ nodal domains of u} \leq N \tag{2}$$

My aim is to investigate the case when the Lipschitz condition is dropped or relaxed. The results are:
(I) When $n = 2$ and $A \in L^\infty$ then (2) continues to hold.
(II) When $n \geq 3$, $A \in C^\alpha$ and $N > 1$ then (2) is replaced by

$$\sharp \text{ nodal domains of u} \leq 2(N-1).$$

Fred Weissler:
Exact Selfsimiliar Blow-Up for Semilinear Parabolic Equations with Nonlinear Gradient Term

In a joint work with S.Tayachi we consider self - similar solutions of

$$\partial_t \psi = \Delta \psi - b \mid \nabla \psi \mid^q + \mid \psi \mid^{p-1} \psi$$

where $\psi = \psi(t,x), x \in I\!R^n, t < 0, b > 0, p > 1, q = \frac{2p}{p+1}$. More precisely

$$\psi(t,x) = (-t)^{\frac{-1}{p-1}} u(\frac{\mid x \mid}{\sqrt{-t}}),$$

where

$$u'' + (\frac{n-1}{r} - \frac{r}{2})\, u' - b \mid u' \mid^q - \frac{1}{p-1} u + \mid u \mid^{p-1} u = 0$$

We prove that for $n = 1$, there exist $b_0 > 0, p_0 > 1$ such that if $0 < b < b_0$ and $1 < p < p_0$, then there is a regular solution $u(r)$ such that $u(r) > 0, u'(r) < 0$ for all $r > 0$ and $\lim_{r\to\infty} r^{\frac{2}{p-1}} u(r) = c > 0$. It follows that $\lim_{t\to 0^-} \psi(t,x) = c \mid x \mid^{\frac{-2}{p-1}}$.

Friedemann Brock:
Continuous Steiner Symmetrisation and Applications to Problems in the Calculus of Variations

For each function $u : I\!R^n \to I\!R$ having the property that all level sets $\{u > c\}$, $c > \inf u$ have a finite Lebesgue measure we construct a scale of functions $u^t, 0 \leq t \leq \infty$, which is a continuous homotopy between u and u^* (the Steiner symmetrisation of u), i.e.:

$$u^0 = u \quad ; \quad u^\infty = u^*$$

$t \to u^t$ continuous in $L^p_+(I\!R^n)$ and rightcontinuous in $W^{1,p}_+(I\!R^n)$ for $1 < p < \infty$.

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We prove various integral inequalities which are "continuous" analogues of well known inequalities for Steiner symmetrisation.
As an application we show that smooth solutions of the semilinear problem:

$$\begin{aligned} -\nabla\cdot(\nabla u \mid \nabla u \mid^{p-2}) &= 0 \quad \text{in} \quad \Omega \\ u &> 0 \quad \text{in} \quad \Omega \\ u &= 0 \quad \text{in} \quad \partial\Omega \end{aligned}$$

with $\Omega = \Omega^*$ bounded, f continuous in u, $f(\cdot,u) = f^*(\cdot,u)$ satisfies some weak - *local* - kind of symmetry.

Olga Oleijnik:
On Asymptotics of Solutions to Some Nonlinear Elliptic Equations
Many problems of Mathematical Physics lead to consider the asymptotic behaviour at infinity of solutions of semilinear second order elliptic equations in unbounded domains and, in particular, in cylindrical domains. Questions of this kind occur in problems of the theory of travelling waves, homogeneization theory, stationary states, boundary layer theory, biology, flame propagation, in probability theory (branching processes) and so on.
For equations of the form

$$\sum_{i,j=1}^{n} \partial_i(a_{ij}(x)\partial_j u(x)) + \sum_{j=1}^{n} a_j(x)\partial_j u(x) - f(x,u) = 0$$

in the domain $S = \{x : x' \in \omega, 0 < x_n < \infty\}$ with the boundary condition $u = 0$ on $\sigma = \{x : x' \in \partial\omega, 0 < x_n < \infty\}$ or $\partial u/\partial\nu = 0$ on σ, where ω is a bounded smooth domain, $x' = (x_1, ..., x_{n-1})$, ν is a conormal direction, under some conditions on a_{ij}, a_j and f the asymptotic behaviour for $u(x)$ as $x_n \to \infty$ is given.
Two new approaches to investigate these problems are given (the method of subsolutions and supersolutions and the method of mean value functions).

Michael Wiegner:
Blow Up of Solutions of Some Degenerate Parabolic Equations
We study the degenerate parabolic equation

$$\begin{aligned} u_t &= u^p\Delta u + u^{p+1} \quad \text{on} \quad \Omega\times(0,T) \\ u &= 0 \quad \text{on} \quad \partial\Omega\times(0,T) \end{aligned}$$

and $u(x,0) = \phi(x)$. Here $\Omega \subset I\!R^n$ is a smoothly bounded domain, $p > 1$, and $0 < c_0 \leq \phi(x)/dist(x, \partial\Omega) \leq c_1$. If $\lambda_1 < 1$, where λ_1 denotes the first eigenvalue of $-\Delta$ on Ω with Dirichlet conditions, we prove:
There is a unique solution, positive and smooth inside, with $\max(u(t,x)) \to \infty$ for $t \to T_0$, and the lifespan T_0 can be estimated by $c_1^{-p} c(n,p,\Omega) \leq T_0 \leq c_0^{-p} \tilde{c}(n,p,\Omega)$. Some numerical calculations in the one dimensional symmetric case ($\Omega = (-a,a), a > \frac{\Pi}{2}, xu_x \leq 0$ for all t) give hints, that the scaled asymptotic profile $w(x) = \lim_{t \to T_0} u(t,x)/u(0,t)$, which is easily seen to fulfill $w(x) \geq \cos(x)$, may be different from $\cos(x)$.

Michel Chipot:
Elliptic Equations Involving Critical Exponents and Nonlinear Neumann Boundary Conditions
Let $n \geq 3$, $H = \{x = (x_1, ..., x_n) : x_1 > 0\}$, a, b constants. We describe all the nonnegative solutions to

$$\begin{aligned} -\Delta u &= a u^{\frac{n+2}{n-2}} \quad \text{in} \quad H; \\ \frac{\partial u}{\partial \nu} &= b u^{\frac{n}{n-2}} \quad \text{in} \quad \partial H \end{aligned}$$

where $\nu = -e_1$. There are two types of solutions depending on a and b:

$$\begin{aligned} (I) \quad u &= \alpha(|x - x^0|^2 + \beta)^{-\frac{n-2}{2}} \quad \text{where} \quad \alpha > 0 \\ x_1^0 &= -\frac{b}{n-2}\alpha^{\frac{2}{n-2}}, \quad \beta = \frac{a}{n(n-2)}\alpha^{\frac{4}{n-2}} \\ (II) \quad & \text{solutions depending on} \quad x_1 \quad \text{only} \end{aligned}$$

This is joint work with M.Fila (Bratislava), I.Shafrir (Metz).

