

Mathematisches Forschungsinstitut Oberwolfach

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Funktionalanalysis und partielle Differentialgleichungen

1.1. - 6.1.1996

The conference was organized by Wolfgang Arendt (Ulm) and Jan Prüß (Halle). There were 44 participants from 14 countries. The main subjects of the meeting were the following:

- a) H^∞ -Calculi of Operators and Maximal Regularity
- b) Heat Kernel Estimates and Gaussian Bounds
- c) Integrodifferential Equations and Viscoelasticity
- d) Nonlinear Evolution Equations

To present the state of the art in each of these fields, four survey lectures were given, namely by A. McIntosh (a), E. B. Davies (b), M. Renardy (c), and H. Amann (d). 16 further plenary talks on very recent results showed the dynamic development in these subjects and the interesting interplay. The long breaks between the talks and ample free time were used for direct communication, research, and many interesting discussions. There were also a few spontaneously organized informal sessions on special subjects and problems.

Abstracts of Plenary Lectures

H. AMANN

Remarks on Evolution Equations and Fourier multipliers

In this survey we indicate the usefulness of the theory of Banach-space-valued distributions for the study of linear and nonlinear evolution equations. In particular we present a Fourier multiplier theorem for operator-valued symbols that applies to general Besov spaces of Banach-space-valued distributions without any restriction on the underlying Banach space. Applications to maximal regularity results are indicated.

C. J. K. BATTY

Local spectrum, asymptotic behavior and Tauberian theorems

Let $f : \mathbb{R}_+ \rightarrow X$ be bounded and uniformly continuous, and suppose that the Laplace transform of f has only countably many singular points in $i\mathbb{R}$, and for each singular point $i\eta$,

$$\lim_{t \rightarrow \infty} \frac{1}{t} \left\| \int_0^t e^{-i\eta u} f(s+u) du \right\| = 0,$$

uniformly for $s \geq 0$. It is shown that $\|f(t)\| \rightarrow 0$ as $t \rightarrow \infty$.

This result is derived from a local version of a theorem of Arendt and Batty, Lyubich and Vũ concerning stability of uniformly bounded C_0 -semigroups. In turn, the result can be applied to extend that local theorem to certain orbits of unbounded semigroups.

Related results have been obtained by Vũ, Ruess and de Laubenfels.

(Joint work with Jan van Neerven and Frank Rábiger.)

P. BENILAN

Singular limit of semigroups

We consider the abstract general problem:

X Banach space

$(A_m)_{m=1,2,\dots,\infty}$ m -accretive operators in X , $A_m \rightarrow A_\infty$ in the sense of graphs as $m \rightarrow \infty$

$(f_m)_{m=1,2,\dots}$ with $f_m \in \overline{D(A_m)}$ and $f_m \rightarrow f$ in X as $m \rightarrow \infty$

If $f \in \overline{D(A_\infty)}$, the Trotter-Kato-Brézis-Pazy theorem states that

$$e^{-tA_m} f_m \rightarrow e^{-tA_\infty} f \quad \text{in } X \text{ uniformly for } t \geq 0 \text{ bounded.}$$

Problem If $f \notin \overline{D(A_\infty)}$, what can be said about $\lim_{m \rightarrow \infty} e^{-tA_m} f_m$?

Remark In general there is no limit for $t > 0$ as shown by the example :

$$X = \mathbb{R}^2 = \mathbb{C}, \quad A_m = -mi, \quad D(A_\infty) = \{0\}, \quad A_\infty 0 = X.$$

In the linear case, one has convergence of the function $u_m(t) = e^{-tA_m} f_m$ to $\frac{dU_\infty}{dt}$ in $\mathcal{D}'(]0, \infty[; X)$ where U_∞ is the mild solution of $\frac{dU_\infty}{dt} + A_\infty U_\infty \ni f$ on $(0, \infty)$, $U_\infty(0) = 0$. In particular, the following are all equivalent:

- i) for some $\delta > 0$, $(u_m(t))$ converges in X for a.a. $t \in (0, \delta)$
- ii) for any $\delta > 0$, there exists $t \in (0, \delta)$ such that $\{u_m(t); m \in \mathbb{N}\}$ is relatively compact in X and $\bigcap_{m \in \mathbb{N}} \{u_k(t); k \geq m\} \subset \overline{D(A_\infty)}$
- iii) $U_\infty \in C^1(]0, \infty[; X)$ and $u_m \rightarrow U'_\infty$ in $C(]0, \infty[; X)$.

In the nonlinear case, no general result seems to be known. We consider the particular problem

$$(P_m) \quad \begin{cases} u_t = \Delta u^m + g(u) & \text{on } Q =]0, T[\times \Omega \\ u = 0 & \text{on } (0, T) \times \partial\Omega \\ u(0, \cdot) = f & \text{on } \Omega \end{cases}$$

where Ω is a bounded open set in \mathbb{R}^N , $f \in L^\infty(\Omega)$, $f \geq 0$, $g : \mathbb{R}^+ \rightarrow \mathbb{R}$ is continuous with $g(0) \geq 0$, $dg/dr \leq K$ with $K \in C(\mathbb{R}^+)$ and the solution of $M = g(M)$, $M(0) = \|f\|_\infty$ is defined on $[0, T)$. (P_m) has a unique weak solution, denoted by u_m . One has

