Tagungsbericht 15/1995

Orders in Arithmetic and Geometry

16.04. – 22.04.1995

Die Tagung fand unter der Leitung von J. Ritter (Augsburg) und M.J. Taylor (Manchester) statt. Die Vorträge und Diskussionen betrafen vor allem

- die Galoisstrukturbestimmung der ganzen Zahlen, der Einheiten und der Klassengruppe in einem Zahlkörper K, der als galoissche Erweiterung eines Zahlkörpers k gegeben ist,
- das Isomorphieproblem und die Zassenhaus-Vermutung,
- ganzzahlige Realisierungen von Gruppendarstellungen.

Tatsächlich hat es in letzter Zeit gerade in diesen Anwendungsbereichen der Darstellungstheorie von Gruppen und Algebren große Fortschritte gegeben, über die zu berichten die Tagung eine einmalige Gelegenheit bot. Im folgenden sind die Vortragsauszüge in der Reihenfolge der obengenannten Fachgebiete angeordnet.

Vortragsauszüge

C. GREITHER :

The second Chinburg conjecture for absolutely abelian fields

To any Galois extension of number fields L/K with group G, Chinburg attached three invariants $\Omega(L/K,i)$ $(i=1,2,3)$ in the locally free class group $Cl(\mathbb{Z}G)$. If L/K is tame, $\Omega(L/K,2)$ specializes to $[\mathfrak{O}_L]$. Chinburg's second conjecture postulates that always $\Omega(L/K,2) = W_{L/K}$, the (generalized) root number class. We review the evidence in favour of this (results of Kim, Snaith and Holland). The main part of the talk was devoted to a discussion of the following result:

THEOREM.

For any absolutely abelian field L of odd conductor, the invariant $\Omega(L/\mathbb{Q},2)$ vanishes. (In other words: The second Chinburg conjecture is true for $L/\mathbb{Q}$).

The starting point is the so-called "local" sum formula which expresses $\Omega(L/K,2)$ in the form $[X] + \sum_{\mathfrak{p}\,\text{wild}} \text{ind}_{G_\mathfrak{p}}^G \Omega_\mathfrak{p}$ where X is a certain lattice and the $\Omega_\mathfrak{p}$ are defined using local Class Field Theory, and also using the choice of X. We actually show that in our setting, with the "canonical" choice of X involving Leopoldt's theory, *all* terms in the sum above vanish. A very detailed analysis of the Galois module structure of $L_\mathfrak{P}^*$ is required. This makes heavy use of Coleman theory.

K. GRUENBERG :

Recognizing S-units

This is a report on a joint work with A. Weiss. Let K/k be a finite Galois extension of algebraic number fields with group G and S a finite G-stable set of primes of K containing all infinite and all ramified primes and large enough so that the S-class group is 1. We address the problem of the $\mathbb{Z}G$-module structure of the group E of S-units of K. The set S yields a $\mathbb{Z}G$-permutation lattice $\mathbb{Z}G$ and the augmentation sequence $\Delta S \rightarrowtail \mathbb{Z}S \twoheadrightarrow \mathbb{Z}$. Let $L = \Delta G \otimes \Delta S$. We describe an arithmetically defined homomorphism $H^1(G, \text{Hom}(L,E)) \to \mathbb{Q}/\mathbb{Z}$ and hence an arithmetically defined homomorphism $\varepsilon : H^1(G, \text{Hom}(L,\mu)) \to \mathbb{Q}/\mathbb{Z}$ where μ is the torsion group of E (i.e. the roots of unity in K). Work of Tate yields an exact sequence $E \rightarrowtail A \twoheadrightarrow L$ where A is a cohomologically trivial $\mathbb{Z}G$-module and this provides the Chinburg invariant $\Omega_m = [A] - r[\mathbb{Z}G]$ where $r = |G|(|S|-1)$. If Chinburg's conjecture (that Ω_m is the root number class in $Cl(\mathbb{Z}G)$) is true, then Ω_m is also an arithmetic invariant.

THEOREM.

The $\mathbb{Z}G$-module μ, the G-set S, the Chinburg invariant Ω_m and the character ε determine the stable isomorphism class of E.

To prove this one begins by constructing an envelope of μ, $\mu \rightarrowtail C \twoheadrightarrow \overline{C}$ (so C is cohomologically trivial and $\overline{C}$ is a lattice) and using ε to find $\overline{f} : \overline{C} \twoheadrightarrow L$. If f is the composite $C \twoheadrightarrow \overline{C} \overset{\overline{f}}{\twoheadrightarrow} L$, then $M \rightarrowtail C \overset{f}{\twoheadrightarrow} L$ is an envelope of M, μ is the torsion module in M and (by choosing C large enough) $C \simeq A \oplus P$ for some projective P. Now comes the basic module theoretic result: *Given envelopes $M \rightarrowtail C \twoheadrightarrow L$, $E' \rightarrowtail A' \twoheadrightarrow L$ both linked to ε and with $[C] = [A']$ in $K_0(\mathbb{Z}G)$, then M is stably isomorphic to E'.* Applying this with $E' = E \oplus P$ (and $A' = A \oplus P$) gives M stably isomorphic to $E \oplus P$, whence (more module theory) $M \simeq E \oplus P$. Finally, for any decomposition $M = M' \oplus P$, then M' must be stably isomorphic to E.

