Tagungsbericht 12/1995

Constructive Methods in Complex Analysis

26.3. bis 1.4.1995

Die Tagung fand unter der Leitung von D. Gaier (Universität Gießen) und R. Varga (Kent State University) statt. Sie wurde von 38 Teilnehmern besucht, wobei 33 Vorträge gehalten wurden.

Ein wesentliches Merkmal dieser Tagung scheint mir die Kontinuität zu den letzten Tagungen zum gleichen Thema zu sein. Viele Fragestellungen wurden aufgenommen, und Fortschritte wurden so sichtbar gemacht. Darüberhinaus gab es neue Schwerpunkte. Hervorzuheben ist hier die Teilnahme einiger Mathematiker aus den ehemaligen Ostblockstaaten.

Im Mittelpunkt des Interesses standen Fragen der konstruktiven Funktionentheorie, mit den Schwerpunkten:

- Approximation analytischer Funktionen mit Hilfe von Polynomen oder rationalen Funktionen.
- Konstruktion und Berechnung konformer Abbildungen, sowie damit zusammenhängender Größen.

Ich möchte im folgenden kurz die auf der Tagung vorgestellten Resultate und Diskussionsbeiträge zu jedem dieser beiden Schwerpunkte zusammenfassen. Einige Vorträge sollen als Beispiele für die beschrieben Themengebiete hervorgehoben werden. Dabei beschränke ich mich auf die Beiträge zu den Hauptgebieten der Tagung. Wir hörten darüberhinaus jedoch auch einige interessante Vorträge zu Randthemen.

Im Vordergrund der Berichte über Approximation im Komplexen stand diesmal der Themenkreis der Padé-Approximation. Wir hörten Vorträge über die Konvergenztheorie von Padé-Approximationen. Um ein Beispiel zu nennen, soll hier der Beitrag von H. Wallin erwähnt werden, in dem das Padé-Schema um einen ganz oder teilweise vorgegebenen Nenner erweitert wird (Padé type approximation), wobei für gewisse Klassen von ganzen Funktionen geometrische Konvergenzraten hergeleitet werden können. Andere Vorträge berichteten über die Lage der Nullstellen und Pole, und damit zusammenhängend, über die Konvergenzgebiete von Padé-Approximationen. P. Borwein zeigte Ergebnisse über die Approximierbarkeit von Funktionen mit gewichteten Padé-Approximationen, wobei ältere Resultate von Saff und Varga als Grenzfälle wiedergewonnen werden. Wir hörten außerdem über multivariate Padé-Approximation und über die Numerik bei der Berechnung spezieller Funktionen.

Mit der Approximation von Funktionen auf einem oder mehreren Intervallen befaßten sich mehrere Vorträge. So lieferte D. Gaier eine Erweiterung und einen neuen Beweis für ein Resultat von Saff, Totik und Ivanov, das die Approximation von in einem Intervall stückweise analytischen Funktionen durch Polynome betrifft.

Ein weiterer Themenschwerpunkt kann grob als Approximationstheorie im Komplexen bezeichnet werden. Die klassische Frage nach der Konvergenzgeschwindigkeit von polynomialen und rationalen Approximationen wurde in den Vorträgen von V. Prokhorov und V. Andrievskii aufgenommen. Es wurde auch über die asymptotische Lage der Nullstellen von besten rationalen Approximationen berichtet.

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In engem Zusammenhang mit dem eben genannten Thema steht die Konstruktion von minimalen Polynomen auf Teilkompakta der komplexen Ebene. F. Peherstorfer gab asymptotische Resultate für Tschebyscheff-Polynome im Fall mehrerer Intervalle an. Minimalpolynome treten auch im Zusammenhang mit Iterationsalgorithmen bei der Lösung von Gleichungssystemen auf. Verschiedene Vorträge gingen auf diese numerisch orientierte Fragestellung ein.

Das zweite zentrale Thema der Tagung war wiederum die Konstruktion konformer Abbildungen. Dieses Thema hat bekanntlich Anwendungen in den Ingenieurwissenschaften. Einige Vorträge gaben daher Berichte über die explizite und schnelle Berechnung dieser Abbildung. Im Zusammenhang mit der elektrischen Widerstandberechnung steht die Berechung des Moduls von Vierecken. Für gewisse in der Praxis auftretende "lange" Vierecke kann man Unterteilungen angeben, so daß sich der Modul mit großer Genauigkeit approximieren läßt.

