MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 22/1995

Freiformkurven und Freiformflächen

4. – 10. 6. 1995

Die Tagung fand unter der Leitung von R. E. Barnhill (Arizona State University, Tempe), W. Böhm (TU Braunschweig) und J. Hoschek (TH Darmstadt) statt. Im Mittelpunkt der Vorträge standen Fragen der Konstruktion und Darstellung von Kurven und Flächen im Bereich des Computer Aided Geometric Design. Dabei wurden unter anderem folgende Schwerpunkte gesetzt:

Anwendung von Methoden aus der algebraischen Geometrie, Konstruktion und Einsatz spezieller Kurvenklassen, Erzeugung von Kurven und Flächen durch Unterteilungsalgorithmen, Anwendung von Ergebnissen der Approximationstheorie zur Beschreibung geometrischer Objekte, Rationale B–Spline–Kurven und –Flächen, Einsatz von Variationsmethoden bei der Approximation mit B–Spline–Kurven und –Flächen, Globale Modellierung, Anwendung von Methoden der Differentialgeometrie im CAGD.

Unter den Teilnehmern befanden sich neben Mathematikern auch zahlreiche Ingenieure und Informatiker sowie Vertreter der Industrie. Trotz des dicht gedrängten Vortragsprogramms kam es zu zahlreichen intensiven Diskussionen, die allen Beteiligten eine Fülle von Anregungen vermittelten. Als fruchtbar erwies sich dabei auch der Kontakt zwischen den Anwendern aus der Industrie und den Vertretern der universitären Forschung.

Vortragsauszüge

R. Andersson

Shape Preserving Surface Conversion

Within CAGD, many different methods for representing curves and surfaces have been proposed and used. It seems likely that different forms of representations are optimal for different tasks. On the other hand, there are a few de facto standard representation methods for communication of geometry, understood by CAD systems from several different vendors. In this situation, conversion from the representation suitable for a given purpose to a standard one is inevitable.

In the talk, we will consider the problem of converting a smooth parametrized surface of any, possibly unknown, origin into a standard B-spline surface, while keeping its shape close to the original surface. In general, this is hard to attain by just fitting a B-spline surface to the given one.

In the method proposed, instead of fitting the B-spline surface itself, we find a B-spline surface whose first- and second fundamental forms are close to those of the given surface. This is accomplished by approximately solving the system of partial differential equations appearing in the fundamental theorem of surface theory.

R. E. Barnhill

John Gregory's Research: from Computable Error Bounds through Gregory's Square to Convex Combinations

In this paper I present three of John's intellectual feats. The topics involved are computable error bounds, compatibly corrected bicubic patches and convex combination patches. These early themes signalled his special talent and they recurred throughout his career. His later work on geometric continuity, subdivision methods and monotonic approximations is discussed by Nira Dyn in this Volume. John's first published work was on the topic of computable error bounds for surface interpolants. Perhaps his best known research involves the patch scheme known as "Gregory's Square" which solved an outstanding problem in the use of networks of bicubic patches. The concept of "convex combinations" provides, among other things, a variety of patch schemes including n–sided patches. The last two topics are especially useful for the design of objects such as automobiles and airplanes as well as geometric modelling in general. Below I present computable error bounds for a triangle, the first non–rectangular case. I illustrate this with the example of linear interpolation of a triangle. I show the connection between finite element error analysis and surface interpolation remainder theory. I then move to his best known work, Gregory's Square. I present the problem of incompatible twists and present John's method for interpolating to them. I allude to the important special case of polynomial boundary data. Third, I define convex combinations and use the unifying concept of convex combinations to frame his two classes of smooth vector–valued triangular interpolants and his pentagonal interpolant.

R. E. Barnhill

A Geography Application of CAGD

The prediction of solar radiation onto the earth is modelled by a set of integral differential equations in Geography. Triangular interpolants from CAGD permit the visualization of this radiation so that global climatic trends can be predicted.

