MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Quadratische Formen

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This conference was the seventh one on quadratic forms in Oberwolfach. It was organized by M. Knebusch (Regensburg), A. Pfister (Mainz) and W. Scharlau (Münster). The lectures covered many topics from the theory of quadratic forms mainly over fields but over algebraic varieties and schemes, too.

Although many interesting new results were reported, perhaps some main emphasis was laid on the results on index reduction formulas (confer the talks of Merkurjev, Panin and Wadsworth) and on the isotropy of quadratic forms over function fields of a quadric, which was dealt in the lectures of Hoffmann, Kahn and Laghribi. This topic got new stimulation by a recent result of Izhboldin, showing the non-excellence of n-fold multiplicative forms for $n \geq 3$.

Vortragsauszüge

Jon Arason

The Witt Ring of an Elliptic Curve over a Local Field (joint with R. Elman and B. Jacob)

Let X be an elliptic curve over a field $K, \mathrm{char}(K) \neq 2$. In an earlier work of ours we constructed a natural epimorphism $\bigoplus_T W(K(T)) \to W(X)$, where T runs through the closed points of the kernel ${}_2X$ of multiplication by 2 on X. In fact, we showed that the natural morphism $\tilde{W}(X) \to W(X)$ is an epimorphism, where $\tilde{W}(X)$ is the Witt ring of spaces over X that become diagonalizable after an algebraic base field extension, modulo the hyperbolic ones. We then constructed a natural isomorphism $\tilde{W}(X) \cong \bigoplus_T W(K(T))$.

Let $\tilde{I}(X)$ be the kernel of $\tilde{W}(X) \to W(X)$. We also constructed a natural epimorphism $\bigoplus_P W(K(P)) \to \tilde{I}(X)$, where P runs through the closed points of X.

We consider the case that K is a local field.

Theorem: $I^3(X) \neq 0$ if and only if X has split multiplicative reduction. In this case $| I^3(X) | = 2$. $(I^3(X) := W(X) \cap I^3(K(X)).)$

Theorem: If X has three rational points of order 2 then $W(X)$ is diagonalizable. Furthermore, $\tilde{I}(X) \neq \tilde{I}_v(X)$ if and only if K is dyadic and X can be described by a Weierstraß equation $y^2 = x(x-1)(x-e)$ with $v(e-1) = 4v(2) - 1$. (v is the normalized valuation on K.)

Theorem: If X has no rational point of order 2 then $\tilde{I}(X) = \tilde{I}_v(X)$. Furthermore, $W(X)$ is diagonalizable if and only if K is not dyadic.

We also have a complete description when X has exactly one rational point of order 2, except for a few sporadic cases where the residue class field has 2 or 4 elements.

R. Baeza

Positive Definite Quadratic Forms over Number Fields (joint with M.I. Giasa)

$K/\mathbf{Q}$ totally real number field, $m = [K : \mathbf{Q}], d_K$ = discriminant, O_K = integers. A Humbert form of rank n is a m-tuple $S = (S_1, \ldots, S_m)$, where each S_i is $n \times n$ pos. def. symmetric real matrix. Let $P_{m,n} \subset \mathbf{R}^{\frac{1}{2}mn(n+1)}$ be the space of such forms. $GL(n, O_K)$ acts on $P_{m,n}$ by: $U \in GL(n, O_K), S \in P_{m,n}$ define $S[U] = (S_1[U^{(1)}], \ldots, S_m[U^{(m)}])$, where $U^{(1)}, \ldots, U^{(m)}$ are the conjugates of U under the embeddings $\sigma_1, \ldots, \sigma_m : K \to \mathbf{R}$ (here $A[B]$ means $B^t AB$ whenever it is defined). Then $\det S = \prod \det S_i$, and $m(S) = \min\{S[u] \mid 0 \neq u \in O_K^n\}$ are class-invariants of S (here $S[u] := \prod_{i=1}^{m} S_i[u^{(i)}]$ for $u \in O_K^n$. The number $\gamma_K(S) = m(S/d(S))^{1/n}$ is bounded for all S and it holds $\gamma_K(S) \leq 4^m w_n^{-\frac{2m}{n}} \mid d_K \mid$ (w_n = vol. of n-sphere). Define $\gamma_{K,n} = \sup \gamma_K(S)$ = Hermite Humbert constant of K, n. Then $\gamma_{K,n} \leq 4^m w_n^{-\frac{2m}{n}} \mid d_K \mid$ (for $m = 1$ we get Minkowski's bound). Let $M(S) = \{u \in O_K^n \mid S[u] = m(S)\}$ be the set of minimal vectors of S. We define a constant $M_{K,n}$ with the prospects: $M_{K,n} = 1 \iff$ each S has unimodular minimal vector. For $u \in M(S)$ set $N(u) = \mid N_{K/\mathbf{Q}}(\prod_{u_j \neq 0} u_j) \mid, N(S) := \inf_{u \in M(S)} N(u), N[S] := \inf_{T \simeq S} N(T)$. Then

<u>Prop.</u>: There is a constant $C = C(K, n)$ with $N[S] \leq C$ for all $S \in P_{m,n}$.
We define then: $M_{K,n} = \sup_S N[S]$. With this notations we have

<u>Theorem</u>: $\gamma_{K,n}^{n-2} \leq (M_{K,n})^{\frac{2(n-1)}{2}} \gamma_{K,n-1}^{n-1}$.

If $h(K) = 1$, then $M_{K,n} = 1$, and we get Mordell's theorem: $\gamma_{K,n}^{n-2} \leq \gamma_{K,n-1}^{n-1}$.