Vortragsauszüge

A constructive characterization of some classes of functions on quasiconformal arcs

V. V. Andrievskii

We introduce three classes of functions given and continuous on a Jordan arc in the complex plane without cusps (more precisely quasismooth in the sense of Lavrentiev arcs) and study the relations between them. The main idea of our approach is to represent the function as a Cauchy transformation and to describe its properties by giving the restrictions on the density of the Cauchy transformation.

Sign changes in L_p-approximation

H.-P. Blatt

Let $f \in L_p[-1,1]$, $1 < p < \infty$, $B_{n,p} \in \Pi_n$ the best approximating polynomial of degree $\leq n$ with respect to the L_p-norm

$$\|g\|_p = \left[\int_{-1}^{1} |g(x)|^p w(x)\,dx\right]^{\frac{1}{p}},$$

where $w(x)$ is a weight function such that a Nicholski-inequality $\|p_n\|_\infty \leq n^\kappa \|p_n\|_p$ holds for all $p_n \in \Pi_n$, $\kappa > 0$ fixed.

Then the following result is proved: There exists a subsequence $\{n_k\}$ of $\mathbb{N}$ such that the error function of $B_{n,p}$ has $n+1$ sign changes at

$$-1 < y_1 < \ldots < y_{n+1} < 1$$

and the discrepancy $D[\nu_n - \mu]$ between the normalized counting measure ν_n of the points y_i and the equilibrium distribution μ on $[-1,1]$ satisfies

$$D[\nu_n - \mu] \leq c\sqrt{\frac{\log n}{n}}$$

for $n = n_k$, $k = 1, 2, \ldots, \infty$, where c is an absolute constant.

Rational approximation in Hardy spaces and Morse theory

L. Baratchart

We use Morse theory to analyse critical points index in H_2 rational approximation. We derive the index theorem asserting that, generically:

$$\sum_{x \text{ critical}} (-1)^{\varepsilon(x)} = 1,$$

where $\varepsilon(x)$ is the Morse index of x.

We then apply this to the uniqueness problem, and discuss cases where uniqueness prevails by analysing the signature of the Hessian matrix as linked to a certain interpolation error.

The numerical condition of the monomial basis: Estimates for the condition number of Vandermonde, Krylov and positive definite Hankel matrices

B. Beckermann

Matrices of the classes of matrices mentioned above are known to be notoriously ill-conditioned (e.g., the Hilbert matrix). In the last years, several lower bounds for the condition number of such matrices have been given (Gautschi 1972-1988, etc.). It is one aim of this talk to present best possible bounds.

The problem of optimal lower bounds for the condition number of Vandermonde matrices of order n with nodes located in some $M \subset \mathbb{C}$ is shown to be closely related with a Markov type approximation problem: under all polynomials p of degree at most n, $|p(z)| \leq 1$ for $z \in M$, find the one with maximal $|p^{(j)}(0)|, j = 0, \ldots, n$.

We will also study the case of weighted maximum norms in the complex plane, corresponding to Krylov and Hankel matrices.

Incomplete rationals in the complex plane

P. Borwein

We consider the problem of when

$$\left\{ z^{\theta n} \frac{p_n(z)}{q_n(z)} : p_n, q_n \in \Pi_n \right\}$$

is dense in the analytic functions $A(K)$. Certain natural regions are constructed. The analysis requires a detailled description of the zeros and poles of Padé approximants to functions of the form $(1+z)^{\alpha n}$. The "classical" results of Saff and Varga on the location of zeros and poles of Padé approximants to e^z are recovered as limiting cases.

Homogeneous or non-homogeneous multivariate Padé approximation?

A. Cuyt

In the past 20 years many definitions for multivariate Padé approximants were launched and all of them can be grouped in 2 main categories. We compare these two fundamentally different approaches with respect to unicity, consistency and convergence of the multivariate Padé approximant. It appears that both approaches have some appealing properties as well as obvious disadvantages.

The numerical solution of the biharmonic equation by conformal mapping

T. K. DeLillo

The solution of the biharmonic equation in a simply-connected region Ω in the plane is computed in terms of the Goursat functions. The boundary conditions are conformally transplanted to the disk with a numerical conformal map. A linear system is obtained for the Taylor coefficients of the Goursat functions. The coefficient matrix A of the linear system is roughly the identity plus a Hankel matrix. We show that if the boundary Ω is analytic, then A is, in fact, a low rank plus small norm perturbation of the identity matrix. Hence, if conjugate-gradient-like methods are applied to solve the linear system, the convergence rate is superlinear. We remark that multiplication of the $N \times N$ matrix A times an N-vector x can be done in $0(N \log N)$ by using the fast Fourier transform. Numerical results are given to illustrate the fast convergence. (This is joint work with Raymond H. Chan, Chinese University of Hong Kong and Mark A. Horn, Wichita State University.)

