Tagungsbericht 25 / 1995

Bifurkation und Symmetrie

Die Tagung fand unter der Leitung von E. Allgower (Colorado State University, Ft. Collins), K. Böhmer (Philipps Universsät Marburg, und M. Golubitsky (University of Houston, Houston) statt. Ein detaillierter Bericht folgt ab der nächten Seite.

Berichterstatter: Martin Golubitsky

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Report: *Bifurcation and Symmetry*
June 25, 1995 — July 1, 1995

The conference took place under the direction of:

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The central points of interest were the interrelation of symmetry and bifurcation in dynamical systems and in applications. The main topics discussed and tools used in different combinations included: analysis (differential equations, group and representation theory), numerical (modern iteration and continuation methods, numerical linear algebra, K-L decompositions, computer algebra), and applications (convection, combustion, neural systems, fluid mechanics).

During the conference there were many private discussions of current work strongly stimulated by the lectures of most of the paticipants. These discussions were aided by the relaxed atmosphere of Oberwolfach and the conference schedule which left afternoons and evening free for discussion. In addition lectures were scheduled for twenty-five (25) minutes with an additional discussion period of ten (10) minutes after each lecture.

We include a copy of the program and a list of the abstracts. With the abstracts we have included affiliations and e-mail addresses of the participants. Finally, at the end is a list of particpants who did not speak.

Report submitted on June 30, 1995 by:

Eugene Allgower — Eugene L. Allgower

Klaus Böhmer — Klaus Böhmer

Martin Golubitsky — Martin Golubitsky

Conference Program

Monday, June 26

9:00-9:25	John Guckenheimer
9:35-10:00	Bernie Matkowsky
10:30-10:55	Fritz Busse
11:05-11:30	Ian Melbourne
11:40-12:05	Stephan van Gils
16:00-16:25	Willy Govaerts
16:35-17:00	Alois Steindl
17:10-17:35	Reiner Lauterbach

Tuesday, June 27

9:00-9:25	Bernold Fiedler
9:35-10:00	Tim Healey
10:30-10:55	Kurt Georg
11:05-11:30	Tudor Ratiu
11:40-12:05	Mike Field
16:00-16:25	Michael Dellnitz
16:35-17:00	Oliver Mihatsch
17:10-17:35	Martin Krupa

Wednesday, June 28

9:00-9:25	Edgar Knobloch
9:35-10:00	Zhen Mei

10:25-10:50 Jay Alexander

11:00-11:25 Ralph Sebastian

11:35-12:00 Mary Silber

Thursday, June 29

9:00-9:25 Pascal Chossat

9:35-10:00 Karin Gatermann

10:30-10:55 Dieter Armbruster

11:05-11:30 Pete Ashwin

11:40-12:05 John David Crawford

16:00-16:25 Alistair Spence

16:35-17:00 Gabriela Gomes

17:10-17:35 Jeroen Lamb

Friday, June 30

9:00-9:25 Gerhard Dangelmayr

9:35-10:00 Philip Aston

10:30-10:55 Floris Takens

11:05-11:30 Alain Bossavit

11:40-12:05 Vladimir Janovsky

13:30-13:55 Stanislaus Maier-Paape

14:05-14:30 André Vanderbauwhede

14:40-15:05 Marty Golubitsky

Jay Alexander
University of Maryland/NSF — jca@math.umd.edu
Riddling: a phenomenon in dynamical systems

Riddled and intermingled basins of attraction are a relatively recently noticed phenomenon in dynamical systems and differential equations. Although not necessary for its occurrence, riddling seems to occur most naturally in systems with some kind of (discrete) symmetry. A chaotic system exhibits sensitive dependence to initial conditions; however, usually the long-term average behavior is robust, except for initial conditions near basin boundaries. In a riddled system, the long-term average behavior is also infinitely sensitive to initial conditions. The name comes from the fact that the basin of an attractor is infinitely riddled with points which do not tend to the attractor. Riddled basins have no open sets–every point is a basin boundary. Intermingled basins are basins for different attractors which are dense in each other. Over the past several years, riddling has been observed in several scientific contexts: visually (originally), mathematically (rigorously), numerically (simulations), and experimentally (bench experiments).

Dieter Armbruster
Arizona State University — dieter@math.la.asu.edu
Dynamics of cellular flames

Video data from experiments on the dynamics of two dimensional flames are analyzed. The Karhunen Loeve analysis is used to identify the dominant spatial structures and their temporal evolution for several dynamical regimes of the flames. A data analysis procedure to extract and process the boundaries of flame cells is described. It is shown how certain spatial structures are associated with certain temporal events. The existence of small scale, high frequency, turbulent background motion in almost all regimes is revealed.

