# MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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## Mathematische Stochastik 5.3. - 11.3. 1995

Zu dieser Tagung unter der gemeinsamen Leitung von Dietrich Stoyan (TU Bergakademie Freiberg) und Hans-Georg Müller (University of California) trafen sich Spezialisten der Stochastik und der mathematischen Statistik aus verschiedenen Ländern mit einem breiten Spektrum an Interessen, wobei das Hauptinteresse den Fragen der räumliche Statistik, der räumlichen Datenanalyse (statistische Bildverarbeitung) sowie Problemen der stochastischen Geometrie galt.

Diese Themenkreise erfuhren in den letzten zehn Jahren eine enorme Entwicklung, die durch Erfordernisse aus verschiedenartigen Anwendungsgebieten beschleunigt und teilweise bestimmt wurde. Der teils gut ausgebaute mathematische Apparat (z.B. Punktprozesse, zufällige abgeschlossene Mengen, Integralgeometrie) mußte und muß dabei weiter verfeinert werden, und gerade Verfahren aus der "klassischen" Statistik sind bei räumlichen (ebenen) Problemen erheblich zu modifizieren und führen auf neue, zum Teil recht komplizierte theoretische Fragestellungen. Eine derartige fruchtbringende Wechselwirkung zeigte sich in den außerordentlich regen Diskussionen nach den Vorträgen und in den vortragsfreien Zeiten. Neben Vorträgen zu einigen anderen Fragen der Statistik (wie Bayessche Verfahren und Change-Points Problemen) waren wichtige Themen dieser Tagung:

- Modelle, Statistik und Stereologie von zufälligen abgeschlossenen Mengen, insbesondere bei Booleschen Modellen und zufälligen Mosaiken,
- Räumliche Markov-Modelle deren Statistik und Simulation,
- Probleme der statistischen Gestaltsanalyse,
- Grenzwertsätze für Funktionale von räumlichen und abhängigen Größen,
- (Markierte) Punktprozeßmodelle und Bildverabeitung,
- Kaplan-Meier Schätzer und dessen Verallgemeinerungen
- Survival Analysis und Analyse ein-und mehrdimensionale Zeitreihen

Insgesamt hatte die Tagung 37 Teinehmer, die 31 Vorträge hielten. Ein Abend war dem gegenseitigen Vorstellen gewidmet, was mit einer gelungenen musikalischen Umrahmung (getragen von Herrn Mammitzsch und Herrn Schmitt) verbunden wurde.

# Vortragsausszüge - Abstracts

#### V. Beneš

# Total projections of random surfaces

For a given stationary random surface process in  $\mathbb{R}^d$  we define the projection measure by means of its weights (normal orientations). We derive characteristics of the random projection measure including intensity, weight distribution, pair correlation function, Palm distribution. In connection with the formula for the pair correlation function the two-point weight distribution is studied and the inversion of the two-fold cosine transform is derived. Finally, the variance of the projection measure is evaluated explicitly for an anisotropic Poisson process. An application for the computation of variances of stereological estimators of surface intensity is given.

### J. Chadœuf

# Modelling random surfaces with Boolean randon functions

A non-parametric estimation procedure for Boolean random functions having a half-sphere as primary grain is proposed when a non-stationarity is present. It is assumed that this non-stationarity depends only on one known direction and that no longe range trend is present. We assume also that a multiplicative decomposition of the intensity function between a spatial term and a mean intensity term exists. The method is applied to a case study in soil science.

## S.N. Chiu

# Estimators of distance distributions for spatial point patterns: Hanisch vis Kaplan-Meier

Statistical estimators of the empty space function for spatial point patterns are compared. It is shown that a Hanisch type estimator is closely connected with a Kaplan-Meier type estimator introduced by Baddeley and Gill (1993,1994). Corresponding density estimators can be interpreted as minus-sampling estimators. For practical use the Hanisch type estimator is recommended.