J. COUGNARD :

Ring of integers stably free and not free

A. Fröhlich proved that if $N/\mathbb{Q}$ is a Galois extension with Galois group Γ quaternionic of order 2^n $(n \geq 4)$, the ring of integers is $\mathbb{Z}[\Gamma]$-stably free. Unfortunately when $n \geq 5$ that doesn't allow us to say that there is a normal integral basis. In this talk, following R.G. Swan we give a description of isomorphism classes of rank 1 projective $\mathbb{Z}[\Gamma]$-modules (for $n = 5$). Thanks to that description one is able to construct explicitly an example with a ring of integers stably free and not free.

J. MORALES :

Gaussian periods mod p and Vandiver's conjecture

Let p be an odd prime number and let A be the p-Sylow subgroup of the ideal class group of the cyclotomic number field $\mathbb{Q}(\zeta_p)$. Using Kummer's Complementary Reciprocity Law, we define a $\mathrm{Gal}(\mathbb{Q}(\zeta_p)/\mathbb{Q})$-equivariant representation $\rho : (A/A^p)^- \to U$, where U is a group of circulant orthogonal matrices with coefficients in $\mathbb{F}_p$. This representation turns out to be faithful if and only if the Vandiver Conjecture holds. We describe the cokernel of ρ in terms of Stickelberger relations. This description, in combination with the Mazur-Wiles theorems on A^- sheds new light on Vandiver's conjecture.

L. CHILDS :

Kummer extensions of formal groups

Let (R, π) be the valuation ring of a finite extension K of $\mathbb{Q}_p$. Let $f : F \to F'$ be an isogeny of commutative dimension n formal groups defined over R. Then 1) $H_f = R[[x]]/(f)$ is an R-Hopf algebra of finite rank over R, 2) the group of H-Galois objects $\mathrm{Gal}(H) \cong F'(R)/f(F(R))$, and 3) for any c in $F'(R) = \mathfrak{m}^{(n)}, \mathfrak{m} = \pi R$, the corresponding element of $\mathrm{Gal}(H)$ is the Kummer extension $S_c = R[[t]]/(f - c)$. These results are particularly attractive when F, F' are polynomial formal groups. For $n = 1$, F is polynomial iff $F = F_b$, $F_b(x,y) = x+y+bxy$ for some $b \in R$ ($b = 0 \Longrightarrow F_b = \mathbb{G}_a$; $b = 1 \Longrightarrow F_b = \mathbb{G}_m$). If $p/b^{p-1} \in R$, then $f_p = \frac{1}{b^{p-1}}[p]_{F_b}(x)$ is an isogeny from F_b to F_{b^p} and $H_{f_p} \cong R[\frac{\sigma-1}{b}] \subseteq KC_p$; in this way we get all Hopf orders over R in KC_p. Assume F is a dimension n, degree 2 polynomial formal group over R so that over K, $\theta(F(\theta^{-1}x, \theta^{-1}y)) = \mathbb{G}_m^n(x,y)$ for some $\theta \in M_n(R) \cap Gl_n(K)$. Call $F = F_\theta$. Letting $\theta^{(p)} = (\theta_{ij}^p)$, then $[p] \equiv \theta^{-1}\theta^{(p)}x^{(p)} \pmod p$; if $p(\theta^{-1}\theta^{(p)})^{-1} \in \pi M_n(R)$, then $(\theta^{-1}\theta^{(p)})^{-1}[p]_F = f_p$ is an isogeny from F_θ to $F_{\theta^{(p)}}$ and $H_{f_p} = R[x]/(f_p)$ is a Hopf order in KC_p^n. We can assume θ is lower triangular, thus H_{f_p} depends on $\frac{n(n+1)}{2}$ parameters in R. For $n = 2$, adapting work of Greither shows that the Hopf orders obtained from such isogenies form a non-trivial but proper subset of all Hopf orders over R in KC_p^2.

B. EREZ :

On the module theoretic interpretation of epsilon factors

Let X be a projective regular variety over a finite field k supporting an action of a finite group G. For any complex representation V of G one defines an epsilon constant $\varepsilon(X/G, V)$ in $\overline{\mathbb{Q}}$. For g in G consider the expression $S(g) = -\sum_{V \text{ irr}} v_p(\varepsilon(X/G,V))\chi_V(g)$ with $p =$ char k , $\chi_V =$ character of V. We shall give an expression for $S(g)$ in terms of the Brauer traces at g of a combination of total direct images of powers of $\Omega^1_{X/k}$. More precisely, let $\mathfrak{H}$ be a coherent O_X-G-sheaf on X and let $f : X \to \mathrm{Spec}\ k$ be the structure morphism; put $f_*(\mathfrak{H}) = \sum_i (-1)^i [H^i(X, \mathfrak{H})]$ in $G_0(kG)$. $S(g)$ defines an element in $G_0(kG) \otimes \mathbb{Q}$ and one can show directly for G abelian that if g is not in G_{reg} (elements of G of order prime to p), then $S(g) = 0$. So we need classes in $G_0(kG)$ coming from $K_0(kG)$. Now, *if the action is tame* (i.e. inertia groups of order prime to residue characteristic) then for all $\mathfrak{H}$ as above $f_*(\mathfrak{H})$ can be lifted to $G_0(kG)$. For $[M]$ in $G_0(kG)$ let $BTr_g([M])$ denote the Brauer trace of g in G_{reg} on M. Let $\Psi(X,G) = \sum_{i=0}^{\dim X - 1}(-1)^i(\dim X - i) f_*(\Omega^i_{X/k})$. One then shows:

$BTr_g(\mathrm{Res}^k_{\mathbf{F}_p}(\Psi(X,G))) \overset{(*)}{=} S(g)$. This is due to Chinburg (see Annals 1994). The purpose of the talk was to explain a strategy of proof for surfaces not using crystalline cohomology to determine the $v_p(\varepsilon)$'s. This strategy carries over to handle curves over $\mathbb{Z}$. To evaluate the LHS of $(*)$ we use a Lefschetz-Riemann-Roch theorem, to evaluate the RHS of $(*)$ we use a result of T. Saito which expresses the ε-factors in terms of Gauss sums and apply Stickelberger's Theorem to compute the valuations.

This is joint work with T. Chinburg, G. Pappas and M.J. Taylor.

T. Chinburg :

Galois structure of K-groups and de Rham cohomology

Suppose N/K is a finite Galois extension of number fields, $G = \mathrm{Gal}(N/K)$ and $n > 0$. In the first part of my talk I sketched the construction by M. Kolster, G. Pappas, V. Snaith and myself of an invariant $\Omega_n(N/K)$ in the class group $Cl(\mathbb{Z}G)$ of G. If $n = 1$, the image of $\Omega_n(N/K)$ in $G_0(\mathbb{Z}G)$ is $(K_{2n}(\mathfrak{O}_N)) - (K_{2n+1}(\mathfrak{O}_N)) - (Y_{2n+1})$, where $K_i(\mathfrak{O}_N)$ is the i^{th} K-group of the integers of N, and Y_{2n+1} is a certain natural permutation module. The same is true if $n > 1$ up to classes of finite modules of 2-power order if the Quillen - Lichtenbaum conjecture is true. The present approach does not use conjectural Lichtenbaum complexes (as did the one I reported on here last summer), and it differs from the one used by D. Burns and M. Flach to construct related invariants when G is abelian.

In the last half of the talk I reported on work with B. Erez, G. Pappas and M. Taylor on the Galois structure of the de Rham cohomology of tame G-covers $X \to Y$ of projective flat regular schemes over $\mathrm{Spec}(\mathbb{Z})$. I report on a higher dimensional generalization of the statement of Fröhlich's conjecture, and on the proof of this generalization when $\dim(X) = 2$ and Y has bad fibres over $\mathbb{Z}$ which have strictly normal crossings.

A. Agboola :

Intersection theory on elliptic curves and Galois module structure

Let E/F be an elliptic curve with everywhere good reduction over a number field F. Write $\mathfrak{O}_F$ for the ring of integers of F and let $\mathfrak{E}/\mathfrak{O}_F$ denote the Néron minimal model of E/F. Fix a rational prime $p > 3$. Write $\mathfrak{G}_i/\mathfrak{O}_F$ for the $\mathfrak{O}_F$-group scheme of p^i-torsion on $\mathfrak{E}$. Then we may set $\mathfrak{G}_i = \mathrm{Spec}(\mathfrak{A}_i)$, where $\mathfrak{A}_i$ is an $\mathfrak{O}_F$-Hopf algebra.
Suppose that $Q \in E(F)$. We define a scheme $\mathrm{Spec}(\mathfrak{C}_Q(i))$ via the following pullback diagram:

$$\begin{array}{ccc} \mathfrak{E} & \longleftarrow & \mathrm{Spec}\,(\mathfrak{C}_Q(i)) \\ p^i \downarrow & & \downarrow \\ \mathfrak{E} & \xleftarrow{Q} & \mathrm{Spec}\,(\mathfrak{O}_F) \end{array}$$

Then $\mathfrak{C}_Q(i)$ is a locally free $\mathfrak{A}_i$-module whose module structure is intimately related to the Galois module structure of fields obtained via dividing the point Q on E.
We thus obtain a map

$$\begin{array}{rccc} \psi_i : & E(F) & \to & Cl(\mathfrak{A}_i) \\ & Q & \mapsto & (\mathfrak{C}_Q(i)) \end{array}$$

This map was first introduced and studied by M.J. Taylor in connection with his deep investigations of the relative Galois module structure of rings of integers of abelian extensions of imaginary quadratic fields.

The purpose of the lecture was to report on the following theorem:

THEOREM.

$$E(F)_{\text{tors}} \subseteq \ker \psi_i.$$

This generalises a result of Srivastav and Taylor, who proved the theorem in the case that E has complex multiplication. Their main technique was the use of modular functions and the q-expansion principle to obtain integrality results concerning certain resolvent elements that arise as special values of elliptic functions on $\mathfrak{E}$. In our proof we replace all of this by intersection theory on $\mathfrak{E}$, and this enables us to obtain a more general result.