Comparison theorems for the convergence factor of a compact set

M. Eiermann

Let $\Omega \subset \mathbb{C}$ be compact. With

$$K_m := \{\|p\|_\Omega : p \text{ is a polynomial of degree at most } m \text{ with } p(0)=1\},$$

the limit $K(\Omega)$ of the sequence $\{K_m^{1/m}\}$ exists. $K(\Omega)$ plays a certain role in the asymptotic convergence analysis of polynomial based iterative methods for linear systems - which explains the name "convergence factor". If Ω possesses a Green's function with singularity in ∞, then $K(\Omega)$ can be expressed in terms of G. Here, we consider the question how the asymptotic convergence factor is effected by a rational transformation of Ω.

On some explicitly solvable complex Chebyshev approximation problems

B. Fischer und F. Peherstorfer

When applying a polynomial iteration method

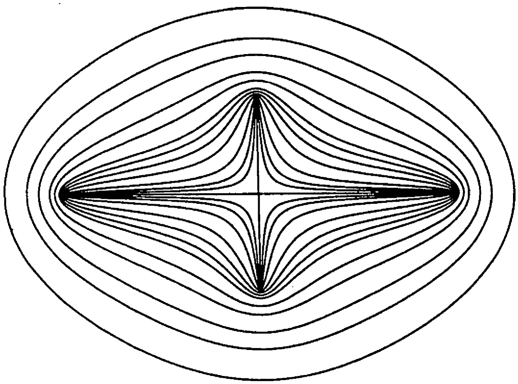
$$r_n = p_n(A)r_0, \quad p_n(0) = 1,$$

to the iterative solution of a linear system $Ax = b$, the norm of the nth residual r_n may be estimated in terms of the solution of a complex Chebyshev approximation problem

$$||r_n||_2 \leq C \min_{p_n(0)=1} \max_{z \in \Omega} |p_n(z)|.$$

Here Ω is a subset of the complex plane which contains all eigenvalues of A.

Typically, an iterative scheme is not directly applied to the given system but to a preconditioned version. When using a particular preconditioner for the Stokes-equations it turns out that the eigenvalues of the preconditioned system are located in "cross-shaped" regions.



In this note we will explicitly compute the Chebyshev polynomial on these sets and on related geometries. In particular we will present a variety of examples.

A block Lanczos algorithm and matrix Padé approximants

R.W. Freund

Let $H : \mathbb{C} \mapsto (\mathbb{C} \cup \{\infty\})^{p \times m}$ a matrix-valued transfer function of the form

$$H(z) = L^T(I - zA)^{-1}R$$

where $A \in \mathbb{C}^{N\times N}$, $R \in \mathbb{C}^{N\times m}$, and $L \in \mathbb{C}^{N\times p}$. In many applications, N is very large and one is interested in approximating H by a rational function H_n of order $n << N$. For the case $p = m = 1$, it is well known that the Padé approximant $H_n = [n-1\,/\,n]$ can be computed efficiently and stably by exploiting the connection beteen the classical Lanczos process and Padé approximation. In this talk, we present an extension of this result to the general case $m, p \geq 1$. First, we describe a novel Lanczos-type algorithm for multiple starting vectors. Then we show that this algorithm can be used to compute n-th matrix Padé approximants to H.

Polynomial approximation of piecewise analytic functions

D. Gaier

Let f be continuous on $I = [-1,1]$ and belong to a certain smoothness class, typically $f^{(k)} \in \text{Lip}\,\alpha$. By well known theorems, there exist polynomials of degree n such that (*) $|f(x) - P_n(x)| \leq M \cdot n^{-k-\alpha}$ for $x \in I$. This global estimate can, however, be improved in the neighborhood of a point x_0 where f is regular. In fact, there exist P_n satisfying (*) and $|f(x_0) - P_n(x_0)| = O(q^n)$ for some $q = q(x_0) < 1$. A result of this type was first given by Saff, Totik and Ivanov (1989). We improve their result somewhat and give a simpler and more constructive approach.

