Tagungsbericht 2/1996

Set Theory Meeting

07. - 13.01.1996

The meeting was organized by Ronald Jensen (Berlin), Menachem Magidor (Jerusalem), and Ernst Jochen Thiele (Berlin). The subject of the meeting was set theory, and talks were given on many subjects within set theory as well as connections of set theory to other branches of mathematics.

Vortragsauszüge

SY D. FRIEDMAN

Generic Saturation

A forcing over L is a p.o. P, definable with parameters over $\langle L, A, \rangle$ for some amenable A such that P-generics preserve ZFC. P is *relevant* if P has a generic definable over $L[0^\#]$ and *almost relevant* if it has one definable over a set-generic extension of $L[0^\#]$.

Relevance Conjecture If $0^\#$ exists and P has a generic then P is almost relevant.

Thm 1 *If $0^\#$ exists and P has a generic G s.t. there is a set X of strong indiscernibles for $\langle L[0^\#, G], G\rangle$, o.t. $X > \omega$ then P is almost relevant.*

P is *codable* if it has a generic G s.t. for some real R in $L[0^\#]$, G is definable in $L[R]$, R generic over L. P is *almost codable* if the same holds with "in $L[0^\#]$" replaced by "in a set-generic extension of $L[0^\#]$".

Thm 2 *P is almost codable iff P has a generic G s.t. for some $\lambda_0, \lambda : I_{\lambda_0\lambda} = \{i_\alpha \mid \alpha = \lambda_0 + \lambda \cdot \beta$ some $\beta\}$ is a class of strong indiscernibles for $\langle L[G], G, A\rangle$, where $i_0 < i_1 < \ldots$*

is the increasing enumeration of I = Silver indiscernibles. P is codable iff λ_0, λ can be chosen to be countable in $L[0^\#]$.

Periodicity Conjecture If $0^\#$ exists and P has a generic then P is almost codable, i.e., P has a generic G s.t. for some λ_0, λ, $I_{\lambda_0\lambda}$ is a class of strong indiscernibles for $\langle L[G], G, A\rangle$.

Thm 3 *If $0^\#$ exists and P has a generic G s.t. for some X, X is a set of strong indiscernibles for $\langle L[0^\#, G], G\rangle$, o.t. $X > \omega$ then P is almost codable.*

Cor 4 *If there is an $(\omega + 1)$-Erdös cardinal then the Periodicity Conjecture is true.*

Andreas Blass

Subgroups of $\mathbb{Z}^\omega$

This talk was about some applications of Baire category to the study of subgroups of

$$\Pi := \mathbb{Z}^\omega.$$

To fix notation, let

$$\begin{aligned} \Sigma &:= \{x \in \Pi \mid (\forall^\infty n)\, x(n) = 0\}, \\ D &:= \{x \in \Pi \mid (\forall q \in \omega \setminus \{0\})(\forall^\infty n)\, q \text{ divides } x(n)\}, \text{ and} \\ B &:= \{x \in \Pi \mid \text{ Range of } x \text{ is finite}\}. \end{aligned}$$

I began with a quick review of some known results: All countable subgroups of Π are free but Π itself is not; no uncountable pure subgroup of D is free; all homomorphisms $\Pi \to \mathbb{Z}$ are finite $\mathbb{Z}$-linear combinations of projections, and similarly for $D \to \mathbb{Z}$; and B is free. Then I presented the following new results.

Π is not the union of any directed system of fewer than $\mathrm{cov}(B)$ proper, analytic subgroups. ($\mathrm{cov}(B)$ means the covering number for Baire category.)

D is not the union of any directed system of fewer than $\mathrm{add}(B)$ pure, proper, analytic subgroups.

The minimum number of isomorphs of Π whose union is D equals the dominating number, and so does the minimum number of isomorphs of Π whose union is Σ^ω.

Every Borel homomorphism $B \to \mathbb{Z}$ is a finite $\mathbb{Z}$-linear combination of projections.

Π has a pure, non-free subgroup H such that $H \cap G$ is free for every analytic subgroup G of Π except for G's that include a group of the form

$$V_{k,q} := \{x \in \Pi \mid (\forall n < k)\, x(n) = 0 \text{ and } (\forall n)\, q \text{ divides } x(n)\}.$$

(The exception is unavoidable. If $H \cap V_{k,q}$ is free for even one $k \in \omega$, $q \in \omega \setminus \{0\}$, then H is free.)