A. WEISS :

Toward 'strong Stark' for abelian fields

Let K/k be a finite Galois extension of number fields with Galois group G, and S a finite set of primes of K containing all archimedean primes and G-stable. If S is *large* (i.e. contains all ramified primes of K/k and enough primes to generate the class group of K) then Tate has defined a regulator $R_\varphi(\chi)$ and a 'q-index' $q_\varphi(\chi)$ for irreducible characters χ of G. Here φ is a G-homomorphism $\Delta S \to E$ inducing $\mathbb{Q} \otimes \Delta S \simeq \mathbb{Q} \otimes E$ where ΔS is the kernel of the augmentation $\mathbb{Z}S \to \mathbb{Z}$ and E is the S-units of K. Letting $c(\chi)$ denote the leading term of the Taylor expansion at $s = 0$ of the Artin L-series $L(s, \chi)$ with Euler factors for primes in S deleted, then $A_\varphi(\chi) = R_\varphi(\chi)/c(\chi)$ satisfies

$$A_\varphi(\chi^\sigma) = A_\varphi(\chi)^\sigma \quad , \ \sigma \in \mathrm{Aut}_{\mathbb{Q}}(\mathbb{C})$$

according to the Stark conjecture. Tate proved this for $\mathbb{Q}$-valued χ by showing that then

$$(A_\varphi(\chi^*)) = q_\varphi(\chi) \ ,$$

which assertion for arbitrary χ was conjectured by Chinburg: this is the 'strong Stark' of the title.

When K/k is contained in a real cyclotomic extension of $\mathbb{Q}$ then 'strong Stark' is true except possibly for the behaviour of primes above 2 (due to the present status of the Main Conjecture of Iwasawa theory). The purpose of the talk was to describe the proof of this result, which depends on a generalization of the Tate sequence and of the Tate regulator to small sets S.

This is joint work with J. Ritter.

M. J. TAYLOR :

ε-constants for varieties over finite fields

This was a report on joint work with T. Chinburg, B. Erez and G. Pappas; to study the relationship between Euler characteristics of sheaves of differentials and the ε-constants of a regular, projective variety $X/\mathbb{F}_p$, which supports a tame action by a finite group G. Two main results were discussed: the first, following an idea of Serre, was to show that, if $\dim(X)$ is even and if V is a symplectic representation, then $\varepsilon(X/G, V)$ is a power of p. The sign of $\varepsilon(X/G, V)$, when $\dim(X)$ is odd, was then determined in terms of Euler characteristics of a stratification of X (via inertia groups), which were analysed in Fröhlich's hermitian classgroup.

A. FRÖHLICH :

Galois Gauss Sums and Factorization

We define a diagram (Γ a finite group)

$$\begin{array}{ccccc} & & \mathrm{Hom}_\Omega(R_\Gamma, J_f) & \xrightarrow{c} & Cl(\mathbb{Z}\Gamma) \\ & & \downarrow T & & \\ K(\Gamma) & \xrightarrow{r} & \mathfrak{M}(\Gamma) & & \end{array}$$

$K(\Gamma)$ is the relative Grothendieck group of pairs of $\mathbb{Z}\Gamma$-lattices spanning the same $\mathbb{Q}\Gamma$-module, Ω = Gal $(\mathbb{Q}^c/\mathbb{Q})$, J_f = finite ideles, $R\Gamma$ = virtual characters, c comes from the Hom-description.
If $f \in \mathrm{Hom}_\Omega(R_\Gamma, J_f)$, $(X, Y) \in K(\Gamma)$, where X, Y are $\mathbb{Z}\Gamma$- lattices spanning the same $\mathbb{Q}\Gamma$-module, we call f a factorisation of (X, Y) if $T(f) = r(X, Y)$; f is principal if actually $f \in \mathrm{Hom}_\Omega(R_\Gamma, P_f)$, where P_f is the principal ideles made finite.
Now let N/K be a Galois extension of number fields, $\mathfrak{O}_N$, $\mathfrak{O}_K$ the respective ring of integers, $a \in N$, $aK\Gamma = N$, $\mathcal{N}(a|\chi)$ the norm resolvent, $\tau(\chi)$ the Galois Gauss sum, $h_a(\chi) = (\tau(\chi)/\mathcal{N}(a|\chi))$. Then

THEOREM.

1. $T(h_a) = r(\mathfrak{O}_N,\ a\mathfrak{O}_K\Gamma)$, h_a *principal*
2. $c(h_a) = W_{N/K}$.

D. BURNS :

de Rham structure invariants associated to motives over number fields

Let L/K be a finite Galois extension of number fields with group G. Let X be a smooth projecture variety over K and let M denote a motive of the form $h^n(X \times_K L)(r)$. We describe an element $\Omega^{\mathrm{dR}}(L/K, M)$ of $Cl(\mathbb{Z}[G])$ which measures canonical 'integral structures' in the de Rham realisation of M. The definition is cohomological in nature (using the p-adic exponential map of Bloch and Kato) and uses the same approach as in S. Kim's description of Chinburg's element $\Omega(L/K, 2)$. By using the formula of Kim one can show that $\Omega^{\mathrm{dR}}(L/K, h^0(\text{ Spec }(L))(1)) = \Omega(L/K, 2)$. By results of Deligne one can define an element $w(L/K, M)$ of $Cl(\mathbb{Z}[G])$ using epsilon constants attaches to 'symplectic pieces' of M. In the case $M = h^0(\text{ Spec }(L))(1)$ this element $w(L/K, M)$ conincides with the classical Cassou-Noguès-Fröhlich root number class. It is thus natural to ask if the elements $\Omega^{\mathrm{dR}}(L/K, M)$ and $w(L/K, M)$ should coincide, and for certain classes of quaternionic extensions of $\mathbb{Q}$ one can prove that this is indeed the case.