Tom Craven (University of Hawaii)

Witt groups of hermitian forms over Baer ordered *-fields

Let D be a skew field with involution * and assume the space of Baer orderings, Y_D, of D is nonempty. Define $WS(D, *)$ to be the subring of $C(Y_D, \mathbf{Z})$ [the ring of continuous functions from Y_D to the integers] generated by the image of the Witt group of anisotropic hermitian forms $W(D, *)$.

One can then see that the Baer orderings of $(D, *)$ are naturally bijective with the ring homomorphisms $WS(D, *) \to \mathbf{Z}$ and with the group homomorphisms $\tau : W(D, *) \to \mathbf{Z}$ such that $\sigma(< 1 >) = 1$ and $\sigma(< a >) \in \{\pm 1\}$ for any symmetric element $a \neq 0$.

We can also give a nice characterization of the kernels of the group homomorphisms.

When $[D : Z_D]$ is odd, we can reduce the computation of $WS(D,*)$ to the ring $WS(F)$, where F is the field of central symmetric elements.

Define $(D,*)$ to be pythagorean if for any symmetric r, any $d_i \in D$, there exists $d \in D$ such that $\sum d_i r d_i^* = drd^*$. Then a formally real $(D,*)$ is pythagorean iff $W(D,*) \to C(Y_D, \mathbb{Z})$ is H.

Jean-Louis Colliot-Thélène

Hasseprinzip und schwache Approximation für homogene Räume über reellen Funktionenkörpern

Sei $K = \mathbf{R}(Y)$ der Funktionenkörper einer glatten projektiven Kurve $Y/\mathbf{R}$. Für $P \in Y(\mathbf{R})$, sei K_P die Komplettierung von K im Punkt P. Sei G/K eine zusammenhängende lineare Gruppe über K, und sei X/K ein homogener Raum für G. Anhand von Ergebnissen von Witt (1934, 1937), J. Knight (1969), Thaing (1993) hatte ich Anfang 1995 folgende Vermutungen gemacht:

Vermutung 1. (starkes Hasseprinzip) Falls $X(K_P) \neq \emptyset$ für fast alle $P \in Y(\mathbf{R})$, dann ist $X(K) \neq \emptyset$.

Vermutung 2. (schwache Approximation) Falls $X(K) \neq \emptyset$, dann liegt $X(K)$ dicht im Produkt der $X(K_P), P \in Y(\mathbf{R})$ (Bewertungstopologie auf K_P).

Diese Vermutungen könnte ich für prinzipielle homogene Räume von Tai beweisen. Für beliebige $X = G$ könnte ich Vermutung 2 auch beweisen.
Vermutung 1 für prinzipiell homogene Räume könnte ich auf den Fall von einfach zusammenhängenden halbeinfachen Gruppen reduzieren, und in einigen Fällen beweisen. Inzwischen hat A. DUCROS die Vermutung für prinzipiell homogene Räume von klassischen Gruppen bewiesen, und C. SCHEIDERER hat dann eine einheitliche Methode entwickelt, und beide Vermutungen ganz allgemein bewiesen. Durch ein Gegenbeispiel wird gezeigt, daß die Vermutungen z.B. für Varietäten, die ein Büschel von homogenen Räumen besitzen, falsch sind.

Dorothea Diers, Münster

Counter-Examples to the Strong Hasse-Principle

The strong Hasse-Principle is an important local-global principle in the theory of quadratic forms. It holds over global fields of characteristic not 2 for quadratic forms of dimension ≥ 1 as well as over algebraic function fields of transcendence degree one over real closed and algebraically closed fields for quadratic forms of dimension ≥ 3. In other cases it probably does not hold, however not many counter-examples are known.

In my talk counter-examples to the strong Hasse-Principle over algebraic function fields F of transcendence degree one over discretly valued fields K of characteristic not 2 are constructed. Some counter-examples of Hsia and Johnson over rational function fields $K(X)$, K some quadratic number field, are given and generalized. By Elman and Prestel, there exist counter-examples of arbitrary dimension ≥ 4 over $K(X)$, if K is a global field with at least two orderings. If F is an arbitrary function field in one variable over some base field K then we can construct some counter-examples over F if their Hasse-numbers satisfy certain conditions.

If K is a finite extension of $\mathbb{Q}_p$ or $\mathbb{F}_p((t))$, $p > 2$ prime and $< 1, 1 >$ is anisotropic over K then the form

$$f := < -x, -x, 1, \pi, -\pi(x^2+1) >, \quad \text{where} \quad \pi \quad \text{is the prime element of} \quad K,$$

is a 5-dimensional counter-example to the strong Hasse Principle over $K(X)$.

Martin Epkenhans (Paderborn, Germany)

Trace forms of algebraic number fields and of Galois extensions

We consider the trace form of a finite field extension L/K which is given by $< L >$: $L \to K : x \to \text{trace}_{L/K} x^2$. If ψ is a quadratic form over the number field K, then there is a field extension L/K with $< L > \simeq \psi$ if and only if $\psi \simeq < 1 >, \psi \simeq < 2, 2D >, D \notin K^{*2}, \psi \simeq < 1, 2, D >, D \in K^*$, or the dimension of ψ is ≥ 4 and all signatures are non-negative. For $\dim \psi \geq 4$ we improve this result as follows: Let ψ be a quadratic form with non-negative signatures and set $\text{ram}(\psi) := \{p, p$ is a finite spot of K which ramifies in $K(\sqrt{\det_K \psi})/K$ or for which the local Hasse invariant satisfies $H_p\psi \neq (2, \det_K \psi)p\}$. Let J_u, J_s be finite sets of finite spots of K with $J_u \cap J_s = \emptyset$ and $(J_u \cup J_r) \cap \text{ram}(\psi) = \emptyset$, then there is a field extension L/K with $\psi \simeq < 2 >$ and all $p \in J_r$ ramify in L/K and all $p \in J_u$ are unramified in L/K.