Thomas Lachland-Robert:
Some Application of the Monotone Rearrangement to Elliptic Equations in Cylinders
We study the problem

$$\begin{aligned} -\Delta u &= f(x', u) \quad \text{on} \quad \Omega = (0,h) \times \omega & (1) \\ \frac{\partial u}{\partial \nu} &= 0 \quad \text{on} \quad \{0,h\} \times \omega & (2) \\ u &= 0 \quad \text{on} \quad \{0,h\} \times \partial\omega & (3) \end{aligned}$$

where $h > 0, \omega \subset I\!R^{N-1}$ is a bounded domain, $x = (x_1, x') \in \Omega$, and f is superlinear and subcritical.
We prove, that if $h >$ a critical value h^*, then there exists a solution u of (1) - (3) satisfying $\partial_1 u < 0$. (This implies, that there are many solutions if h is large).
We use the montone decreasing rearrangement in order to exhibit this solution. In the simple case $f(x', u) = a(x')u^p, a > 0$, bounded and $p \in (1, \frac{N+2}{N-2})$ an analysis of the equality $\int | Du |^2 = \int | Du^* |^2$ suffice.
In the more general case we use the mountain pass theorem (f has to satisfy the corresponding assumptions) and the continuity of the rearrangement in the H^1- norm for a dense subset of H^1.

Hans - Christoph Grunau:
Positive Solutions to Semilinear Poyharmonic Dirichlet Problems Involving Critical Sobolev Exponent
We are concerned with the semilinear polyharmonic model Dirichlet problem $(-\Delta)^K v = \lambda v + v | v |^{s-1}$ in B and $D^\alpha v |_{\partial B} = 0$ for $| \alpha | \leq K - 1$. Here K is a positive integer, B is the unit ball in $I\!R^n$, $n > 2K$, $s = \frac{n+2K}{n-2K}$ is the critical Sobolev exponent. Let Λ_K denote the first Dirichlet eigenvalue of $(-\Delta)^K$ in B. The existence of a positive radial solution v is shown for

- $\lambda \in (0, \Lambda_K)$, if $n \geq 4K$
- $\lambda \in (\bar{\Lambda}, \Lambda_K)$, for some $\bar{\Lambda} \in \bar{\lambda}(n, K) \in (0, \Lambda_K)$ if $2K + 1 \leq n \geq 4K - 1$.

The crucial point is to show the positivity of a solution v with the help of the positivity of Green's function for $(-\Delta)^K$ in balls

Vincenzo Ferone:
Convex Symmetrization
Let $H(\xi) : I\!R^n \to I\!R$ be nonnegative, positively homogeneous of degree one and convex such that the measure of the set $\{x : H(x) \leq 1\}$ is equal to the measure of the unit ball in $I\!R^n$. One can define the perimeter of a set E with respect to H as

$$P_H(E) = \sup \{ \int_E div\phi : \phi \in C_0^1(I\!R^n; I\!R^n), H^0(\phi(x)) \leq 1 \}$$

where $H^0(x) \sup_{H(\xi)\leq 1} < x, \xi >$ is the polar function of H. The following isoperimetric inequality can be proven: $P_H(E) \geq n\kappa_n^{\frac{1}{n}} \mid E \mid^{\frac{n-1}{n}}$ where $\kappa_n = \mid \{\xi : H^0(\xi) \leq 1\} \mid$. Using this inequality it is possible to show, that for any nonnegative function compactly supported in $I\!R^n$ such that $\int_{R^n} H^2(\nabla u)dx < \infty$, the following "generalized" Polya-Szegö principle holds:

$$\int_{R^n} H^2(\nabla u)dx \geq \int_{R^n} H^2(\nabla \tilde{u})dx$$

where $\tilde{u}$ is the rearrangement of u such that its level sets are homothetic to the set $\{\xi : H^0(\xi) \leq 1\}$. Using (1) one can also obtain comparison results for solutions of the Dirichlet problem:

$$-\nabla \cdot (a(\nabla u)) = f, \qquad u \in W_0^{1,2}(\Omega)$$

where $\Omega \subset I\!R^n$ is a bounded open set and $a(\xi)$ satisfies $(a(\xi), \xi) \geq H^2(\xi)$. The above results are contained in a joint paper with A.Alvino, P.L.Lions and G.Trombetti.

Neil Trudinger:
Symmetrization for Fully Nonlinear Elliptic Equations
We apply our isoperimetric inequalities for quermassintegrals on non-convex domains to obtain symmetrization results for nonlinear operators of the form $F_m[u] = [D^2u]_m$ = sum of the $m \times m$ principle minors of the Hessian matrix D^2u, acting on $m-$ admissable functions $u \in C^2(\Omega), \Omega \subset I\!R^n$, i.e. functions u satisfying $[D^2u + \xi]_m \geq [D^2u]_m, \forall \xi \geq 0, \xi \in S^n$. Here $m = 1, ..., n$, the case $m = 1, m = n$ corresponding respectively to the Laplacian and the Monge Ampere operators, being already well studied. Two examples of our results are:
(1) If $F_m[u] = \psi \geq 0$ in Ω, u m-admissable, $u = 0$ on $\partial\Omega$, then $u^*_{m-1} \geq u^0$ in B_R, where u^*_{m-1} is the $m-1$ - symmetrand of u as determined by the quermassintegral V_{n-m+1}, $R = (\frac{1}{\omega_n} V_{n-m+1}(\Omega))^{\frac{1}{n-m+1}}$, and u^0 is the solution of the radial problem $F_m[u^0] = \psi^*$ in B_R, $u^0 = 0$ on ∂B_R, ψ^* is the spherical rearrangement of ψ.
(2) $I_m[u] := \int_\Omega F_m^{ij}\partial_i u \partial_j u \geq I_m[u^*_{m-1}]$, for all m - admissable u in Ω, $u = 0$ on $\partial\Omega$, where $F_m^{ij}(r) = \frac{\partial}{\partial r_{ij}} F_m(r)$.