If G is abelian then $\Omega^{\mathrm{dR}}(L/K, M)$ can be reinterpreted as the class (in $\mathrm{Pic}(\mathbb{Z}[G]) \simeq Cl(\mathbb{Z}[G])$) of a canonical invertible $\mathbb{Z}[G]$-lattice. In this way one can use the Artin-Verdier duality theorem to relate $\Omega^{\mathrm{dR}}(L/K, M)$ to the classes $\Omega(L/K, M)$ and $\Omega(L/K, M^*(1))$ discussed in previous joint work with Matthias Flach. In addition, one can relate the supposed vanishing of $\Omega^{\mathrm{dR}}(L/K, M)$ to the 'local epsilon conjecture' of Kato. By these means one can show that if G is abelian and $K = \mathbb{Q}$ then for all integers r the class $\Omega^{\mathrm{dR}}(L/\mathbb{Q}, h^0(\text{ Spec }(L))(r))$ vanishes in $Cl(\mathbb{Z}[\frac{1}{2}][G])$.

B. Köck :

The Adams-Riemann-Roch Theorem in Geometry and Representation Theory

After putting the Galois module structure problem into the framework of Riemann-Roch theory the equivariant Adams-Riemann-Roch theorem was stated. Roughly speaking this theorem says that for any G-projective morphism $f : X \to Y$ between G-schemes X and Y (G a flat group scheme) the associated equivariant Euler characteristic $f_* : K_0(G, X) \to K_0(G, Y)$ commutes with the Adams operation ψ^k after multiplying with the so-called k-th Bott class $\theta^k(f)$. As a corollary we obtained that induction commutes with Adams operations. Finally a relation between this Adams-Riemann-Roch theorem and a formula of Burns and Chinburg was discussed. In doing so Adams operations on the Grothendick group of projective RG-modules were constructed.

W. Bley :

Associated orders; local and global freeness

Let N/M be an abelian extension of number fields with group G. Let $E \subseteq M$ be a subfield with class number $h_E = 1$. Let $X \subseteq N$ be a full G-lattice. We define the associated order of X with respect to E by

$$\mathfrak{A} = \mathfrak{A}(E[G], X) = \{\lambda \in E[G] / \lambda(X) \subseteq X\} .$$

The aim of the talk was to give an algorithmic approach to study the structure of X as a module over $\mathfrak{A}$. The problem of explicitly computing $\mathfrak{A}$ and answering the question, whether or not X is locally/globally free over $\mathfrak{A}$, was reduced to hard, but classical problems of computational algebraic number theory.

S.M.J. Wilson :

Factorizability and Species

A general framework was given to describe the several theories of factorization and factorizability proposed in recent years by Fröhlich, Nelson, Burns, Holland and Wilson. The theory of *strict factorizability* of Fröhlich (see his talk) and that of Hecke factorizability of Holland and Wilson were described in this framework and were shown to be equivalent. Tools used in the proof included the theory of species of permutation projective modules and the Conlon induction theorem.

B. de Smit :

On a conductor discriminant formula of McCulloh

Let E be a finite commutative ring. Assume that E is Gorenstein, i.e., that $\check{E} = \mathrm{Hom}(E, \overline{\mathbb{Q}}^*)$ is a free E-module of rank 1. For such E McCulloh has given a canonical construction of an order $T(E)$ within a Galois algebra over $\mathbb{Q}$ with Galois group E^*. We have $E \subset T(E)^*_{\mathrm{tors}}$ and $T(E) = \mathbb{Z}[\zeta_n]$ when $E = \mathbb{Z}/n\mathbb{Z}$. McCulloh conjectured that the following generalization of the conductor discriminant formula for cyclotomic fields holds:

$$\Delta_{T(E)/\mathbf{Z}} = \prod_{\chi : E^* \to \mathbf{C}^*} [E : f_\chi] \, .$$

Here the conductor f_χ of χ is the largest E-ideal I such that χ factors through $(E/I)^*$. In this talk it is shown that the right side divides the left side and that we have equality if and only if E is a principal ideal ring.

G. Lettl :

Galois module structure of abelian fields

Let N be a finite, abelian extension of $\mathbb{Q}_p$ with ring of integers $\mathfrak{O}_N$ and K some subfield, $\mathbb{Q}_p \subset K \subset N$ with $\Gamma = \mathrm{Gal}(N/K)$. The associated order of the extension N/K is defined by $\mathfrak{A} = \{\alpha \in K\Gamma | \alpha\mathfrak{O}_N \subset \mathfrak{O}_N\}$. Let $\Gamma_0 \leq \Gamma$ be the inertia group and $\mathfrak{M} \subset K\Gamma_0$ the maximal $\mathfrak{O}_K$-order. Then we have:

Theorem.

For $p \neq 2$, one has $\mathfrak{A} = \mathfrak{M} \otimes_{\mathfrak{O}_K\Gamma_0} \mathfrak{O}_K\Gamma$, and there exists some (explicitly given) $T \in \mathfrak{O}_N$ with $\mathfrak{O}_N = \mathfrak{A}T$.