Joint work with Antonio Palacios and Eric Kostelich, ASU, and Emily Stone, Utah State University

Peter Ashwin
Institut Non-Lineare de Nice, Sophia-Antipolis
Symmetry of attractors from observations

I shall discuss the recent definition of symmetry detectives of Barany, Dellnitz, Golubitsky and Nicol. By introducing an observation space and then averaging within this I show how one can overcome problems of needing to integrate over high dimensional phase spaces. I shall also discuss the use of symmetry detectives on parametrised systems and an example application to a systen of four oscillators with full permutation symmetry. Joint with Matt Nicol and Joerg Tomes.

Philip J. Aston
University of Surrey
Symmetry breaking bifurcations of chaotic attractors

In arrays of coupled oscillators, ***synchronous chaos*** occurs in which all the oscillators behave identically but chaotically. When a parameter is varied, this fully symmetric state can lose its stability and we consider the type of behaviour which occurs after such a transition. This is done by classifying the Lyapunov exponents by the symmetry of the system. This classification leads to analytical results and also leads to methods for the efficient numerical computation of the dominant Lyapunov exponent associated with each symmetry type. We show how these dominant exponents determine the stability of invariant sets which lie in fixed point spaces and this leads to the idea of ***symmetry breaking bifurcations of chaotic attractors***. These results are illustrated for several systems of coupled oscillators. This is joint work with Michael Dellnitz.

Alain Bossavit
Electricité de France, Clamart — Alain.Bossavit@der.edf.fr
Homogenization and symmetry

When solving Maxwell equations in a periodic medium of small "granularity" (small spatial period), it is often useful to first look for some equivalent "homogenized" material. There are standard methods in this respect, but most fail to take into account the dependence of the result on frequency, which is mandatory in many cases. (In other words, homogenization should be performed in timespace, not only in space.) We show that, when properly done, homogenization explains the phenomenon of "chirality", by which some mixtures of ordinary materials become capable of rotating the polarization plane of waves. In a first part, the approach is heuristic, and leads to the definition of the so-called "cell-problem", whose solution yields the homogenized constitutuve laws. In a second part, it is made rigorous by appealing to the standard theory about the exploitation of symmetry in case of equivariance with respect to some group action. There are two group actions to consider in that particular instance, and the interplay between them helps clarify the nature of the cell-problem.

Fritz Busse
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Bifurcation sequences in fluid flow

Fluid systems with high degrees of symmetry in the external conditions often exhibit sequences of bifurcations, such that with increasing control parameter an evolution occurs in discrete steps from simple to complex forms of fluid flow. The Rayleigh-Bénard layer and the Taylor-Couette system are but the most famous of such systems. A general approach for the analysis of fluid systems that are homogeneous with respect to two spatial dimensions and in time is outlined and results from the case of Rayleigh-Bénard convection are reviewed[1,2,3]. Quarternary solutions such as oscillatory bimodal convection can be compared with experimental observations made with the shadowgraph method and good agreement can be found[3,4]. New results have been obtained recently for a horizontal convection layer of an electrically conducting fluid permeated by a vertical magnetic field[5]. Surprisingly, the first supercritical bifurcation from steady rolls to steady three-dimensional flows as well as the second one to oscillatory three-dimensional flows lead to a strong decrease of the convective heat transport. Another numerical analysis of convection in a layer heated from below and bounded from above by a thermally nearly insulating plate indicates a transition from rolls to a hexaroll pattern, which is unusual since only rolls or hexagonal cells are expected to be stable in a horizontally isotropic layer. There could well be a connection here with recent results for higher bifurcations of solutions on the hexagonal lattice presented by M. Silber at this Oberwolfach meeting in extension of earlier work by Buzano and Golubitsky[6].