Polynomials in the complex plane

M. v. Golitschek

A theorem of Bernstein states that if an algebraic polynomial P_n of degree $\leq n$ satisfies $|P_n(x)| \leq 1$, $-1 \leq x \leq 1$, then

$$|P_n(z)| \leq \rho^n \quad \text{for all } z \in E_\rho,\ \rho > 1,$$

where E_ρ is the ellipse with foci ± 1 and the sum of half-axis ρ.

Let $A \subset \mathbb{R}$ be the finite union of compact intervals and $w \in C(A)$ be non-negative on A. We describe the largest set $D_\rho := D_\rho(w, A)$, independent of n and P_n, with the following property:

If P_n satisfies $w(x)^n |P_n(x)| \leq 1$, $x \in A$, then

$$|P_n(z)| \leq \rho^n \quad \text{for all } z \in D_\rho.$$

On segment approximation

R. Grothmann, Eichstätt

We discuss best segment approximation (with free knots) by polynomials to piecewise analytic functions on a real interval. It is shown that, if the degree of the polynomials tends to infinity, and the number of knots is the same as the number of singularities of the function, then the optimal knots converge geometrically fast to the singularities. When the degree is held fixed and the number of knots tends to infinity, we study the asymptotic distribution of the optimal knots.

This is a joint work with H. Mhaskar.

Pre-adapted quadrature rules for boundary integrals and Donaldson-Elliot error estimation

D. Hough

This talk describes the quadrature strategy that is implemented in the CONFPACK numerical conformal mapping package. In particular, the Donaldson-Elliot representation of quadrature errors for n-point Gaussian rules is combined with certain asymptotic expansions for large n in order to produce computable quadrature error estimates for the type of integrals, which arise in the simple layer boundary integrals of classical two-dimensional potential theory. These error estimates enable efficient adaptive and pre-adapted composite rules to be constructed. Efficiency comparisons are made with the QUADPACK public domain quadrature package.

Computation of special functions in the complex domain by Padé approximants

W. B. Jones

The talk is on the computation of complex valued functions of a complex variable defined by Stieltjes transforms

$$F(z) = \int_a^b \frac{d\psi(t)}{z+t}, \qquad -\infty \leq a < b \leq +\infty.$$

Use is made of both one-point and two-point Padé approximants expressed in terms of three families of continued fractions: real J-fractions, Stieltjes fractions and positive T-fractions. The primary goal is to describe methods for determining the exact number of significant digits $SD(\hat{f}_n(z), F(z))$ in a computed approximation $\hat{f}_n(z)$ of the function value $F(z)$. The methods take into account the *machine unit* ν of the computer, ***rigorous bounds for the truncation error***

$|F(z) - f_n(z)|$ where $f_n(z)$ denotes the true value of the nth approximant, and ***rigorous bounds of the roundoff error*** $|f_n(z) - \hat{f}_n(z)|$ in the computed approximation $\hat{f}_n(z)$ of $f_n(z)$. Conformal mapping plays a basic role in the determination of both types of error bounds. Results from machine computations are given to illustrate the methods.

The role of the endpoint in weighted polynomial approximation with varying weights

A. Kuijlaars

For a weight function $w : [a, b] \to (0, \infty)$, we consider weighted polynomials of the form $w^n P_n$ where the degree of P_n is at most n. The class of functions that can be approximated with such weighted polynomials depends on the behavior of the density of the extremal measure associated with w. We show that every approximable function must vanish at the endpoint a, if the density behaves like $(t-a)^\beta$ near a with $\beta > -1/2$. On the other hand, if $\beta = -1/2$ then there exist approximable function that do not vanish at a. We also present an analogous result for interval points.

Weighted Zolotarev problems

E. Levin

Given two disjoint compacts E_1, E_2 in the complex plane, consider the ration $Z_{mn}(r) = \sup_{E_1} |w^{m+n} r| / \inf_{E_2} |w^{m+n} r|$, where w is a weight function and r is a rational function of type (m, n). Let $Z_{m,n}$ denote the infimum (over all such r) of $Z_{mn}(r)$. For any "ray sequence" (m, n), that is $m/n \to \lambda$, $m + n \to \infty$, $Z_{mn}^{1/(m+n)}$ approaches a limit $L(\lambda)$. We study the behavior of zeros and poles of a sequence r_{mn} for which $\{Z_{mn}(r_{mn})\}^{1/(m+n)} \to L(\lambda)$. Application to minimal Blaschke products are also given.