R. Boltje :

On class group relations

Let L/K be a Galois extension of number fields with Galois group G. For $H \leq G$ let $Cl(H)$ denote the ideal class group of the corresponding intermediate field. Using the cohomological Mackey functor structure on the set $Cl(H), H \leq G$, of finite abelian groups we show that for every subgroup $H \leq G$ which is not p-hypoelementary (resp. not hypoelementary) one has a relation

$$\sum_{U \leq H} |U|\mu(U,H)[Cl(U)_p] = 0 \text{ (resp. } \sum_{U \leq H} |U|\mu(U,H)[Cl(U)] = 0 \text{)}$$

in the Grothendieck group of the category of finite abelian groups with respect to direct sums. Here μ denotes the Möbius-function of the poset of subgroups of G, and $Cl(U)_p$ denotes the p-part of $Cl(U)$ for a prime p.

N. Byott :

Realisable classes over Hopf orders

Let G be a finite abelian group and K a number field. L. McCulloh (Crelle, 1987) characterised the subgroups $R(\mathfrak{O}_K G), R_{\mathrm{nr}}(\mathfrak{O}_K G)$ in $Cl(\mathfrak{O}_K G)$ given by the classes of the rings of integers in tame (unramified) G-Galois algebras. We describe a generalisation where $\mathfrak{O}_K G$ is replaced by a Hopf order $\mathfrak{A}$. Let $\Omega = \mathrm{Gal}(K^c|K)$ act on G and let $\mathfrak{A}$ be a Hopf order in $A = (K^c G)^\Omega$. For any principal homogeneous space $\mathfrak{C}$ over the dual $\mathfrak{B}$ of $\mathfrak{A}$, define $\psi(\mathfrak{C}) = (\mathfrak{C})(\mathfrak{B})^{-1} \in Cl(\mathfrak{A})$. Using resolvents, one can give a description of $\mathrm{im}(\psi)$ analogous to McCulloh's description of $R_{\mathrm{nr}}(\mathfrak{O}G)$. By considering orders $\mathfrak{C}$ which are tame over $\mathfrak{A}$ in a suitable sense, we also obtain an analogue of McCulloh's description of $R(\mathfrak{O}G)$.

L. McCulloh :

On class groups of integral p-group rings and Stickelberger modules

Let G be a finite group and R_G the virtual character ring of G. We define a $\mathbb{Q}$-pairing ("Stickelberger pairing"): $\langle\ ,\ \rangle : \mathbb{Q}R_G \to \mathbb{Q}G$ as follows. If G is abelian and $\chi \in \hat{G}, s \in G$, we define $\langle \chi, s\rangle \in \mathbb{Q}$, $0 \le \langle \chi, s\rangle < 1$ by $\chi(s) = e^{2\pi i \langle \chi, s\rangle}$ and extend by $\mathbb{Q}$-linearity. If G is arbitrary, and $\chi \in R_G$, define $\langle \chi, s\rangle = \langle \mathrm{res}^G_{(s)}\chi, s\rangle$. The "Stickelberger" map $\Theta : \mathbb{Q}R_G \to \mathbb{Q}G$ is defined by $\Theta(\chi) = \sum_{s\in G}\langle \chi, s\rangle s$, with values in the center $c(\mathbb{Q}G)$. The "Stickelberger" module is

$$S_G = \Theta(R_G) \cap c(\mathbb{Z}G) \ .$$

Conjecture: If G is a p-group, p on odd prime, $Cl(\mathbb{Z}G) =$ the locally free classgroup, then

$$|Cl(\mathbb{Z}G)^-| = [c(\mathbb{Z}G)^- : S_G^-]$$

where the minus sign refers to the skew-symmetric part with respect to the canonical involution induced by the map $\tau : G \to G$ given by $\tau(s) = s^{-1}$.

Question: To what extent does this hold for arbitrary G?

Evidence:

1. $[c(\mathbb{Z}G)^- : S_G^-] < \infty$ for all G.
2. The conjecture holds for G abelian of homogeneous type $(p^n, p^n, \dots, p^n)$, p odd.
3. The conjecture holds for the two nonabelian groups of order p^3, p odd.

V. Snaith :

Quaternionic Exercises in Galois structure of K-groups

In this talk I described a computation of the invariant $\Omega_1(N/K, 3)$, in some quaternion cases. This is part of a programme undertaken with Chinburg, Kolster and Pappas. The totally real quaternion extensions discussed, $N/\mathbb{Q}$, are constructed as follows: $p \equiv 3 \pmod 8$ is a prime and $N = \mathbb{Q}(\sqrt{2}, \sqrt{p}, \beta)$ with $\beta^2 = \sqrt{p}(\sqrt{2}/u + v\sqrt{p})(1+\sqrt{2})$, where $u, v \in \mathbb{Q}$ and $-2 = u^2 - v^2 p$. In this case $\Omega_1(N/\mathbb{Q}, 3)$ is the Euler characteristic of a 2-extension,

$$K_3^{\text{ind}}(\mathfrak{O}_{N,S}) \to A \to B \to K_2(\mathfrak{O}_{N,S})'$$

where $S = \{2, p\}$ and the prime denotes the kernel of the map to $\bigoplus_{v \in \Sigma_\infty(N)} K_2(N_v) \cong \bigoplus_1^8 \mathbb{Z}/2$. The construction of $\Omega_1(N/\mathbb{Q}, 3)$ is simple – being a totally real case – and is described in [V. Snaith, Galois Module Structure; Fields Institute Monographs # 2(1994) pub. by AMS]. In general $\Omega_1(N/\mathbb{Q}, 3)$ is conjectured to be the root number class, $W_{N,\mathbb{Q}} \in Cl(\mathbb{Z}(H_8]) \cong (\mathbb{Z}/4)^*$. In our example $W_{N/\mathbb{Q}} = 1$.
I proved:

THEOREM.