In the second part of the talk we investigate trace forms of Galois extensions with cyclic Galois group over an arbitrary field of $\mathrm{char}(K) \neq 2$ with $\dim \psi = 2^l \geq 4$.

Detlev Hoffmann

Non-excellence of function fields of quadratic forms

Let F be a field with $\mathrm{char} F \neq 2$, and let K/F be a field extension. Let φ be an anisotropic form over F. Knebusch showed that if $F(\varphi)/F$ is excellent then φ is a Pfister neighbour. The converse is known to be true always if $\dim \varphi \leq 4$ ($\dim \varphi = 3, 4$ due to Arason). Recently, Izhboldin proved that if φ is an anisotropic Pfister neighbour, $\dim \varphi \geq 5$, then there exists a field extension E/F such that $E(\varphi)/E$ is not excellent. We conceptualize and generalize Izhboldin's examples and obtain an interesting class of anisotropic n-fold Pfister forms σ, τ, an anisotropic m-fold Pfister form π, such that σ and π are $(m-1)$-linked $(m < n)$ and such that $\varphi \simeq (\tau \perp -\pi)_{an}$ becomes isotropic over $F(\tau)$. We give necessary and sufficient conditions for $(\varphi_{F(\tau)})_{an}$ to be defined over F. These examples have further interesting consequences. If φ is a neighbour of γ then $F(\varphi)$ and $F(\gamma)$ are (place-) equivalent over F. One might conjecture that if φ and ψ are such that $F(\varphi)$ and $F(\psi)$ are equivalent, then there exists an anisotropic form γ such that φ and ψ are neighbours of γ. Using examples of the type described above, one obtains counter examples to this somewhat naive conjecture (cf. also Bruno Kahn's talk).

Bruno Kahn

Quadratic forms isotropic over the function field of a quadric: a survey

This "survey" talk also wanted to provoke some conceptualization of the classical problem: let q, q' be two anisotropic quadratic forms over a field F of characteristic $\neq 2$. When is q' isotropic over the function field of q?

Consider the Witt ring $W(F)$ simply as the set of classes of anisotropic quadratic forms. The relation

$$q \leq q' \quad \text{if} \quad q' \quad \text{becomes isotropic over the function field} \quad F(q) \quad \text{of} \quad q$$

defines a preorder on $W(F)$ (it is transitive). We denote by $\approx$ the associated equivalence relation

$$q \approx q' \quad \text{if} \quad q \leq q' \quad \text{and} \quad q' \leq q \,.$$

Understanding the relation $\leq$ in elementary terms means finding a complete set of conditions implying $\leq$, such that for $q, q' \in W(F), q \leq q'$ if and only if one can pass from q to q' by applying these conditions a finite number of times. In the talk, we gave some examples of elementary conditions, implying $\leq$ and $\approx$.

One can ask: do these conditions suffice to generate $\leq$ (resp. $\approx$)? The answer is no, thanks to constructions used by Izhboldin in his disproof of excellence of function fields of higher Pfister forms. However, the answer is yes for $q \leq q'$ with $\dim q' \leq 6$ (Leep, Hoffmann) and $\dim q' = 8, d_{\pm} q' = 1$ (Laghribi), except perhaps in the remaining open cases.

Nikita Karpenko (Universität Münster)

Cycles of codimension 4 on a quadric

Over a field of char $\neq 2$ take a non-degenerate quadratic form φ and consider the projective quadric X_φ defined by φ. One likes to compute for any p the p-th Chow group $CH^p(X_\varphi)$, i.e. the group of p-codimensional algebraic cycles on X_φ modulo rational equivalence. Take the embedding $X_\varphi \subset \mathbf{P}$ of X_φ into the projective space (as a hypersurface) and call the image of the pull-back

$$\begin{array}{ccc} CH^p(\mathbf{P}) & \longrightarrow & CH^p(X_\varphi) \\ \| & & \\ \mathbf{Z} & & \end{array}$$

the "elementary part" of $CH^p(X_\varphi)$.

Question: is it true that for a fixed p the group $CH^p(X_\varphi)$ is elementary (i.e. coincides with its elementary part) if $\dim \varphi$ is large enough?
The answer is known to be positive for $0 \leq p \leq 3$.
We are going to answer the question for $p = 4$:

Theorem: if $\dim \varphi > 24$ then the group $CH^4(X_\varphi)$ is elementary.

Remark: the "biggest" known example of φ with non-elementary $CH^4(X_\varphi)$ has dimension 16; so, there is a "gap" between dimension 16 and dimension 24.

Remark: if $\dim \varphi \neq 7, 8$ then the non- elementary part of $CH^4(X_\varphi)$ is finite.

M.-A. Knus (ETH Zürich)

Central simple algebras of degree 3 and their trace forms (joint with M. Rost, D. Haile and J.-P. Tignol)

Let B be a central simple algebra of degree 3 over a field K of characteristic $\neq 2,3$. The trace form $Q_\sigma : x \longmapsto Trd_B(x^2)$ is a nonsingular quadratic form with values in F, the fixed field of σ in K. We show:

(1) Q_σ is of the form $< 1,1,1 > \perp < 2 > \cdot << \alpha >> \cdot < -b, -c, bc >$ for some $b, c \in F^*$ and $K = F(\sqrt{\alpha})$.