Congming Li:
Some Applications of the Method of Moving Planes

1) Jointly with Wenxiong Chen, we seek metrics conformal to the standard ones on S^n having prescribed Gaussian curvature in case $n = 2$ (the Nirenberg Problem), or prescribed scalar curvature for $n \geq 3$ (the Kazdan-Warner problem). There are well-known Kazdan-Warner and Bourguignon-Ezin necessary conditions for a function $R(x)$ to be the scalar curvature of some conformally related metric.
We show that, in all dimensions, if $R(x)$ is rotationally symmetric and monotone in the region where it is positive, then the problem has no solution at all. It follows that, on S^2, for a non-degenerate, rotationally symmetric function $R(\theta)$, a necessary and sufficient condition for the problem to have a solution is that R' changes signs in the region where it is positive. This condition, however, is still not sufficient to guarantee the existence of a rotationally symmetric solution. We also consider similar necessary conditions for non-symmetric functions.
2) I generalize and simplify the work of L. Caffarelli, B. Gidas and J. Spruck about local asympototics of non-negative solutions to semilinear elliptic equations.
3) Jointly with J. Bebernes and Y. Li, we studied the existence, uniqueness and stability of the travelling front with exponetial decay to some combustion problems.

T.A. Shaposhnikova:
Homogenization of Differential Operators in Partially Perforated Domains

In the lecture we consider the problem of homogenization for the Laplace operator in a partially perforated domain Ω_ϵ in the case when the size of the holes is much smaller than the size of the cell. The boundary of the holes are denoted by S_ϵ and the exterior boundary is denoted by Γ_ϵ. We study the following boundary value problems:

$$(I) \quad \Delta u_\epsilon = f \quad \text{in} \quad \Omega_\epsilon, u_\epsilon = 0 \quad \text{on} \quad \partial\Omega_\epsilon;$$

$$(II) \quad \Delta u_\epsilon = f \quad \text{in} \quad \Omega_\epsilon, \frac{\partial u_\epsilon}{\partial \nu} + b_\epsilon u_\epsilon = 0 \quad \text{on} \quad \partial S\epsilon, \quad u_\epsilon = 0 \quad \text{on} \quad \Gamma_\epsilon;$$

$$(III) \quad \Delta u_\epsilon = f \quad \text{in} \quad \Omega_\epsilon, \frac{\partial u_\epsilon}{\partial \nu} = 0 \quad \text{on} \quad \partial S\epsilon, \quad u_\epsilon = 0 \quad \text{on} \quad \Gamma_\epsilon;$$

We study the behaviour of u_ϵ when $\epsilon \to 0$ and for every case we obtained estimates for $u_\epsilon - u$, where u is a solution of a limit problem. We also consider the corresponding spectral problems

Peter Laurence:
3-D MHD Equilibrium: The Inverse Problem
Both, the equations of ideal state MHD equilibria and inviscid, incompressible, steady rotational flow, can be written as

$$\begin{aligned} \nabla \times u \times u &= \nabla p \quad \text{in} \quad \Omega \\ \nabla \cdot u &= 0 \quad \text{in} \quad \Omega \end{aligned}$$

where in MHD, u is the magnetic field and in fluids the fluid velocity.
The physically relevant boundary condition is $u \cdot \nu = 0$ on $\partial\Omega$. Until recently no solutions were known to exist in domains Ω, that do not possess a symmetry. In the case $\Omega = T =$ a 3 - D torus, close to axissymmetry, we establish the existence of such tori by solving an appropiate inverse problem.

Steven J. Cox:
The Design and Identification of Dissipatve Structures
With respect to the dissipative wave equation

$$u_{tt}(x,t) - u_{xx}(x,t) + a(x)u_t(x,t) = 0 \quad \text{for} \quad 0 < x < 1$$

with fixed ends, we show, under the assumption that $a \in BV(0,1)$, that the rate at which energy decays coincides with the spectral abscissa of the generator of the associated semigroup.
We show that this decay rate assumes its (negative) minimum over those $a(x)$ whose total variation does not exceed a given constant. We show, that $a(x) = \pi$ is a weak local minimum and provide numerical evidence in support of the conjecture that π is in fact the local minimizer.

Steffen Heinze:
Wave Solutions for Reaction Diffusion Systems in Perforated Domains
Consider a system of reaction - diffusion equations in the ϵ - periodicallly perforated $\mathbb{R}^n$, with Neumann boundary conditions at the holes. Travelling

waves in this domain satisfy

$$u(t - \frac{k_i\epsilon}{c}, x + \epsilon e_i) = u(t,x) \qquad i = 1, ..., n$$

where $(k - 1, ..., k_n)$ is the direction and c is the unknown velocity of the wave. As the period of the domain tends to zero the homogenized system admits a nondegenerate wave solution. It is shown, that then the original problem also admits a wave solution. Furthermore error estimates in powers of ϵ are given. This problem has applications, e.g. in combustion processes.

Vladimir Oliker:

Some Results on Mean and Gauss Curvature Flow

We consider an evolution that starts as a flow of smooth nonparametric surfaces propagating in space with normal speed equal to mean curvature. The boundaries of the surfaces are assumed to remain fixed. The domains over which such flows are considered are not required to be "mean convex". Consequently, singularities may develop on the boundary. We show, that such singularities will disappear in finite time and the solution will become smooth up to the boundary. We also investigate the asymptotic behaviour of such flows as $t \to \infty$. These results are obtained jointly with N.N. Uraltseva.

Martin Flucher:

Vortex Motion in Two Dimesional Hydrodynamics and The Core Energy Method

We study the dynamics of point vortices in an inviscid, incompressible fluid in a bounded container. This is a dynamical system of infinite kinetic energy. Still we can derive a finite integral of motion by means of the core energy method. The resulting renormalized is a Hamiltonian reflecting the interaction between different vortices via a special Green function and the self interaction of each vortex with the boundary in terms of a Robin function (regular part of the Green function). A list of qualitative statements on the dynamics of the vortex centers can be derived from conservation of the Hamiltonian using results of [BF].

[FG] Flucher M., Gustafsson B.: Vortex motion in two dimensional hydrodynamics (in preparation)

[BF] Bandle C., Flucher M.: Harmonic radius and concentration of energy, hyperbolic radius and Liouville equation $\Delta u = \exp u$ and $\Delta u = u^{\frac{n+2}{n-2}}$, (to

appear in SIAM Review).