In the quaternionic examples, $F = \mathbb{Q}(\sqrt{2}, \sqrt{p})$,

$$\Omega_1(N/\mathbb{Q}, 3) = \quad -\chi_+(\zeta_F(-1)(1-p^2)_{\text{odd}})\sqrt{(\tfrac{\zeta_N(-1)}{\zeta_F(-1)})_{\text{odd}}} \in (\mathbb{Z}/4)^*$$

where n_{odd} is the odd part of $n \in \mathbb{Q}^*$ and $\chi_+ : (\mathbb{Z}/8)^* \to (\mathbb{Z}/4)^*$ has kernel $\{\pm 1\}$.

For small p we have

p	$\zeta_F(-1)_{\text{odd}}$	$\zeta_N(-1)_{\text{odd}}$
3	1	71^2
11	$7 \cdot 23/3$	$3 \cdot 7 \cdot 11^2 \cdot 23 \cdot 109^2$
19	$19 \cdot 41/3$	$3^3 \cdot 19 \cdot 41 \cdot 1993^2$

and we obtain

THEOREM.

For $p = 3, 11, 19$, $H_8 = Gal(N/\mathbb{Q})$,
$\Omega_1(N/\mathbb{Q}, 3) = W_{N/\mathbb{Q}} = 1 \in (\mathbb{Z}/4)^* \cong Cl(\mathbb{Z}[H_8])$

K. ROGGENKAMP :

A counterexample to the isomorphism problem for polycyclic groups

For a finite group G and a ring of algebraic integers R, the natural map

$$\Phi \ : \ \text{Out}(G) \to \text{Out}(RG)$$

is injective for a p-group G (Coleman), or if $O_{p'}(G) = 1$ for p odd (Kimmerle), or if the Sylow 2-subgroup is normal (Jakovski - Marciniak). Here we present an example, where $|\ker \Phi| = 2$. Using an observation of M. Mazur, the group ring $R(G \times C_\infty)$ and $R(G \rtimes_\alpha C_\infty)$ are isomorphic when $\langle \alpha \rangle = \ker(\Phi)$.

This is joint work with A. Zimmermann.

F.M. Bleher :

Group bases of integral group rings

For a finite group G a certain conjecture of Zassenhaus is examined:
(ZC) Let X, Y be two group bases of $\mathbb{Z}G$, i.e. subgroups of the group of units of $\mathbb{Z}G$ with augmentation 1 which have the same order as G. Then there exists a unit $u \in \mathbb{Q}G^\times$ such that $X = u^{-1}Yu$.
If G is a group for which the integral isomorphism problem has a positive solution then (ZC) is equivalent to $(ZC\text{Aut})$: Let $\sigma \in \text{Aut}_n(\mathbb{Z}G)$, i.e. σ is an augmentation preserving ring automorphism of $\mathbb{Z}G$. Then $\sigma = \alpha \cdot \tau$ where $\alpha \in \text{Aut}(G)$ and $\tau \in \text{Aut}_n(\mathbb{Z}G)$ preserves all class sums of $\mathbb{Z}G$.

THEOREM.

(ZC) is valid for all Weyl groups.

Modular representation theory is used to obtain further results. Let (K, R, k) be a p-modular system with K sufficiently large relative to G.

Cyclic blocks: (joint work with Hiss and Kimmerle)
Let B be a block of kG with cyclic defect. Let σ be an autoequivalence of B-mod. Then σ fixes the isomorphism classes of all finitely generated B-modules if σ fixes the isomorphism class of one indecomposable finitely generated B-module.

Tensor products:

Let G be a finite group, let $\sigma \in Aut_n(\mathbb{Z}G)$ an let ξ, ζ be two ordinary or two Brauer characters, respectively. Then $\sigma(\xi \otimes \zeta) = \sigma(\xi) \otimes \sigma(\zeta)$.

With the aid of these results it can be shown that (ZC) is valid for 15 of the 26 sporadic simple groups. Moreover, (ZC) is valid for some series of finite groups of Lie type of small rank which proves the following theorem:

THEOREM.

(ZC) is valid for all minimal simple groups, for all simple Zassenhaus groups and for all simple groups with abelian Sylow 2-subgroups.

Generic Hecke orders: (joint work with Geck and Kimmerle)

The results obtained for the Weyl groups extend naturally to the corresponding generic Hecke orders: *Let H be the generic Hecke order of type A_n over $\mathbb{Z}[q, q^{-1}]$. Then $Aut(H) = Aut_{\text{cent}}(H) \rtimes C_2$.*

A. Zimmermann :

Conjugacy class structure of torsion units of certain dihedral group rings

Let $D_{2^n} = \langle a, b | a^{2^n} = b^2 = 1,\ baba = 1\rangle$ be the dihedral group of order 2^{n+1} and let $S_{2^n} = \langle a, b | a^{2^n} = b^2 = 1,\ baba = a^{2^{n-1}}\rangle$ be the semidihedral group of order 2^{n+1}. Then, let $V(\mathbb{Z}G)$ be the group of units of augmentation 1 and let ζ_{2^n} be a primitive 2^n-th root of unity.