Let $f_3(\sigma) = [\alpha] \cup [b] \cup [c] \in H^3(F, \mu_2)$ be the Arason invariant of the Pfister form $<< \alpha, b, c >>$. Then

(2) If σ, σ' are involutions of second kind on B, $\sigma \simeq \sigma' \iff Q\sigma \simeq Q\sigma' \iff << \alpha, b, c >> \simeq << \alpha, b', c' >> \iff f_3(\sigma) = f_3(\sigma')$.

Involution σ with $f_3(\sigma) = 0$ form a distinguished class. We prove that

(3) any c.s.a. of degree 3 over K, which admits an involution of second kind, admits a distinguished involution.

We sketch three proofs of (3), with Jordan algebra techniques, using a crossed-product construction and using cohomological techniques. Finally we give

(4) a complete set of cohomological invariants for c.s.a. of degree 3 admitting involutions of the second kind.

M. Krüskemper

Bihomogeneous Nullstellensatz for p-fields

Let p be a prime number and K a p-field, that is there exist only finite field extensions of p-power-degree. Then any system of forms $f_1, \ldots, f_r \in K[X_0, \ldots, X_r]$ of degrees $d_1, \ldots, d_r, (d_i, p) = 1$ for $i = 1, \ldots, r$ is isotropic (Pfister, Terjanian). An elementary proof was given by Fendrich. This result can be generalized for systems of bihomogeneous polynomials. As an application one can show that over K there exist only division algebras of p-power-degree (where non-associative division algebras are also considered). Modifying the arguments of Fendrich (and van der Waerden) it is possible to give an elementary proof of the bihomogeneous Nullstellensatz. The proof can be modified to obtain some elementary intersection theory. Furthermore, other results of Hopf (1940) can be generalized by applying the result above.

Ahmed Laghribi

The isotropy of quadratic forms of dimension 8 over the function field of a quadric

Let F be a field of characteristic not 2 and q be an anisotropic quadratic form in I^2F of dimension 8. Question: For which quadratic form ψ, q becomes isotropic over $F(\psi)$? If we take K be the function field of the Severi-Brauer-Variety of $\mathcal{C}(q)$, we can summarize the answer as follows:

Theorem: For a quadratic form ψ of dimension ≥ 5, we have:

1. If ind $\mathcal{C}(q) = 1$, then ($q_{F(\psi)}$ is isotropic) $\Longleftrightarrow$ ($a\psi < q, a \in F^*$).
2. If ind $\mathcal{C}(q) = 2$, then ($q_{F(\psi)}$ is isotropic) $\Longleftrightarrow$ - either ψ is a Pfister neighbour of a 3-fold Pfister form q' and q contains a Pfister neighbour of q' - or ψ is not and $a\psi < q, a \in F^*$).
3. If ind $\mathcal{C}(q) = 4$, then ($q_{F(\psi)}$ is isotropic) $\Longleftrightarrow$ - either ψ is a Pfister neighbour of a 3-fold Pfister form q' and q contains a Pfister neighbour of q' - or ψ is not and there exists ψ' such that $\psi' > \psi, \dim\psi' = 8, d_\pm\psi' = 1$ and $q \perp a\psi' \in I^4F$ for some $a \in F^*$).
4. If ind $\mathcal{C}(q) = 8$, then ψ is not a Pfister neighbour and:
   - i) if $q_K \not\sim 0$: ($q_{F(\psi)}$ isotropic) $\Longleftrightarrow$ there exists ψ' such that $\dim\psi' = 8, d_\pm\psi' = 1, \psi' > \psi$ and $q \perp a\psi' \in I^4F$ for some $a \in F^*$).
   - ii) if $q_K \sim 0$:
     - a) if ψ is an Albert form, then: ($q_{F(\psi)}$ isotropic) $\Longleftrightarrow$ ind $(\mathcal{C}(q) \otimes_F \mathcal{C}(\psi)) = 2$)
     - b) if not, then ($q_{F(\psi)}$ is isotropic) $\Longleftrightarrow$ there exists ψ' such that $\psi' > \psi, \dim\psi' = 8, d_\pm\psi' = 1$ and ind $(\mathcal{C}(q) \otimes_F \mathcal{C}(\psi')) = 1$.).

David Lewis

Scaled trace forms of central simple algebras

Let A be a central simple algebra of degree n over a field F of characteristic $\neq 2$.
Let $\mathrm{tr} : A \longrightarrow F$ be the reduced trace map. Let $z \in A, z \neq 0$.
Define the **scaled trace form** $q_z : A \longrightarrow F$ by $q_z(x) = \mathrm{tr}(zx^2)$.
Necessary and sufficient conditions on z are given for q_z to be a non-singular quadratic form.
A diagonalization is given for q_z when

(i) A is a quaternion algebra

(ii) $A = M_n F$ and z is a triangular matrix.

It is shown in general that if q_z is non-singular then its determinant is $(-1)^{\frac{n(n-1)}{2}} nr(z)(\mathrm{mod} F^2)$ where $nr : A \longrightarrow F$ is the reduced norm map. Some formulae for the signature of q_z can be obtained for F formally real.
Finally scaled trace forms are related to transfer theory of quadratic forms and some results about Witt kernels and Witt images are obtained.

