MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 23/1995

Differentialgeometrie im Großen

11.06. bis 17.06.1995

Die Tagung fand unter der Leitung von W. Ballmann (Bonn), J.P. Bourguignon (Palaiseau) und W. Ziller (Philadelphia) statt.

Wie üblich waren zu der Tagung so viele Geometer von fern und nah angereist, daß es nicht schwer viel, ein hochkarätiges Programm zusammenzustellen — zu bedauern wäre lediglich, daß wir nicht noch eine weitere Woche bleiben durften.

So wurden etwa auf dem Gebiet der positiven Krümmung zahlreiche spektakuläre neue Resultate dargestellt, die sowohl mit analytischen Methoden aus dem Bereich der partiellen Differentiagleichungen als auch mit verfeinerten Vergleichskonstruktionen erzielt worden waren. Darüber hinaus wurden seit langer Zeit auch einmal wieder neue Beispiele positiv gekrümmter Mannigfaltigkeiten konstruiert.

Viele Vortäge beschäftigten sich mit dem Zusammenspiel spezieller geometrischer Eigenschaften wie homogenen Kählermannigfaltigkeiten, 3-Sasake Mannigfaltigkeiten oder Kontaktstrukturen, wobei bemerkenswerte Klassifikationsresultate erzielt wurden.

Das interessante Tagungsprogramm wurde dadurch aufgelockert, daß jeweils der erste Nachmittagsvortrag als Überblicksvortrag angesetzt war.

Vortragsauszüge

M. KAPOVICH

Flats in 3-manifolds

We prove that if a compact aspherical Riemannian manifold M^3 contains a 2-flat in the universal cover $\tilde{M}^3$ then $\pi_1(M)$ contains $\mathbb{Z} \oplus \mathbb{Z}$. This generalizes the Buyalo-Schroeder theorem to the case of indefinite sectional curvature.

D.V. ALEKSEEVSKY & V. CORTÉS

Isometry groups of homogeneous quaternionic Kähler manifolds

A general method for calculation of the full isometry group of a Riemannian solvemanifold is presented.

Using it the full isometry group of the non-symmetric quaternionic Kähler solvemanifolds M is determined: $\tau-, \omega$- and γ-spaces.

As an application, it is proved that the isometry group acts transitively on the twistor space and on the $SO(3)$-principal ("3-Sasakian") bundle of M and that the manifold M does not admit quotients of finite volume.

Finally, a new, simple description of the Alekseevsky spaces

$$\tau(p),\ \omega(p,q),\ \gamma(l,k)\ (k \not\equiv 0(4)),\ \gamma(p,q;k)\ (k \equiv 0(4)))$$

in terms of a certain spinorial module S of the group $\mathrm{Spin}(3, 3+k)$ is given.

C. CROKE

A survey of manifolds without conjugate points

We survey the current status of the subject with special attention to the recent proof by Burago and Ivanov of the E. Hopf conjecture: "Every metric on an n-torus without conjugate points is flat". We also discuss results and open questions concerning the topology of such manifolds as well as results and open questions of rigidity type.

Open Questions: Topology of compact NCP

(1) Do all compact NCP manifolds admit a metric with $K \leq 0$?

(2) (Tits alternative) Does every subgroup of $\pi_1(M)$ contain either a free group of two generators or is virtually abelian?

(3) (Quadratic isoperimetric inequality) Does there exist $C(M)$ such that for all closed $\tilde{\gamma}$ in $\tilde{M}$, $\tilde{\gamma}$ spans a surface of *area* $\leq C(M)L^2(\tilde{\gamma})$?

(4) Do there exist infinitely many geometrically distinct closed geodesics ($\pi_1(M) \neq \mathbb{Z}$) ?

Open Questions: Rigidity

(1) (Mañé conjecture) If M is a compact NCP manifold and $h_{top} = 0$, must M be flat?

(2) For M a non-compact NCP manifold of higher rank, does it admit any nontrivial compactly supported NCP perturbation?

(3) For M a compact NCP of higher rank, must any NCP metric also have higher rank? (special case: must the universal cover of $\Sigma_2 \times S^1$ with NCP metric split off an $\mathbb{R}^1$?)

(4) Are compact NCP manifolds C^0 conjugacy rigid?

(5) Are there any analogue of the splitting theorem?

(6) (Parallel postulate) If M^2 is 1-connected, NCP such that for all geodesic γ and every $p \notin \gamma$ there is a unique geodesic τ sucht that $p \in \tau$ and $\tau \cap \gamma = \emptyset$ then must $M^2 = E^2$?

T. COLDING

Aspects of Ricci curvature

We survey various geometric and topological results of manifolds with a given lower Ricci curvature bound. In particular we discuss a new estimate of the distance function for such manifolds and discuss applications of this to volume convergence, Gromov's conjecture generalizing Bochner's theorem to manifolds of almost nonnegative Ricci curvature, splitting theorem for singular spaces and volume cone imply metric cone.

K. GALICKI

We survey geometric and topological results on Riemannian manifolds admitting a 3-Sasakian structure. These spaces were introduced by Kuo and Udriste in 1970 and, in particular, are Einstein manifolds of positive scalar curvature. We describe some old results concerning this type of geometry obtained in the 70's as well as some recent developements. These include a simple quotient construction of a large class of families of compact, simply connected 3-Sasakian manifolds out of 3-Sasakian spheres $S^{4n+3}(1)$. This new construction yields examples that are not regular and the Riemannian metric is inhomogeneous. We conclude with a discussion of several open problems concerning: moduli of 3-Sasakian structures on a given smooth manifold, rigidity, Betti numbers, and classification of 3-Sasakian spaces in dimension 7.