P. Barry

Geometrically Continuous Tchebycheffian B-spline Curves

Geometrically continuous Tchebycheffian B-spline curves generalize both geometrically continuous piecewise polynomial spline and Tchebycheffian splines. Many of the desirable geometric and algorithmic properties of B-spline curves extend to this more general representation. In this talk, I will define the space of geometrically continuous Tchebycheffian spline, mention briefly some of the properties, make a few observations, and list some open questions.

R. Bartels

Preliminaries to the Design of a Higher-Order Basis for Global Illumination

In this presentation we review the general illumination model used in image synthesis, known as the *rendering equation*. Three approximations to the full model, the *ambient/diffuse/specular* version of classical computer graphics, the *radiosity* version, and the *radiance* version are progressively more complicated computational approaches to using the model. Recent work on radiosity and radiance computations develop solutions in terms of scale–and–wavelet expansions of the illumination, often using piecewise constant bases. Higher–order bases have also been considered, but computational complexity increases rapidly with order. In the belief (and with some evidence) that higher–order bases do, indeed, provide more appealing results, we present a design approach currently in progress to produce spline scales and wavelets, which are tuned to the particular characteristics of the radiosity approximation in order to economise on computations. The requirements present an interesting exercise in wavelet design. Speculations on extensions to radiance computations close the presentation.

M. Bercovier

Energy Methods and Duality in Surface Editing/Constructing

We define a energy type minimization over patches whose planar basis can be an unstructured mesh or even non standard joints (like two patches connected to the same boundary curve of a third one). Continuity conditions whether C^1, C^2 or G^1, G^2 are introduced by constraint equations. G^1 and G^2 conditions are controlled by linear external definitions (i.e. the normal to the tangent plane in the G^1 case). Lagrange multipliers for the constraints define a complete dual problem. To be precise, from the energy + constraints one has to solve

$$\left.\begin{aligned} M\,\underline{b} - C^{\mathsf{T}}\,\underline{\lambda} &= F \\ C\,\underline{b} &= \underline{g} \end{aligned}\right\} \tag{1}$$

which is reduced to

$$C\,M^{-1}\,C^{\mathsf{T}}\,\underline{\lambda} = M^{-1}\,F - \underline{g}. \tag{2}$$

M is decomposed once only and (2) can be modified to construct surfaces that will be G^1 or C^0 or C^2 etc. by part. A final example is given of the strength of the method by constructing G^1 "hole" filling patches for an odd number of boundary curves.

P. Bonitz

Global Modelling with ICEM SURF

The capability has been developed of modelling complete surfaces, such as a complete car body or large parts, such as doors.

In the past, direct modelling operations were limited to single surface elements, typically Bézier or B-spline patches. The alteration of an arbitrarily structured collection of parametric patches was a time consuming procedure.

The new global modelling facility allows the CAD-user to pick a surface point or a row of control points of a governing control patch, and modify it. This can influence many patches automatically, so that the new shape of the complete body or part can be redrawn.

W. Dahmen

Multiresolution on Surfaces

This talk is concerned with applications involving the analysis or the numerical treatment of functions defined on a surface Γ which typically describes the boundary of some (3D) geometric object. Examples are the computation of electrostatic fields from charge distributions on Γ, scattering from obstacles or the computation of light intensities based on radiosity concepts. In these cases one has to solve a (singular) integral equation on the surface Γ. The solution requires a representation of Γ which suits also the construction of appropriate trial spaces on Γ. The main difficulty in this context is the tremendous computational complexity caused by the densely populated matrices which typically arise from such discretizations. The key to overcoming these problems is to adapt multiresolution concepts to the geometry representations. We formulate the essential requirements arising in this context and indicate ways of constructing trial spaces and corresponding wavelet bases which meet these requirements.

W. Degen

Heigh Accuracy Approximation of Curves

We start the talk with a survey on various approaches to the subject recently given by several authors. Introducing the notion of a "geometric contact element" (at a certain point **P** in $\mathbb{R}^d$ and with a certain order k), a general theory of "geometric Hermite interpolation" will be outlined. Thereby, an arbitrary finite set of geometric contact elements at different points and of various orders are prescribed and a (polynomial or rational) approximant with degree n is wanted so that it realizes these contact elements. This new theory comprises many special cases, recent as well as classical ones. A general estimate of the approximation order will be derived. More details will be given for the two-point Hermite interpolation problem for $d = 2$ and $n = 4$.