Theorem (P. Benilan, N. Igbida) As $m \rightarrow \infty$, $u_m \rightarrow u_\infty$ in $C(]0, T[; L^1(\Omega))$ where u_∞ is

the unique solution of variational problem:

$$\begin{cases} u_\infty \in L_{loc}^\infty([0, T] \times \bar{\Omega}) \cap C([0, T]; L^1(\Omega)), & 0 \leq u_\infty \leq 1, \\ \text{there exists } w_\infty \in C([0, T]; H_0^1(\Omega)), & w_\infty \geq 0, \\ w_\infty(u_\infty - 1) = 0, & \frac{\partial u_\infty}{\partial t} = \Delta w_\infty + g(u_\infty) \text{ in } \mathcal{D}'(Q), \\ u_\infty(0, \cdot) = \underline{f} = f\chi_{\{w>0\}} + \chi_{\{w=0\}}, \end{cases}$$

where w is the unique solution of

$$\begin{cases} w \in H_0^1(\Omega), \Delta w \in L^\infty(\Omega), & 0 \leq f + \Delta w \leq 1, \\ w \geq 0, & w(f + \Delta w - 1) = 0. \end{cases}$$

This result is obtained by combining the result in the case $g \equiv 0$ (cf. P. Benilan, L. Boccardo, M. Herrero 1989) and an abstract perturbation result (cf. P. Benilan, N. Igbida 1996).

Corollary (P. Benilan, N. Igbida) If $g(1) \leq 0$, then $u_m \rightarrow \underline{v}$ in $C([0, T]; L^1(\Omega))$, where \underline{v} is the solution of the o.d.e. $\dot{\underline{v}} = g(\underline{v})$ on Q , $\underline{v}(0, \cdot) = \underline{f}$ given above.

D. BOTHE

A reaction-diffusion system with moving boundary

We study a mathematical model describing the regeneration of exhausted ion exchangers. In this chemical process we have a number of pellets carrying a chemical B with uniform concentration c_B , placed in the (liquid) bulk phase of a reactor. The bulk phase carries a chemical A with concentration c_A^b which diffuses into the pellets, where it reacts with B to produce a certain substance. Since this is a fast reaction, we study the limit case of an instantaneous reaction in which case A and B cannot coexist, i.e. $c_A \cdot c_B = 0$ where c_A is the concentration of A in the pellets. Consequently, A and B are separated by a moving interface and, since the pellets are of spherical shape, the interface has spherical shape too. Moreover, the model can be reduced to one space dimension. We assume the bulk phase to be ideally mixed except near the surface of the pellets, where we use the so-called film theory. Finally, balance of mass for the bulk phase yields a dynamical boundary condition at the film surface. This leads to the following mathematical formulation.

$$\begin{aligned} \partial_t c_A &= D \frac{1}{r^2} \partial_r (r^2 \partial_r c_A) & t > 0, \varrho(t) < r < R & & c_A(t, r) = 0 \text{ on } [0, \varrho(t)] \\ & & & & \text{if } \varrho(t) > 0, \partial_r c_A(t, 0) = 0 \\ \partial_t c_A &= D' \frac{1}{r^2} \partial_r (r^2 \partial_r c_A) & t > 0, R < r < R + \delta & & c_A(t, R-) = c_A(t, R+)/H_A \\ c_B \partial_t \varrho^3 &= -3D \varrho^2 \partial_r c_A(t, \varrho(t)+) & t > 0 & & D \partial_r c_A(t, R-) = D' \partial_r c_A(t, R+) \\ V_L \partial_t c_A^b &= \dot{V}_L^i (c_A^i - c_A^b) - AD' \partial_r c_A(t, R + \delta) & t > 0 & & c_A(t, R + \delta) = c_A^b(t) \\ c_A(0, r) &= 0 \text{ on } [0, R], c_A(0, r) = c_{A,0}(t) \geq 0 \text{ on } [R, R + \delta], & \varrho(0) = R, c_A^b(0) = c_{A,0}^b \geq 0. \end{aligned}$$

Here $\varrho(t)$ is the position of the moving boundary at time t and its dynamic behavior is modeled by the third equation where we used ϱ^3 instead of ϱ , since this is the right formulation to have conservation of mass to be incorporated into the model.

We write this as a nonlinear evolution equation and the canonical setting is to work in L^1 -spaces, in order to be able to exploit conservation of mass. In fact, it turns out that the corresponding operator is dissipative and also satisfies the range condition. Consequently, we

get existence of a unique global mild solution from the Crandall-Liggett-theory. To obtain further regularity we characterize the generalized domain and prove that solutions immediately enter this set, given that the initial values (corresponding to c_A) belong to L^2 .

Concerning the asymptotic behavior we finally show that every mild solution tends to the unique stationary solution $u_\infty(t) \equiv (c'_A/H_A, c'_A, 0, c'_A)$

(Joint work with J. Prüf, Universität Halle.)