1. Busse, F.H., Nonlinear properties of thermal convection, *Rep. Progr. Phys.* 41, 1929-1967, 1978
2. Clever, R.M., and Busse, F.H., Three-dimensional knot convection in a layer heated from below, *J. Fluid Mech.* 198, 345-363, 1989
3. Clever, R.M., and Busse, F.H., Steady and oscillatory bimodal convection, *J. Fluid Mech.* 271, 103-118, 1994
4. Busse, F.H., and Whitehead, J.A., Oscillatory and collective instabilities in large Prandtl number convection, *J. Fluid Mech.* 66, 67-79, 1974
5. Busse, F.H., and Clever, R.M., Three-dimensional convection in the presence of strong vertical magnetic fields, submitted to Eur. J. Mech B: Fluids, 1995
6. Buzano, E., and Golubitsky, M., Bifurcation on the hexagonal lattice and the planar Bénard problem, *Trans. Roy. Soc. London* A308, 617-677, 1983

Pascal Chossat
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Heteroclinic cycles in the spherical Bènard problem

The existence of large classes of structurally stable heteroclinic cycles in differential systems with O(3) symmetry has been proved when irreps of degrees l and l+1 interact, with l ¿ 0. This existence is subjected to a certain number of conditions, one of which being a quadratic degeneracy in the bifurcation equations. It is remarkable that, for most of these heteroclinic cycles, the conditions of existence are satisfied by the spherical Benard problem (onset of convection in a self-gravitating spherical shell). This fact is due to a single property of the model equations for this problem, namely the Navier-Stokes equations coupled with the heat equation in the Boussinesq approximation. When the gravitational field and temperature field in the absence of convection have identical dependance in the radial coordinate, the linear part of the equations is self-adjoint. In this case, we can prove that indeed the conditions for the existence of the heteroclinic cycles are fulfilled. If there is a small departure from self-adjointness (which then corresponds to "generic" gravity and temperature), these conditions persist. (This is joint work with F. Guyard.)

John David Crawford
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Bifurcation from a continuum: the effect of neutral modes

Amplitude equations, derived by expanding in the amplitudes of the unstable modes, are an important tool for analyzing the nonlinear behavior of a weak instability. If, in addition to the unstable mode, there are neutrally stable modes (the center subspace E^c is not empty) then these expansions may involve singular coefficients because nonlinear effects are very strong even in the regime of weak instability and small amplitudes. This feature is present in various settings: unstable electrostatic waves in collisionless plasma (Vlasov-Poisson equations), unstable modes in ideal shear flows (Euler equations), and mean field models of oscillator populations. In each of these cases the neutral modes correspond to a continuous spectrum and $\dim E^c = \infty$; however this behavior can also be studied in simpler settings where $\dim E^c < \infty$. Two examples are considered; in both cases the unstable mode corresponds to a complex conjugate pair of eigenvalues. In the first example, there is only a single neutral mode represented by a zero eigenvalues (the steady state - Hopf normal form) and the problem may be solved exactly in addition to using amplitudes equations. The scaling behavior of the exact solution can be recovered by analyzing the singularities in the coefficients of the amplitude equation. The Vlasov equation is the second example, and again the coefficient singularities can be absorbed by introducing an appropriately scaled amplitude variable. In the Vlasov case this rescaling leads to expansion coefficients that are surprisingly independent of the underlying equilbrium.

Gerhard Dangelmayr
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Parity breaking bifurcation in inhomogenous systems

Parity breaking instabilities of spatially periodic patterns are considered. In homogeneous systems such instabilities produce steadily drifting patterns. We investigate here the effects of small imperfections, that break the translation invariance but retain the reflection invariance, on such instabilities. These imperfections may, e.g., originate from spatial inhomogeneities. By means of invariant manifold calculations the imperfect bifurcation problem can be reduced to a perturbed normal form defined in a cylindrical phase space. Typically, a broken translation invariance leads to pattern pinnig. Analysis of the perturbed normal form shows that the transition from pinned patterns to drifting ones may be surprisingly complex. An example is described containing infinite cascades of heteroclinc bifurcations. The values of the bifurcation parameter at which these occur obey a somple scaling law. The predicted dynamics provide a qualitative understanding of recent experiments on binary fluid convection in an annulus. (Joint work with E. Knobloch and J. Hettel.)

Michael Dellnitz
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A subdivision algorithm for the computation of unstable manifolds and global attractors

Each invariant set of a given dynamical system is part of the global attractor. Therefore the global attractor contains all the potentially interesting dynamics, and, in particular, it contains every (global) unstable manifold. For this reason it is of interest to have an algorithm which allows to approximate the global attractor numerically. In this article we develop such an algorithm using a subdivision technique. We prove convergence of this method in a very general setting, and, moreover, we describe the qualitative convergence behavior in the presence of a hyperbolic structure. The algorithm can successfully be applied to dynamical systems of moderate dimension, and we illustrate this fact by several numerical examples.