#### N. Cressie

# Spatial statistical analysis with partially ordered Markov models

Consider the spatial process  $\{Z(u): u \in D\}$ , where  $D = \{s_1, \ldots, s_n\}$  is a finite set of spatial locations in  $\mathbb{R}^d$ . It is easy to see that (D, <), a partially ordered set equipped with the partial order <, is a one-to-one correspondence with a minimal directed acyclic graph. We define a partially ordered Markov model (POMM) as having the property that: The conditional probability of  $Z(s_i)$ , given Z(.) at all sites  $s^* < s_i$  and Z(.) at all sites

The conditional probability of  $Z(s_i)$ , given Z(.) at all sites  $s^* < s_i$  and Z(.) at all sites unrelated to  $s_i = \frac{1}{2} \sum_{i=1}^{n} \frac{$ 

The conditional probability  $Z(s_i)$ , given Z(.) at only the adjacent lower neighbours of  $s_i$ , for all  $s_i$  that are not minimal elements of (D, <).

It is shown that the joint probability of  $\{Z(u): u \in D\}$  can be written in closed product form and that  $\{Z(u): u \in D\}$  is a Markov random field with neighbours generated from





the adjacent lower neighbourhood sets of (D, <). It is also seen that POMM's can be generated by imposing a particular local Markov property on the (finite) directed acyclic graph obtained from (D, <); the latter construction is due to Lauritzen & Spiegelhalter (1988, J.Royal Statist. Soc. B). Our results give formulas for the likelihood-based inference on unknown POMM parameter and for fast simulation. We have used the POMM's in image texture analysis, Baysian hierarchical modeling, and spatio-temporal statistical analysis.

## I. Dryden

## Statistical shape analysis

Statistical shape analysis has a wide variety of applications, for example in image analysis, medical imaging, computer vision, archaeology, biology and geography. With increasingly large amounts of image data being routinely collected the automatic interpretation of such images becomes very important. Tasks such as object recognition by a robot, location of defects in factory goods, and disease diagnosis from muscle biopsies would all involve the use of shape analysis. The shape of an object consists all of its geometrical properties that are invariant under translation, scaling and rotation - and statistical shape analysis concerns models and methods for inference where this invariance is taken into account. Probability distributions for shape will be explored and these will be used for practical inference. Many methods for analyzing the size and shape of landmark data are in principle as straightforward for 3-D data as for 2-D data. For example, Procrustes analysis provides a general framework for shape analysis in m-dimensions. Alternatively, procedures such as edge superimposition are also easily generalized. However, the geometry of the shape space for 3-D data is complicated and distributions for 3-D shape are not easily obtained. These issues will be explored in some biological datasets.

#### T. Gasser

#### Estimating shift functions by dynamic time warping

Our interest in shift functions comes from the analysis of samples of curves (Kneip & Gasser, Ann. Statist., 1992). In structural analysis we try to align individual functions to an average dynamic via smooth strictly monotone shift functions. The latter have been determined on a discrete grid by common features in the curves (such as extrema, inflection points) followed by strictly monotone interpolation. Dynamic time warping has been developed for speech analysis and consists in minimizing a cost function, without "features". We can show that an appropriately modified cost function determines the "right" shift function. Further, expressions for bias and variance can be derived.

#### F. Götze

## Rates of convergence in functional limit theorems and lattice point problems

We consider the distribution of quadratic forms  $Q_n = \sum_{1 \leq i < j \leq n} a_{ij} X_i X_j$  in i.i.d. r.v.'s  $X_1, \ldots, X_n$ . It can be approximated by the distribution of  $G_n = \sum_{1 \leq i < j \leq n} a_{ij} Y_i Y_j$ , where  $Y_1, \ldots, Y_n$  are i.i.d. normal r.v.'s which have zero mean and variance 1 similarly as the X's. Assuming  $\mathbb{E}|X_1|^3 < \infty$  we prove

$$\sup_{x \to \infty} |\mathbb{P}\{Q_n < x\}| - |\mathbb{P}\{G_n < x\}| = O(\gamma_n),$$



where  $\gamma_n = \max_{1 \le i \le n} \sum_{j=1}^n |a_{ij}|$ .

If the sixth largest eigenvalue of  $(a_{ij})_{i,j=1,\dots,n}$  is bounded away from zero as  $n \to \infty$ , we prove this approximation up to an error  $O(\max_{1 \le i \le n} \sum_{j=1}^{n} |a_{ij}^2|)$ .