D. DANERS

Domain perturbation for parabolic equations

We study convergence properties of solutions of the nonlinear parabolic equation

$$(*) \quad \begin{cases} \partial_t u_n - A(x, t)u_n = f(x, t, u_n) & \text{in } \Omega_n \times (0, T] \\ u_n = 0 & \text{on } \partial\Omega_n \times (0, T] \\ u_n(0, \cdot) = u_{0n} & \text{in } \Omega_n \end{cases}$$

as $\Omega_n \rightarrow \Omega$ und $u_{0n} \rightarrow u_0$ weakly in L_2 . Here $A(x, t)$ is a uniformly strongly elliptic operator of second order in divergence form with real bounded and measurable coefficients. Convergence of domains is strongly singular and includes cutting small holes or a dumbbell with shrinking handle. We show that the existence time $t^+(u_0)$ of the solution is lower semicontinuous with respect to the domain and initial values, and that the solution of $(*)$ converges to the solution of $(*)$ with n deleted in L_q ($q \in [1, \infty)$) uniformly with respect to compact subintervals of $(0, t^+(u_0))$. In a periodic setting one can prove results on the existence of periodic solutions of $(*)$ near a periodic solution of the unperturbed problem. This can be used to construct examples of parabolic equations having an arbitrary number of periodic solutions.

E. B. DAVIES

Heat kernels of self-adjoint elliptic operators

If $K(t, x, y)$ is the heat kernel of a self-adjoint elliptic operator of order $2m$ on $L^2(\mathbb{R}^N)$ one can often prove an upper bound of the type

$$|K(t, x, y)| \leq c_1 t^{-\frac{N}{2m}} \exp \left[-c_2 \frac{|x-y|^{\frac{2m}{2m-1}}}{t^{\frac{1}{2m-1}}} \right]$$

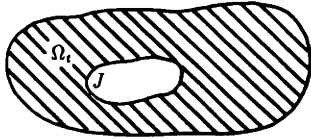
for all $t > 0$ and $x, y \in \mathbb{R}^N$. This type of estimate is standard for uniformly elliptic divergence form self-adjoint operators of order 2 with real measurable symmetric coefficients.

For higher order uniformly elliptic operators with measurable coefficients, or for second order operators with complex coefficients one needs a dimensional restriction $N \leq 2m$ for the bound to be valid. For strictly elliptic second order operators with L^p coefficients the natural distance associated with the coefficients may be identically zero, so no Gaussian bound need exist. For Laplace operators on manifolds the long time behaviour of $K(t, x, y)$ for fixed x, y is independent of x, y as far as the order of magnitude is concerned, but the actual rate of decay is not simply related to the geometry of the manifold.

J. ESCHER

Classical Solutions to some Multi-Dimensional Moving Boundary Problem

Let J and Γ_0 denote two compact disjoint hypersurfaces in \mathbb{R}^n and consider the following moving boundary problem: Find a function u and a family of hypersurfaces $\Gamma = \bigcup \Gamma_t$ such that:



$$\left. \begin{aligned} \Delta u &= 0 && \text{in } \Omega_t \\ \partial_\nu u &= b && \text{on } J \\ u &= \sigma \kappa && \text{on } \Gamma_t \\ V &= -\partial_\nu u && \text{on } \Gamma_t \\ \Gamma_{t=0} &= \Gamma_0 && . \end{aligned} \right\} (*)$$

Here, $b \in C^1(J)$ is given, $\sigma \in \mathbb{R}$ is the surface tension, κ is the mean curvature of Γ_t , V is the normal velocity of Γ . We prove that :

- (i) If $\sigma > 0$ then (*) is classically well-posed.
- (ii) If $\sigma < 0$ then (*) is linearly ill-posed.
- (iii) If $\sigma = 0$ and $b \geq 0$ then (*) is classically well-posed.
- (iv) If $\sigma = 0$ and $b < 0$ then (*) is ill-posed.

(Joint work with G. Simonett, Vanderbilt University, Nashville.)

G. R. GOLDSTEIN

Smoothing for Nonlinear Parabolic Problems

Of concern are equations of the form

$$\begin{aligned} \frac{\partial u}{\partial t} &= \sum_{i=1}^n \frac{\partial}{\partial x_i} [\psi_i(x, \nabla u)] && (x, t) \in \Omega \times (0, T], \\ -\vec{\psi}(x, u) \cdot \vec{\nu} &= \beta(x, u) && (x, t) \in \partial\Omega \times (0, T], \\ u(x, 0) &= f(x) && x \in \Omega. \end{aligned}$$

Here Ω is a bounded domain in \mathbb{R}^n with smooth boundary, $\vec{\nu}$ is the outward unit normal to Ω and $\vec{\psi} = (\psi_1, \dots, \psi_n)$. Under suitable conditions it is shown that for $f \in L^1(\Omega)$ and $t > 0$, one has $u(\cdot, t) \in L^\infty(\Omega)$ and

$$\begin{aligned} \|u(\cdot, t)\|_\infty &\leq C(T)t^{-\frac{n}{2}} \|f\|_1 \\ \|u(\cdot, t)\|_2 &\leq C(T)t^{-(\frac{n}{2}+1)} \|f\|_1 \end{aligned} \quad \text{for } n \geq 3.$$

Analogous estimates are obtained with other powers of t in dimensions $n = 1, 2$. On addition our estimates show that $\sup_{(x,t) \in \Omega \times (0,T]} |u(x,t)| < \infty$, and $\|u_t(\cdot, t)\|_2 \rightarrow 0, \|\nabla_x u(\cdot, t)\|_2 \rightarrow 0$ as $t \rightarrow \infty$.