Joint work with Andreas Hohmann, Konrad-Zuse-Zentrum für Informationstechnik Berlin

Bernold Fiedler
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Coalescence of reversible homoclinics

Consider a time reversible system with two degrees of freedom ($\mathbf{R}^4$), for example

$$\frac{d^2u}{dt^2} + g(u) = 0, \quad u \in \mathbf{R}^2.$$

Time reversal R is an involution on $\mathbf{R}^4$ fixing a two dimensional subspace Fix(R). reversible homoclinic orbits arise by nontrivial intersections $W^u \cap$ Fix(R), where W^u is the unstable manifold of a hyperbolic equilibrium $\mathcal{O} \in$ Fix(R). Considering one-parameter families, a quadratic tangency of W^u with Fix(R) can occur generically. Varying the parameter $\mu \in \mathbf{R}$ locally, we see two reversible homoclinics coalesce and disappear.

Even in the case where all eigenvalues at $\mathcal{O}$ are real, this bifurcation generates (cascades of) elliptic periodic orbits and subharmonic resonances.

This is joint work with Diman Tronev (Nizhny Novogorod).

Michael Field
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Heteroclinic cycles in symmetrically coupled oscillators: cycling chaos

Using phenomenological models based on the static equivariant bifurcation theory of systems with $\mathbf{Z}_2^N \times \mathbf{Z}_N$-symmetry, it was shown how to construct symmetrically coupled systems of identical oscillators with a heteroclinic cycle connecting N groups of p active (chaotic) oscillators, $1 \leq p \leq N-2 \geq 5$.

This phenomenon is robust with respect to breaking of symmetry if invariant subspaces are preserved. Finally, a brief discussion was given of how (stable) singular intersections of invariant manifolds in the equivariant theory can persist when symmetry is broken.

Some of the work in this talk was joint with Dellnitz, Golubitsky, Holman and Ma.

Karin Gatermann
Automatic classification of normal forms

We study equivarinat bifurcation problems depending on several parameters. Golubitsky, Stewart, Schaeffer have studied this problem class with singularity theory. Similiar phenomena are identified by contact equivalence and the codimension of the tangent space to the manifold of equivalent problems determines the number of unfolding parameters. We start our classification of symmetric bifurcation problems with a classification of possible tangent spaces using Gröbner bases. The weighted ordering is inherited by the degrees of fundamental invariants and equivariants. So we have two ansatzes: one for the tangetn space and one for the bifurcation problem itself. Both depend on unknowns which are determined by demanding that the tangent space of the Ansatz bifurcation problem is included in the first Ansatz. Examples for various finite groups illustrate the type of results which we derive. (Joint with Reiner Lauterbach.)

Kurt Georg
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A Numerical Linear Algebra Package for Exploiting Permutation Symmetries

A numerical linear algebra package is described for square matrices which are equivariant under a group Γ of permutations on the indices. One important aspect is that an index i is allowed to remain fixed under a subgroup $\Gamma_i \neq \{1\}$ for some i since this occurs quite naturally in most applications.

The main aim of the package is to provide a tool for analyzing matrices arising in discretizations of operator equations in a numerically efficient way by exploiting any underlying symmetry structure. Another possible application is given by a class of problems in statistics.

The implementation of the package is based on a (general) Fourier transform with respect to Γ on a block structured representation of the linear system, where the blocks are indexed by the elements of Γ. This leads to a well-known block diagonalization of the linear problem with respect to the irreducible representations which then can be analyzed more efficiently.

In the presence of fixed points, this approach adds a non-trivial kernel to the matrix which can generate difficulties for the analysis (e.g., when solving linear systems). This singularity can be removed by a regularization which is very simple to implement and does not alter the solution.

Stephan van Gils
University of Twente
Modulated waves in perturbations of the Korteweg-de Vries equation

We consider a two-mode approximation for a perturbed Korteweg-de Vries equation. The circle symmetry is divided out by working in the space of Hilbert invariants.

We prove the existence and uniqueness of a global branch of periodic solutions for the reduced equation. We also show that the period along this branch is uniformly bouded. These periodic solutions correspond to quasi-periodic solutions for the two-mode approximation.

This is joint work with E. Soewono, ITB, Bandung, Indonesia.

Martin Golubitsky
University of Houston — mg@uh.edu
Spirals in scalar reaction-diffusion equations

Spiral patterns have been observed experimentally, numerically and theoretically in a variety of systems. It is often believed that these spiral wave patterns can occur only in systems of reaction-diffusion equations. We show, both theoretically (using Hopf bifurcation techniques) and numerically (using direct simulation) that spiral wave patterns can appear in a single reaction-diffusion equation on a disk (in $u(x,t)$) if one assumes 'spiral' boundary conditions ($u_r = mu_\theta$). Spiral boundary conditions are motivated by assuming that a solution is infinitesimally an Archemedian spiral near the boundary. It follows from our bifurcation analysis that for our form of spirals there are no singularities in the spiral pattern (technically there is no spiral tip) and that at bifurcation there is a steep gradient between the 'red' and 'blue' arms of the spiral. This is joint work with Michael Dellnitz, Andreas Hohmann and Ian Stewart.