T. DeRose

Parametrizing Arbitrary Meshes

An algorithm for solving the following problem was presented:

Given: A mesh – that is, a triangulated polyhedron in $\mathbb{R}^3$ – with a large number of faces.

Find: A simplicial complex K with a small number of faces and a homeomorphism $S : K \to M$.

The algorithm is useful in a number of applications, including: finite element analysis, multiresolution approximations of geometry, texture mapping, and B–spline surface fitting. (Joint work with Matthias Eck, Tom Duchamp, Hugues Hoppe, Michael Wunsboy and Werner Stuetzle.)

N. Dyn

John Gregory's Research on Rational Spline Interpolation, Subdivision Algorithms and C^2 Polygonal Patches.

Three of the main research topics in the later part of John Gregory's career are reviewed: (i) Shape preserving interpolation by rational splines; quadratics for monotonic data and cubics for monotonic and convex data. (ii) Construction of interpolatory subdivision schemes with shape control, and their analysis in terms of a general theory for univariate and bivariate schemes. (iii) Constructive methods for C^2 polygonal patches in a complex of C^2 bi–polynomial patches, based on a general theory for C^k-contact between two polygonal patches, and its generalization to C^k patches.

N. Dyn* and D. Levin

Analysis of Hermite–type Subdivision Schemes

The theory of binary subdivision schemes is extended to the case of Hermite–type interpolatory subdivision schemes, which is a special important case of matrix subdivision. In particular the known necessary and sufficient conditions for the convergence of interpolatory subdivision schemes to C^l limit functions are extended to Hermite-type subdivision schemes which interpolate function values and derivatives. A specific family of such schemes is presented as an example.

M. Eck

Applications of Parametrized Meshes

We discuss two applications of a parametrization $\sigma : K^0 \to M$ of a mesh M of arbitrary topology over a simpler base mesh K^0 which is homeomorphic to M.

At first, the multiresolution analysis as described in Lounsbery, DeRose, Warren (1994) can be applied to determine approximations of σ of various resolutions. For example, the lowest resolution would be least-squares approximation to M which has the same connectivity as K^0.

The second application is surface reconstruction using NURBS. Here a given point cloud X is automatically approximated by a collection of NURBS patches. The mentioned parametrization σ is used here to get appropriate parameter values to all data points $X_i \in X$.

G. Farin

Projective Splines for Interpolation and Design

Projective splines are a geometric approach to spline schemes known as rational gamma-splines or rational beta-splines. Those splines need the notion of a knot sequence and of weights: they are replaced by more intuitive handles in the case of projective splines. Projective splines are G^2 in projective space, but one can devise a knot sequence (not needed in their original definition) such that they become C^2. When projected into affine space, they become C^2 piecewise rational cubics over a sequence of simple knots.

We present a method that computes the projective spline shape handles automatically, in order to arrive at "nice" curvature plots. The method produces circles when the input is a regular (closed) n–gon as the control polygon.

We then observe that cubic spline interpolation may be computed by iteratively solving the linear, tridiagonal, system. A geometric interpretation of this is that the solution my be obtained by a sequence of affine maps: for the i–th equation, we move control point no. i to a position such that interpolation occurs for data point no. i. This may be viewed as an affine map, taking control points no. $i-1$ and $i+1$ to themselves, and the current junction point no. i to the desired data point. Next, we say that a sequence of projective maps gives the solution to the projective spline interpolation problem. Projective maps are determined by four image and preimage points. Thus we can now move control point no. i anywhere, and still force interpolation to data point no. i. In particular, we explore the possibility of leaving the control points fixed and still forcing interpolation.