(Joint work with Jerome A. Goldstein.)

P. E. T. JØRGENSEN

Asymptotics of periodic subelliptic operators

We establish that heat diffusion with periodic conductivity is governed by two scales, referring to time, $t \rightarrow 0+$, and $t \rightarrow \infty$. The small time diffusion is described by the geodesic distance d_c where

$$d_c(x, y) := \sup \left\{ \psi(x) - \psi(y) \mid \sum_{i,j} \psi_i c_{ij} \psi_j \leq 1 \right\}$$

with $c_{ij} \in L^\infty$ and $c = (c_{ij})$ real and symmetric defining the 2^{nd} order term in the heat equation. The large time behavior, on the other hand, is dictated by a different and simpler distance associated with a certain homogenized system \hat{c} , and a diffusion semigroup $\{\hat{S}_t\}_{t \in \mathbb{R}_+}$, where $\hat{c} = (\hat{c}_{ij})$ corresponds to a constant coefficient problem, and scaling $c'_{ij}(x) := c_{ij}(x/\epsilon)$. We show $\lim_{\epsilon \rightarrow 0} S'_\epsilon = \hat{S}_t$. Our main result is

$$\lim_{t \rightarrow \infty} \|S_t - \hat{S}_t\|_{p \rightarrow p} = 0$$

where $p \rightarrow p$ refers to the L^p -to- L^p operator norm, and S_t is the diffusion semigroup corresponding to the initially given variable coefficient problem. Our methods are general and apply to stratified Lie groups, and we solve a problem raised by E. B. Davies.

(Joint work with C. Batty, O. Bratteli, and D. Robinson.)

V. LISKEVICH

Dominated semigroups with singular complex potentials

We develop perturbation and approximation theory for C_0 -semigroups on L^p -spaces ($1 \leq p < \infty$) with singular complex potentials.

In the first part of the talk for operators H associated with Dirichlet forms and complex locally integrable potentials q with negative real part in the Kato class we construct an extension of $H+q$ which generates a C_0 -semigroup on L^p and prove an approximation theorem for this semigroup approximating q in the sense of L^1_{loc} -convergence. This result is a joint work with P. Stollmann. The main tool we use in the proof is the Feynman-Kac formula.

In the second part we are in a more general context. Namely, we consider a C_0 -semigroup $(S(t); t \geq 0)$ on $L^p(M, \mu)$, (M, μ) is an arbitrary measure space. The only assumption we make on $S(\cdot)$ is that there exists a positive C_0 -semigroup $(U(t); t \geq 0)$ which dominates $S(\cdot)$. This part is a generalization of the perturbation theory by J. Voigt, which was developed for positive semigroups with real potentials. We construct the perturbed semigroup with the same conditions on the real part of the potential as in Voigt's theory, and with "regular" imaginary part. For this semigroup we prove a dominated convergence theorem assuming that the corresponding approximating sequence of the potentials converges almost everywhere. The main example considered is the Schrödinger operator with singular magnetic field and complex potential. We prove also a new domination criterion for this operator. This is a joint work with A. Manavi.

S.-O. LONDEN

Fractional conservation laws

Estimates of the difference between the entropy solutions of the single conservation law $u_t + \operatorname{div} g(u) = 0$, $u(0, \cdot) = u_0$, and of the evolutionary integral equation

$$(k * (u - u_0))_t + \operatorname{div} g(u) = 0$$

are given in terms of k , g , and u_0 . A corresponding result is obtained for more general evolution equations with an accretive nonlinearity.

A. LUNARDI

Optimal Hölder regularity for elliptic and parabolic equations with unbounded coefficients in \mathbb{R}^n

We consider the problems

$$(E) \quad \lambda u - Au = f, \quad x \in \mathbb{R}^n, \quad (P) \quad \begin{cases} u_t - Au = g(t, x) & 0 \leq t \leq T, \quad x \in \mathbb{R}^n \\ u(0, x) = u_0(x) & x \in \mathbb{R}^n \end{cases}$$

where

$$Au(x) = \sum_{i,j=1}^n q_{ij}(x) D_{ij} u(x) + \sum_{i=1}^n b_i(x) D_i u(x) = \operatorname{Tr}(Q(x) D^2 u(x)) + \langle B(x), Du(x) \rangle$$

$Q(x) = (Q(x))^* \geq 0$, q_{ij} bounded, b_i Lipschitz continuous (possibly unbounded).

We give sufficient conditions in order to get optimal Schauder type estimates for the solutions of (E) and (P), and precisely:

- (1) $\forall \lambda > 0, \forall f \in C^\theta(\mathbb{R}^n)$, ($0 < \theta < 1$), (E) has a unique solution $u \in C^{2+\theta}(\mathbb{R}^n)$ and $\|u\|_{C^{2+\theta}(\mathbb{R}^n)} \leq c \|f\|_{C^\theta(\mathbb{R}^n)}$.
- (2) $\forall g \in C([0, T] \times \mathbb{R}^n)$ such that $g(t, \cdot) \in C^\theta(\mathbb{R}^n)$ and $\sup_{0 \leq t \leq T} \|g(t, \cdot)\|_{C^\theta(\mathbb{R}^n)} < \infty$; $\forall u_0 \in C^{2+\theta}(\mathbb{R}^n)$ ($0 < \theta < 1$) problem (P) has a unique solution $u \in C^{1,2}([0, T] \times \mathbb{R}^n)$; moreover $u(t, \cdot) \in C^{2+\theta}(\mathbb{R}^n) \forall t \in [0, T]$ and

$$\sup_{0 \leq t \leq T} \|u(t, \cdot)\|_{C^{2+\theta}(\mathbb{R}^n)} \leq c \left(\|u_0\|_{C^{2+\theta}(\mathbb{R}^n)} + \sup_{0 \leq t \leq T} \|g(t, \cdot)\|_{C^\theta(\mathbb{R}^n)} \right).$$