Bradley Willms:
An Isoperimetric Inequality for the Buckling of a Clamped Plate
This is joint work with H.F. Weinberger. We consider a homogeneous, isotropc plate covering a smoothly bounded, plane domain, clamped on the boundary ∂D, under a uniform, compressive load proportional to Λ. At the critical buckling load, $\Lambda = \Lambda_1$ is the least eigenvalue of the boundary value problem

$$\begin{aligned} \Delta^2 u + \Lambda_1 \Delta u &= 0 \quad \text{in} \quad D \\ u = \frac{\partial u}{\partial \nu} &= 0 \quad \text{in} \quad \partial D \end{aligned}$$

where $u(x,y)$ is the transverse displacement, and Δ is the Laplacian operator. Let $D^* \equiv$ unit disc. Then a conjecture of Polya and Szegö is that

$$\Lambda_1(D) \geq \Lambda_1(D^*) \quad \forall D \quad \text{such that} \quad |D| = |D^*|$$

They also showed, that $u > 0$ in D is a sufficient condition for the validity of the conjecture in all dimensions. Following Courant & Hilbert we show that a "naive" necessary condition for a minimum of $\Lambda_1(D)$ under domain variation, is that $\Delta u\,|_{\partial D} = c$.
From this boundary condition it follows,that D is a "Schiffer" domain, i.e. a domain for which there exists a non constant solution V to

$$\begin{aligned} \Delta v + \lambda_1 v &= 0 \quad \text{in} \quad D \\ \frac{\partial u}{\partial \nu} &= 0 \quad \text{in} \quad \partial D \\ v &= \frac{-c}{\Lambda_1} \quad \text{in} \quad \partial D \end{aligned}$$

If D is not a disc, then the isoperimetric inequalities of Faber & Krahn of Payne & Polya & Weinberger and of Payne & Weinberger are combined to give the result, $\Lambda_1(D) > \Lambda_1(D^*)$. Existence of a minimizer follows locally in $\mathbb{R}^n$ from a theorem of Pironneau, and global existence for convex domains in $\mathbb{R}^2$ is proved with an idea of B.Kawohl.

Jerrold Bebernes:
Beyond Blow Up
Consider the Cauchy initial value problem

$$u_t - \Delta u = (1-\epsilon)f(u), \quad x \in I\!R^n,\ t \geq 0,\ \epsilon > 0 \tag{1}$$

with $u(x,0) = u_0(x) \geq 0$, where u_0 is radially symmetric, decrasing and $u_0 = u(r) \in C^2$, $r = | x |$ satisfies $u_0 = 0$, $u_0''(r) \leq 0$. For $f(u) = \exp u$ or u^p, $p > 1$, assume u_0 is such that the associated (ignition) problem ($\epsilon = 0$):

$$u_t - \Delta u = f(u) \tag{2}$$

has finite blow up time T. Then the blow up set is $(0, T)$ and the final time blow up profile called a hot spot is precisely known. The question adressed is to determine the behaviour of the solution $Q(x,t)$ of (1), which exists on $I\!R \times [0, \infty)$ and is bounded above by $\frac{1}{\epsilon}$ for $t \geq T$.

Michiel Bertsch:
Fourth Order Parabolic Equations
Consider the equations

$$w_t = -w_{xxxx} + \left(\frac{w_x^2}{w}\right)_{xx} \tag{1}$$

$$u_t = -(u^n u_{xxx})_x \quad (n \in I\!R^+) \tag{2}$$

(1) describes interface fluctuation and $w(x,t)$ has the properties of a probability density: $w(x,t) \geq 0$ and $\int w(x,t)dx = 1$ (see Bleher, Lebowitz, Speer in Comm. Pure Appl. Math, 1994).
(2) is a model for the dynamics of a thin liquid film on a horizontal solid surface driven by surface tension. The lubrification approximation is assumed to be valid and u stands for the thickness of the liquid film. Current values for n are $n = 2$ and $n = 3$.
We show that (1) and (2) possess nonnegative solutions (although the linear equation $u_t + u_{xxxx} = 0$ does not preserve nonnegativity) and study their positivity properties. Both equations exhibit nonuniqueness, and for (2) the qualitative behaviour of the solution depends strongly on the value of n. As a curiosity we observe, that in order to obtain strictly positive approximating solutions, instead of regularizing the equations, we make (1) more singular

and (2) more degenerate.
(1) is in preparation as joint work with R.Dal Passo (Roma), while (2) is joint work with E. Beretta (Roma) and R.Dal Passo and will appear in Archive Rat. Mech. Anal..

Nicola Garofalo:
Smoothing Effects and Monotonicity for Evolution Equations
Consider the Allen - Cahn equation in $I\!R^n \times (0,\infty)$

$$\Delta u - u_t = u^3 - u$$

It models the motion of phase transition layers by surface tension. Solutions to (1) satisfy a monotonicity property which is, in fact, shared by solutions of a large class of equations related to motion by mean curvature. Here is the main result
Theorem: Let u be a smooth solution to

$$\begin{aligned} \Delta u - u_t &= F'(u) \quad \text{in} \quad I\!R^n \times (0,\infty) \\ u(x,0) &= u_0(x), \end{aligned}$$

and suppose that $F(u) \geq 0$ for all $u \in I\!R$. If $|\nabla u_0|^2 \leq 2F(u_0)$, then for any $(x,t) \in I\!R^n \times (0,\infty)$ one has

$$|\nabla u(x,t)|^2 \leq 2F(u(x,t)).$$

Similiar monotonicity results hold for equations of the type

$$\nabla \cdot (\Phi'(|\nabla u|^2)\nabla u) - F'(u) = u_t \Phi'(|\nabla u|^2)$$

under general structural assumptions on Φ.

G.R. Burton:
Rearrangements and Vortices
Let $\Omega \subset I\!R^2$ be a bounded open set and suppose $\partial\Omega$ is a smooth simple closed curve. An ideal fluid, obeying the Euler equations in Ω with tangential velocity on $\partial\Omega$, is studied. The vorticity, which is a vector field perpendicular to $I\!R^2$, may be represented by a scalar field ξ, and the kinetis energy E can be expressed by $E(\xi) = \frac{1}{2}\int_\Omega \xi K \xi$, where $K = (-\Delta)^{-1}$ (zero Dirichlet boundary

conditions). The following variational problem is considered: Find critical points of E subject to the constraint, that ξ is a rearrangement of a prescribed function ξ_0. Solutions with prescribed energy are shown to exist, and to represent steady flows of the fluid.