We explore several ways to take advantage of the new degrees of freedom in this scheme, keeping in mind that they have to be automatic, and not user–specified. We can present a scheme that reproduces circles exactly and yet produces a tight fit to rapidly varying data.

G. Albrecht and R. T. Farouki*

Construction of C^2 Pythagorean–Hodograph Interpolating Splines by the Homotopy Method

The complex formulation of polynomial Pythagorean-hodograph (PH) curves allows the problem of constructing a C^2 piecewise-PH-quintic "spline" that interpolates a given sequence of points $\mathbf{p}_0, \mathbf{p}_1, \ldots, \mathbf{p}_N$ and end-derivatives $\mathbf{d}_0$ and $\mathbf{d}_N$ to be reduced to solving a "tridiagonal" system of N quadratic equations in N complex unknowns. The system can also be easily modified to incorporate PH–spline *end conditions* that bypass the need to specify end–derivatives. Homotopy methods have been employed to compute all solutions of this system, and hence to construct a total of 2^{N+1} distinct interpolants for each of several different data sets. We observe empirically that all but one of these interpolants exhibits undesirable "looping" behavior (which may be quantified in terms of the *elastic bending energy*, i.e., the integral of the square of the curvature with respect to arc length). The remaining "good" interpolant, however, is invariably a *fairer* curve – having a smaller energy and a more even curvature distribution over its extent – than the corresponding "ordinary" C^2 cubic spline. Moreover, the PH spline has the advantage that its offsets are *rational* curves and its arc length is a *polynomial* function of the curve parameter.

T. Sederberg and R. Goldman*

A New Approach to Implicitizing Rational Curves and Surfaces

Resultants are the standard tool for implicitizing rational curves and surfaces. But resultants vanish in the presence of base points. We investigate a new technique for implicitizing rational curves and surfaces based on the method of moving algebraic curves and surfaces. Empirical results show that this method is robust, even in the presence of base points. We explain the new method, examine cases where it succeeds, and show how to recover when it fails. Since our results are still preliminary, we close with a list of open questions for future research.

T. Goodman

Total Positivity and Total Variation

Suppose that a curve is represented in terms of a totally positive basis. Then the total angle turned through by the tangent to the curve (the integral of the magnitude of its curvature) is bounded by the total angle turned through by the control polygon. The *twist* of a curve is the total angle turned through by its binormal (the integral of the magnitude of the torsion). In contrast to the above result, the twist of a curve *bounds* the corresponding twist of its control polygon, provided that the torsion of the polygon keeps the same sign. A bound for the twist of a curve can be given for cubic splines and this reduces, for a planar curve, to a bound on the number of inflections.

G. Greiner

Interpolating Scattered Data using a Variational Approach

A method for interpolating scattered data is presented. It is based on a variational approach, i.e., the resulting function (surface) is the solution to a constrained optimization problem:

$$J(F) = \text{min} \ \& \ F \text{ satisfies the interpolation conditions}$$

Hereby J is a quadratic, data–dependent fairness functional of second order. F varies in the class of all tensor product B–spline functions (surfaces) over an equidistant, rectangular grid in the plane. The advantages of this method, compared to the existing methods, lies in the fact, that the function (surface) is a tensor product B–spline, thus can be easily integrated in a CAD system. Moreover, this method can be extended to the parametric case.

H. Hagen

Stability Concept for Surfaces

In CAD/CAM technologies the design of free form surfaces is the beginning of a chain of operations that ends with the numerically controlled (NC–) production of the designed object. An important part of this chain is shape control. A new aspect of shape control is the stability of a surface.

In this talk stability conditions based on the concept of infinitesimal bendings are presented.

K. Höllig* and J. Koch

Geometric Hermite Interpolation with Maximal Order and Smoothness

We conjecture that splines of degree $\leq n$ can interpolate points on a smooth curve in $\mathbb{R}^m$ with order of contact

$$k - 1 = n - 1 + \lfloor (n-1)/(m-1) \rfloor$$

at every n–th knot. Moreover, this geometric Hermite interpolant has the optimal approximation order $k+1$. We give a proof of this conjecture for planar quadratic spline curves and describe a simple construction of curvature continuous quadratic splines from control polygons.