The simplest case in which (1) and (2) hold was described by Da Prato - Lunardi (JFA 1995), in the case $Q(x) \equiv Q > 0$, $B(x) = B \cdot x$, B any matrix.

A second situation in which (1) and (2) hold is when q_{ij}, b_i are Lipschitz continuous and bounded, q_{ij} are differentiable and moreover $Q(x) \geq \nu I$, $x \mapsto \langle Dq_{ij}(x), B(x) \rangle$ is bounded [Lunardi - Vespi, preprint]. There are similar results also in the degenerate case $\operatorname{Det} Q(x) = 0$. In [Lunardi, preprint] it is studied the case where

$$Q(x) \equiv Q = Q^*, \quad B(x) = B \cdot x, \quad \det Q = 0$$

under the hypoellipticity assumption

$$\operatorname{Rank} [Q^{1/2}, BQ^{1/2}, \dots, B^{n-1}Q^{1/2}] = n.$$

Then it is possible to decompose $\mathbb{R}^n = E_0 \oplus E_1 \oplus \dots \oplus E_k$, $k \leq n - 1$ in such a way that setting

$$C_d^\theta(\mathbb{R}^n) = \left\{ \varphi \in L^\infty(\mathbb{R}^n) : \forall x_0 \in \mathbb{R}^n, \varphi(x_0 + \cdot)|_{E_k} \in C^{\theta/(2k+1)}(E_k) \right\};$$

$$\|\varphi\|_{C_d^\theta(\mathbb{R}^n)} = \sup_{x_0 \in \mathbb{R}^n} \left\| \varphi(x_0 + \cdot)|_{E_k} \right\|_{C^{\theta/(2k+1)}(E_k)}$$

the results (1) and (2) are true with C_d^θ , $C_d^{\theta+2}$ replacing C^θ , $C^{\theta+2}$, respectively (in this case, however, there are distributional solutions, not classical ones).

The proofs are based on the estimates for the semigroup $T(t)$ associated to problem (P),

$$\|T(t)\|_{L(C^\theta(\mathbb{R}^n), C^\alpha(\mathbb{R}^n))} \leq \frac{ce^{\omega t}}{t^{(\alpha-\theta)/2}}, \quad t > 0, \quad 0 \leq \theta \leq \alpha < 3$$

in the nondegenerate case, and

$$\|T(t)\|_{L(C_d^\theta(\mathbb{R}^n), C_d^\alpha(\mathbb{R}^n))} \leq \frac{ce^{\omega t}}{t^{(\alpha-\theta)/2}}, \quad t > 0, \quad 0 \leq \theta \leq \alpha < 3$$

in the degenerate case, which allow to prove (1) and (2) by an interpolation procedure.

A. McINTOSH

Functional calculi, quadratic estimates, and interpolation theory

This is a survey talk about holomorphic functional calculi of operators of type ω in a Hilbert space. We consider the connections with quadratic estimates and interpolation theory. These results will be considered in the context of proving that the operator $-b\Delta$ has a bounded holomorphic functional calculus in $L_p(\Omega)$, $1 < p < \infty$, if $b \in L_\infty(\Omega)$ and $\text{Re } b \geq \kappa > 0$. (The more recent results were obtained in collaboration with Pascal Auscher and Andrea Nahmod.)

M. PIERRE

A tridimensional inverse shaping problem and an Hamilton-Jacobi equation on a closed surface

We discuss a question which arises in the following tridimensional inverse shaping problem: Can one find a distribution of currents around a levitating liquid metal bubble so that it takes a given shape? It leads to the resolution of a Hamilton-Jacobi equation of eikonal type on the surface of the bubble which has a self-contained interest. We answer the question for closed smooth surfaces which are homeomorphic to a sphere. We give a necessary and sufficient condition on the data for existence and uniqueness of a C^1 -solution. It follows that small analytic perturbations of admissible surfaces may not be admissible.

(Joint work with Elisabeth Roisy.)

M. RENARDY

Some Mathematical Issues in Viscoelastic Flows

The lecture reviews a number of topics in viscoelastic flows which present challenges to mathematics and open problems. In particular, the following issues are discussed:

1. Stability of viscoelastic flows

In stability studies, two premises are often taken for granted: First that linear stability implies stability to small disturbances, and second, that linear stability can be determined from the spectrum. There are abstract results which imply this for Newtonian flows. However, viscoelasticity introduces a hyperbolic component to the equations of motion, and there are actually counterexamples of hyperbolic PDEs where linear stability is not determined by the spectrum. The talk shows such a counterexample and also presents some positive results for hyperbolic PDEs in one space dimension and for flows of fluids of Jeffreys type.