This is a joint work with E.B. Saff.

Progress in convergence theory of Padé approximation

D. S. Lubinsky

We survey/discuss recent progress in the convergence theory of diagonal Padé approximants (that is approximants $[n/n]$ with numerator and denominator degree at most n, $n \to \infty$) and some problems that arise in this theory. For example, estimate the size of the lemniscate

$$\{z : |P(z)| \le \varepsilon^n\},$$

P a polynomial of degree $\leq n$, normalized by

$$||P||_{L_\infty(|z|=r)} = 1.$$

Also, analogues of this in several variables.

Complex rational approximation: the DKL problem, Padé-type cuts

A. P. Magnus

The DKL problem: Complex approximation is seen in the following setting: le f be holomorphic in a domain $D \subset \mathbb{C}$, let K be a compact set $\subset D$ where values of f are available (Known), and L a compact set $\subset D$ where one wants to estimate (to Learn) f. How to use n units of information out of K in building an approximation r to minimize the norm of $f - r$ on L? Ideally, this should be investigated with optimal recovery techniques. The solution is asymptotically ($n \to \infty$) known for rational interpolation in 3 cases: $K = L$, K or L = a single point of D (Padé-type approximation).

Padé-type cuts: Moreover, when the boundary of D is allowed to move (cuts linking fixed branchpoints for instance), optimization can be pushed further. This leads to quadratic differentials, already discussed in Padé approximation (H. Stahl).

'1/9' and me: The so-called CF method for approximating $\exp(-x)$ on $(0, \infty)$ by rational functions leads to the study of the eigenvalues and eigenfunctions of the operator on $(0, \infty)$ $(\mathcal{H}f)(x) = 2\pi^{-1/2} \int_0^\infty \exp(-(x+y)^2)f(y)dy$. Any help on asymptotics (eigenvalue $\lambda_n \to 0$ when $n \to \infty$) will be welcome.

On certain aspects of orthogonal polynomials on annular sectors in the complex plane

G. Opfer

Orthogonal polynomials on a compact set S in $\mathbb{C}$ can be generated relatively easily at least for moderate degrees if the scalar products are computable precisely with little amount of work. This is the case for annular sectors S and ordinary line- and area-integrals as scalar products. The orthogonal polynomials p so generated are normalized by $p(0) = 1$ and the size $||p||_\infty^S$ (uniform norm on S) is investigated in order to find out whether these polynomials can be used in so-called polynomial based iterations for solving linear equations. The orthogonal polynomials are computed by a full term recursion. But an artificial k-term-recursion ($k \geq 3$) is also tested with the surprising result that the zeros of these (non orthogonal) polynomials are located on k-2 arcs which are very close to line segments.

Determining crosscuts of subdivision for domain decomposition in numerical conformal mapping

N. Papamichael

Let $Q := \{\Omega; z_1, z_2, z_3, z_4\}$ be a quadrilateral consisting of a Jordan domain Ω and four points z_1, z_2, z_3, z_4 in counterclockwise order on $\partial\Omega$. We consider a domain decomposition method for computing approximations to the conformal module $m(Q)$ of Q in cases where Q is "long" or, equivalently, $m(Q)$ is "large". This method is based on decomposing the original quadrilateral Q into two or more component quadrilaterals $Q_1, Q_2, \ldots$ and then approximating $m(Q)$ by the sum $\sum_j m(Q_j)$ of the modulus of the component quadrilaterals. The specific purpose of the talk is to consider ways for determining appropriate crosscuts of subdivision so that the sum $\sum_j m(Q_j)$ does indeed give a good approximation to $m(Q)$.

On asymptotics for minimal polynomials by Green's functions

F. Peherstorfer

Let $E_l = [a_1, a_2] \cup [a_3, a_4] \cup \ldots \cup [a_{2l-1}, a_{2l}]$, $H(x) = \Pi_{j=1}^{2l}(x - a_j)$ and let $M_n(x) = x^n + \ldots$ be the minimal polynomial on E_l with respect to the sup-Norm and $\tilde{M}_n(x) = M_n(x)/L_n$, where L_n is the minimum deviation. Further let for $z_0 \in \mathbb{R} \setminus E_l$ $\phi(z, z_0) = \exp G(z, z_0)$, where $G(z, z_0)$ the complex Green's function which has purely imaginary modulus of periodicity. First it is shown that a polynomial $P_{n+l-1}(x) = x^{n+l-1} + \ldots$ which can be represented in the form