For arbitrary ellipsoids E in  $\mathbb{R}^k$  the following lattice point approximation result holds:

$$|\#(\mathbb{Z}^k \cap rE) - \operatorname{Vol}_k(rE)| = O(r^{k-2})$$
 as  $r \to \infty$ ,

provided that  $k \geq 9$ . The constants depends on k and the eigenvalues of E only.

## M.B. Hansen

## Estimators for range of vision

Imagine yourself standing in a forest, what is the range of vision in a particular direction? This distance distribution, usually denoted the linear contact distribution function is of interest in many spatial statistical applications. It enables e.g. estimation of: (i) preferred alignment of sets, (ii) parameters of specified spatial models, and (iii) the star volume. In the talk we will focus on regularity properties of the distribution funcion and discuss estimation by means of the reduced sample and the recently introduced Hanisch and Kaplan-Meier estimators. Finally we take a look at an application to images of protein network in yoghurt.

## L. Heinrich

Central limit theorems for the empirical K-function of a homogeneous Poisson process in  $\mathbb{R}^d$ 

The statistical second-order analysis of a stationary (isotropic) point process  $\Psi = \sum_{i\geq 1} \delta_{X_i}$  in  $\mathbb{R}^d$  is mainly based on the estimation of the so-called K-function

$$K(r) := \lambda \mathbb{E}[\Psi(B_r(o) \setminus \{o\} | \Psi(\{o\}) > 0] \quad , 0 \le r \le R < \infty \quad ,$$

where the intensity  $\lambda = \mathbb{E}\Psi([0,1)^d)$  is known or estimated by  $\Psi(A_n)/|A_n|$  from a rectangular or circular sampling region  $A_n$  expanding in all directions. In case of a stationary Poisson process we have  $K(r) = \lambda^2 \omega_d r^d$ , where  $\omega_d$  denotes the volume of the unit ball  $B_1(o)$ . There are several (asymptotically) unbiased estimators of K(r), e.g.

$$K_n(r) = |A_n|^{-1} \sum 1_{A_n}(X_i) \Psi(B_r(X_i) \setminus \{X_i\}) .$$

For these empirical K-functions we prove Berry-Esseen bounds of order  $|A_n|^{-1/2}$ , Edgeworth expansions in the integral and local CLT and large deviation relations in so-called Linnik zones  $[0, |A_n|^{1/6} \log^{\alpha} |A_n|]$  for some  $\alpha > 0$ . A functional limit theorem provides the limiting distribution of the scaled maximal deviation

$$\Delta_n := (|A_n|/\lambda)^{1/2} \sup_{0 \le r \le R} |K_n(r) - \lambda^2 \omega_d r^d| ,$$

which is equal to  $\lim_{n\to\infty} \mathbb{P}(\Delta_n \geq x) =$ 

$$2\left(1-\Phi\left(\frac{x(1-L)}{\sqrt{L}}\right)\right) + 2\sum_{k=1}^{\infty} (-1)^{n+1}e^{2nx^2}\left(\Phi\left(\frac{x(2n+1-L)}{\sqrt{L}}\right) - \Phi\left(\frac{x(2n-1+L)}{\sqrt{L}}\right)\right),$$



where  $L := 2\lambda \omega_d R^d/(1+2\lambda \omega_d R^d)$ . This enables us to construct a goodness-of-fit test for checking the Poisson hypothesis on a given point field. The corresponding problem, when the intensity is estimated, leads to a slightly different result. Most of the presented results remains valid with obvious changes for Poisson cluster processes with uniformly bounded clusters.