2. Corner singularities in viscoelastic flows

Problems with reentrant corners have long caused difficulties for numerical simulations. In contrast to the Newtonian case, where corner behavior is dominated by the Stokes equation, corner singularities in viscoelastic flows are highly nonlinear. Recent results have shed some light on the corner singularity for the upper convected Maxwell fluid, and also on the source of numerical problems. Future challenges include the analysis of other constitutive models as well as the possibility of a more general program of high Weissenberg number asymptotics.

3. Problems with open boundaries

Computational problems often involve truncation of the domain, leading to boundaries which are crossed by the fluid. The memory of viscoelastic fluids leads to the need for extra boundary conditions at inflow boundaries. The talk reviews results on the well-posedness of such boundary value problems for steady flows of differential fluid models of Maxwell or Jeffreys type. There are many open problems such as a more complete characterization of admissible boundary conditions and a satisfactory analysis of time-dependent flows.

D. W. ROBINSON

Complex elliptic operators: Gaussian bounds and Hölder continuity

We review the theory of second-order elliptic operators with complex measurable coefficients.

The theory for real coefficients developed vigorously in the period 1955-70: Di Giorgi and Nash independently proved boundedness properties and Hölder continuity of solutions of the corresponding elliptic and parabolic equations, in 1957; Aronson established Gaussian bounds for the parabolic solution, the heat kernel, in 1967. Then Di Giorgi gave an example of an elliptic system for which boundedness and continuity fail, in 1969.

Since 1980 renewed interest in these problems has developed both in the classical context and for operators on Lie groups or general manifolds. In 1985 Davies developed a perturbation technique for deriving near optimal Gaussian bounds from appropriate cross-norm estimates of the evolution semigroups from L_1 to L_∞ . Hölder continuity inquires similar estimates from L_1 to C^α . The cross-norm estimates on the L_p -spaces have then been established by various techniques: Sobolev, log-Sobolev, or Nash inequalities. (For manifolds the validity of these inequalities, or the cross-norm estimates, are equivalent to geometric growth properties.)

In 1985, an example of Maźya, Nazarov and Plemenskii established that Di Giorgi estimates can fail for complex operators in spatial dimensions five or more; so Gaussian bounds and Hölder continuity also fail. In 1994, Auscher, McIntosh and Tchamitchian proved, however, that the bounds and continuity are valid for dimensions one and two. Then in 1995, Auscher showed that uniform continuity of the principle coefficients suffices for both Gaussian bounds and Hölder continuity. This proof is based on elliptic Di Giorgi estimates and

cross-norm estimates for Morrey spaces.

In 1995, ter Elst and Robinson established Gaussian bounds and Hölder continuity for the heat kernels associated with complex subelliptic operators on Lie groups. Their methods extend Auscher arguments by use of parabolic techniques.

H. SOHR

A perturbation theorem for the sum of two operators with applications to partial differential equations

Consider some Banach space X which possesses the UMD-property and two operators A and B densely defined and with dense ranges in X . Then under some assumptions on their resolvents and imaginary powers it can be shown that the norms

$$\|A^\alpha v\| + \|B^\alpha v\| \quad \text{and} \quad \|(A+B)^\alpha v\|$$

are equivalent for all $0 \leq \alpha \leq 1$. This result yields some new properties of weak solutions of the Navier-Stokes equations in exterior domains in particular concerning their asymptotic behavior as $t \rightarrow \infty$. Furthermore it yields the proof of the Helmholtz decomposition in L^q , $q > 1$, for infinite cylinders.

(Joint work with Y. Giga and M. Giga.)

O. J. STAFFANS

Coprime Factorizations and Optimal Control of well-posed L^2 -systems

We study the infinite horizon quadratic cost minimization problem for well-posed L^2 -systems (= abstract linear control systems). First we show that the transfer function of every jointly stabilizable and detectable L^2 -system has a doubly coprime factorization in \mathcal{H}^∞ . The converse is also true: every function with a doubly coprime factorization in \mathcal{H}^∞ can be realized as the transfer function of a jointly stabilizable and detectable L^2 -system. We then solve the quadratic cost minimization problem in state feedback form, and tie the solution to a coprime factorization with an inner numerator. Moreover, under an extra regularity assumption, we show that the optimal cost operator satisfies an algebraic Riccati equation. This Riccati equation is nonstandard in the sense that the positive definite weighting operator in the quadratic term differs from the expected one, and the computation of the correct weighting operator is a nontrivial task.

Definition of well-posed L^2 -system:

$u = \text{control}$, $x(t) = \mathcal{A}(t)x_0 + \overline{BT}(t)u = \text{state}$; $y = Cx_0 + D\pi_+ u = \text{observation}$;
 $\mathcal{A}(t+s) = \mathcal{A}(t)\mathcal{A}(s)$, $\mathcal{A}(0) = I$; $\mathcal{A}(t)B = \overline{BT}(t)\pi_-$; $C\mathcal{A}(t) = \pi_+ T(t)C$; $T(t)D = D\overline{T}(t)$,
 $\pi_- D\pi_+ = 0$; $\pi_+ D\pi_- = CB$.