Murray Marshall (University of Sask., Canada)

Axioms for abstract real spectra

Let T be a proper preordering in ring A and denote by X_T the set of all orderings P of A with $P \geq T$. For $a \in A$ define $\bar{a}_T : X_T \longrightarrow \{-1, 0, 1\}$ by $\bar{a}_T(P) = 1$ if $a \in P \setminus -P$, 0 if $a \in P \cap -P$, -1 if $a \in -P \setminus P$, and let $G_T := \{\bar{a}_T \mid a \in A\}$. The structure of the pair (X_T, G_T) can be axiomatized, generalizing the axioms for a space of orderings in the field case. The resulting objects (X, G) are called abstract real spectra. An axiomatization will appear in "Constructible sets in real geometry" by Andradas, Bröcker, and Ruiz. Here we indicate two other axiomatizations equivalent to this one. Various local-global principles are known in this abstract setting relating (X, G) with the residue spaces $(X_\mathfrak{p}, G_\mathfrak{p})$, $\mathfrak{p}$ a prime in G, and these residue spaces are spaces of orderings. In particular, results on minimal generation of constructible sets carry over. Finite abstract real spectra (more generally, real spectra of finite chain length) are classified. (X_T, G_T) has a P- structure respecting specialization coming from the real places. Any finite (X, G) having such a P-structure can be realized as (X_T, G_T). There are two sets of examples known of abstract real spectra which cannot be realized as (X_T, G_T).

Alexander Merkurjev (St. Petersburg University)

Maximal indexes of Tits algebras

Let G be an adjoint semisimple group defined over an arbitrary field F, $\varphi : \tilde{G} \longrightarrow G$ be the universal covering, $Z = \ker\varphi =$ center of $\tilde{G}, C = \hom(Z_{sep}, \mathbb{G}_m)$ be the character group of the center. Tits has constructed a homomorphism

$$\beta : C(F) \longrightarrow BrF, \chi \longmapsto [A_{X,G}]$$

and algebras $A_{\chi,G}$ are known as Tits algebras. It follows from the description of β that ind$A_{\chi,G}$ divides the number $n_\chi = gcd(\dim V)$ where V runs over all representations ρ of corresponding inner quasi-split form $\tilde{G}^q$ such that the restriction of ρ on Z^q coincides with χ.

The goal of the talk is to present the notion of a "generic" algebraic group of a given inner type and prove that for any inner type of adjoint groups over an arbitrary field F the generic group G defined over the function field L of corresponding classifying variety satisfies the following property: for any $\chi \in C(L)$ the index of the Tits algebra $A_{\chi,G}$ coincides with n_χ.

I. Panin (St. Petersburg)

K-theory of twisted flags and a general index reduction formula

F is a field, D is a CSA/F.
L/F is a field extension, $\quad \text{ind}_L(D \otimes_F L) =?$.

We give an answer for the case, when L is the function field of a G-homogeneous projective variety X for a simply-connected s.-s. algebraic group G.

Tits constructed certain algebras associated with such a group G. This algebras are parametrized by the character group Ch of the center Z of G. So every element $\chi \in Ch$ determines $CSA \quad A_{\chi,G}$, which is unique up to the Brauer equivalence.

Let $R_{\bar{F}}(G)$ be the representation ring of the group G over the algebraic closure $\bar{F}$ of F. Let $R_{\bar{F}}^{\chi}(G)$ be the direct summand of $R_{\bar{F}}(G)$ respecting to the representations V s.t. the center Z acts on V by the character χ.

Let $x \in X(\bar{F})$ be an $\bar{F}$-point on a G-homogeneous projective variety X. Let $P = \mathrm{Stab}_{G_{\bar{F}}}(x)$. Then set

$$n_{X,P} = g.c.d.\{\dim_{\bar{F}} W \mid W \in \mathrm{Rep}^X_{\bar{F}}(P)\}$$

Theorem 1: $\mathrm{ind}(D_{F(x)}) = g.c.d._X(n_{X,P} \cdot \mathrm{ind}(D \otimes_F A_{X,G}))$.

This theorem is based on

Theorem 2: $K_0^G(X) \longrightarrow K_0(X)$ is surjective,
$K_0^G(X, D) \longrightarrow K_0(X, D)$ is surjective.

Merkurjev, Panin and Wadsworth deduced from 2 explicite and "best possible" index reduction formulas for all G-homogeneous projective varieties.

R. Parimala

Hasse Principle for the classical croups over fields of virtual cohomological dimension two (joint with E. Bayer)

Let k be a field of characteristic not 2. Let G be a semi-simple simply connected linear algebraic group defined over k. Let k_s be a separable closure of k and set $\Gamma_k = \mathrm{Gal}(k_s/k), H^1(k, G) = H^1(\Gamma_k, G(k_s))$.
If k is a number field, the Hasse principle holds for k; i.e. the natural map

$$H^1(k, G) \longrightarrow \prod_{v \quad real} H^2(k_v, G)$$

is injective (Kneser - Harder - Chernousov).
If k has no real places, Hasse principle implies that $H^1(k, G) = 0$. More generally, Serre conjectured in 1962 that for any perfect field $k, cdk \leq 2, H^1(k, G) = 0$. We recently proved this conjecture for classical groups.

We say that a field k has virtual cohomological dimension $\leq n$ if $cdk(\sqrt{-1}) \leq n$. In the talk, we present the following

Theorem: Let k be a perfect field of virtual cohomological dimension ≤ 2. Let G be a semi-simple simply connected group of classical type defined over k or of type G_2 or F_4. Then the natural map

$$H^1(k, G) \longrightarrow \prod_v H^1(k_v, G)$$

is injective, as v runs over the real orderings of k.

Colliot-Thélène conjectured that this holds for all semi-simple simply connected groups.