Xu-Yan Chen:
Strong Unique Continuation and Local Asymptotics for Parabolic Equations
Local behaviour of solutions of second order linear parabolic equations near zero points is investigated. The main results are:
(i) strong unique continuation theorem: a solution having a zero point of infinite order vanishes identically;
(ii) a complete classification of local forms of asymptotics in terms of poynomial solutions of the classical heat equation;
(iii) upper bound estimates for the Hausdorff dimension of nodal sets.
Applications include:
(iv) a complete long - term description of spatio - temporal structure for bounded nonegative solutions of a nonlinear parabolic problem on a ball;
(v) the bang - bang principle and a description of oscillations for optimizers in a control problem involving one - dimensional parabolic equations.

Wolfgang Reichel :
Symmetry for an Overdetermined Boundary Value Problem in Potential Theory
We consider a bounded smooth domain $\Omega \subset I\!R^n$ with a constant source distribution $\phi > 0$ on $\partial\Omega$. Let Ψ be the induced single layer potential. We prove the following conjecture of P. Gruber: Ω is a ball if and only if Ψ is constant in Ω. As Ψ satisfies the overdetermined elliptic boundary value problem (ν is the outer unit normal)

$$\begin{aligned} \Delta\Psi &= 0 \quad \text{in} \quad I\!R^n - \Omega, \\ \frac{\partial\Psi}{\partial\nu} &= -\phi < 0 \quad \text{on} \quad \partial\Omega, \\ \Psi &= const. > 0 \quad \text{on} \quad \partial\Omega, \end{aligned}$$

we can use the moving plane method of Alexandroff and Serrin to show the radial symmetrey of Ω and Ψ. In fact, the linear problem from potential radial symmetrey of Ω and Ψ. In fact, the linear problem from potential theory turns out to be a very special case of a large class of nonlinear overdetermined boundary value problems

$$\begin{aligned} \Delta u + f(u, |\nabla u|) &= 0 \quad \text{in} \quad I\!R^n - \Omega, \\ \frac{\partial u}{\partial \nu} &= const. \leq 0 \quad \text{on} \quad \partial\Omega, \\ u &= const. > 0 \quad \text{on} \quad \partial\Omega, \\ u &= 0 \quad \text{at} \quad \infty \end{aligned}$$

Under suitable hypotheses on f, the moving plane method is used again to show, that all solutions and the underlying domain are radial.

Friedmar Schulz:
Symmetrization with Respect to a Measure
The spherical symmetric rearrangement u^* of a nonnegative measurable function u on $I\!R^n$ with respect to a measure given by a nonhomogeneous density distribution p was studied. Conditions on u were given, which guarantee that u^* is continuous or absoltely continuous on lines, i.e., Sobolev regular.
The energy inequality was proven in $n = 2$ dimensions by employing a Carleman type isoperimetric inequality if $\log p$ is subharmonic. The energy equality was settled, as well as the case of $n > 2$ dimensions considered, when it was assumed, that $p = Jh$ with a $K-$ quasiconformal map h.

APPENDIX: OPEN PROBLEMS

Problem 1: Consider a capillary surface

$$\operatorname{div} \frac{\nabla u}{\sqrt{1 + |\nabla u|^2}} = \kappa u \qquad \text{in} \quad D \subset I\!R^2$$

with prescribed contact angle

$$\frac{\nabla u \cdot n}{\sqrt{1 + |\nabla u|^2}} = \cos\gamma \qquad \text{on} \quad \partial D \quad .$$

Let $\alpha(D) = \min_D u$, $\beta(D) = \max_D u$. If D^* is a disc of same area as D, show that

$$\alpha(D) \geq \alpha(D^*) \quad \text{and} \quad \beta(D) \geq \beta(D^*) \quad .$$

(Suggested by C.Bandle; from the last page of R.Finn's book.)

Problem 2: Consider the P-function associated to $\Delta u + \lambda u = 0$ in D, i.e. consider $\phi := |\nabla u|^2 + \lambda u^2$. This function satisfies

$$\Delta\phi - \frac{\nabla Q}{Q}\nabla\phi = 0 \quad \text{in } D, \text{ where} \quad Q = |\nabla u|^2 \quad .$$

If $D \subset I\!R^2$ is convex and $\phi = \text{const}$ on ∂D, **show** that Q is quasiconcave. This would lead to a proof of Schiffer's (or the Pompeiu) problem. (Suggested by B.Willms.)

Problem 3: Consider the variational problem $\min R(v) = \int_D dx/(1+|\nabla v|^2)$ over the following set A of admissible functions: $A := \{v \in W^{1,\infty}_{loc}(B) \mid 0 \leq v \leq M,\ v \text{ is concave}\}$. Here D is a convex domain in $I\!R^n$. It is known that a solution to this problem exists, and what it looks like if D is a ball in $I\!R^2$ and if the solution is radial. **Prove or disprove** that for $D = B_1(0) \subset I\!R^n$ the solution is radial. (Suggested by B.Kawohl.)

Problem 4: Let $D \subset I\!R^2$ be convex and consider the solution u of $u_t - \Delta u = 0$ in $D \times I\!R^+$, under initial condition $u(x,0) \equiv 1$ in D and boundary condition $u(x,t) = 0$ on ∂D. It is known that for every positive t the function u has exactly one spatial maximum $x_m(t)$, the hot spot. Suppose that the hot spot does not move in time. Does this imply some sort of symmetry of D? Partial answers to this questions can be found in SIAM Reviews. (Suggested by B.Kawohl, originally from M.Klamkin.)

Problem 5: Let $D \subset I\!R^2$ and consider $\Delta u + \lambda_2 u = 0$ in D, $u = 0$ on ∂D. Can it happen, that a nodal surface $\{x \in D \mid u(x) = 0\}$ has positive distance to ∂D? The answer is negative (Alessandrini/Melas) if D is convex. (Suggested by B.Kawohl, originally from L.Payne)

Problem 6: Let $D \subset I\!R^2$ and consider $\Delta u + \nu_2 u = 0$ in D, $\partial u/\partial n = 0$ on ∂D. Here $\nu_1 = 0$ is the first eigenvalue. **Show** that $|u|$ attains its maximum only on ∂D. (Suggested by B.Kawohl, originally from J.Rauch.)