B. LEEB

Rigidity of nonpositivly curved spaces

We report on joint work with M. Kapovich and B. Kleiner about quasiisometry invariants of nonpositively curved spaces. Quasi-isometries play an important role in geometric group theory and rigidity problems. Concerning symmetric spaces of noncompact type, we show that irreducible higher-rank spaces are quasi-isometrically rigid (each quasi-isometry is within bounded distance of an isometry) and that the product structure of reducible spaces is preserved. This extends Mostow's rigidity theorem and confirms a conjecture of Margulis. A finitely generated group quasi-isometric to a symmetric space of noncompact type is a finite extension of a uniform lattice. We mention some results about universal covers of smooth nonpositively curved Riemannian manifolds: the geometric rank and the de Rham product decompositions are preserved. For universal covers of non-geometric Haken 3-manifolds, the lift of the canonical (Jaw-Shalen-Johannson) decomposition is preserved.

J. BLOCK

Positive Scalar curvature on manifolds of non-positive sectional curvature

We show that a locally symmetric space of non-positive curvature and finite volume X has a metric of positive scalar curvature $K \geq \varepsilon > 0$ if and only if (when $\Gamma = \pi_1(x)$ is arithmetic) the $\mathbb{Q}$-rank X is ≥ 3. If the $\mathbb{Q}$-rank is ≥ 3 these metrics can also be made of finite volume. The obstructions in $\mathbb{Q}$-rank 1 and 2 are proved by and index theorem involving $\pi_1(X)$ and $\pi_1^\infty(X)$ which is the fundamental group of the end.

H. PEDERSEN

Survey of Einstein-Weyl geometry

We shall consider a conformally invariant version of the Einstein equations. A manifold with conformal structure and compatible symmetric connection is called a Weyl manifold. If, furthermore, the trace-free symmetric part of the Ricci curvature vanishes, the geometry is called Einstein-Weyl.

Einstein metrics are special examples of Einstein-Weyl solutions. We shall present more examples on compact spaces as well as obstructions to solutions. In dimension four we give a classification in presence of symmetry. These symmetric examples give some insight into the moduli space near Einstein metrics as well as new minimal submanifolds in compact Riemannian manifolds with positive Ricci curvature. Also, we shall see that Einstein-Weyl geomety in the Gauducthon gauge gives a solution to a problem in Besse concerning examples satisfying $\nabla_x r(x,x) = 0, \nabla r \neq 0$ and being of high cohomogeneity.

J. LOHKAMP

On the geometry of multiple connected sums

Using a construction generalizing the connected sum of manifolds (we entitled "multiple connected sums") we obtain new metrics which have a lot of unexpected properties.

Beside others, this is the proof of existence of metrics with prescribed eigenvalues and volume and of metrics with ergodic geodesic flows on arbitrary manifolds of dimension ≥ 3. The effect of these constructions might be understood as a kind of "producing" volume and ergodicity, respectively. This also allows to sharpen results to the end that we find dense sets of such metrics and other flexibility results.

C. BÄR

Existence of metrics with harmonic spinors

Classical Hodge-deRham theory tells us that on a closed manifold the dimension of the kernel of the Laplace operator acting on k-forms is a topological invariant, the k^{th} Betti number. On a Riemannian spin manifold there is another natural operator, the Dirac operator. The question arises whether $h = \dim \ker(\text{Dirac})$ is also topologically invariant. But this is not true, in fact, in many dimension existence of harmonic spinors is not topologically abstructed.

Thm. On every closed spin manifold of dimension $n \equiv 3 \pmod 4$ there exists a Riemannian metric s.t. $h > 0$.

The proof uses a comparison theorem for eigenvalues on connected sums and an explicit computation of Dirac-eigenvalues on Berger spheres.

C. MARGERIN

A smooth sphere theorem, sharp

We give a sharp (local) geometric characterization of the (standard quotient of the) standard smooth spheres in all dimesnions evend and *odd*, but a finite number ($n \geq 64$). The curvature invariant we consider is weak-pinching, a properly scaled norm of the trace free part of the curvature: we prove that w.r.t. this invariant there is no other diffeomorphism type than the standard $S^n/_\Gamma$ before we meet geometries of type $S^1 \times S^{n-1}$. We also discuss the rigidity statement that these are the only ones on the boundary. We sketch a few of the ideas in the proof, which involves deforming such metrics along the integral curves of the Ricci curvature field (on the space of metrics) towards a metric of constant sectional curvature. We finally suggest possible generalizations toward a more systematic higher dimensional "geometrization".

I. TAIMANOV

New examples of positively curved 13-dimensional manifolds

We talk about infinite series of closed Riemannian positively curved manifolds with dimension equal to 13. These examples were constructed by our student Ya. V. Bazaikin

(1994, Novosibirsk University) by using the Eschenburg method. They can be considered as deformations of the Berger normally homogeneous manifold $SU(5)/Sp(2) \times S^1$ in the same sense as Eschenburg's 7-dimensional examples are deformations of the Aloff-Wallach spaces. Some perspectives of further investigations are also discussed.

U. ABRESCH

Injectivity Radius Estimates and Sphere Theorems

(joint work with W.T. Meyer)

For odd-dimensional manifolds the pinching constant that is required to prove injectivity radius estimates and sphere theorems can be improved as follows:

Theorem A: Let $\delta_{inj} := \frac{1}{4(x+\varepsilon_{inj})^2}$ where $\varepsilon_{inj} := 10^{-6}$, and let (M^n, g) be a compact Riemannian manifold with $\delta_{inj} < K_m \leq 1$ and $\pi_1(M^n) = 0$. Then $inj(M^n) = conj(M^n) \geq \pi$. The Berger metrics on the odd dimensional spheres provide examples where the equality $inj(M^n) = conj(M^n)$ fails provided that one allows for pinching constants < 0.117223.