K. T. STURM

Dirichlet forms, diffusion processes and geodesic spaces

Every regular Dirichlet form $(H^{\frac{1}{2}}u, H^{\frac{1}{2}}v)$ defines in an intrinsic way a metric ρ on the underlying state space. This metric turns out useful to describe several properties of the heat semigroup e^{-Ht} or of the Markov process associated with the Dirichlet form. For instance, if the volume growth $r \mapsto m(B_r(x_0))$ of balls in this metric is $\leq C \cdot r^2$ (for $r \rightarrow \infty$) then the semigroup is recurrent.

If the volume growth is $\leq e^{C \cdot r^2}$ then the semigroup is conservative, in particular,

$$\|e^{-Ht}\|_{1,1} = \|e^{-Ht}\|_{\infty,\infty} = 1.$$

If the volume growth is subexponential, then $\|e^{-Ht}\|_{2,2} = 1$.

Other topics are Gaussian heat kernel estimates of the type

$$(e^{-Ht} 1_A, 1_B) \leq \sqrt{m(A)} \cdot \sqrt{m(B)} \cdot \exp\left(-\frac{\varrho^2(A, B)}{4t}\right)$$

and capacity estimates of the type

$$\text{cap } F \leq \left(\int_0^\infty \frac{r \, dr}{m(B_r(F))}\right)^{-1}.$$

The latter can be applied to the problem of hitting the nodal lines $\{\varphi = 0\}$ of the weight function $\varphi \geq 0$ on \mathbb{R}^n by the diffusion process associated with the operator $H = -\Delta + 2\frac{\nabla\varphi}{\varphi}\nabla$.

A. F. M. TER ELST

Weighted subcoercive operators on Lie groups

Let U be a continuous representation of a Lie group G on a Banach space \mathcal{X} and $a_1, \dots, a_{d'}$ an algebraic basis of the Lie algebra \mathfrak{g} of G , i.e., the $a_1, \dots, a_{d'}$ together with their multi-commutators span \mathfrak{g} . Let $A_i = dU(a_i)$ denote the infinitesimal generator of the continuous one-parameter group $t \mapsto U(\exp(-ta_i))$ and set $A^\alpha = A_{i_1} \dots A_{i_n}$ where $\alpha = (i_1, \dots, i_n)$ with $i_j \in \{1, \dots, d'\}$. We analyze properties of m -th order differential operators

$$dU(C) = \sum_{|\alpha| \leq m} c_\alpha A^\alpha$$

with coefficients $c_\alpha \in \mathbb{C}$.

If L denotes the left regular representation of G in $L_2(G)$ then $dL(C)$ satisfies a Gårding inequality on $L_2(G)$ if and only if the closure of each $dU(C)$ generates a holomorphic semigroup S on \mathcal{X} in an open representation independent subsector of the sector of holomorphy, the action of S_z is determined by a smooth, representation independent, kernel K_z which, together with its derivatives $A^\alpha K_z$, satisfies m -th order Gaussian bounds and, in case U is unitary, $(S_z)_z$ is quasi-contractive on a subsector.

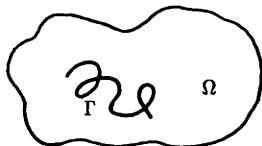
Alternatively, $dL(C)$ satisfies a Gårding inequality on $L_2(G)$ if, and only if, the closure of $dL(C)$ generates a holomorphic, quasi-contractive, semigroup satisfying bounds $\|A_i S_t\|_{2 \rightarrow 2} \leq c t^{-1/m} e^{\omega t}$ for all $t > 0$ and $i \in \{1, \dots, d'\}$.

These results extend to operators for which the directions $a_1, \dots, a_{d'}$ are given different weights. The unweighted Gårding inequality is a stability condition on the the principal part, i.e., the highest order part, of $dL(C)$ but in the weighted case the condition is on the part of $dL(C)$ with the highest weighted order.

(Joint work with D.W. Robinson.)

H. TRIEBEL

Fractals and spectra



Omega bounded, smooth domain in R^n
Gamma compact set in R^n, Gamma subset Omega

Gamma is said to be a *d*-set, $0 < d < n$, if $\mathcal{H}^d|_{\Gamma}$ (Hausdorff measure restricted to Gamma) has the property $(\mathcal{H}^d|_{\Gamma})(B(x, r)) \sim r^d, 0 < r < 1, \forall x \in \Gamma$.

$(\text{tr}^\mu u)(\varphi) = \int_{\Gamma} (\text{tr}_{\Gamma} u)(\gamma)(\varphi|_{\Gamma})(\gamma) \mathcal{H}^d(d\gamma)$ generates on Gamma an operator from $\mathcal{D}(\Omega)$ in $\mathcal{D}'(\Omega)$. The extension of tr^μ from $B_{2,1}^{\frac{n-d}{2}}(\Omega) \rightarrow B_{2,\infty}^{-\frac{n-d}{2}}(\Omega)$ is possible. tr^μ generalizes the multiplication operator $Au = (-\Delta)^{-1} \circ \text{tr}^\mu$, where $-\Delta$ means the Dirichlet Laplacian with respect to Omega.

Theorem: Let A be ≥ 0 , selfadjoint, compact in $H^1(\Omega)$. Let $\mu_k > 0, k \in \mathbb{N}$ be the positive eigenvalues. There are two positive constants c_1, c_2 such that $c_1 k^{-\frac{2-(n-d)}{2}} \leq \mu_k \leq c_2 k^{-\frac{2-(n-d)}{2}}$.