Problem 7: Consider the Monge-Ampere-type equation

$$\det[D^2 u] = (-\lambda u)^n \quad \text{in} \quad \Omega \subset I\!R^n \quad , \quad u = 0 \quad \text{on} \quad \partial\Omega \quad ,$$

where Ω is uniformly convex. It is known that there exists a convex solution u and a positive "eigenvalue" λ such that $u \in C^\infty(\Omega) \cap C^{1,1}(\overline{\Omega})$. **Show** that $u \in C^\infty(\overline{\Omega})$. The same problem can be posed for other curvature type operators, namely $F_m[D^2u] = (-\lambda u)^m$ with $1 \le m \le n$. (Suggested by N.Trudinger.)

Problem 8: Pompeiu's problem: Characterize those sets $\Omega \subset I\!R^2$ such that ($f \in C(I\!R^2)$ and $\int_{\sigma(\Omega)} f(x)dx = 0$ for every rigid motion σ) implies $f \equiv 0$. This question is known to be equivalent to Schiffer's conjecture: Consider

$$\Delta u + \lambda u = 0 \quad \text{in} \quad \Omega \subset I\!R^2 \quad , \quad \partial u/\partial n = 0 \quad \text{on} \quad \partial\Omega \quad ,$$

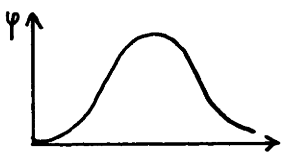
and suppose that in addition u = const on $\partial\Omega$. Show that Ω must be a ball. There have been many attempts to prove this conjecture; a more recent contribution states: If $\partial\Omega$ can be parametrized by a finite Fourier series, then Ω must be a ball. Show that the same is true if $\partial\Omega$ can be parametrized by an infinite Fourier series. (Suggested by N.Garofalo, originally by Pompeiu.)

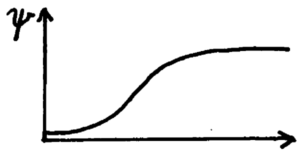
Problem 9: Consider the Cauchy-problem

$$u_t = (\varphi(u)u_x(1+u_x^2)^{-1/2})_x = \psi(u,u_x)_x) \quad \text{for} \quad x \in I\!R \quad , \quad t > 0$$

$u(x,0) = u_0(x)$ for $x \in I\!R$, with smooth u_0 and ψ. It is known that for sufficiently steep initial data solutions can develop discontinuities in finite time. Approximate by $\psi_\varepsilon(u,p) = \psi(u,p) + \varepsilon p$. Show that the approximate solutions u_ε satisfy: $u_{\varepsilon x} : I\!R \to I\!R^+ \to [-\infty, +\infty]$ are (locally) equicontinuous. This would imply that u behaves like a first order conservation law $u_t = \varphi(u)_x$ near the shocks. (Suggested by M.Bertsch.)

Problem 10: The equation $u_t = \varphi(u_x)_x$ (with φ and ψ as drawn below) is regularized by $u_t = \varphi(u_x)_x + \tau\psi(u_x)_x$. Here $\tau << 1$ is a small parameter. As in Problem 9 one knows that u can become discontinuous in finite time. Give a constructive proof for this. (Suggested by M.Bertsch.)





Problem 11: Given $D \subset I\!R^2$ find the location and shape of $A \subset D$ such

that there exists a function u, harmonic in $D\backslash A$, and satisfying $u = 0$ on ∂D, $u = 1$ on ∂A and $\partial u/\partial n = Q$ on ∂A. As $Q \to \infty$, A becomes small. **Show** that it approaches the shape of a ball. (Suggested by M.Flucher.)

Problem 12: Consider the problem $\Delta u - e^u = 0$ in the semi-infinite cylinder $S = \omega \times I\!R$, where $\omega \subset I\!R^{n-1}$ is bounded and smooth,
a) under boundary condition $u = 0$ on $\partial\omega \times I\!R^{\times}$. Then it is known that $u(x, x_n)$ can only have the following two types of asymptotic behaviour as $x_n \to +\infty$
i) $u(x) = cz(x')\exp\{\sqrt{\lambda_1}x_n\} + O(1)$ or
ii) $u(x) = u_0(x') + O(\exp\{-\sqrt{\lambda_1}x_n\})$.
Here z, λ satisfy $\Delta z + \lambda_1 z = 0$ in ω, $z > 0$ in ω and $z = 0$ on $\partial\Omega$, and $c < 0$; while u_0 solves

$$\Delta u_0 - e^{u_0} = 0 \quad \text{in} \quad \omega \quad , \quad u_0 = 0 \quad \text{on} \quad \partial D \quad ,$$

b) under boundary condition $\partial u/\partial\nu = 0$ on $\partial\omega \times I\!R^+$.
Then $u(x', x_n)$ can only have the asymptotics
iii) $u(x) = cx_n + o(x_n)$ with $c < 0$, or
iv) $u(x) = -2\ln x_n + o(\ln x_n)$.
Prove or disprove existence of solutions with behaviour i), iii), iv). What can be said about asymptotics and existence on the infinite cylinder $\omega \times I\!R$? (Suggested by O.Oleinik.)

Problem 13: Consider the equation $\Delta u - |u|^{p-1}u = 0$ in the semi-infinite cylinder from problem 9, under Neumann boundary conditions $\partial u/\partial\nu = 0$ on $\partial\omega \times I\!R^+$. Then the following asymptotic representation is known:

$$u(x) = \pm K_p(x_n + h)^{2/(1-p)} + \sum_{i=1}^{m} A_i v_i(x')e^{-\sqrt{\nu_2}x_1} + o(e^{-\sqrt{\nu_2}x_n})$$

as $x_n \to \infty$, where h, A_i are constants, ν_2 is the second eigenvalue from Problem 6 in $D = \omega$, m is the multiplicity of ν_2, and $v_i(x')$, $i = 1, \ldots, m$ are the orthogonal eigenfunctions from the eigenspace to ν_2. Moreover either $K_p = 0$ or $K_p = \left(\frac{2(1+p)}{(p-1)^2}\right)^{1(p-1)}$.
How does $u(x', 0)$ influence the behaviour of u so that $K_p = 0$, $K_p \neq 0$ or $u(x) \to \pm\infty$ as $x_n \to T < 0$? (Suggested by O.Oleinik.)

Problem 14: Consider $L(u) - |u|^{p-1}u = 0$ in Ω, $\frac{\partial u}{\partial n} = 0$ on $\partial\Omega \cap \{x : |x| > N\}$.