Combining Theorem A with the limiting arguments that are used in the proof of Berger's pinching below $-\frac{1}{4}$ theorem, one obtains

Theorem B: for any $n \equiv 1(2)$ there exists some $\delta_n \in (0, \frac{1}{4})$ such that any compact Riemannian manifold M^n with $\delta_n < K_M \leq 1$ and $\pi_1(M^n) = 0$ is homeomorphic to S^n. In contrast to Theorem A we find that Theorem B relies on pinching constants that depend on the dimension and are not effectively computable because of the way the proof relies on Gromov's compactness theorem. The question whether there is a different approach by more direct methods of comparison geometry can be answered positively, and thus we obtain a result with an explicite pinching constant:

Theorem C: Let $\delta_{odd} := \frac{1}{4(1+\varepsilon_{odd})}$ where $\varepsilon_{odd} = 10^{-6}$, and let M^n be a compact odd dimensional manifold with $\delta_{odd} < K_M \leq 1$ and $\pi_1(M^n) = 0$. Then M^n is homeomorphic to S^n.

The analogous techniques, when applied to even dimensional manifolds, however, do not show that M^n must be diffeomorphic to a projective space unless it is homeomorphic to S^n, as one would expect in view of Berger's pinching below $-\frac{1}{4}$-theorem. So far, we can only recover the cohomology ring for coefficients in $\mathbb{Z}_2$ or in $\mathbb{Q}$. The proof of Theorem C relies on the following result:

Horse Shoe Theorem: Suppose that M^n is an compact Riemannian manifold with $\delta_{hs} < K_M \leq 1$ and $\pi \leq inj(M^n) \leq diam(M^n) \leq \pi \cdot (1 + \varepsilon_{hs})$ where $\delta_{hs} := (1 + \varepsilon_{hs})^{-2}$ and $\varepsilon_{hs} := \frac{1}{27000}$. Then for any $p \in M^n$ and any $v \in T_pM^n$ with $|v| = 1$ one has

$$dist(exp_p(-\pi v), exp_p(\pi v)) < \pi$$

The reduction of Theorem C to the Horse Shoe Theorem is done by means of standard arguments in algebraic topology, whereas the proofs of Theorem A and of the Horse Shoe

Theorem make use of serveral new comparison techniques. The starting point for the injectivity radius estimate is a new lifting construction for shortly null-homotopic curves which can be controlled better. The key ingredient in the proof of the Horse Shoe Theorem are new types of Jacobi field estimates, which make use of simultaneously provided initial and boundary data.

J.M. SCHLENKER

Generalization of Efimov's theorem

Let (Σ, σ) be a complete Riemannian surface with curvature $K \leq K_1 < 0$ and (M, μ) a 3-manifold with sectional curvature $K \in [K_2, K_3)$ with $K_1 < K_2$ and $(K_3 - K_2)^2 < 16(K_3 - K_1)(K_2 - K_1)$. Suppose that $\|\Delta K_\Sigma\|, ; \|\Delta K_M\|$ are bounded. Then there exists no isometric immersion of Σ into M. A similar result holds when M is a Lorentz space form. The presented methods and results also extend to more general situations involving hyperbolic Monge-Ampère operators over surfaces.

A. MOROIANU

Kählerian killing spinors, contact structures and twistor spaces (II)

Manifolds with Kählerian Killing spinors are spin Kähler manifolds with the least possible eigenvalue for the square of the Dirac operator among all Kähler manifolds of positive curvature. They are necessarily Kähler-Einstein of odd complex dimension $m = 2k + 1$.

Theorem: Let M^{4k+2} admit Kählerian Killing spinors. If k is even then M is isometric to $\mathbb{C}P^m$. If k is odd then M is the twistor space of a quaternionic Kähler manifold of positive scalar curvature.

Proof: A Kählerian Killing spinor on M induces a Killing spinor on E, a maximal root of the canonical bundle of M, $\Lambda^{m,0}M$. A closer analysis of this spinor shows that E is a regular 3-Sasakian manifold, and finally M has to be the twistor space of the associated quaternionian Kähler manifold.

F. WILHELM

A generalization of Berger's $\frac{1}{4}$-pinched rigidity theorem

Recall that the radius of a compact metric space $(X, dist)$ is given by

$$radX = \min_{x \in X} \max_{y \in X} dist(x, y).$$

In this talk we generalize Berger's 1/4-pinched rigidity theorem and show that a closed Riemannian n-manifold with sectional curvature ≥ 1 and radius $\geq \pi/2$ is either homeomorphic to S^n or isometric to a compact rank one symmetric space.

R. GORNET

Spectral geometry on Nilmanifolds and the marked length spectrum

In this talk, we review a construction for producing pairs of nilmanifolds of arbitrary step, and present the properties of resulting mew examples:

Ex of 3-step isosp. nilmfd	isosp. on functs	isosp. on p-forms $\forall p$	represent. equiv. fund.gps	same length spec.	same marked length spec.	isomorphic fund.gps
$\overset{7-\text{dim}}{I}$	Yes	Yes	Yes	No	No	No
$\overset{5-\text{dim}}{II}$	Yes	Yes	Yes	Yes	No	Yes
$\overset{7\backslash 5-\text{dim}}{III\backslash IV}$	Yes	No	No	No	No	No
$\overset{7-\text{dim}}{V}$	Yes	No	No	Yes	Yes	Yes

Note that in $E \times V$, the pairs of nilmanifolds have the same marked length spectrum, Laplace spectrum, but *not* the same spectrum on one-forms. In contrast:
Theorem: for a large class of 3-step nilmanifolds, the same marked length spectrum $\Longrightarrow$ isosp. on functions.

The large class is all 3-step nilpotent Lie groups, strictly nonsingular, with cocompact, discrete subgroups with the same intersection with center. This partially extends and partially contrasts with a result of Eberlein, who showed that for pairs of two-step nilmanifolds, the same marked length spectrum implies isospectral on functions and on forms.

J. CAO (joint work with J. Escobar)

An isoperimetric comparison theorem for PL-manifolds of non-postive curvature

We derive an optimal isoperimetric comparison theorem for manifolds with non-positive curvature.

A piecewise flat manifold M^n is said to have non-positive curvature in the sense of Gromov if M^n satisfies the $CAT(0)$ inequality.