Chr. M. Hoffmann

Geometric Constraint Solving, the 3D Case

We investigate how to solve geometric constraint systems between points, lines and planes in 3–space. The general approach is a graph–constructive one: A constraint graph abstracts the constraints and the geometric objects. A clustering phase aggregates geometric structures that can be solved sequentially, and a merging phase combines clusters that interconnect in particular patterns. Specific problems include how to combat combinatorial explosion, and how to devise uniform computations for difficult geometric constructions.

B. Jüttler

Least-Square Approximation by Parametric Curves using Parameter Variation

Least-square approximation by B-Spline curves and surfaces is of fundamental importance in Computer Aided Geometric Design, e.g., for the construction of curves or surfaces from measured data or for the approximate conversion between different curve or surface descriptions. Using the dual basis of the Bernstein polynomials, we are able to construct an explicit representation of the approximation of (vector-valued) functions by (vector-valued) polynomials in Bernstein-Bézier form. But the approximant of a curve or surface depends on its special parametric representation. With help of the explicit representation of the least-square approximant we introduce the idea of parameter variation: The optimal parameterization of the given curve is found by minimizing an appropriate functional. Several functionals and their influence to the result of the approximation scheme are discussed. The whole scheme can be said to be a geometric formulation of least-square approximation.

P. D. Kaklis*, A. I. Ginnis and I. R. Sarantidis

Cubic Spline Interpolation under Unit-Tangent-and-Curvature Boundary Conditions

This work addresses the problem of constructing a C^2 cubic spline, which interpolates a given planar set of points, with given parametrization, and possesses fixed unit–tangent and curvature at the two boundary points. It is shown that this problem can be equivalently formulated as a quartic equation, with respect to the Euclidean norm of either of the two boundary tangent vectors, plus an inequality constraint. Necessary conditions are derived and numerical results are also presented.

P. D. Kaklis* and K. G. Pigounakis

Fairing Three-Dimensional C^2 Cubic B-splines

In this work we present an extension of the Sapidis–Farin technique for fairing three-dimensional C^2 cubic B-splines. The technique is local and essentially employs the following strategy: move the "offending" control point onto its projection on the line (or plane) along which the derivative of the curvature (or the torsion) of the curve at the "offending" knot becomes continuous. An algorithm is proposed and its behaviour is illustrated for three data sets.

L. Kobbelt

A Variational Approach to Interpolatory Refinement

Interpolatory refinement is a simple and intuitive method for the iterative generation of smooth curves and surfaces. The motivation for doing a variational approach is the observation that in most of the literature only sufficient or necessary conditions for the convergence of a *given* scheme are proven, but one can hardly find any general methods for the construction of interpolatory schemes which satisfy these conditions. Further: the analytical proofs for the convergence do not illustrate what really happens when refinement is done. The variational approach gives a simple answer to why interpolatory refinement can produce smooth curves: because the refinement can be considered as strain energy minimization.

In this talk, I present how the concept of energy minimization can be exploited to derive refinement schemes for curves and surfaces which produce very well-shaped objects. I explain how the convergence analysis of such schemes can be done and give some examples for schemes which produce interpolating curves with high order of continuity.

D.–Y. Liu* and J.–Y. Wang

Modeling of Rubber Curves and its Applications

In this paper, a method of creating a curved polygon based on the cubic NURBS is presented. To create or modify a curve does not need to manipulate the control points one by one, a group of control points will be generated or modified simultaneously instead. These curve displayed on the screen are rubber-like.

The method described in this paper has been applied in apparel pattern making system developed by GINTIC of Nanyang Technology University, Singapore. Also it can be extended to 3D surface case and applied to computer animation.