Gabriela Gomes
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Pulse Coupled Oscillators with Nearest Neighbour Coupling

Pulse coupling is common in biology: fireflies, crickets chirping, networks of neurons, pacemaker cells of the heart. Synchrony is the most familiar mode of organization for pulse coupled oscillators. Mirollo and Strogatz created a model of fireflies and under the assumption of all-to-all coupling proved that the oscillators will always become synchronized. Changing the model to keep only connections between nearest neighbours makes better physical sense but makes the mathematics much more complicated. As a result computer simulations were carried out to investigate the effect of nearest neighbour coupling.

The phase of each oscillator increases in time until a threshold is reached whereupon it "fires". When an oscillator fires the phase of its neighbours is increased by an amount ϵ. If this jump in phase brings the neighbour past threshold then two possible variations of the model are:

1. Competition: the neighbour is not allowed to fire and its phase is reset to zero;

2. Cooperation: the neighbour fires immediately. In this case a possibility exists of a chain reaction of additional firing which will continue until no other oscillators are brought to threshold.

With all-to-all coupling synchronization occurs for both competition and cooperation conventions. However with nearest neighbour coupling the computer simulations showed that synchronization only rarely occured when operating under the competition convention but always occured when operating with the cooperative convention.

W. Govaerts
University of Gent
Defining functions for multiple Hopf bifurcations

Let $A(u,\alpha) = F_u(u,\alpha)$ ($u \in R^n, \alpha \in R^k$) be a family of real n by n matrices arising as the Jacobian matrices of equilibrium solutions to the dynamical system $\dot{u} = F(u,\alpha)$.

An equilibrium point is called a Hopf point if A has a conjugate pair of pure imaginary eigenvalues $\pm i\omega$, $\omega > 0$. It is called a double Hopf point if there are two such pairs $\pm i\omega_1, \pm i\omega_2$ and a 1:1 resonant double Hopf point if in addition $\omega_1 = \omega_2$.

A method is described for the numerical detection, computation and continuation of Hopf, double Hopf and 1:1 resonant double Hopf points. A combination of matrix biproduct methods and bordered matrix methods leads to the definition of families of defining functions for these types of points.

We also study the stratified set of Hopf points near a 1:1 resonant double Hopf point in a generic three - parameter unfolding and we draw conclusions for the numerical computation of curves of Hopf points near a resonant point and of the curve of double Hopf points through that resonant point.

Example computations are done in a fairly realistic and complicated neural model problem with $n = 13$ and $k = 29$.

(joint work with J. Guckenheimer and A. Khibnik)

John Guckenheimer
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Bifurcation analysis of neural models

We are interested in fitting observed patterns of dynamical behavior in multiparameter models for complex dynamical phenomena. Our approach to this task is to compute the partitioning of the multidimensional parameter spaces into regions in which different types of dynamical behavior is present.

This lecture will discuss algorithmic approaches to this problem and their application to the study of the dynamics within the stomatogastric ganglion of crustaceans.

Tim Healey
Cornell University

No abstract given.

Vladimir Janovsky
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A note on the recursive projection method

The Recursive Projection Methods (RPM) aim to stabilize fixed point iterations via an adaptive dimensional reduction.

The method will be discussed in the context of pathfollowing steady states and periodic orbits of equivariant vector fields. Techniques for a numerical detection and aposteriori analysis of symmetry-breaking bifurcation will be presented.

For particular dissipative systems, the dimensional reduction could be made robust and RPM could be interpreted as the construction of an approximate inertial manifold.

Edgar Knobloch
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Dynamics of travelling waves in large aspect ratio containers

This seminar described recent work at Berkeley by the author and two students (P. Hirschberg and A. Landsberg), trying to understand (a) the relation between the theories describing the onset of oscillatory convection in unbounded and large aspect ratio containers, and (b) the origin of burst-like convection observed near onset in binary fluid convection. In the large-aspect ratio limit the dynamics is described by the Hopf bifurcation with brokn $D_4 \times S^1$ symmetry. The D_4 symmetry is generated by left/right reflections and the (approximate) interchange symmetry between odd/even modes present in that limit. As a result the dynamics differ from those expected by treating the system as a perturbation of an unbounded system which leads to a Hopf bifurcation with broken $O(2)$ symmetry. In particular the new approach can describe the burst-like convection. The difference between the two treatments is attributed to the presence in finite containers of large scale recirculation generated by the interaction of odd/even modes, which is absent in the formulation of the unbounded problem.