Main Theorem: Let M^n be a simply-connected, complete and piecewise flat manifold. Suppose that M^n has non-positive curvature in the sense of Gromov. Then

$$vol_{n-1}(\partial\Omega) \geq c_n[vol_n(\Omega)]^{\frac{n-1}{n}}$$

for any compact domain $\Omega \subseteq M^n$ with rectifiable boundary $\partial\Omega$, where $c_n = \frac{vol_{n-1}[S^{n-1}(1)]}{(vol_n[B^n(1)])^{\frac{n-1}{n}}}$ is the optimal constant in the n-dimensional Euclidean space $\mathbb{R}^n$.

P. GAUDUCHON

Existence and uniqueness of complex structure on some class of compact Riemannian manifolds

We address the general problem of the existence and the uniqueness of an *integrable* almost-complex structure orthogonal with respect to some given Riemannian metric g on some oriented, even-dimensional manifold M. The problem is well unterstood in dimension 4, when a possible orthogonal structure is entirely encoded, up to a two-fold choice, by the half-Weyl tensor w^+ at any point where w^+ does not vanish; it follows that an n-dimensional Rimannian manifold with non-vanishing w^+ admits at most two orthogonal complex structures (up to conjugacy). Examples with exactly two (non conjugate) orthogonal complex structures have been given recently by P. Kobak.

In general, for given g, the problem appears as an overdetermined , elliptic problem, equivalent to looking for sections of the so-called "twistor space" $Z(M)$, stable with respect to the canonical almost-complex structure J of $Z(M)$.

The latter geometric picture has been used by F. Burstal, O. Muškanov, G. Grantchanov, J. Rawnsley to prove a rigidity theorem for the invariant complex structure of compact Hermitian symmetric spaces. We prove a similar result for compact quotients of irreducible symmetric spaces of non-compact type by using the following general criterion for compact manifolds of (even) dimension $n > 2$:

For any Riemannian metric g, consider the following expression:

$$S(g) = \frac{1}{V^{\frac{n-2}{n}}} \int_M \left(\frac{Scal^g}{n(n-1)} - \frac{1}{(n-2)} \lambda^g_{min}(W) \right) v_g$$

where $Scal^g$ denotes the scalar curvature, $\lambda^g_{\min}(W)$ the smallest eigenvalue of the Weyl tensor W (viewed as a symmetric operator on $\Lambda^2 M$), v_g the volume-form, $V = \int_M v_g$ the total volume. Then,

(1) if $S(g) < 0$ (for some metric g is the conformal class), then is no $[g]$-compatible complex structure.

(2) If $S(g) = 0$, and there exists some compatible complex structure J, then:

(i) $\frac{scal^g}{n(n-1)} - \frac{1}{(n-2)} \lambda^g_{min}(W) \equiv 0$;

(ii) the Kähler form F of J belongs to the eigenspace of W with respect to $\lambda^g_{min}(W)$;

(iii) the Hermitian structure (g, J) is semi-Kähler (i.e. F is co-closed).

E. Calabi

On singular symplectic structures

The problem of the existence of a symplectic structure in a given, closed, $2n$-dimensional manifold M^n may be studied by analysing the deformation of the singularities in a one-parameter family of closed 2-forms. By a singularity of a 2-form we understand any point where the rank of the coefficient matrix of a closed 2-form ω is discontinuous; the rank of a 2-form $\omega(x)$ at any point $x \in M$ is an even integer $2k$, where k is the highest power of $\omega(x)$ that is not zero.

We donote by $w^{[k]}$ the divided powers of a 2-form w

$$w^{[0]} = 1, w^{[k]} = k^{-1} w \wedge w^{[k-1]} (k \geq 1)$$

A (non-singular) symplectic structure on $M^{(2n)}$ is the geometric structure defined by a closed 2-form ω such that $\omega^{[n]} \neq 0$ everywhere. The existence of such a form requires (trivially) two necessary conditions: 1) the existence of an almost complex structure; 2) a 2-dimensional, real cohomology class (the deRham class of ω) whose n^{th} cup-power (represented by $\omega^{[n]}$) is non-trivial.

In general a (closed) 2-form ω in $M^{(2n)}$ has a non-empty *singular set*, namely the set where $\omega^{[n]} = 0$. An open, dense set of closed forms ω in the space of all such forms (with a C^∞ topology) reduces the singular set to a set that is at most $(2n-1)$-dimensional. A deformation of ω (usually considered are only deformations leaving ω in a fixed deRham class) displaces the singular set and modifies other invariants attached to it. The question that arises naturally is to find invariants of the singularity that are preserved under deformations of ω. Some of these invariants are described here.

The singular set has a stratified structure, where the different strata are indexed by two independent indices at least. A preliminary study of these singularities was initiated by J. Martinet in his thesis and later described in an expositiry work by R. Roussarie. Consider the sheaf of germs of closed 2-forms, Ω^2_c, of class C^∞ (resp. C^ω) and its filtration by the bundle of jets of order k, $(k = 0, 1, 2, \ldots)$.

The sheaf of germs of diffeomorphisms near the identity acts on the sheaf Ω^2_c, and the $(k+1)$-jets of diffeomorphism acts on the k-jets of closed forms. The orbits in Ω^2_c under the action of the sheaf of diffeomorphism are classified by their codimension and by the least order of jet that distinguish them from the neighbouring orbits. In considering single (closed) 2-forms ω in M^{2n} one may limit one's consideration only to the dense set of cross-sections in Ω^2_c that meet orbits in strata that have codimension $2n$; in the case of one-parameter families of forms, one should look further into strata of codimension as high as $2n + 1$.

The only strata that are identifiable at the 0-jet level in Ω^2 are determined just by the rank: the 2-forms of *rank* $\leq 2k$ constitute a set of codimension $(n-k)(2n-2k+1)$: thus in a 4-dimensional M, for instance one may ignore closed forms ω that vanish completely at any point, since they have codimension 6.