A. Prestel (Konstanz)

On Solèr's characterization of Hilbert spaces

We gave a sketch of the proof of the following characterization of Hilbert spaces obtained by M.P. Solèr (see: Communications in Algebra 23 (1995), 219 - 243 or manuscripta math. 86 (1995), 225 - 238):

Let K be a division ring, $^* : K \longrightarrow K$ an involution, E an infinite dimensional K-vector space and $<,>: E \times E \longrightarrow K$ a hermitian form on E. Then $(E, < \;\; >)$ is a Hilbert space with $K = \mathbf{R}, \mathbf{C}$ or $\mathbf{H}$ and the obvious involution on K if and only if axioms (1) and (2) hold:

(1): if a subspace U of E satisfies $U = (U^\perp)^\perp$ then $E = U \oplus U^\perp$,

(2): there exists a sequence $(l_n)_{n \in \mathbf{N}}$ in E such that $l_n \perp l_m$ for $n \neq m$, and $< l_n, l_n >= 1$ for all $n \in \mathbf{N}$.

(here $x \perp y$ is clearly defined by $< x, y >= 0$).

We also mentioned some applications of this characterization in infinite dimensional projective geometry, orthomodular lattices and Quantum Mechanics (see: S. Holland, Bull AMS 32 (1995), 205 - 234).

S. Pumplün

Composition algebras over rings of genus zero

In contrast to composition algebras over fields little is known about composition algebras and their norm forms over arbitrary (commutative) rings. The only promising line of attack here seems to be considering special classes of rings. In particular one can look at composition algebras over a ring R where Spec$R$ is an open dense subscheme of some curve X of genus zero over k (k a field). With the help of results from algebraic geometry, the theory of composition algebras over locally ringed spaces by Petersson, the theorem of Hurwitz by van Geel as well as with some elementary lattice theory the structure of these algebras is characterized. They are classified for certain base fields k, when $\mathrm{Spec} R = X - \{P_0, \ldots, P_n\}$, $\sum_{i=0}^{n} \deg P_i = 2$.

Carl Riehm

Absolutely irreducible orthogonal representation of finite groups

K is a field of char 0, $b : V \times V \longrightarrow K$ a non-degenerate symmetric bilinear form, and $\varphi : G \longrightarrow O(b)$ an orthogonal representation of the finite group G.

It was shown that if φ is absolutely irreducible as a linear representation, then the orthogonal equivalence classes of orthogonal representations which are linearly equivalent to φ, are in bijective correspondence with K^*/K^{*^2}, while those whose bilinear form is equivalent to be correspond to the subset $M(b)/K^{*^2}$ where $M(b)$ is the group of multipliers $\{\alpha \in K^* : \alpha b \sim b\}$.

It follows at once that if the degree of φ is odd, orthogonal equivalence is the same as linear equivalence (assuming absolute irreducibility).

Assume the degree of φ is even. Then it was shown that in many cases, the Clifford invariant of A. Fröhlich distinguishes the orthogonal representations linearly equivalent to φ; finally these results were applied to the case $G = S_n$.

Markus Rost

On THE H^3-invariant for simply connected groups

Let G be a simply connected algebraic group over some field k. Then there is an invariant

$$\Theta : H^1(k, G) \longrightarrow H^3(k, \mathbb{Q}/\mathbb{Z}(2)')$$

with the following property: For $x \in H^1(k, G)$ let P_x/k be the corresponding G-torseur. Then the kernel of the restriction map

$$H^3(k, \mathbb{Q}/\mathbb{Z}(2)') \longrightarrow H^3(k(P_x); \mathbb{Q}/\mathbb{Z}(2)')$$

is generated by Θ (in case when the Lie algebra of G is simple). There are various definitions of Θ due to J.-P. Serre and the lecturer (see also Serre's Bourbaki-talk in March '94). In the lecture we discussed the possibility of defining Θ using motific cohomology. This way gives an immediate interpretation of Θ as a torsion-variant over fields of the standard $H^4(\_,\mathbb{Z})$-characteristic class, wellknown e.g. in topology.

C. Scheiderer

Sums of squares and étale cohomology

Let A be a ring with $1/2 \in A$, let $H^*(A) = \oplus_{i=0}^{\infty} H^i_{ét}(A, \mathbb{Z}/2)$. For $n \in A^*$ let (n) be the image of n under the boundary $A^* \longrightarrow H^1(A)$ coming from $1 \to \mu_2 \to \mathbb{G}_m \xrightarrow{2} \mathbb{G}_m \to 1$. Let α be an element in the symbolic part of $H^*(A)$ (which is the subring generated by the $(n), n \in A^*$). It is known that $\alpha \cup (-1)^N = 0$ for $N \gg 0$ iff $\alpha_K = 0$ in $H^*(K)$ for every homomorphism $A \longrightarrow K$ into a real closed field K. We give some quantitative versions of this fact. For example, if $u \in A^*$ satisfies an equation $u\bar{m} = 1 + \bar{n}$ in A (where $\bar{m}$ stands for a sum of m squares) then $(u)^{m+n} = 0$. At least in the case $n = 0$, this bound is best possible in general. If the sums of squares satisfy suitable "transversality" conditions, the general bounds can be improved. All these upper bounds are obtained using the existence of (étale) Stiefel-Whitney classes of quadratic forms (over rings). We also have localization theorems like the following: If $n \in A^*$ satisfies $u = \bar{n} \bmod I, I \subset A$ being an ideal, then $(u)^n$ annihilates kernel and cokernel of the restriction maps $H^q_{ét}(A) \longrightarrow H^q_{ét}(U), q \geq 0, U :=$ complement of Spec(A/I) in Spec$A$.