T. Lyche

Multiresolution Analysis Based on Quadratic Hermite Interpolation – Part 1: Piecewise Polynomial Curves

We discuss a multiresolution analysis based on C^1 quadratic Hermite interpolation and the L^∞ norm. The use of the L^∞ norm is natural in many CAGD applications and it leads to schemes which are faster and simpler to implement than the wavelet schemes based on the L^2 norm. We have chosen to discuss quadratic Hermite interpolation because (i) it is a C^1 scheme with nice shape preserving properties, (ii) we have a certain sup norm stability in the wavelet spaces, (iii) there are local support bases for these spaces, (iv) the decomposition coefficients can be determined explicitly in real time, (v) it generalizes to splines over triangulations. In the talk we give several examples of decomposition of parametric curves. This is joint work with M. Dæhlen, G. Holm, K. Mørken, and H.-P. Seidel.

A. McEntee and H. McLaughlin*

The Shape of Noisy Data

An algorithm is presented which accepts two sets of noisy curve data and compares their shapes. The output of the algorithm is one of the following three statements: (1) the two data sets have the same shape, or (2) the two data sets do not have the same shape, or (3) it is not possible to determine the shape of at least one of the data sets. The input of the algorithm consists of two unordered sets of planar points: each point is given by a pair of x-y coordinates. The notion of shape is a qualitative one: it is defined by a resident catalogue.

A. Le Méhauté

Knot Removal for Scattered Data

We present a review of some strategies recently developed for reducing the number of knots for the representation of a piecewise polynomial approximation of a function defined on scattered data, without perturbing the approximation more than a given tolerance. The method removes some (or all) of the interior knots. The number and location of these knots are determined automatically. Applications are in approximation of data, data storage and image reconstruction.

H. Nowacki

Toward a Synthesis of Fair Free–Form Curves and Surfaces

A synthesis process is described in which a fair free–form surface is built up by consecutively fairing curves, curve meshes, and surfaces. At each stage of this process, an explicit fairness measure is used as an objective function and appropriate constraints are applied. The shape elements have more free shape parameters than constraints so that the shape is free to respond to the fairing objectives. At the final stage, surfaces of regular or irregular mesh topology are faired based on a flexible choice of the fairness measure, including isotropic and direction dependent fairing criteria. The quality of the resulting surface and of its shape elements can be quantified in terms of their fairness measures.

J. Peters

Decompositions of the Identity and the Construction of Smooth Surfaces

Geometric modelers typically define surfaces as images of closed polygonal regions under polynomial or rational maps, called patches. The images, also called patches, do not overlap but join along curves in $\mathbb{R}^3$. Differential topologists define surfaces as domains of invertible maps, also called patches, from $\mathbb{R}^3$ to open sets in $\mathbb{R}^2$. The patches cover the surface by overlapping in open subsets. This paper develops a surface model that reconciles the apparent discrepancy between the constructive and the analytic approach by defining and characterizing maps that link the domains and ranges of the various types of patches. Of particular interest are families of maps whose composition matches the Taylor expansion of the identity map. Such families are named decompositions of the identity and its members roots of the identity.

H. Pottmann

Applications of the Cyclographic Map and Laguerre Geometry in CAGD

We briefly review Euclidean Laguerre geometry and its different models, namely the standard Euclidean model, cyclographic model, Blaschke cylinder and isotropic model. Based on that, applications in CAGD are studied: the geometry of the medial axis transform, geometrical optics and, in particular, rational PH curves and surfaces. These curves and surfaces, which are characterized by the rationality of all their offsets, can be elegantly handled within Laguerre geometry. In the isotropic model, they appear as arbitrary rational curves or surfaces, but also the other models are appropriate to study their properties and derive algorithms for modelling with them. Furthermore, we address rational canal surfaces and the invariance of PH curves and surfaces under Lie transformations.

M. J. Pratt

Quartic Supercyclides

Various classes of algebraic surfaces have been examined as to their suitability for CAGD purposes. This paper contributes further to the study of a class of quartic surfaces recently investigated by Degen, having strong potential for use in blending, and possibly also in free-form surface design. These surfaces are here put in the context of a classification of quartic surfaces originally given more than one hundred years ago. An algebraic representation is provided for them, and a simple geometric interpretation given for their rational biquadratic parametric formulation. Their theory is established from an analytic geometry viewpoint which is more straightforward than Degen's original approach and gives further useful geometric insight into their properties. A major subclass of Degen's surfaces consists of projective transforms of the Dupin cyclides; for this reason (and others, explained in the text) the name *supercyclides* is proposed for them.