Given a 2-form ω that has everywhere rank either $2n$ (in an open set) or $(2n-2)$ (in a set of co-dimension 1) the singular set of the set where ω has rank $2n-2$ has codimension $2n+1$ in the full sheaf and will be ignored here. Hence we shall consider only forms that have rank $2n$ or $2n-2$, and the latter value only on a smooth hypersurface. Introduce locally a non-vanishing, smooth $2n$-form σ and define a corresponding, smooth, real valued function $z = z_\sigma$ defined by $\omega^{[n]} = z\sigma$. The singular set of ω is determined by the hypersurface defined by $z = 0$, in which case $dz \neq 0$.

For each point in the singular set M, we have the 2-dimensional vector subspace N of $T(M^{2n})$ consisting of the vectors L such that $L\lfloor\omega$ (the inner product) vanishes. The intersection of N with the tangent space of the singular set M_1 determines a stratification of the latter, in the following sense. Generically in an open, dense subset of M, the plane $N \subset TM^{2n}$ is transversal to the hyperplane $TM_1 \subset TM^{(2n)}$ (i.e. have a line in common and $N \subset TM_1$ on a closed subset.

More precisely, N is locally generated by two vector fields denoted by X, Y, defined as follows: choose two functions u, v such that $du \wedge dv \wedge \omega^{[n-1]} = \sigma$ and set

$$\begin{array}{lll} X & \text{uniquely defined by} & X\lfloor\sigma = du \wedge \omega^{[n-1]} \\ Y & \text{uniquely defined by} & Y\lfloor\sigma = dv \wedge \omega^{[n-1]} \\ Z & \text{uniquely defined by} & Z\lfloor\sigma = dz \wedge \omega^{[n-1]} \end{array}$$

Then $X\lfloor\omega = zdu, Y\lfloor(\omega) = zdv, Z\lfloor\omega = zdz$ and $Z = Y(z)X - X(z)Y + z[X,Y]$. This shows that Z lies in $N \cap T(M_1)$ (and hence generates it) whenever $z = 0$ (i.e. everywhere in M_1) and N is transversal to TM_1, while $Z = 0$ whenever $z = 0$ and $N \subset T(M_1)$: in this case X, Y and $[X, Y]$ are linearly independent and the next stratum M_2 is defined by the equations $z = X(z) = Y(z) = 0$; its smooth part is therefore the $(2n-3)$-dimensional submanifold of M, defined by the additional conditions

$$dz \wedge dX(z) \wedge dY(z) \wedge \omega^{[n-2]} \neq 0$$

Additional strata are defined recursively, represented in their regular parts by submanifolds $M_3, M_4, \ldots$, where each succesive M_j is defined by a non-transversal intersection of N with a previously determined $M_{j'}$ but here we deal only with the decomposition of M_2.

The previous construction of X, Y and Z around M_2 result in the identity $[X,Y](z) = 0$. The second derivatives of z along the plane N therefore have an invariant meaning as the "Hessian form" of z along N, denoted by $H_{M_2}[z] = \{(\zeta,\eta) \to X(X(z))\zeta^2 + 2X(Y(z))\zeta\eta + Y(Y(z))\eta^2)$. One distinguishes points of M_2 in which this Hessian is positive definite, negative definite or non-degenerate indefinite: they are characterized by geometric properties of the vector field Z in a neighborhood of a point where $Z = 0$. A further sub-stratum is then defined by points where the Hessian form $H_{M_2}(z)$ is degenerate (of rank 1 or zero), which can not be discussed in the present report.

Gefördert durch DFG Deutsche Forschungsgemeinschaft

Each of the types of singularities above allows representation of ω in "standard model" coordinates, as follows.

1) For non-singular points, from DarbouX's theorem we have coordinates $p_1, p_2, \dots, p_n$; $q_1, q_2, \dots q_n$ such that

$$\omega = \sum_{j=1}^{n} dp_j \wedge dq_j$$

2) For points in $M_1 \backslash M_2$ (i.e. where $z = 0$, $dz \wedge \omega^{[n-1]} = Z\lfloor \sigma \neq 0)$ the standard coordinates are obtained by choosing z, u, v as above with $z = u$, so that

$$\omega = zdz \wedge dv + \sum_{j=2}^{n} dp_j \wedge dq_j$$

3) For points on the smooth part of M_2, where the Hessian form $H_{M_2}(z)$ is non-degenerate we replace p_1, p_2, q_1 and q_2 by four special coordinates u, v, w, z and write ω in the form

$$\omega = d\left((z + \frac{1}{2} f(u,v))(udv - vdu) + 2(z + f(u,v))dw \right) + \sum_{j=3}^{n} dp_j \wedge dq_j$$

where $f(u,v)$ is a "standard" quadratic form, either $\pm u^2 \pm v^2$, or $2uv$. Thus, for $n = 2$, the matrix of ω in terms of the prefered coordinates (u, v, w, z) is

$$\begin{pmatrix} 0 & z + f(u,v) & -\frac{\partial f}{\partial u} & \frac{v}{2} \\ -z - f(u,v) & 0 & -\frac{\partial f}{\partial v} & -\frac{u}{2} \\ \frac{\partial f}{\partial u} & \frac{\partial f}{\partial v} & 0 & 1 \\ -\frac{v}{2} & \frac{u}{2} & -1 & 0 \end{pmatrix}$$

H. GLUCK & Liu-Hua PAN

Knot theory and differential geometry

We introduce and develop a curvature-sensitive version of knot theory, dealing with the embedding and knotting in 3-space of simple closed curves with nowhere vanishing curvature, and of compact orientable surfaces with nonempty boundary and positive curvature.

Theorem 1. Any two smooth simple closed curves in 3-space, each having nowhere vanishing curvature, can be deformed into one another through a one-parameter family of such curves if and only if they have the same knot type and the same self-linking number.

This result holds as well for links in place of knots. It also follows from the "C^1-dense one-parametric h-principle" proved by Eliashberg and Gromov in 1971; see Gromov's book: "Partial Differential Relations".

It was previously known to Bill Pohl, though never published, according to his former students John Littele and James White.