#### E.B. Vedel-Jensen

## A simpler proof of the Blaschke-Petkantschin formula

The Blaschke-Petkantschin formula is a formula in integral geometry, giving a geometric measure decomposition of a q-fold product of Hausdorff measures. In this talk, I present a new and simpler proof of this formula, which avoids a product version of the coarea formula and makes extensive use of multilinear algebra. The proof is based on induction. In the classical version of this formula, the special case of decomposition of a q-fold product of Lebesque measures is considered. The special case q=1 is polar decomposition of Lebesgue measure while the induction step  $q-1 \to q$  involves in addition to the induction assumption the commuting relation  $dL^n_{q(q-1)}dL^n_{q-1}=dL^q_{q-1}dL^n_q$ . This type of induction cannot be used in the general case where Hausdorff measures are considered, but still an inductive proof is possible. In the talk I illustrate this by proving the case where one set is involved by induction on p and r of the subsets  $\mathcal{L}^n_{n(r)}$  involved.

#### H.R. Künsch

## Hidden Markov random fields

We show that we can estimate the law of a stationary random field with finite state space by using the sieve of hidden Markov random fields, i.e. of many one-to-one functions of nearest neighbour Markov fields on a larger state space. The cardinality of this state space is the regularization parameter of the sieve. It has to increase together with the observation window at a suitable rate. We also discuss an algorithm for maximum likelihood estimation of the parameters of a hidden Markov random field. It uses Markov chain Monte Carlo and convex majorization of minus log-likelihood.

# M.-C. von Lieshout (joint work with A. Baddaley)

# A non-parametric measure of interaction in spatial point patterns

The stength and range of interpoint interactions in a spatial point process can be quantified by a function J=(1-G)/(1-F), where G is the nearest-neighbour distance distribution function and F the empty space function of the process. J(r) is identically equal to 1 for a Poisson process; values of J(r) smaller or larger than 1 indicate clustering or regularity, respectively. We show that, for a large class of point processes, J(r) is constant for distances r greater than the range of spatial interaction. Hence, both the range and type of interaction can be inferred from J without parametric model assumptions. We show that the J-function of the superposition of independent point processes is a weighted sum of those of the components. This property can be used to study interactions in multivariate point patterns.



# I. Molchanov (joint work with L. Heinrich)

# Limit theorems for random measures associated with germ-grain models

We consider germ-grain models with i.i.d. compact (convex) grains and define a class of stationary random measures associated with them. These measures are determined by the "visible" parts of the shifted grains, for example, by their exposed boundaries. The intensity and higher-order moment measures of these random measures are explicitly calculated and the corresonding ergodic theorem is proved. Assuming that the underlying stationary point process is absolutely regular or  $\beta$ — mixing with some polynomial mixing rate (which is automatically satisfied for Poisson processes), a corresponding CLT for the empirical intensity is proved and its asymptotic variance could be obtained in a closed form. It turns out that in the special case of a Boolean model the assupmtions to hold the CLT coincide with those ensuring the finiteness of the variance, i.e. they are actually optimal. The class of associated random measures under consideration comprises, in particular, the positive extensions of the Minkowski measures. The derived limit theorems are applied to find the approximate distributions of estimators for parameters in stationary Boolean models.

## J. Møller

### Markov chain Monte Carlo methods and spatial point processes

The objective in this talk is both to review those Markov chain Monte Carlo methods, which are used for finite point processes with either a fixed or a random number of points, and to discuss the problem of modelling spatial point processes which exhibit regular or clustered patterns. We consider a general set-up which covers ordinary spatial point processes and marked point processes as used in spatial statistics and stochastic geometry. First we study general Metropolis-Hastings type algorithms and compare these with the usual birth-death process techniques. The algorithms are used in a discussion on the effect of conditioning on the number of observed points when performing likelihood inference for the interaction structure of the model. Secondly, we discuss some practical aspects of using the algorithms in cases where different types of Markov point processes are used as models for either regular or clustered platterns. Especially, in the case of clustering, it is demonstrated that the usual Gibbsian point processes or Ripley and Kelly (1977) Markov point processes do not provide very flexible and satisfactory models and most algorithms become very inefficient. However, the larger class of nearest-neighbour Markov point processes introduced in Baddeley and Møller (1989) may provide mucl better models which in turn are feasible for simulation. We illustate this by a particular example of a nearest-neighbour Markov model for a disc process which can clustered as well as regular patterns.