$$P_{n+l-1}^2 - HQ_{n-1}^2 = g_{(n)}^2$$

where $g_{(n)} \in P_{l-1}, Q_{n-1} \in P_{n-1}$ and at the zeros of $g_{(n)}$, denoted by $w_{j,n}$

$$P_{n+l-1}(w_{j,n}) = -(\sqrt{H}Q_{n-1})(w_{j,n}),$$

can also be represented with the help of the Green's functions in the following way

$$\frac{P_{n+l-1}(z)}{g_{(n)}(z)} = \frac{1}{2}\left(\frac{\phi(z,\infty)^n}{\prod_{j=1}^{l-1}\Phi(z, w_{j,n})} + \frac{\prod_{j=1}^{l-1}\phi(z, w_{j,n})}{\phi(z,\infty)^n}\right)$$

This gives, in conjunction with the assertion, that

$$\frac{P_{n_k+l-1}(x)}{g_{(n_k)}(x)} = \tilde{M}_{n_k}(x) + \varepsilon_k(x) \qquad \text{with } \varepsilon_k \to 0 \text{ on } [-1,1] \text{ uniformly,}$$

if $w_{j,n_k} \in [a_{2_j} + \delta, a_{2_j+1} - \delta]$, $\delta > 0$, $j = 1, \ldots, l-1$ for $k \geq k_0$, an asymptotic representation of the minimal polynomial in terms of Green's function. The representation coincides with that one conjectured by Widom. For a wide set of intervals the above rational function, which gives asymptotically the minimal polynomial, can even be determined explicitely.

Behavior of Laurent-type rational functions with applications to conformal mapping

I. E. Pritsker

This is a joint work with N. Papamichael, E. B. Saff and N. S. Stylianopoulos.

Consider the annular region A bounded by Jordan curves Γ_e and Γ_i, with Γ_i interior to Γ_e. Let the domain G be the interior of Γ_i, $0 \in G$, and Ω be the exterior of Γ_e.

We consider the problems of convergence and asymptotic zero distribution of sequences of Laurent-type rational functions of the form:

$$R_{m,n}(z) = \sum_{j=-n}^{m} a_j^{m,n} z^j.$$

Under certain assumptions on the asymptotic behavior of the leading coefficients of $R_{m,n}$, the weak* limit of the normalized counting measures in zeros of $R_{m,n}$ is the linear convex combination of two harmonic measures $\mu_e(\cdot) := \omega(\infty, \cdot, \Omega)$, supported on Γ_e, and $\mu_i(\cdot) := w(0, \cdot, G)$, supported on Γ_i.

We answer the question about asymptotic proportion between m and n for $R_{m,n}$ to achieve the best geometric rate of convergence in approximation of a function analytic on the closure of annular region A. The result suggests an adaptive version of the orthonormalization method in approximate conformal mapping of doubly connected domains. The estimates of the error in ONM for the domains with piecewise analytic boundary without cusps are also given.

Finally, we present some numerical results for the adaptive ONM.

Rational approximation of analytic functions

V. Prokhorov

Let E be an arbitrary compact set belonging to the complex plane $\mathbb{C}$, and let f be entire function of finite order $\sigma \geq 0$. For each nonnegative n and m denote by $R_{n,m}$ the class of the rational functions of order (n, m):

$$R_{n,m} = \{r : r = p/q,\ \deg p \leq n,\ \deg a \leq m, q \not\equiv 0\}.$$

The deviation of f from $R_{n,m}$ (in the uniform metric on E) is denoted by $\rho_{n,m}$

$$\rho_{n,m} = \inf_{r \in R_{n,m}} ||f - r||_E,$$

where $||\cdot||_E$ is the supremum norm on E.

We suppose that the sequence $m(n), n = 0, 1, 2, \ldots$, satisfies the following conditions:

$$m(n) \leq n, \quad \lim_{n\to\infty} m(n)/n = \theta, \quad 0 \leq \Theta \leq 1.$$

The investigation of the asymptotic behavior of the sigular numbers of the Hankel operator constructed from the function being approximated enables us to prove the following result in the theory of rational approximation of analytic functions.