The hypothesis of nowhere vanishing curvature is the standard one in the geometric theory of curves in 3-space, always achievable by slight perturbation, and enabling the construction along the curve of the moving Frenet frame, consisting of the tangent, principal normal and binormal vectors, the self-linking number of the curve is then defined to be its linking number with its own displacement along the principle normal.

Self-linking numbers were introduced in 1959 by Călugăreanu via an integral formula derived as a limiting case of Gauss' integral formula for the linking number of two space curves. They were studied extensively by Bill Pohl in 1968, and further by James White the next year.

Self-linking numbers in 3-space can be viewed as akin to winding numbers in the plan, and Theorem 1 regarded as a natural generalization of the Whitney-Graustein theorem to 3-space, in a knot-theoretic setting. A homotopy version of Theorem 1 for immersed closed curves with nowhere vanishing curvature, with a mod 2 invariant in place of the self-linking number, was proved in 1968 by Edgar Feldman.

Surfaces of positive curvature in 3-space are closely connected with self-linking of knots:

- If a smooth knot lies on a surface of positive curvature, then its own curvature never vanishes, and hence its self-linking number is defined.
- If two smooth knots on a surface of positive curvature are isotopic on that surface, then their self-linking numbers are equal.
- If a smooth knot is the boundary of a compact orientable surface of positive curvature, then its self-linking number is zero.
- If two smooth knots together bound a compact orientable surface of positive curvature, then their self-linking numbers are equal.

The next two ötheorems deal with the existence and classification of compact orientable surfaces in 3-space having nonempty boundary and positive curvature.

Theorem 2. In 3-space, any compact orientable surface with nonempty boundary can be deformed into one with positive curvature.

Theorem 3. In 3-space, any two compact orientable surfaces with nonempty boundary and positive curvature can be deformed into one another through surfaces of positive curvature if and only if they can be deformed into one another through arbitrary surfaces.

In 1990, S.-T. Yau asked which knots bound positive curvature surfaces? A necessary condition is that the knot have nowhere vanishing curvature and slef-linking number zero. But this is not enough.

Theorem 4. In 3-space, there exists simple closed curves with nowhere vanishing curvature and self-linking number zero, which do not bound any compact orientable surface of positive curvature.

What then is the appropriate version, in the presence of curvature, of Seifert's theorem that every knot in 3-space bounds a compact orientable surface? As a first step, we prove

Theorem 5. In 3-space, any simple closed curve with nowhere vanishing curvature and slef-linking number zero can be deformed through such curves until it bounds a compact orientable surface of positive curvature.

U. SEMMELMANN

Kählerian Killing spinors, contract structures and twistor spaces I (in addition to part II)

Theorem: Let (M^{2m}, g, ∂) be a compact Kähler-Einstein manifold of positive scalar curvature and a complex contact structure. Assume further that $m = 2n+1$ and n odd. Then M is a spin manifold and there are Kählerian Killing Spinors on M.

Using the results of S. Salamon on the twistor spaces of quaternionic Kähler manifolds one obtains manifolds satisfying the assumptions of the theorem. In combination with the theorem of part II we have the following equivalent conditions.

Theorem: Let (M^{2m}, g, ∂) be a compact Kähler-Einstein manifold with positive scalar curvature and $m = 2n + 1, n$ odd. The following conditions are equivalent

(i) M is spin and admits Kählerian Killing spinors.

(ii) M is a twistor space of a quaternionic Kähler manifold with positive scalar curvature.

(iii) M is a complex contact manifold.

In particular, this theorem describes complex contact manifolds with a Kähler-Einstein metric as twistor spaces. This equivalence, without the dimension restriction, was also obtained in a recent work of C. Le Brun.

M. HERZLICH

In 1963, R. Penrose introduced a new way of studying asymptotic behaviour of Riemannian (as well as pseudo Riemannian) non compact manifolds, called "conformal compactification". He noticed that, given a compact Riemannian smooth manifold, suitable rescalings of the metric by a conformal factor which vanishes on a given submanifold gave birth to "nice" non compact manifolds (w.r.t. the behaviour of the metric at infinity). In the context of asymptotically flat Riemannian manifolds, one possible compactification is by a point at infinity.

We address the question of finding some conditions that ensure that a given asymptotically flat manifold is compactifiable by a point (and this gives a smooth (at least C^2) compact Riemannian manifold). We show that this construction can be done if the Weyl and Cottan-York tensors of the asymptotically flat manifold decay faster than $1/r^4$ (resp.$1/r^5$). The proof involves conformal geometry and the use of weighted Hölder spaces analysis.

C. SEARLE (joint work with K. Grove)

Differential Topological Restrictions by Curvature and Symmetry

We consider manifolds of positive sectional curvature admitting a large effective and isometric group action. One way to measure the size of a G-action on M is via the dimension of its orbit space M/G, also called the cohomogeneity of the action. Motivated by the fact that (for nontrivial action) the dimension of M/G is constrained by the dimension of M^G, the fixed point set of G in M, we define the *fixed point cohomogeneity* of a G-manifold M as follows:

$$cohomfix(M,G) = dim(M/G) - dim(M^G) \geq 1$$

(where, by convention $cohomfix(M,G) = cohom(M,G) + 1$ when $M^G = \emptyset$). Then G-manifolds of minimal fixed point cohomogeneity 1 are either (a) homogeneous, or (b) G acts transitively on a normal sphere to some component of M^G. In this last case, we call (M,G) *fixed point homogeneous*.

As one of our main results we obtain a complete classification of fixed point homogeneous manifolds of positive sectional curvature. As a special case, we obtain:

Theorem A: A simply connected fixed point homogeneous manifold of positive sectional curvature is diffeomorphic to $S^n, \mathbb{C}P^m, \mathbb{H}P^k$ or $\mathbb{C}aP^2$.