What is a behaviour of $u(x)$ or $|u(x)|$ as $|x| \to \infty$? See J.I.Draz, O.A.Oleinik, C.R.Acad.Paris **315** (1992), 787-792, suggested by O.Oleinik.

Problem 15: Consider boundary value problems with discontinuous coefficients

$$L_1 u = 0 \quad \text{in} \quad \Omega_1 \quad , \quad L_2 u = 0 \quad \text{in} \quad \Omega_2$$

and suppose that Ω_1 and Ω_2 look like this:

What is the optimal regularity of u near the points marked with arrows. (Suggested by O.Oleinik.)

Problem 16: Prove the isoperimetric inequality (or a related Sobolev resp. Poincare inequality)

$$n\alpha_n^{1/n}\left(\int_\Omega p(x)dx\right)^{(n-1)/n} \leq \int_{\partial\Omega} p(x)^{(n-1)/n} \partial\mathcal{H}^{n-1} \quad ,$$

where α_n is the volume of the unit ball in $I\!R^n$, under a suitable assumption regarding the weight p. For $n = 2$ this is known if $\log p$ is subharmonic (Carleman). See also F.Schulz and V.Vera de Serio, Trans.Amer.Math.Soc **337** (1993). (Suggested by F.Schulz.)

Problem 17: Let $D \subset I\!R^n$ be convex and $f \in C^1(\overline{I\!R^+})$ with $f(0) > 0$ and $f' \geq 0$ (and maybe $f' \leq \lambda_1$). Let $u \in C^2(\Omega) \cap C(\overline{\Omega})$ be the minimal solution of

$$-\Delta u = f(u) \quad u > 0 \quad \text{in} \quad D \quad , \quad u = 0 \quad \text{on} \quad \partial D \quad .$$

Show that u is quasiconcave. (Suggested by F.Brock.)

Problem 18: Let $D \subset I\!R^n$ be convex, $\psi \in C^{1,1}(\Omega)$ a concave obstacle in the obstacle problem

$$\min_{v \in K} \int |\nabla v(x)|^2 dx \quad , \quad K = \{v \in W_0^{1,2}(D) \mid v = \psi \text{ a.e. in } D\}$$

Prove or disprove that u is quasiconcave. (Suggested by F.Brock.)

Problem 19: Is Steiner-symmetrization continuous in $W_+^{1,p}(I\!R^n)$? For $n = 1$ the answer is positive (Coron), for $n \geq 2$ and Schwarz-symmetrization it is negative (Almgren and Lieb). (Suggested by F.Brock)

Problem 20: Suppose there exists $q \in L^\infty(I\!R^n)$ and an "eigenfunction" $\psi \in L^\infty(\Omega)$ solving $-\Delta\psi + q\,\psi = 0$ in $I\!R^n$ and changing sign in $I\!R^n$. Does this imply that $L = (-\Delta + q)$ has some negative spectrum, i.e. there exists $\varphi \in C_0^\infty(I\!R^n)$ with $\int |\nabla\varphi|^2 + q\varphi^2 dx < 0$? The answer is positive for $n = 1, 2$, but open for $n \geq 3$. (Suggested by H.Berestycki.)

Problem 21: On travelling fronts for systems from combustion with Lewis number $\neq 1$. Let $\omega \subset I\!R^{n-1}$ and $\Sigma = I\!R \times \omega$. Consider the system

$$-\Delta T + (c + \alpha(y))\partial T/\partial x_1 = f(T)Y \quad , \tag{1}$$
$$-L^{-1}\Delta T + (c + \alpha(y))\partial Y/\partial x_1 = -f(T)Y \tag{2}$$

in the cylinder Σ under homogeneous Neumann conditions $\partial T/\partial\nu = \partial Y/\partial\nu = 0$ on the lateral boundary $I\!R \times \partial\omega$ and under Dirichlet conditions $T(-\infty, y) = 0$, $Y(-\infty, y) = 0$; $T(\infty, y) = 1$, $Y(\infty, y) = 0$. It is known that solutions exist for constant α or (if L is close to 1) for slowly varying α. Open is a) if the solutions for $\alpha \equiv 0$ and $L \neq 1$ can depend on y and b) what happens for strongly oscillating α. (Suggested by H.Berestycki.)

Problem 22: Let $D \subset I\!R^n$ be bounded and suppose the functional $F = \int_D f(|\nabla u|)dx$ has a minimizer under $u = g$ on ∂D. Moreover let $f'' > 0$, f coercive. If $f'(0) = 0$, then F is Frechetdifferentiable and u solves the Euler equation $\mathrm{div}(f'(|\nabla u|)\nabla u/|\nabla u|) = 0$. Suppose $f'(0) > 0$. Then the functional is not differentiable at functions with critical points.
a) What is a meaningful definition of a solution?
b) Show that critical points of solutions to the (degenerate) Euler equation are not isolated but are located along circles or lines. (Suggested by G.Talenti.)

Berichterstatter: A. Wagner

Tagungsteilnehmer

Prof.Dr. Giovanni Alessandrini
Dipartimento di Scienze Matematiche
Universita di Trieste
Piazzale Europa 1

I-34100 Trieste (TS)

Prof.Dr. Friedemann Brock
Mathematisches Institut
Universität zu Köln
Weyertal 86-90

50931 Köln

Prof.Dr. Catherine Bandle
Mathematisches Institut
Universität Basel
Rheinsprung 21

CH-4051 Basel

Dr. Almut Burchard
Department of Mathematics
Princeton University
Fine Hall
Washington Road

Princeton , NJ 08544-1000
USA

Prof.Dr. Jerrold Bebernes
Dept. of Mathematics
University of Colorado
Campus Box 395

Boulder , CO 80309-0395
USA

Dr. Geoffrey R. Burton
School of Mathematical Sciences
University of Bath
Claverton Down

GB-Bath Avon BA2 7AY

Prof.Dr. Henri Berestycki
Universite Paris VI
Laboratoire d'Analyse Numerique
4, place Jussieu

F-75252 Paris

Prof.Dr. Xu-Yan Chen
School of Mathematics
Georgia Institute of Technology

Atlanta , GA 30332-0190
USA

Prof.Dr. Michiel Bertsch
Dipartimento di Matematica
Universita degli Studi di Roma
Tor Vergata
Via della Ricerca Scientifica

I-00133 Roma

Prof.Dr. Michel Chipot
Mathematiques
Universite de Metz
Faculte des Sciences
Ile du Saulcy