H. Prautzsch

G^k Surfaces of Minimal Degree

One can build G^k surfaces of arbitrary topology with tensor product patches. While it is possible to join most patches with simple C^k joints the surface has in general n sided patch configurations with general C^k joints (G^k joints).

In the talk it was shown that these n–sided patch complexes can be built so as to lie on an arbitrarily chosen polynomial. If this polynomial is of degree r, then the patches lying on it are parametrized as tensor product patches of degree $r\,k + r$.

Secondly Catmull/Clark type subdivision schemes of arbitrary higher degree were considered where the scheme around an extraordinary point is described by a matrix. The differentiability of the limiting surface was related to the spectral properties of the matrix. These smoothness conditions are both sufficient and necessary and also apply to hypersurfaces of any degree. Further these conditions also show how one can control C^k schemes producing surfaces with polynomial patches of in general minimal degree.

U. Reif

TURBS – Topologically Unrestricted Rational B-Splines

A new method for constructing free form surfaces of arbitrary topological genus is presented. It fits in the standard tensor product NURBS framework by providing the following features:

- Representation by real-valued B-splines
- Shape control by spatial control points and rational representation
- Subdivision formulas
- Arbitrary order of smoothness

The bi–degree for generating C^k-surfaces is $2k + 2$. On one hand, this is a substantial improvement of all existing methods which require polynomials of order $O(k^2)$. On the other hand, this result is optimal in the sense that no further degree reduction is possible if subdivision formulas are assumed to exist.

M. Sabin

Discrete Geometry

In CAGD we typically use floating point numbers to hold 'reals', and ignore the effects of inaccuracies of arithmetic. The solid modelling community do worry about errors and jump through hoops with tolerances.

In fact, floating point numbers form a discrete set, and any code which uses them in fact implements approximations to the algorithms.

This presentation explores the possibility of being explicit about this discreteness and knowing exactly what we are doing.

Because of the novelty and difficulty involved, it addresses only the 2D geometry of points, lines and conics.

- Some Abstract Ideas
- Luby line theory
- Towards a theory of conics

L. H. T. Chang and H. B. Said*

A C^2 Triangular Patch for Scattered Data Interpolation

Given a set of data points $V_i = (x_i, y_i, z_i), i = 1, \ldots, n$, and their positional and up to second derivative values, we wish to construct a C^2 surface that interpolates these values. Firstly these data points are triangulated, and then a triangular patch is defined over each triangle. We propose a triangular patch which consists of a convex combination of three "local" triangular patches. Each of these "local" patches satisfies C^2 continuity over a boundary of the triangle.

R. Sarraga

Optimization Methods for Shaping Trimmed B–spline Surfaces

This talk discusses results obtained by applying shape–optimization functionals on several geometric configurations. The functionals are of the family introduced by Günther Greiner, and the surface is composed of C^1 quintics.

L. L. Schumaker

Splines on the Sphere

We describe a natural set of barycentric coordinates associated with spherical triangles and use them to construct spherical Bernstein–Bézier polynomials which have almost all of the properties of the classical BB–polynomials on planar triangles, including a de-Casteljau algorithm, subdivision, smooth joins, etc. This leads to a new class of spline functions defined on the sphere (and on sphere-like objects). After briefly describing theoretical properties of these spaces, we discuss a variety of new methods for interpolating and fitting scattered data on the sphere. These include macro element methods (Powell–Sabin, Clough Tocher, etc.), minimal energy methods, least squares, and penalized least squares. The talk is based on joint work with Peter Alfeld and Mike Neamtu.