#### L. Muche

#### Distributional properties of the Poisson Voronoi tessellation

For the Voronoi tessellation generated by a stationary Poisson point process in  $\mathbb{R}^d$  a method for the determination of contact distributions is given. In particular, a tractable form of the spherical and linear contact distribution function of the planar and spatial Poisson Voronoi tessellation and the spherical contact distribution function of a planar



section through a spatial Poisson Voronoi tessellation (being neither a planar Poisson Voronoi nor a another planar Voronoi tessellation) is obtained. By means of the linear contact distribution function an expression for the chord length distribution function in terms of double integrals is derived. Further results are proved by combining the distributional properties of the generating Poisson process and a well-known formula of R.E. Miles describing the configuration of points around the neighbourhood of the typical vertex in a Poisson Voronoi tessellation. This enables us to determine the distribution function of the angles around the typical vertex and the distribution function of the typical edge length. Planar sections through a higher-dimensional Poisson-Voronoi tessellations and the paircorrelation function of the point process of vertices are also discussed in some detail.

# W. Nagel

# Estimation of the Euler-Poincaré characteristic from pairs of parallel sections

Consider a standard model of stochastic geometry - a stationary random closed set (RACS) in  $\mathbb{R}^d$ , e.g. a Boolean model, having the property that a.s. the intersection with any compact convex set is a finite union of convex compact set. For this model the intensity  $\chi_V$  of the Euler-Poincaré characteristic (briefly EPC) can be defined as the mean EPC per unit volume.

A stereological problem consists in estimating  $\chi_V$  from observations from lower-dimensional sections. DeHoff and Gundersen have suggested heuristic methods which are based on a pair of parallel (hyperplane) sections - the so-caled disector - and which are derived from Hadwiger's inductive definition of the EPC. As our main result we present an estimator for  $\chi_V$  and give conditions ensuring its unbiasedness.

# Y. Ogata

# Period-age-cohort decomposition of incidence rate from incompletely detected retrospective data

A statistical point-process model is introduced to decompose an intensity retrospective incidence data on (time,age)-coordinates into three risk factors of period, age and cohort, taking missing incidence into consideration. For the objective decomposition a Bayesian estimation method with a smooth prior is applied. Analysis of onset data of diabetes in a local district is carried out. We examine the goodness-of-fit in comparison with other types of smoothed intensity models of the (time,age)-coordinate.

#### A. Penttinen

# Markov chain Monte Carlo method in Bayesian and and likelihood inference for Gibbsian point processes

The first problem consists in practical aspects in modelling spatial point patterns. In their statistical analysis the data, usually a point map, is given. The quality of such a data set is discussed and some suggestions are made for statistical modelling in connection of Gibbs point processes. Especially, models with measurement errors are considered.

The second problem is an algorithmic one. Various possibilities to create MCMM algorithms for solving maximum likelihood equations in connection of Gibbs point processes



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are suggested.

The third part gives a short outline for estimation of pair potential function using Bayesian approach. The method suggested is a "semiparametric" one.

## G. Roussas

## Estimation under dependence

Let  $Y_{ni}, i = 1, ..., n$ , observables taken at points  $x_{ni}$ , selected at will be the experimenter from the interval [0, 1], and suppose that the  $Y_{ni}$ 's are related to  $x_{ni}$ 's through the relationship  $Y_{ni} = g(x_{ni}) + \varepsilon_{ni}$ . Here g is an unknown real-valued continuous function on [0, 1], and the errors  $\varepsilon_{ni}, i = 1, ..., n$ , satisfy the requirement  $(\varepsilon_{n1}, ..., \varepsilon_{nn}) = (\xi_1, ..., \xi_n)$  in distribution for each  $n \geq 1$ ;  $(\xi_i), t = 0, \pm 1, \pm 2, ...$ , is a stationary general linear time series with  $\mathbb{E}\xi_0 = 0$  and  $\text{Var}(\xi_0) = \sigma^2 \in (0, \infty)$ . The problem is that of estimating g(x) for each  $x \in [0, 1]$ . The proposed estimate is  $g_n(x) = \sum_{i=1}^n w_{ni}(x)Y_{ni}$ , where the weights  $w_{ni}$  are subject to certain "regularity" conditions. If the  $\xi$ - process is also strong mixing, then, under some additional assumptions, it is shown that, as  $n \longrightarrow \infty$ :

$$\mathbb{E}g_n(x) \longrightarrow g(x) \ , \ g_n(x) \xrightarrow{\text{a.s.}} g(x) \ , \ (\sigma_n(x))^{-1} \left(g_n(x) - \mathbb{E}g_n(x)\right) \xrightarrow{\text{d}} N(0,1) \ ,$$

where  $(\sigma_n(x))^2 = \operatorname{Var}(g_n(x))$ .