F-57045 Metz Cedex 1

Prof.Dr. Steven Cox
Dept. of Computational & Applied
Mathematics
Rice University
P.O. Box 1892

Houston , TX 77251
USA

Dr. Steffen Heinze
Sonderforschungsbereich 123
"Stochastische math. Modelle"
Universität Heidelberg
Im Neuenheimer Feld 294

69120 Heidelberg

Prof.Dr. Vincenzo Ferone
Dipartimento di Matematica e Appl.
Universita di Napoli
Complesso Univ. Di Monte S. Angelo
Via Cintia

I-80126 Napoli

Prof.Dr. Gerhard Huisken
Mathematisches Institut
Universität Tübingen
Auf der Morgenstelle 10

72076 Tübingen

Dr. Martin Flucher
Mathematisches Institut
Universität Basel
Rheinsprung 21

CH-4051 Basel

Prof.Dr. Bernhard Kawohl
Mathematisches Institut
Universität zu Köln
Weyertal 86-90

50931 Köln

Prof.Dr. Nicola Garofalo
Dept. of Mathematics
Purdue University

West Lafayette , IN 47907-1395
USA

Prof.Dr. Thomas Lachand-Robert
Laboratoire d'Analyse Numerique,
Tour 55-65
Universite P. et M. Curie(Paris VI)
4, Place Jussieu

F-75252 Paris Cedex 05

Dr. Hans-Christoph Grunau
Fakultät für Mathematik und Physik
Universität Bayreuth

95440 Bayreuth

Prof.Dr. Peter Laurence
Dipartimento di Matematica
Universita degli Studi di Roma I
"La Sapienza"
Piazzale Aldo Moro, 2

I-00185 Roma

Prof.Dr. Congming Li
Program in Applied Mathematics
University of Colorado at Boulder
Campus Box 426

Boulder , CO 80309-0426
USA

Wolfgang Reichel
Mathematisches Institut I
Universität Karlsruhe

76128 Karlsruhe

Prof.Dr. Alexander Mielke
Institut für Angewandte Mathematik
Universität Hannover
Postfach 6009

30060 Hannover

Prof.Dr. Jose-Francisco Rodrigues
CMAF & University of Lisboa
Centro de Matematica Aplicacoes
Fundamentais
Av. Prof. Gama Pinto, 2

P-1699 Lisboa Codex

Prof.Dr. Stefan Müller
Mathematik Department
ETH Zürich
ETH-Zentrum
Rämistr. 101

CH-8092 Zürich

Prof.Dr. Friedmar Schulz
Abteilung für Mathematik I
Universität Ulm

89069 Ulm

Prof.Dr. Olga A. Oleinik
Department of Mathematics
Moscow University
Korpus "K"
App. 133

Moscow 117 234
RUSSIA

Prof.Dr. James Serrin
School of Mathematics
University of Minnesota
127 Vincent Hall
206 Church Street S. E.

Minneapolis , MN 55455
USA

Prof.Dr. Vladimir Oliker
Dept. of Mathematics and
Computer Science
Emory University

Atlanta , GA 30322
USA

Prof.Dr. Tanya A. Shaposhnikova
Moscow University
Korpus "K", app. 133

117234 Moscow B-234
RUSSIA

Prof.Dr. Giorgio Talenti
Istituto Matematico
Universita degli Studi
Viale Morgagni, 67/A

I-50134 Firenze

Prof.Dr. Fred B. Weissler
Departement de Mathematiques
Institut Galilee
Universite Paris XIII
Av. J.-B. Clement

F-93430 Villetaneuse

Prof.Dr. Neil S. Trudinger
Centre for Mathematics and its
Applications
Australian National University

Canberra ACT 0200
AUSTRALIA

Prof.Dr. Michael Wiegner
Fakultät für Mathematik und Physik
Universität Bayreuth

95440 Bayreuth

Alfred Wagner
Mathematisches Institut
Universität zu Köln
Weyertal 86-90

50931 Köln

Prof.Dr. Bradley Willms
School of Mathematics
University of Minnesota
127 Vincent Hall
206 Church Street S. E.

Minneapolis , MN 55455
USA

Prof.Dr. Wolfgang Walter
Mathematisches Institut I
Universität Karlsruhe

76128 Karlsruhe

e-mail Adressen **Oberwolfachtagung 12.-18. Februar 1995**

Alessandrini, G.	alessang@univ.trieste.it
Bandle, C.	bandle@math.unibas.ch
Bebernes, J.	bebernes@newton.colorado.edu
Berestycki, H.	beres@dmi.ens.fr
Bertsch, M.	bertsch@mat.utovrm.it
Brock, F.	brock@mathematik.uni-leipzig.d400.de
Burchard, A.	burchard@math.princeton.edu
Chen, Xu Yan	xchen@math.gatech.edu
Chipot, M.	chipot@ciril.fr
Cox, S.	cox@rice.edu
Ferone, V.	ferone@matna1.dma.unina.it
Flucher, M.	flucher@math.unibas.ch
Burton, G.R.	grb@uk.ac.bath.maths
Grunau, H.	Heinz-Christoph.Grunau@uni-bayreuth.de
Heinze, S.	heinze@espresso.iwr.uni-heidelberg.de
Huisken, G.	huisken@commlink.zdv.uni-tuebingen.de
Kawohl, B.	kawohl@mi.uni-koeln.de
Lachand-Robert, T.	lachand@ann.jussieu.fr
Laurence, P.	laurence@mat.uniroma1.it
Li, Congming	cli@newton.colorado.edu
Mielke, A.	mielke@ifam.uni-hannover.de
Müller, St.	sm@mathematik.uni-freiburg.de
Oleinik, O.	shamaev@ipm.msk.su
Oliker, V.	oliker@mathcs.emorgy.edu
Rodriguez, J.	rodrigue@lmc.fc.ul.pt
Schulz, F.	friedmar.schulz@mathematik.uni-ulm.de
Serrin, J.	serrin@math.umn.edu
Shaposhnikova, T.	shamaev@ipm.msk.su
Talenti, G.	talenti@udini.math.unifi.it
Trudinger, N.	nst851@huxley.anu.edu.au
Weissler, F.	weissler@math.univ-paris13.fr
Wiegner, M.	michael.wiegner@uni-bayreuth.de
Willms, B.	willms@math.umn.edu
Reichel, W.	wolf@mailwst.mathematik.uni-karlsruhe.de