HEIKE MILDENBERGER

Non-existence of Borel morphisms

The combinatorical core of this talk was

Thm 1: $n > m \geq 1$. *There is no Baire measurable* $\alpha : n^\omega \to m^\omega$, *such that there is a* $\beta : [\omega]^\omega \to [\omega]^\omega$ *s.th.*
$\forall f \in n^\omega \forall X \in [\omega]^\omega$: *If* $\alpha(f)$ *is constant on* X, *then* f *is constant on* $\beta(X)$.

We conjecture that an analogous theorem for "almost constant" instead of "constant" is also true. Yet we have

Thm 2: $n \geq 2, m \geq 1$. *There is no Baire measurable* $\alpha : n^\omega \to m^\omega$ *such that there is a* $\beta : [\omega]^\omega \to [\omega]^\omega$ *s.th.*
$\forall f \in n^\omega \forall X \in [\omega]^\omega$: *If* $\alpha(f)$ *is almost constant on* X, *then* f *is constant on* $\beta(X)$.

A theorem maybe closely related to the conjecture is

Thm 3: $n > m > 1$. *There is no Baire measurable* $\alpha : n^\omega \to m^\omega$, *such that there is some* $\beta : [\omega]^\omega \to [\omega]^\omega$ *s.th.*
$\forall f_0, f_1 \in n^\omega \forall X \in [\omega]^\omega$: *If* $\alpha(f_0) \restriction X = \alpha(f_1) \restriction X$, *then* $f_0 \restriction \beta(X) =^* f_1 \restriction \beta(X)$.

ALESSANDRO ANDRETTA (joint work with John Steel)

Iterability for non-tame mice

Tame mice were introduced by Steel in "Inner models for many Woodin cardinals". They are structures of the form $J_\alpha^{\vec{E}}$ and although they can have many Woodins they cannot satisfy
"There is κ which is $\delta + 1$-strong and δ is Woodin".

We introduce an iteration game $\mathcal{G}(M)$, for coarse premice M, and prove that II wins $\mathcal{G}(M)$ if M is countable and $M \prec V_\mu$, some μ. The game $\mathcal{G}(M)$ is stronger than the weak iteration game and allows us to prove a comparison theorem for non-tame mice containing strong cardinals and Woodin cardinals above. Recently we proved a comparison process for mice satisfying the so-called $AD_{\mathbb{R}}$-hypothesis:
$\exists(\delta_n)\, \exists(\kappa_n)\, \kappa_0 < \delta_0 < \kappa_1 < \delta_1 < \ldots$ such that κ_n is $< \delta$-strong and $\delta = \sup \delta_n$
(but the details of the proof have not been checked carefully!)

JAMES CUMMINGS

Collapsing successors of singulars

Let κ be singular of cofinality ω, κ a cardinal. A *good scale* for κ is a sequence $\langle f_\alpha : \alpha < \kappa^+ \rangle$ where $f_\alpha \in \prod_{i<\omega} \kappa_i$ (κ_i an increasing sequence of regular uncountable

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cardinals, $\kappa_i \rightarrow \kappa$) which is increasing and cofinal in $(\prod \kappa_i, <^*)$ with the additional property (*goodness*) that for $\alpha < \kappa^+$, cf $\alpha > \omega$, there exists $A \subseteq \alpha$ unbounded and $m < \omega$ s.t. $\beta, \gamma \in A$, $\beta < \gamma$ and $m < n \rightarrow f_\beta(n) < f_\gamma(n)$.

Thm *If $cf(\kappa) = \omega$ and there is a good scale for κ then there is no extension of the universe in which κ_V^+ is the successor of an uncountable regular cardinal.*

JEAN A. LARSON

Multicolored graphs

In the notation of Erdős and Rado, the partition relation

$$\omega^n \longrightarrow (\omega^3, \ell_1, \ell_2, \ldots, \ell_m)^2$$

holds if and only if for every graph on a vertex set ω^n whose edges are colored with m colors, either there is an independent set $A \subseteq \omega^n$ of type ω^3 (one with no edges) or for some q, there is a set B of size ℓ_q all of whose pairs are joined by edges of color q.

In joint work with Carl Darby, the following theorems have been proved:

Theorem: *Suppose $\ell_1, \ell_2, \ldots, \ell_m$ are positive integers with $2^{t_i} < \ell_i$ for $i = 1, \ldots, m$ and suppose $t = t_1 + t_2 + \cdots + t_m$. If $n \leq t + 2$, then $\omega^n \not\longrightarrow (\omega^3, \ell_1, \ell_2, \ldots, \ell_m)^2$.*

Theorem: *Suppose $\ell_1, \ell_2, \ldots, \ell_m$ are positive integers with $2^{t_i} < \ell_i \leq 2^{t_i+1}$ for $i = 1, \ldots, m$ and suppose $t = t_1 + \cdots + t_m$. If $n > t+2$, then $\omega^n \longrightarrow (\omega^3, \ell_1, \ell_2, \ldots, \ell_m)^2$.*

T. JECH (joint work with Saharon Shelah)

A complete Boolean algebra without complete atomless subalgebras

We show that there are forcing conditions which add a definable real of minimal degree to a ZFC model. By a theorem of McAloon this gives a Boolean algebra as above.