Berichterstatter: Matthias Weber

Differentialgeometrie im Großen
Oberwolfach, 11.06.95–17.06.95

Alekseevsky, Dimitri V. (Bonn)	daleksee@mpim-bonn.mpg.de
Abresch, Uwe (Münster)	abresch@math.uni-muenster.de
Ballmann, Werner (Bonn)	ballmann@rhein.iam.uni-bonn.de
Bär, Christian (Freiburg)	baer@arcade.mathematik.uni-freiburg.de
Berger, Marcel (Bures-sur-Yvette)	berger@ihes.fr
Berndt, Jürgen (Köln)	berndt@mi.uni-koeln.de
Block, Jonathan (Philadelphia)	blockj@math.upenn.edu
Bourguignon, Jean-Pierre (Bures-sur-Yvette)	jpb@ihes.fr
Bunke, Ulrich (Berlin)	ubunke@mathematik.hu-berlin.de
Calabi, Eugenio (Philadelphia)	calabi@math.upenn.edu
Cao, Jianguo (Ithaca)	cao@math.cornell.edu
Colding, Tobias Holck (New York)	Colding@cims.nyu.edu
Cortés, Vicente (Bonn)	V.Cortes@uni-bonn.de or: vicente@rhein.iam.uni-bonn.d
Croke, Chris B. (Philadelphia)	ccroke@math.upenn.edu
Galicki, Krzysztof (Albuquerque)	galicki@math.unm.edu
Gauduchon, Paul (Palaiseau)	pg@orphee.polytechnique.fr
Gluck, Herman (Philadelphia)	gluck@math.upenn.edu
Gornet, Ruth (Lubbock)	gornet@math.ttu.edu
Gromoll, Detlef (Stony Brook)	detlef@math.sunysb.edu
Grove, Karsten (College Park)	kng@math.umd.edu
Hamenstädt, Ursula (Bonn)	ursula@rhein.iam.uni-bonn.de
Heber, Jens (Augsburg)	Heber@uni-augsburg.de
Heintze, Ernst (Augsburg)	Heintze@uni-augsburg.de
Herzlich, Marc (Palaiseau)	herzlich@orphee.polytechnique.fr
Hingston, Nancy (Princeton Junction)	Hingston@math.Trenton.edu
Hulin, Dominique (Palaiseau)	hulin@orphee.polytechnique.fr
Kapovich, Misha (Salt Lake City)	kapovich@math.utah.edu
Karcher, Hermann (Bonn)	unm416@ibm.rhrz.uni-bonn.de
Kleiner, Bruce (Philadelphia)	bkleiner@math.upenn.edu
Knieper, Gerhard (Augsburg)	g.knieper@uni-augsburg.de
Lang, Urs (Zürich)	lang@math.ethz.ch
Leeb, Bernhard (Bonn)	leeb@rhein.iam.uni-bonn.de
Liu, Xiabo (Philadelphia)	liu@math.upenn.edu
Lohkamp, Joachim (Bures-sur-Yvette)	Lohkamp@ihes.fr
Margerin, Christoph (Palaiseau)	margerin@orphee.polytechnique.fr
Meyer, Wolfgang T. (Münster)	meyerw@math.uni-muenster.de
Moroianu, Andrei (Palaiseau)	moroianu@clipper.ens.fr or: am@orphee.polytechnique.fr
Pedersen, Henrik (Odense)	henrik@imada.ou.dk
Petersen, Peter (Los Angeles)	Petersen@math.ucla.edu
Schlenker, Jean Marc (Palaiseau)	jms@orphee.polytechnique.fr
Schroeder, Viktor (Zürich)	vschroed@math.unizh.ch
Schüth, Dorothee (Hanover/Bonn)	d.schueth@ibm.rhrz.uni-bonn.de or: schueth@dartmouth.edu
Schwachhöfer, Lorenz (Bonn)	lorenz@mpim-bonn.mpg.de
Searle, Catherine (Mexico)	csearle@math.cinvestav.mx
Semmelmann, Uwe (Berlin)	uwe@rhein.iam.uni-bonn.de
Taimanov, Iskander A. (Novosibirsk)	taimanov@math.nsk.su
Thorbergsson, Gudlaugur (Köln)	gthorbergsson@mi.uni-koeln.de
Ville, Marina (Vandœvre les Nancy)	ville@orphee.polytechnique.fr
Weber, Matthias (Bonn)	weber@rhein.iam.uni-bonn.de
Wilhelm, Fred (Stony Brook)	wilhelm@math.sunysb.edu
Ziller, Wolfgang (Philadelphia)	wziller@math.upenn.edu

Tagungsteilnehmer

Prof.Dr. Uwe Abresch
Mathematisches Institut
Universität Münster
Einsteinstr. 62

48149 Münster

Dr. Jürgen Berndt
Mathematisches Institut
Universität zu Köln
Weyertal 86-90

50931 Köln

Prof.Dr. Dimitri V. Alekseevsky
Max-Planck-Institut für Mathematik
Gottfried-Claren Str. 26

53225 Bonn

Prof.Dr. Jonathan Block
Department of Mathematics
University of Pennsylvania
209 South 33rd Street

Philadelphia , PA 19104-6395
USA

Prof.Dr. Werner Ballmann
Mathematisches Institut
Universität Bonn
Beringstr. 6

53115 Bonn

Prof.Dr. Jean-Pierre Bourguignon
Institut des Hautes Etudes
Scientifiques
Le Bois Marie
35, route de Chartres

F-91440 Bures-sur-Yvette

Prof.Dr. Christian Bär
Mathematisches Institut
Universität Freiburg
Hebelstr. 29

79104 Freiburg

Prof.Dr. Ulrich Bunke
Mathematisches Institut
Universität Göttingen
Bunsenstr. 3-5

37073 Göttingen

Prof.Dr. Marcel Berger
Institut des Hautes Etudes
Scientifiques
Le Bois Marie
35, route de Chartres

F-91440 Bures-sur-Yvette

Prof.Dr. Eugenio Calabi
Department of Mathematics
University of Pennsylvania
209 South 33rd Street

Philadelphia , PA 19104-6395
USA

Prof.Dr. Jianguo Cao
Dept. of Mathematics
Cornell University
White Hall

Ithaca , NY 14853-7901
USA

Prof.Dr. Paul Gauduchon
Centre de Mathematiques
Ecole Polytechnique
Plateau de Palaiseau