Tara Smith (University of Cincinnati)

Subgroups of W-groups and Witt rings

Let F be a field of characteristic $\neq 2$, and let $F^{(3)}$ denote the composition over F of all quadratic, cyclic of order 4, and dihedral of order 8 extensions of F. Let $G_F = \text{Gal}(F^{(3)}/F)$, the so-called W-group of F. Then it is known that $WF \cong WK \Longrightarrow G_F \cong G_K$, and $G_F \cong G_K \Longrightarrow WF \cong WK$ under the additional assumption that if $< 1, 1 >_F$ is universal, then $s(F) = s(K)$. Thus one would expect that the subgroup structure of G_F would yield information about the quadratic structure of F.

A first question is which groups can be subgroups of G_F. It is not hard to show that the quaternion group cannot be. If one consider only "essential subgroups", i.e. H s.t. $\Phi(H) = H \cap \Phi(G_F)$ (Φ = frattini subgroup), then no group having $\mathbb{Z}/2\mathbb{Z}$

as a direct factor occurs as a subgroup. So far, all known essential subgroups can themselves be realized as W-groups. Observe that if K is the fixed field of H, then H is also a quotient of G_K. H is the W-group realization of the image of WF in WK. The question when H is itself a W-group is related to when the image of WF is again an abstract Witt ring in the sense of Marshall.

R. Sujatha, TIFR, Bombay

On the Hasse principle for Witt groups (joint with R. Parimala)

Let X/k be a smooth projective curve over a number field k of char $\neq 2$. Assume that $X(k) \neq \emptyset$. Let $W(k(X))$ denote the Witt group and $P(k)$ the set of places of k. For $v \in P(k)$, let k_v denote the completion of k at v. We prove that the kernel

$$K = \operatorname{Ker}(W(k(X))) \longrightarrow \prod_{v \in P(k)} W(k_v(X))$$

is isomorphic to ${}_2Ш(J(\bar{X}))$. Here ${}_2Ш(J(\bar{X}))$ is the 2-torsion subgroup of the Tate-Shafarevich group of the Jacobian $J(\bar{X}), \bar{X} = X \times_k k$. This group $({}_2Ш(J(\bar{X})))$ is finite.

As an application, we show that if E is an elliptic curve over a number field: elements in the kernel K are quaternion norm forms of the type $\ll x, a \gg$ or $\ll x - a\tau, \tau \gg$ where $a, \tau \in k^*$ and $a \mid D$, the discriminant of the curve.

Finally, we note that the hyothesis $X(k) \neq \emptyset$ is necessary by considering the example of an anisotropic conic over a number field.

Nguyen Quoc Thang

On certain corestriction maps

Let G be a linear algebraic group defined over a field k of char 0. If G is commutative one can define corestriction maps for all finite extensions k' of k.

Cor : $H^i(k', G) \longrightarrow H^i(k, G)$, where $H^i(k, G), \ldots$ stands for usual Galois cohomology groups of $G, (i \geq 0)$. However, if G is not commutative, perhaps there is no functorial definition of such corestriction maps for cohomology sets $H^i(k, G)(0 \leq i \leq 1)$.

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Here we consider the following weaker variant for the corestriction "situation": Assume we are given a map $\alpha_k : H^p(k, G) \longrightarrow H^q(k, T)$ which is functorial in k, where G is a non commutative k-group, T a commutative k-group. One has similar maps $\alpha_k : H^p(k', G) \longrightarrow H^q(k', T)$ for all finite extension k' of k, and also corestriction maps $H^q(k', T) \xrightarrow{Cor} H^q(k, T)$.

Question: When does $\mathrm{Cor}(Im(\alpha_{k'})) \subset Im\alpha_k$? If the question has an affirmative answer for all k', we say that the Corestriction Principle holds for the Im of map α_k. One can define similar notion for Kernel of a map $\beta_k : H^p(k, T) \longrightarrow H^q(k, G)$, which is functorial in k.

In this talk we show that over local or global fields, the Corestriction Principle holds for the image or kernel of maps α_k. Also a reduction theorem for the case of arbitrary fields of char 0 has been stated.

Jean-Pierre Tignol

The Witt kernel of a finite field extension

Let F be a field of characteristic $\neq 2$. For any monic separable polynomial $\pi \in F[t]$ of degree $2n$, define an F- algebra M_π as the quotient of the free algebra in $2n$ variables $f\{a_1, \ldots, a_n; b_1, \ldots, b_n\}$ by the relations which make $\pi = (t^n + a_1 t^{n-1} + \ldots + a_n)(t^n + b_1 t^{n-1} + \ldots + b_n)$. This algebra carries an involution σ_π such that $\sigma_\pi(a_i) = b_i$. Using a generalization of the Cassels-Pfister theorem for algebras with involution, we prove that an involution σ on a central simple algebra A over F becomes hyperbolic under scalar extension to $F[t]/(\pi)$ if and only if there is a homomorphism of F-algebras with involution $(M_\pi, \sigma_\pi) \longrightarrow (A, \sigma)$. This result is used to parametrize the 2-fold Pfister forms which become hyperbolic under a scalar extension of degree 4.

Adrian Wadsworth (University of California, San Diego)

Index Reduction Formulas and Discriminant Algebras

This is a report on joint work with A.S. Merkurjev and I.A. Panin. Let G be a semisimple connected linear algebraic group over a field F, and let X be a projective variety on which $G/Z(G)$ acts over F, with the action transitive over F_s. Let $C =$ character group of the center of $G \times_F F_0$, and K_G the smallest field extension of F so that the absolute Galois group of K_G acts trivially on the Dynkin diagram of G.