H.-P. Seidel

Multiresolution Analysis Based on Quadratic Hermite Interpolation – Part 2: Piecewise Polynomial Surfaces over Triangles

This talk continues the talk given by T. Lyche and covers piecewise polynomial surfaces over triangles. We first construct a nested sequence of spaces using Powell-Sabin splits. We then construct a locally supported wavelet basis and prove stability in the L^∞-norm. Similar to the curve case, the resulting scheme has the following attractive features: (i) C^1-scheme with nice shape preserving properties, (ii) stability of the wavelet basis, (iii) there are local support bases for these spaces, (iv) the decomposition coefficients can be determined explicitly in real time.

H.-P. Seidel

Convergence of B-Spline Subdivision and Degree Elevation

We outline a short proof for the convergence of both subdivision (quadratic convergence) and degree elevation (linear convergence) for B-splines, based on the de Boor-Fix dual functionals and/or polar forms. The approach naturally extends to surfaces. For subdivision of B-spline curves this approach goes back to the classical work by C. de Boor.

J. Warren

An Efficient Algorithm for Evaluating Polynomials in the Pólya Basis

A new $O(n)$ algorithm is given for evaluating univariate polynomials of degree n in the Pólya basis. Since the Lagrange, Bernstein, and monomial bases are all special instances of the Pólya basis, this technique leads to efficient evaluation algorithms for these special bases. For the monomial basis, this algorithm is shown to be equivalent to Horner's rule.

F.-E. Wolter

Geodesic Offsets, Computations and Applications

Geodesic offset curves are obtained as end points of a family of geodesic segments which emanate orthogonally from a given progenitor curve. (All geodesic segments of this family have equal length.) The talk describes how methods from Riemannian geometry can be used to compute the curvature of geodesic offset curves and how to estimate the distance of offset singularities to the progenitor curve. Further methods are explained to compute generalized medial curves on a surface. A medial curve on a surface contains surface points that are (in geodesic distance) equidistantial with respect to two given surface curves. Finally we sketch an idea for a potential contribution of geodesic offsets (and of the offsets tangent vector field) to model a deformation procedure in a die–less metal forming process. This forming process is supposed to generate a curved metal shell (with a user defined Gauss curvature) from a flat metal plate with a plastic deformation behaviour.

A. Worsey

Reparameterizing Rational Curves for Applications

Rational Bézier curves are typically parameterized so that the first and last weights are unity. We consider the problem of reparameterizing these curve so as to optimize certain properties which are useful in applications. Specifically, our results lead to an improved method for polynomial approximation using hybrid curves, and more importantly, generate parameter values for "optimal" subdivision, as well as bounding box estimates that are often sharper than the standard convex hull of the curve's control points.

Y. Yamaguchi

Bézier Normal Vector Surface and its Application

A Gauss map, which is a representation of normal vectors of a surface, and a visibility map, which is a representation of locally visible directions of a surface, have wide varieties of applications such as offset surface generation, surface–surface intersection, tool orientation for NC machining, workpiece setup, and so on. However, it is usually very expensive to calculate a Gauss map and a visibility map. Therefore, the previous works approximate their bound either by bounding cones, by rectangular pyramids, or by hexagonal pyramids. We will propose a Bézier normal vector surface, which is a Bézier form representation of non–unit normal vectors of a surface. It gives a much tighter bound of the surface normals. Normal vectors of both a tensor product Bézier surface and a triangular Bézier surface can be represented as a tensor product Bézier surface and a triangular Bézier surface respectively.

We will also discuss how a Bézier normal vector surface can be used for surface–surface intersection. A relatively robust algorithm to calculate intersections is a marching method. However, it is difficult to find all initial points of intersection branches and to trace a curve near singular points. Critical points which have parallel normals on both surfaces are key points to solve those problems. We will explain a new algorithm to find all critical points efficiently by using Bézier normal vector surfaces.

J. J. Zheng

Generating Surfaces over Non–four–sided Areas

This talk will present mathematical expressions with control points for 3–, 5– and 6–sided surface patches whose boundary curves may be Bézier curves of arbitrary degrees. These patches can meet surrounding rectangular surface patches of arbitrary degrees with C^1 continuity.

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