Remark: Classical Weyl exponent : $2/n$. Fractal Weyl exponent : $(2 - (n - d))/d$.
Extension to other (non-symmetric) PDE, ψ DO is possible.

Informal Session on Schrödinger Semigroups

P. STOLLMANN

Perturbation of semigroups with applications to spectral theory

Starting from trace norm estimates of the effect of an obstacle in terms of the capacity of the obstacle we present results dealing with absence of absolutely continuous spectra for Schrödinger operators with barriers.

After a discussion of the relevance of such spectral behaviour in connection with random models we briefly touch upon recent results establishing localization near the band edges for Anderson type random perturbations of periodic operators.

El-M. OUHABAZ

Absence of the maximum principle for complex elliptic operators

Consider on $L^2(\mathbb{R}^N)$ the elliptic operator

$$A = - \sum_{k,j=1}^N D_k(a_{kj} D_j)$$

with coefficients satisfying $a_{kj} \in L^\infty(\mathbb{R}^N, \mathbb{C}), 1 \leq k, j \leq N$. Consider the Cauchy problem

$$\frac{\partial u}{\partial t} = -Au, \quad u(0) = f.$$

We are concerned with the following maximum principle

$$(PM) \quad |f(x)| \leq 1 \text{ a.e.} \implies |u(t, x)| \leq 1 \text{ (a.e.) } \forall t \geq 0.$$

It is well known that if a_{kj} , $1 \leq k, j \leq N$ are real-valued then (PM) holds.

We show that if (PM) is satisfied then the operator A has real-valued coefficients.

Informal Session on Singular Interaction Problems

G. LUMER

Singular interaction problems (of parabolic type), and applications

We treat in Banach space and classical context, via asymptotic solutions, the singular parabolic interaction problems (equations) of the type

$$\begin{aligned}
 u' &= \hat{A}u + F(t) \\
 u(0_-) &= f \\
 (si\ u)(0) &= \sigma \\
 Bu(t) &= \varphi(t), \quad t > 0,
 \end{aligned}
 \tag{*}$$

where X is a Banach space (\hat{A} = Laplacian in the typical classical situation), " $(si \cdot)(0)$ " means "singular interaction - at time $t = 0$ ", $\sigma \in \mathcal{E}'_0$ (i.e. \mathcal{E}' with $\text{supp}(\cdot) = 0$ or \emptyset) on $B_0 = \{X\text{-valued hyperfunctions on } \mathbb{R} \text{ with support } 0 \text{ or } \emptyset\}$. ($\sigma = 0$ for "mild" singular transitions without "interaction" such as heat shocks, while for example $\sigma = c\delta$ ($c \in X$, $\delta = \delta(t)$) for "heat explosions". The solution u of (*) is obtained as limit of regular solutions u_η ($\eta \rightarrow 0$). The situation where $\sigma \in \mathcal{E}'_0$ is mathematically well understood by now, and has a number of physical and engineering applications, in particular $(f, \sigma) \mapsto$ solution of (*) $u(\cdot, f, \sigma)$ is injective, (but problems remain on fully understanding certain related physical aspects). On the other hand it was very recently shown that the just above mentioned injectivity may fail to hold when σ is allowed to be a hyperfunction in $B_0 \setminus \mathcal{E}'_0$, although a unique solution also exists in the latter general situation. There are indeed at this time many interesting open problems in the situation where $\sigma \in B_0 \setminus \mathcal{E}'_0$.

Informal Session on H^∞ -Calculi

S. MONNIAUX

Analytic generators

In 1987, G. Dore and A. Venni proved their famous theorem on maximal regularity of the sum of two operators A and B . They used, in particular, the imaginary powers of those operators. The classical way to define these objects is to consider sectorial operators for which we can apply a functional calculus giving the complex powers.

The approach here is slightly different. For a C_0 -group $(U(s))_{s \in \mathbb{R}}$ on a Banach space X , we define its analytic continuation $(C_\alpha)_{\alpha \in \mathbb{C}}$, consisting in - unbounded - operators in X . It turns out that those operators are closed, densely defined and verify a semigroup property. The operator $C = C_1$ is called the analytic generator of U ; its spectral properties are remarkable in the case where the type of U is less than π . Moreover, if the space X has the UMD-property, the analytic generator C of U is sectorial and verifies $C^{is} = U(s)$ for all $s \in \mathbb{R}$. This theory was developed by I. Ciorănescu and L. Zsidó (1976) in the case of bounded C_0 -groups. Our - more general - case allows us to prove, quite easily, the theorem of Dore-Venni.

G. SIMONETT

Bounded H^∞ -calculus for elliptic differential operators with non-smooth coefficients

Let $A = \sum_{|\alpha|=m} a_\alpha D^\alpha$ be an elliptic operator with constant coefficients and consider a small perturbation $B = \sum_{|\alpha|=m} b_\alpha D^\alpha$ with L_∞ -coefficients.

Then $A + B$ has a bounded holomorphic functional calculus on $L_p(\mathbb{R}^n)$ for $1 < p < \infty$. The proof uses Caldéron-Zygmund theory, multilinear expansion and the $T(1)$ -theorem.
(Joint work with X. T. Duong)

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