If the  $\xi$ - process is not strong mixing, then the asymptotic normality above still holds, under suitable conditions.

Now let  $X_1, \ldots, X_n$  be random variables which are positive (negative) quadrant dependent, coming from a stationary sequence, and having (marginal) distribution function (briefly d.f.) F and probability density function f. F may be estimated by the empirical d.f.  $F_n$  and also by a smooth kernel-type estimate  $\hat{F}_n$ . For  $n \ge 1$  and each real t, define i(n) = i(n, t) by:

$$i(n) := \{ \min\{k \ge 1 : MSE(F_k(t)) \le MSE(\hat{F}_n(t)) \},$$

where  $\mathrm{MSE}(F_k(t))$  is the mean squared error of  $F_k(t)$ . Under suitable regularity conditions, it is seen that the optimal (in the MSE sense) bandwidth  $\hat{h}_n$  is the same as that in the independent identically distributed case. Next,  $\limsup_{n\to\infty} i(n)/n \le 1$  and  $\lim_{n\to\infty} i(n)/n = 1$  iff  $n[\mathbb{E}\hat{F}_n(t) - F(t)]^2 \to 0$ . If, however,  $n[\mathbb{E}\hat{F}_n(t) - F(t)]h_n^{-1} \to 0$ , then  $[i(n)-n]h_n^{-1}$  tends to a finite constant  $\theta(t) \ne 0$ . Should this constant be positive (which happens for a suitable choice of the kernel), then i(n) is substantially bigger than n, and in fact,  $i(n)-n\to\infty$ . In this sense,  $F_n$  is deficient as it compares to  $\hat{F}_n$ .

#### M. Rudemo

## Point process analysis with image data

Three examples of analysis of marked point processes from image data are discussed. In [3] we estimate local weed densities from photos of small rectangles suitable sampled from a field - each photo covering a ground area of the order  $15 \times 22cm^2$ . From such a colour image of crop and weed plants against a soil background dijoint segments corresponding to green parts of the image are constructed. This set of segments, viewed as a marked point process, is used to estimate the density of the weed plants. The main problem in



obtaining good etimates are caused by the edge effects, partial cover of weed plants by crop and overlapping weed plants.

In the second example, a digitized aerial photo of a plot of an even-aged stand of the Norway spruce is smoothed by a two-dimensional kernel, for instance an isotropic Gaussian kernel. The stem number per hectar in 6 differently managed subplots (varying thinning degrees) is estimated from the number of maxima of the smoothed image. For the crucial bandwidth estimation problem an iterative method based on analyses at several resolutions for each subplot is suggested in [1].

The third example [2] covers a sequence of weather radar intensity images. Each image is modelled as a mixture of two-dimensional Gaussian distributions corresponding to rain cells. A Kalman filter is used to track the evolution in time of the rain cells. The filter is tested for its ability to track the rain cells in the presence of a strong rotational component of motion.

- [1] Dralle, K. & Rudemo, M. (1995) Stem number estimation by kernel smoothing of aerial photos. Manuscript under preparation.
- [2] Larsen, M. (1995) A Kalman filter for tracking rain cells. To appear in *Proc. 9th Scand. Conf. Image Analysis*, Uppsala, June 1995.
- [3] Rudemo, M., Sevestre, S. & Andreasen, C. (1995) Marked point process models cropweed images. To appear in *Proc. 9th Scand. Conf. Image Analysis*, Uppsala, June 1995.