Theorem. Suppose that E is an arbitrary compact set in $\mathbb{C}$, and f is an entire function of finite order. Then

$$\overline{\lim_{n\to\infty}} \frac{\ln(\rho_{n-m(n),0}\,\rho_{n-m(n)+1,1}\cdots\rho_{n,m(n)})}{n^2 \ln n} \leq -\frac{\theta}{\sigma}.$$

ADI iterative methods and application to the restoration of noisy images

L. Reichel

A generalized Alternating Direction Implicit (ADI) iterative method is introduced in which strict alternation is not required. Analysis based on potential theory shows that the rate of convergence of this method can be higher than for the standard ADI iterative method, which requires strict alternation. An application of the generalized ADI iterative method to the restoration of images corrupted by noise using a Wiener filter is described. The talk presents joint work with D. Calvetti and N. Levenberg.

The summation formulae of Poisson, Plana, Euler-Maclaurin and their relationship

G. Schmeisser

It has been pointed out by P. L. Butzer and his collaborators that the generalized sampling theorem for nonbandlimited functions, the Possion summation formula, the Euler-Maclaurin formula and the functional equation of Riemann's zeta function are essentially equivalent. Communicating a problem of J. P. Dowling, Butzer asked if an equivalence between the Euler-Maclaurin formula and that of Plana could also be established. A severe difference between the two formulae is that the former applies to functions defined on an interval while the latter requires the functions to be holomorphic in a half-plane. Nevertheless we show that each of the two formulae can be deduced from the other if we admit the following tools: Cauchy's contour integration for rectangles, Weierstrass' theorem on approximation by polynomials and a classical result from the theory of summability. We also demonstrate that Plana's formula is a direct consequence of Poisson's summation formula (joint work with Q. I. Rahman).

Rational approximation: A result by Dumas revisited

H. Stahl

In a dissertation (defended in 1908 and supervised by Adolf Hurwitz) Dumas investigated the convergence of the continued fraction development of functions of the form $\sqrt{P(z)}$ with P a quadratic polynomial. He showed that the development diverges at infinitely many points and in general this set of points is dense in the complex plane. On the other hand he also showed that the continued fractions converge at infinitely many points.

We shall reconsider Dumas' problem and will show how with modern techniques the convergence and divergence of the continued fractions can be described. There are sets with essenti divergence. In other areas the divergence is the result of so-called spurious poles of the approximants. These poles may cluster everywhere in the domain of convergence. The phenomenon is related to periodic and nonperiodic behavior of the continued fractions. The results are extended to the case of polynomials P of higer degree.

Estimating the accuracy of certain engineering approximations for conformal modules of quadrilaterals.

N. S. Stylianopoulos

We consider certain engineering rules for approximating the conformal modules of quadrilaterals, due to P.M. Hall [Thin Solid Films **1** (1967/68), 277-295]. By making use of the theory of a domain decomposition method for conformal mapping (N. Papamichael and N.S. Stylianopoulos [Numer. Math. **62** (1992), 213-234]) and, in particular, of certain results for a special class of quadrilaterals, due to D. Gaier and W. Hayman [Constr. Approx. **7** (1991), 453-467], we derive error estimates for the above rules. This estimates show that the rules are in, fact, remarkably accurate.

Markoff constants for Cantor sets

V. Totik

Let K be a Cantor-type set such that during its construction at level n we have 2^n intervals of length $e^{-\Lambda_n}$. K is said to have the Markoff property if for some L and all polynomials P_n we have $||P_n'||_K \leq (\deg P_n)^L ||P_n||_K$. It is proven that the following are equivalent:

(i) K has the Markoff property

(ii) The Green function $g_{\mathbb{C}\setminus K}(z, \infty)$ is Lipschitz continuous

(iii) $\Lambda_n = 0(n)$.

It is also shown that if M_n is the smallest constant in

$$||P_n'||_K \le M_n ||P_n||_K, \quad \deg P_n \le n,$$

then $M_{2n} \ge \frac{1}{4}e^{\Lambda_n}$. In particular, no matter how $\gamma_n \searrow 0$, there is a **regular** Cantor set K such that $M_n \ne 0(e^{\gamma_n n})$.

The latest results on lower bounds for the Bruijn-Newman constant Λ

R. S. Varga

We discuss the latest results on lower bounds for the de Bruijn-Newman constant Ω (where the truth of the Riemann Hypothesis is equivalent to ($\Lambda \le 0$).