Martin Krupa
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Transverse bifurcations from robust homoclinic cycles

In the context of symmetric dynamical systems a heteroclinic cycle joining equilibria on the same group orbit is referred to as a homoclinic cycle. In symmetric dynamical systems homoclinic cycles can occur in a robust manner. In four dimensions there is a clasification of such cycles into three types distinguished by their symmetry properties. Robust heteroclinic cycles may be asymptotically stable. Their stability is determined by a coefficient involving the real parts of some eigenvalues and the requirement that other eigenvalues corresponding to directions transverse to the cycle have negative real partWe study transverse bifurcations occuring as a transverse eigenvalue passes through 0. We show the existence of a unique branch of periodic orbits for one type of cycles and the existence of a branch of homoclinic cycles for the two other types. We investigate the stabilities of solutions. Our analysis indicates the existence of a stable dynamo for rotating convection of an electrically conducting fluid.

This is joint work with P. Chossat, I. Melbourne and A. Scheel

Jeroen S.W. Lamb
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k-Symmetry in dynamical systems

In dynamical systems with discrete time it may happen that symmetry properties occur only on specific time scales: a map $f : R^n \mapsto R^n$ may possess less symmetry than its kth iterate f^k. If k is the smallest integer for which γ is a (reversing) symmetry of f^k then γ is called a (reversing) k-symmetry of f.

In this talk, two mechanisms are discussed by which k-symmetry is understood to arise in (resonantly) driven systems. Finally, with some examples of local bifurcations and periodic orbit structures it is illustrated that certain dynamical phenomena in k-symmetric systems differ from dynamical phenomena one finds in symmetric systems.

Reiner Lauterbach
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Bifurcation analysis using invariant theory

The dynamics near a steady state bifurcation with $O(3)$-symmetry for problems of low codimension turned out to be surprisingly simple. In this talk we look at a higher codimension case for the $\ell = 2$, i.e. the 5 dimensional irreducible representation of $O(3)$, which is the same as for the dihedral group D_3. The analysis is based on a reduction to the orbit space. For our problem a minimal set of homogeneous generators for the algebra of invariant functions is well known, however it is possible to do the analysis without using the explicit form of these functions. We study the case of a singularity of topological codimension 3, C^∞ codimension 5. In the universal unfolding we find a secondary branch of steady states, and a tertiary branch of periodic solutions. The branch of periodic solutions disappears through a bifurcation to a heteroclinic cycle involving two equilibria.

These results can be lifted back to the phase space. In our case this is much simpler than in the case of higher ℓ. The case presented here is considered to be a test case for the use of orbit space techniques in spherical problems. Further applications are under consideration. This is joint work with Jan Sanders.

Stanislaus Maier-Paape
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Heteroclinic cycles and forced symmetry-breaking

We consider solutions $u = u(t,x)$ of the semilinear parabolic equation

$$u_t = \Delta u + \lambda u + f(u), \quad x \in B_1 \subset \mathbf{R}^3 \tag{1}$$

on the unit ball B_1 with $O(3)$−equivariant boundary conditions ($\lambda \in \mathbf{R}$, $f : \mathbf{R} \to \mathbf{R}$). Assuming (1) has an axisymmetric equilibrium u_α, the group orbit of u_α gives a whole (invariant) manifold M of equilibria for (1). Under generic conditions we have that, after perturbing (1) by a (small) $L \subset O(3)$−equivariant perturbation, M persists as an invariant manifold $\widetilde{M}$ slightly changed. However, the flow on $\widetilde{M}$ is in general no longer trivial. Indeed, we find heteroclinic orbits on $\widetilde{M}$ and, in case $L = \mathbf{T}$ (the tetrahedral subgroup of $O(3)$), even heteroclinic cycles.

Bernard J. Matkowsky
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Bifurcation and symmetry in combustion

We present the results of both analytical and computational methods for a simplified model of combustion due to Zeldovich. The results exhibit a rich variety of patterns and dynamics. We describe solutions and their symmetries, including transitions from one solution type to another, which exhibits greater spatiotemporal complexity, as symmetries are broken.