# F.J. Samaniego (joint work with Nieth)

On the efficacy of Baysesian estimation in nonidentifiable models, with applications to medical screening tests and competing risks problems

While classical estimation approaches are inapplicable in the presence of nonidentifiability, Bayes estimations are feasible and interpretable in many such problems. Through examination of a simple prototype - the estimation of the pair  $(p_1, p_2)$  from a binomial observation  $X \sim B(n, p_1 + p_2)$  - a template is suggested for the evaluation of the efficacy of Bayesian estimators of nonidentifiable parameters. The class of prior Dirichlet distributions for which the limiting Bayes estimator  $(n \longrightarrow \infty)$  improves on the prior estimator under squared error loss is characterized. In competing risks, the limiting Bayes estimator of the multiple decrement function is obtained and its efficacy is examined in similar ways. Among the results obtained is that the limiting Bayes estimator of the maximal survival probability based on the prior Dirichlet process with multivariate exponential mean is uniformly superior to the prior estimate when the true multiple decrement function is multivariate exponential.

#### I. Saxl

# Boolean cluster models: Spherical contact distances and induced tessellations

A Boolean cluster model X is the union set of a stationary Poisson process in the space Z of all finite subsets of  $\mathbb{R}^d$ . Three particular cases are treated in detail:

- (i) deterministic clusters, i.e.  $Z = \{z_1, \ldots, z_m\}$  is a fixed m- tuple of points,
- (ii) Poisson clusters, i.e.  $Z \subset B_R$  (globular or Matérn cluster) or  $Z \subset \partial B_R$  (spherical





cluster), where  $B_R$  is the d-ball of radius R centred in the origin and the number of cluster members is Poisson distributed with mean  $\bar{m}$ ,

(iii) Binomial (globular and spherical) clusters as in (ii) with fixed  $m = \bar{m}$ .

First, the "size" of the clusters  $\mathbb{E}\nu_d(\text{conv}Z)$  is discussed, then the mean dilation  $\psi_Z(r) := \mathbb{E}(Z \oplus B_r)$  is computed and the spherical contact distribution function of X

$$H_X(r) := \mathbb{P}(X \cap B_r \neq \emptyset) = 1 - \exp\{-\lambda \psi_Z(r)\}, r > 0,$$

is investigated for all the above mentioned cases, especially with respect to variable R > 0. In the second part, the effect of the cluster parameters R, m on the distribution of cell area in the Voronoi tessellation generated by point clusters of the type (i) is examined.

## M. Schmitt

Inference of the Boolean model: A new formula for the Choquet capacity of the primary grain

We analyse the foolowing points concerning the inference of the Boolean model (= union of randomly distributed independent grains):

- (i) which parameters can actually be inferred in the stationary and non-stationary case
- (ii) we experimentally compare three different estimators of the intensity (Steiner Weil Schmitt). In the convex case, Weil's estimator turns out to be the most accurate.
- (iii) we present two new formulae for estimating the density in the non-stationary case and the Choquet capacity of the primary grain (provided it is uniformly bounded).

(The discussion after the presentation has shown that a continuation of Weil's and Schmitt's estimators can be used to estimate the mean number of connected components of the primary grain.)

# K.S. Song

# Change - points problem

In this talk, the problem of computing the exact value of the asymptotic efficiency of maximum likelihood estimators of a discontinuous signal in Gaussian white noise is considered. A method based on constructing difference equations for the appropriate moments is presented and used to obtain the exact variance of the Pitman estimator. Other related unsolved problems are also discussed.

### U. Stadtmüller

Spatial smoothing of geographically aggregated data with application to the construction of incidence maps

The starting point of this investigation is the commonly encountered situation in spatial statistics where the data like counts of indices of a certain desease are only available in geographically aggregated form. We develop a fairly general model for this situation to propose a smoothing method to recover the unknown smooth spatial function which is assumed to generate the observations. In the case of count data, the target function is the intensity function, conditionally to the total number of observations. Our proposed method is based on a modified version of the locally weighted least square method. It





uses the shape and size of the aggregation areas and avoids the arbitrariness of selecting a point in each area to which the data are attached. We derive asymptotic properties of the method and propose some numerical procedures for calculating the estimates.