THEOREM

1) In $V(\mathbb{Z}D_{2^n})$ there are $2^{n-1} \cdot \prod_{k\le n} |Cl(\mathbb{Z}[\zeta_{2^k} + \zeta_{2^k}^{-1}])|$ conjugacy classes of involutions rationally conjugate to b.
2) In $V(\mathbb{Z}D_{2^n})$ there are 2^{n-1} conjugacy classes of involutions rationally conjugate to b whose elements are contained in group bases.
3) $\mathrm{Outcent}(\mathbb{Z}D_{2^n}) = \langle \mathrm{Conj}(1+ab)\ ,\ \mathrm{Conj}(1+b(a+a^{-1}))\rangle$
4) In $V(\mathbb{Z}S_{2^n})$ there is an involution not contained in a group basis.

COROLLARY: Every unit in $V(\mathbb{Z}D_{2^n})$ is contained in a group basis if and only if $h_{2^n}^{+} := |Cl(\mathbb{Z}[\zeta_{2^n} + \zeta_{2^n}^{-1}])| = 1$; this is a conjecture of H. Cohn.

S. SEHGAL :

On a problem of Brian Hartley

Let FG be the group algebra of a *torsion group* G over a field F. Let $U = U(FG)$ be the group of units of FG. We say that U satisfies a group identity if there exists a non-trivial word $w = w(x_1, \dots, x_m)$ so that $w(\alpha_1, \dots, \alpha_m) = 1$ for all $\alpha_i \in U$. We say that FG satisfies a polynomial identity if there exists a polynomial $0 \neq f(\zeta_1, \dots, \zeta_n) \in F[\zeta_1, \dots, \zeta_n]$ in noncommuting variables so that $f(\beta_1, \dots, \beta_n) = 0\ \forall \beta_i \in FG$. Brian Hartley conjectured: U satisfies a group identity $\Longrightarrow$ FG satisfies a polynomial identity. A proof of this conjecture was given under the assumption that F has an element which is transcendental over its prime field.

W. PLESKEN :

Classifying finite rational matrix groups

The report is on three papers; the first two among others classify the maximal finite subgroups of $GL_n(\mathbb{Q})$ for $n \le 23$ and are joint work with Gabriele Nebe, the third deals with $n = 24$ and is her thesis. The maximal finite groups of degree $p-1$ whose order is divisible by p are fully classified for all primes p. In the course of this a purely character theoretic property of $PSL_2(p)$ turned out to have the consequence for $\mathbb{Q}[\sqrt{-p}]$ to have class number > 1. The exceptional cases $p = 7, 11$ are connected to the classical fact that $PSL_2(p)$ has a permutation representation of degree p in these cases. Group theory and orders can be used to get various restrictions for the normal subgroups of maximal finite primitive rational matrix groups. In the last part the prime divisors of primitive G-invariant forms on G-lattices are investigated. They need not be divisors of $|G|$, once the space of G-invariant form has dimension bigger than 2, however they can be controlled in terms of the commuting algebra of G, in case it is a field.

E. Kleinert :

Handling integral p-group rings

A method is presented to express an integral p-group ring $\mathbb{Z}P$ as an iterated fibre product; this reflects the ring structure of its simple factors. More precisely, these factors carry a crossed product structure in a natural way.

O. Neisse :

Integral representations of groups of odd order

Let G be a finite group and χ be an irreducible complex character. We define the conductor of χ by $f_\chi = \min\{n \in \mathbb{N} | \mathbb{Q}(\chi) \subseteq \mathbb{Q}(\zeta_n)\}$. The main theorem is: *If G is solvable and χ has odd degree, then there exists a representation $D : G \to Gl_n(\mathbb{Z}[\zeta_{f_\chi}])$ with character χ.* We show this by using properties of k-primitive characters and Dade's character analysis. Furthermore we explain where there are the problems with characters of even degree and how one can try to solve them.

This is joint work with M. Cram.

J. Brzezinski :

A generalization of Eichler's trace formula

Let R be a Dedekind ring whose quotient field K is global and let Λ be an R-order in a separable K-algebra A. We say that Λ is a CM R-order if the quotient of the unit groups Λ^*/R^* is finite. If K is a totally real field and A is a totally definite quaternion algebra over K, then Eichler's formula for traces of the Brandt-Eichler matrices relates the number of (left) principal ideals with given norm in orders representing different isomorphism classes in the genus of Λ to the class numbers of some maximal suborders of Λ and the numbers of orbits for the action of unit groups in the completions of Λ on the completions of these suborders.

We consider a generalization of Eichler's trace formula to CM-orders in arbitrary central simple algebras of prime index over K and discuss some applications of it to computations of class numbers and type numbers of CM-orders in central simple algebras, class number relations for (commutative) CM-fields and the numbers of representations by specific ternary and quaternary quadratic forms whose coefficients are algebraic integers. In particular, we get elementary proofs of classical results about representations by sums of 3 and 4 squares over the integers in algebraic number fields.

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