Zhen Mei
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Mode interactions on a homotopy from Neumann to Dirichlet problems

Connecting the Neumann and Dirichlet boundary conditions with a homotopy, we examine behaviour of solutions of a nonlinear elliptic PDE with repect to the homotopy parameter and influence of boundary conditions on the bifurcation scenario. At a mode interaction point, we show that the D_4-mode is involved in all the bifurcating solution branches, in particular, steady state and Hopf secondary bifurcations occur. Path following of solution branches with respect to the homotopy parameter shows that primary bifurcation branch are connected to those Dirichlet problem continuously, while some secondary branches are absorbed by the D_4-branch and some of them go to the trivial solution curve.

Ian Melbourne
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Symmetric attractors for diffeomorphisms and flows

Let $\Gamma \subset \mathbf{O}(n)$ be a finite group acting on $\mathbf{R}^n$. In this work we describe the possible symmetry groups that can occur for attractors of smooth (invertible) Γ-equivariant dynamical systems. In case $\mathbf{R}^n$ contains no reflection planes and $n \geq 3$, our results imply there are no restrictions on symmmetry groups. In case $n \geq 4$ (diffeomorphisms) and $n \geq 5$ (flows), we show that we may construct attractors which are Axiom A.

(Joint work with M. Field and M. Nicol)

Oliver Mihatsch
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Recognition of critical states in chemical reaction systems using a normal form approach combined with neural nets

The behavior of a continuous chemical reactor changes qualitatively and quantitatively, if some system parameter drifts and passes a bifurcation point. Then, for instance, the temperature suddenly increases or starts oscillating, which may cause a loss of production or even reactor accidents.

A new hybrid approach for the on-line recognition of bifurcation points in chemical reaction systems is presented. It does not require any global mathematical model of the underlying process. The recognition algorithm bases on local models of process dynamics that are created and adapted to measurements by solving nonlinear programs. The local model consists of a composition of a *normal form*, and a *neural network*. The normal form describes

the local dynamics inside a center manifold using reduced coordinates. The Neural Net approximates the *a priori* unknown parameter representation of the center manifold that maps the reduced normal form coordinates onto the real world state variables.

The model is constructed in a way that system stability as well as the type of the bifurcation being about to happen can be easily read off on model parameters. The chemical reaction can therefore be appropriately manipulated in time before the reactor loses stability and gets uncontrollable.

The method that is described in the sequel is not restricted to chemical systems but may also be applied to other systems that exhibit bifurcations.

Tudor Ratiu
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Momentum map convexity and bifurcation theory

The convexity property of the image of the momentum map can serve as a guide to a possible classification of Hamiltonian symmetric bifurcations. It appears that this property together with the the theory of singular reduction may serve as the framework in the study of Hamiltonian dynamics in the presence of symmetry. This talk will present an overview of some of the key theorems in this area and will concentrate on the statement of a recent theorem obtained jointly with H. Falschka on the the convexity properties of the momentum map in the category of Poisson Lie groups. It turns out that this theroem is itself a corollary of an even more general convexity theorem closely related to bifurcation theory. will be emphasized.

Ralph Sebastian
Universität Marburg
Detection and computation of singular points using Krylov-methods

A standard technique to numerically treat singularities is the generalized Ljapunov-Schmidt-Reduction. We present a numerical realization that uses a modification of the original system rather than an extension, and show how the linear systems that arise in the procedure can be solved using Krylov-Methods. For the detection of singular points and the choice of the projections and parameterizations in the definition of the reduced equations we suggest the computation of a few interesting eigenvalues if these lie on the right-hand-side of the spectrum of the Jacobian. A new scheme by Sorensen which is based on Arnoldi's Method and can be interpreted as three specific versions of different methods in one algorithm provides an efficient way to compute these eigenvalues and corresponding eigenvectors.

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Some stability results for spatially doubly-periodic steady planforms

In a variety of PDE's, such as those describing hydrodynamic convection and reaction-diffusion systems, the spatially-uniform equilibrium state loses stability to steady disturbances of wavenumber $k_c \neq 0$ when an external control parameter exceeds a critical value. For systems that are equivariant with respect to the Euclidean group, this instability produces a plethora of spatially doubly-periodic steady states; each of which is periodic with respect to some square, hexagonal or rhombic lattice. These consist not only of simple rolls, squares, hexagons and rhombs that have a spatial period $\ell = 2\pi/k_c$, but also periodic planforms with $\ell > 2\pi/k_c$. For example, there is a countably-infinite family of square symmetric patterns of the form: $e^{ik(\alpha x+\beta y)} + e^{ik(-\beta x+\alpha y)} + e^{ik(\beta x+\alpha y)} + e^{ik(-\alpha x+\beta y)} + c.c. +$ harmonics, where $\alpha > \beta > 0$ are co-prime integers, and $k = k_c/\sqrt{\alpha^2+\beta^2}$ ($\lambda \equiv 2\pi/k$). We use methods of equivariant bifurcation theory to determine the possible relative stability between the simple and more complicated square states that are guaranteed to exist by the equivariant branching lemma. A similar analysis of hexagonal states is also performed. We find that most, but not all, of the stability properties are determined by a cubic truncation of the bifurcation problem. The dependence of the stability assignments on high-order resonant interaction terms is described, as is the possibility of secondary transitions between states. (This talk is based on joint work with B. Dionne and A. Skeldon.)