The general index reduction formula says that for any central simple algebra D over F,

$$\mathrm{ind}(D \otimes_F F(x)) = \gcd\Big(\gcd_{\psi \in C} \; \gcd_{E\, field, F_\psi \leq E \leq K_G} \; n_{\psi,E} \quad [E:F] \quad \mathrm{ind}(D \otimes_F A_G(\psi)_E)\Big).$$

The talk concerned determining the algebras $A_G(\psi)$ (the Tits algebras) and the integers $n_{\psi,E}$ appearing in this formula for specific groups G and varieties X. We have determined these for all the classical almost simple groups G and all their associated varieties X. This was illustrated with the following example: Let L/F be a Galois field extension of degree 2, let A be a central simple L-algebra of even degree $n = 2s$, and G an L/F involution of the second kind on A. Let

$$G = SU(A,\sigma) = \{a \in A^* \mid \sigma(a)a = 1 \quad \text{and } \mathrm{ind}(a) = 1\},$$

and

$$X = \{I \mid I \quad \text{is a right ideal of} A, \dim_L(I) = \frac{n^2}{2}, \quad \text{and} \quad \sigma(I)I = 0\}.$$

For this case, the index reduction formula becomes

$$\mathrm{ind}(D \otimes_F F(x)) = \gcd(\mathrm{ind}(D), \mathrm{ind}(D \otimes_F \mathcal{D}(A,\sigma)),$$
$$\gcd_{1 \leq j \leq n-1}\big(\tfrac{2\Delta}{\gcd(i,j)} \mathrm{ind}(D \otimes_F A^{v,j}))\big).$$

Here $\mathcal{D}(A,\sigma)$ is the "discriminant algebra of (A,σ)".

L. Walter

Reduced forms of higher level on a commutative ring

The reduced theory of quadratic forms over a formally real field K is the study of the reduced Witt ring $W(K)/\mathrm{Nil}(W(K))$ and its natural embedding as a subring of $C(X(K),\mathbb{Z})$, the ring of locally constant integer-valued functions on the space of orderings of K. One may identify $X(K)$ with the closed set of characters $\sigma : K \longrightarrow S^1$ of order 2 whose kernels are additively closed. The evaluation maps $\hat{a} : K^* \rightarrow \{\pm 1\}, a \in K^*$, defined by $\sigma \longmapsto \sigma(a)$, generates the reduced ring as a subring of $C(X(K),\mathbb{Z})$.

By replacing the condition "σ has order 2" with "σ has finite even order", one defines the notion of a signature of higher level of K. Reduced Witt rings of higher level are then defined to be a subring of $C(X,\mathbb{Z})$ generated by the evaluation maps

$\hat{a}, a \in K^*$, taking X to be the set of characters with additively closed of order $2s$, where s divides a given positive integer n. All of the notions, methods and results of the reduced theory of quadratic forms have been successfully extended to this higher level setting.

One can consider quadratic forms over semilocal rings. In this case there is a natural embedding of the reduced Witt ring in $C(X(A), \mathbb{Z})$ where $X(A)$ is the space of maximal orders of A. Recently, it has been shown that the reduced theory of quadratic forms can be extended to an arbitrary commutative ring by replacing the space $X(A)$ with the subspace $X_T(A)$ of maximal orders lying over a given preorder which is "well-behaved".

In this paper, we continue these two directions, developing a higher level theory for commutative rings, where maximal orders are replaced by the notion of a maximal signature of higher level.

Roger Ware

On the Quaternion Group as Galois Group

Let H_8 denote the quaternion group of order 8. A well-known theorem of Witt states that H_8 can be realized as a Galois group over a field $F(\mathrm{char} F \neq 2) \Longleftrightarrow$ the form $< 1, 1, 1 > \cong < u, v, uv >$ where u, v represent independent square classes over F. In this talk conditions are given for the non-realizability of H_8 in terms of the structure of the max. pro-2-Galois group $G_F(2) = \mathrm{Gal}(F_q/F)$, where F_q is the quad. closure of F.

Let $\mathcal{A}$ be the class of torsion free abelian pro-2-groups and let ℓ be the class of pro-2-groups generated by involutions. Given pro-2-groups G_1, G_2, let $G_1 *_2 G_2$ denote the free pro-2-product and if G acts on $A \in \mathcal{A}$, let $A \times G$ denote the semidirect product. Define a class J of pro-2-groups as follows: $J_1 = \{\mathbb{Z}/2\mathbb{Z}, \mathbb{Z}_2\}$ and having defined J_i, let

$$\begin{aligned} J_{i+1} = J_i \;&\cup\; \{G_1 *_2 G_2 \mid G_1, G_2 \in J_i, G_1 \quad \text{or} \quad G_2 \in \ell\} \\ &\cup\; \{A \times G \mid A \in \mathcal{A}, G \in J_i\}; \quad J = \textstyle\bigcup_{i=1}^{\infty} J_i. \end{aligned}$$

Theorem: Suppose F has at most a finite number of orderings. Then H_8 does not occur as a Galois group over $F \Longleftrightarrow G_F(2) \in J$.
Cor 1: F is non real and H_8 does not occur $\Longleftrightarrow$ either $G_F(2)$ is torsion free abelian or has generators $\{y_i, x\}_{i \in I}$ with relations $y_i y_j = y_j y_i$ and $x y_i x^{-1} = y_i^{5m}$ for all i, where $m = 2^n, n \geq 0$, is fixed. Here n is the largest integer s.t. F contains a primitive

$2^{n+2}th$ root of 1.
(Examples: $\mathbb{C}((t_1))\ldots((t_n)); \mathbb{Q}_p((t_1))\ldots((t_n)), p \equiv 1 \bmod 4$).

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