## M.L. Stein

# Fixed domain asymptotics for spatial periodograms

The periodogram for a spatial process observed on a lattice is often used to estimate the spectral density. The bases for such estimators are two asymptotic properties that periodograms commonly possess: first, that the periodogram at a particular frequency is approximately unbiased for the spectral density, and second, the correlation of the periodogram at distinct frequencies is approximately zero. For spatial data, it is often appropriate to use fixed domain asymptotics in which the observations get increasingly dense in some fixed region as their number increases. Using fixed domain asymptotics, this work shows that standard asymptotic results for periodograms do not apply and that using the periodogram of the raw data can yield highly misleading results. However, by appropriately filtering the data before computing the periodogram, it is possible to obtain results similar to the standard asymptotic results for spatial periodograms.

## D. Stoyan

# Set-valued means of samples of particles

The aim is to determine set-valued means for samples of objects such as powder-particles or sand grains without respect to their location and orientation. The idea is to translate the coresponding compact sets  $K_1, \ldots, K_n \subset \mathbb{R}^2$  to the origin and to rotate them so that they are "close together" and then to use appropriate definitions of means for random compact sets to the new sample. This method can be justified mathematically by describing the  $K_i$  by functions (indicator, support or others) and by considering them as elements of a suitable Hilbert space on which a group is acting.

## W. Stute

# The statistical analysis of Kaplan-Meier integrals

Let  $\hat{F}_n$  denote the Kaplan-Meier estimator compute from a sample of possibly censored data, and let  $\varphi$  be a given function. In this paper some of the most important properties of the Kaplan-Meier integral  $\int \varphi d\hat{F}_n$  are reviewed. The SLLN, the CLT, the bias and the Jackknife. Since Kaplan-Meier integrals constitute the leading term of more complicated statistical functionals od  $\hat{F}_n$  these results may be viewed as the cornerstones for analyzing right-censored data.

# A. Tsybakov

# Nonparametric estimation of level sets

Let f(x) be a probability density on  $\mathbb{R}^N$ , and let  $X_1, \ldots, X_n$  be a sample from f(x). Consider the problem of estimation of the level set  $G := \{x : f(x) \ge \lambda\}$ , where  $\lambda \ge 0$  is the level. It is assumed that G belongs to a class of subsets of  $\mathbb{R}^N$  (for example, convex sets, or sets with smooth boundaries and star-shaped structure). The estimates of G are constructed, based on the maximization of local empirical excess mass. It is shown that



under the appropriate choice of parameters of the estimates they achieve optimal rates of convergence, and these rates are explicitly given. An interesting consequence is that the zero-level sets, i.e. the support, of the density f(x) can be estimated better than the sets of level  $\lambda > 0$ .

## J.-L. Wang

## Is there a deceleration in mortality?

When data were observed periodically or discretely, it is customary to use lifetable or some smooth version of it to estimate the mortality rate (or hazard rate). However, such methods may induce bias to the oldest segment of the population, the so-called oldest-old group. We discuss this effect and suggest a transformation to correct such bias. A revised locally weighted least squares method is proposed as the smoothing method for the transformed lifetable estimate. The method is illustrated on a recent data set to check the Gopertz law of mortality and whether there is a deceleration in aging or not at older ages.

# W. Weil

# Mean bodies of particle processes and Boolean models

For a stationary point process X of conves bodies, a mean particle M(X) is defined by the support function

$$h(M(X),.) := \gamma \int_{\mathcal{K}_o} h(K,.) dP_o(K) ,$$

where  $\gamma$  is the intensity and  $P_o$  the distribution of the typical grain (being a probability measure on the set  $\mathcal{K}_o$  of bodies with Steiner point at o). If h(K,.) is replaced by the surface area measure S(K,.), in the same manner the Blaschke body B(X) is obtained. Several equivalent descriptions of these mean bodies are given, related to the problem to estimate a mean particle of X from a bounded sampling window or from a random section  $X \cap E$ , where E is a plane.

In particular, it is shown that  $\mathbb{E}_E M(X \cap E) = M_2(B(X))$ , where  $M_2$  is a known transform, which can be inverted. This allows the estimation of B(X) from planar sections  $X \cap E$ .





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