F-91128 Palaiseau Cedex

Prof.Dr. Tobias Holck Colding
Courant Institute of
Mathematical Sciences
New York University
251, Mercer Street

New York NY 10012-1110
USA

Prof.Dr. Herman Gluck
Department of Mathematics
University of Pennsylvania
209 South 33rd Street

Philadelphia , PA 19104-6395
USA

Prof.Dr. Vicente Cortes
Mathematisches Institut
Universität Bonn
Beringstr. 6

53115 Bonn

Prof.Dr. Ruth Gornet
Dept. of Mathematics
Texas Tech. University
Box 41042

Lubbock , TX 79409
USA

Prof.Dr. Chris B. Croke
Department of Mathematics
University of Pennsylvania
209 South 33rd Street

Philadelphia , PA 19104-6395
USA

Prof.Dr. Detlef Gromoll
Department of Mathematics
State University of New York
at Stony Brook

Stony Brook , NY 11794-3651
USA

Prof.Dr. Krzysztof Galicki
Dept. of Mathematics and Statistics
University of New Mexico

Albuquerque , NM 87131
USA

Prof.Dr. Karsten Grove
Department of Mathematics
University of Maryland

College Park , MD 20742
USA

Prof.Dr. Ursula Hamenstädt
Mathematisches Institut
Universität Bonn
Beringstr. 1

53115 Bonn

Prof.Dr. Dominique Hulin
Centre de Mathematiques
Ecole Polytechnique
Plateau de Palaiseau

F-91128 Palaiseau Cedex

Dr. Jens Heber
Institut für Mathematik
Universität Augsburg
Universitätsstr. 8

86159 Augsburg

Prof.Dr. Misha Kapovich
Dept. of Mathematics
University of Utah

Salt Lake City , UT 84112
USA

Prof.Dr. Ernst Heintze
Institut für Mathematik
Universität Augsburg
Universitätsstr. 8

86159 Augsburg

Prof.Dr. Hermann Karcher
Mathematisches Institut
Universität Bonn
Beringstr. 4

53115 Bonn

Marc Herzlich
Centre de Mathematiques
Ecole Polytechnique
Plateau de Palaiseau

F-91128 Palaiseau Cedex

Prof.Dr. Bruce Kleiner
Department of Mathematics
University of Pennsylvania
209 South 33rd Street

Philadelphia , PA 19104-6395
USA

Prof.Dr. Nancy Hingston
Trenton State College
Dept. of Mathematics and Sciences
4 Jeffrey La.

Princeton Junction , NJ 08550-1680
USA

Dr. Gerhard Knieper
Institut für Mathematik
Universität Augsburg
Universitätsstr. 8

86159 Augsburg

Dr. Urs Lang
Forschungsinstitut für Mathematik
ETH-Zentrum

CH-8092 Zürich

Prof.Dr. Wolfgang T. Meyer
Mathematisches Institut
Universität Münster
Einsteinstr. 62

48149 Münster

Dr. Bernhard Leeb
Mathematisches Institut
Universität Bonn
Beringstr. 1

53115 Bonn

Andrei Moroianu
Centre de Mathematiques
Ecole Polytechnique
Plateau de Palaiseau

F-91128 Palaiseau Cedex

Prof.Dr. Xiaobo Liu
Department of Mathematics
University of Pennsylvania
209 South 33rd Street

Philadelphia , PA 19104-6395
USA

Prof.Dr. Henrik Pedersen
Institut for Matematik og Datalogi
Odense Universitet
Campusvej 55

DK-5230 Odense M

Jochen Lohkamp
Mathematik Department
ETH Zürich
ETH-Zentrum
Rämistr. 101

CH-8092 Zürich

Prof.Dr. Peter Petersen
Dept. of Mathematics
University of California
405 Hilgard Avenue

Los Angeles , CA 90024-1555
USA

Prof.Dr. Christoph Margerin
Centre de Mathematiques
Ecole Polytechnique
Plateau de Palaiseau

F-91128 Palaiseau Cedex

Prof.Dr. Jean Marc Schlenker
Centre de Mathematiques
Ecole Polytechnique
Plateau de Palaiseau

F-91128 Palaiseau Cedex

Prof.Dr. Viktor Schroeder
Mathematisches Institut
Universität Zürich
Winterthurerstr. 190

CH-8057 Zürich

Prof.Dr. Iskander A. Taimanov
Institute of Mathematics
Siberian Branch of the Academy of Sciences
Universitetskiy Prospect N4

630090 Novosibirsk
RUSSIA

Dorothee Schüth
Mathematisches Institut
Universität Bonn
Beringstr. 1

53115 Bonn

Prof.Dr. Gudlaugur Thorbergsson
Mathematisches Institut
Universität zu Köln
Weyertal 86-90

50931 Köln

Lorenz Schwachhöfer
Max-Planck-Institut für Mathematik
Gottfried-Claren-Str. 26

53225 Bonn

Prof.Dr. Marina Ville
Departement de Mathematiques
Universite de Nancy I
Boite Postale 239

F-54506 Vandoeuvre les Nancy Cedex

Prof.Dr. Catherine Searle
CINVESTAV-IPN
Dept. de Matematicas
Apartado Postal 14-740

Mexico ,CP 07 000,D.F.
MEXICO

Dr. Matthias Weber
Mathematisches Institut
Universität Bonn
Beringstr. 6

53115 Bonn

Uwe Semmelmann
Institut für Reine Mathematik
Humboldt-Universität Berlin
Ziegelstraße 13a

10117 Berlin

Prof.Dr. Fred Wilhelm
Department of Mathematics
State University of New York
at Stony Brook

Stony Brook , NY 11794-3651
USA

Prof.Dr. Wolfgang Ziller
Department of Mathematics
University of Pennsylvania
209 South 33rd Street

Philadelphia , PA 19104-6395
USA