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Numerical calculation of Hopf bifurcations near a double singular point

The nonlinear equation $f(x, \lambda, \alpha) = 0, f : X \times \mathbf{R}^2 \to X$, where X is a Banach Space and f staisfies a Z_2-symmetry relation is considered. Interest centres on a certain type of double singular point, where the solution x is symmetric and f_x has a double zero eigenvalue, with one eigenvalue symmetric and one antisymmetric.

We show that under certain nondegeneracy conditions there exists a path of Hopf bifurcations passing through the double singular point, and for which x is not symmetric except at the double singular point. Also we show how to jump on to this branch using Keller's pseudo-arclength method.

This is joint work with Philip Aston (Surrey) and Wu Wei (Changchun).

Alois Steindl
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Heteroclinic cycles in the dynamics of a fluid conveying tube

We investigate the motions after loss of stability of a fluid conveying tube, which is supported by a rotational symmetric arrangement of elastic springs. For certain values of the stiffness of the support the loss of stability occurs by a Hopf/Hopf-mode interaction.

By projecting the flow of the quasilinear PDE onto the Center manifold and applying Normal Form theory we reduce the system to a complex 4-dimensional set of equivariant ordinary differential equations. It is shown that for a range of physical parameter values stable heteroclinic cycles connecting the primary branches of planar oscillations ("standing waves") exist.

Floris Takens,
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Time series analysis; estimaticn of dimension and order

We consider time series $\{x_i\}$, i denoting the discrete time.They may be obtained from a deternimistic dynamical system $\varphi : Y \to Y$ through a read-out function $y : Y \to \mathbf{R}$ — an orbit $i \to \varphi^i(p)$ giving rise to a time series $x_i = y(\varphi^i(p))$. They may however also be obtained from some stochastic model.

These time series are analysed in terms of reconstruction vectors $X_i^{(k)} = (x_{i-k+1}, \cdots, x_i) \in \mathbf{R}^k$, and the reconstruction measures μ_k on $\mathbf{R}^k$, defined by the density of the k-dimensionalreconstruction vectors. For time series defined by a dynamical system, the reconstruction measures μ_k are concentrated on low dimensional objects, diffeomorphic with the attractor to which the corresponding orbit is attracted. Even for time series which are not defined by a deterministic dynamical system these dimension can be defined in terms of estimates of the dimension of the reconstruction measures.

A related notion is the order of a time series. This is the minimal number of passed values, like $x_i, \cdots, x_{i-k+1}$ if the order is k, that contain all information of the past regarding the future $\{x_j\}_{j>i}$ assuming the reconstruction measures known. This notion or order, in the context of nonlinear time series, and its estimation in terms of nonlinear regression, was studied by H. Tong.

In the present lecture we discuss how this order can also be estimated in terms of the correlations integrals, in terms of which the dimensions are estimated.

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Hopf bifurcation at k-fold resonances in reversible systems

Consider a reversible system which has a symmetric equilibrium. The reversible version of the Liapunov Center Theorem states that if the linearization at the equilibrium has a pair of simple non-resonant purely imaginary eigenvalues, then there exists a smooth 2-dimensional invariant manifold passing through the equilibrium, tangent to the corresponding eigenspace, and filled with symmetric periodic orbits. We discuss what happens to these families of periodic orbits when under a change of parameters $k \geq 2$ of such pairs of eigenvalues collide and split off the imaginary axis. The tools used to obtain the bifurcation pictures are a general reduction result for periodic orbits at resonances in reversible systems, and a result on linear nilpotent normal forms. The bifurcation sets in parameter space are diffeomorphic to the well known cuspoids from singularity theory. For k=2 the bifurcation picture is similar to that at a Krein collision in Hamiltonian systems. More recent work shows that similar results hold at k-fold resonances in conservative systems, i.e. systems which have a first integral.

This is joint work with Jürgen Knobloch (Ilmenau, Germany).

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