Here, we give the latest result (Csordas, Odlyzko, Smith and Varga) that

$$-0.000\,000\,005\,895 < \Lambda$$

and we give the analysis, involving differential equations, for coupling numerical results on zeros of the Riemann zeta function, in the critical strip. It appears that the conjecture of Newman, namely

$$\Lambda \ge 0$$

may be true, and there is definitely an interesting problem to pursue. (Of course, showing $\Lambda \ge 0$ does not disprove the Rieman Hypothesis, as the Riemann Hypothesis is equivalent to $\Lambda \le 0$.) We also remark that the result of de Bruijn, of 1950, namely that

$$-\infty < \Lambda \le 1/2,$$

is such that the upper bound of 1/2 has not been improved in 45 years. We complete the movement of zeros of $H_\lambda(t)$, as a function to t, to the determination of bounds for Λ.

Padé and Padé type approximation

H. Wallin

Reporting on joint research with Amiran Ambroladze I made a comparison between diagonal PA (Padé approximation) and PTA (Padé type approximation) which is a rational interpolation where some or all of the poles are preassigned. To be more precise, let f be holomorphic at infinity and let p_n and q_n, $q_n \not\equiv 0$, be polynomials in one complex variable z of degree at most

n, and approximate f by p_n/q_n. We get the nth diagonal PA p_n/q_n if we determine p_n and q_n by the interpolation condition $(fq_n - p_n)(z) = 0(z^{-n-1})$ as $z \to \infty$, and we get the nth PTA with preassigned poles at the zeros of q_n if q_n is preassigned and we let p_n be determined by the interpolation $(fq_n - p_n)(z) = 0(z^{-1})$. In a natural way we can also define PTA where some, but not all, of the poles are preassigned. In particular, I reported on the following theorem and its generalizations:

Theorem: Let w be an entire function and let

$$f(z) = \int_{-1}^{1} \frac{w(t)}{z-t} dt.$$

If q_n is the normalized nth Legendre polynomial and p_n/q_n is the PTA with preassigned poles at the zeros of q_n, then

$$\limsup_{n\to\infty} \left| f(z) - \frac{p_n(z)}{q_n(z)} \right|^{1/n} \leq e^{-2g_\Omega(z)}$$

locally uniformly in $\Omega = \bar{\mathbb{C}} \setminus [-1,1]$, where g_Ω is Green's function of Ω with pole at infinity.

Nonlinear boundary value problems for holomorphic functions

E. Wegert

Given a family of curves M_t ($t \in \mathbf{T} := \partial\mathbf{D}$) in the complex plane the corresponding Riemann-Hilbert problem (RHP) consist in finding all functions w, which are holomorphic in the unit disk $\mathbf{D}$, continuous on $\overline{\mathbf{D}}$, and satisfy the boundary condition

$$w(t) \in M_t, \qquad \text{for all } t \in \mathbf{T}$$

Special emphasis of the talk is on RHPs with a non-compact non-orientable target manifold

$$M := \bigcup_{t\in\mathbf{T}} \{t\} \times M_t \subset \mathbf{T} \times \mathbb{C}.$$

The structure of the solution manifold is discussed and an application to the free boundary value problem in electrochemical machining is given. Finally, we propose an efficient Newton-type method for the iterative solution of RHPs with index $-1/2$, which is based on the splitting of Toeplitz matrices into two factors related to discrete linear RHPs.

Fast conformal mapping of an ellipse to a simply connected region

R. Wegmann

The fast iterative method for conformal mapping from the disc to a simply connected region with smooth boundary (Wegmann 1978) is extended and improved.

1) Using a simple representation of the conjugation operator on an ellipse in terms of trigonometric series, the method is reformulated for ellipses as canonical regions. The iteration is still fast using FFT. A proper choice of a canonical ellipse can reduce the crowding and improve the accuracy.

2) The discretized method is modified by cutting off all Fourier terms or order $> m$. It is proved that this smoothed numerical method converges locally if only the number of grid points is large enough. This solves the convergence problems associated with the original method.

These results render the method more flexible, stable and widely applicable.

Rational approximation in the Hardy space H_2

F. Wielonsky

This is a joint work with L. Baratchart and E. B. Saff. Our interest is in the following problem:

Given a function f in the real Hardy space H_2^- of the complement of the closed unit disk, vanishing at infinity, minimize $||f - r||_2$ where r ranges over the set of all rational functions in H_2^- of type $(n-1, n)$. This problem can equivalently be stated in the real Hardy space H_2^+ of the unit disk.

We give a general criterion for uniqueness of critical points and apply it to the particular case $f(z) = \frac{1}{z}e^{1/z}$ in H_2^- (or $g(z) = e^z$ in H_2^+).

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