PETER KOEPKE

Extenders

Elementary embeddings of models of set theory are a central feature in large cardinal theory. To code such embeddings into sets, normal measures and extenders have been used. We suggest to substitute such ultrafilter-based notions by initial segments of the maps to be coded. If the initial segment is chosen properly, the same information is captured. Now the usual extender properties may be transferred to this setting. An extender would now be a specific set-sized elementary embedding. An ultrapower-like construction with a natural Los-theorem is possible, and a closure criterion for extenders ensures the well-foundedness of the image model. We gave the characterisation of measurable, strong and Woodin-cardinals in this setting. Another use of such extenders is in constructibility theory, where the components of the constructing predicate are taken to be embeddings of initial segments of the hierarchy to be constructed. By this method, coherency and amenability of the hierarchy is easily guaranteed.

LEE J. STANLEY (joint work with Saharon Shelah)

Consistent negative and positive partition relations for singular cardinals of uncountable cofinality

By adding Cohen subsets of $\aleph_1$ to a model of $\diamondsuit_{\aleph_1}$, we produce a model where $\aleph_{\omega_1} \not\to (\aleph_{\omega_1}, \omega+1)^2$, thereby answering, negatively, Question 11.4 of [EHMR]. By then adding Cohen reals, we can obtain this simultaneously with, for example, $2^{\aleph_0} = \aleph_{\omega_1}$. This was announced in [419]. We present more general versions of the first construction. Also, starting from situations where $\zeta < \kappa = cf\ \lambda > \omega$ and κ is weakly compact, we show that in the model obtained adding Cohen subsets to κ, the positive relations $\lambda \to (\lambda, \zeta)^2$ hold.

MENACHEM MAGIDOR

Some soft remarks about covering

We try to analyse the situations in which two models of set theory $W \subseteq V$ are similar, for instance have the same cofinalities etc.

Example

Theorem: *$W \subseteq V$, $W \models G.C.H$ V and W agree about cofinalities. Suppose also that every countable set of ordinals in V can be covered by a set in W of cardinality $\leq \lambda$ then every set of ordinals $X \in V$ can be covered by a set $Y \in W$ such that* $|Y| \leq \max(|X|, \lambda)$.

MOTI GITIK (results of joint paper with Saharon Shelah)

Density of box products

Let $d_{<\aleph_1}(\kappa)$ denote the density of the space ${}^{\kappa}2$ with topology generated by g's, $g \in {}^{a}2$, $a \subset \kappa$, $|a| \leq \aleph_0$. We sketch a construction of a model with a strong limit κ of cofinality ω such that (1) $2^{\kappa} > \kappa^+$, (2) $d_{<\aleph_1}(\kappa) = \kappa^+$. The same may hold for $\kappa = \aleph_\omega$.

MENACHEM KOJMAN

ZFC Dowker space in $\aleph_{\omega+1}$ and some open problems

Thm: *(ZFC) There exists a Dowker space in* $\aleph_{\omega+1}$.

Problem: Is there a linearly Lindelöf not Lindelöf normal space? Such space is Dowker.

Thm: There is a Födor Lemma for $\Pi\aleph_n$ which implies non-countable paracompactness of various $X \subseteq \Pi(\omega_{n+1})$.

PATRICK DEHORNOY

Applications of set theory to braids

The canonical well-ordering of the ordinals *leads to* a linear ordering on true algebraic structure involving a left self-distributive operation, which in turn *leads to* a linear ordering on Artin's braid group B_∞. The latter *leads to* a new, very efficient algorithm for comparing braids, improving classical results by Artin, Garside, Morton, Thurston and others.

JOAN BAGARIA

Forcing axioms as generic absoluteness principles

We present some results about the relationship between Forcing Axioms and generic absoluteness, i.e., absoluteness under forcing extensions. Some of the results are: MA is equivalent to the statement that the universe is absolute under ccc generic extensions for Σ_1 sentences with elements of $H(\omega_2)$ as parameters. This is also true for the Bounded Proper Forcing Axiom, and bounded forcing axioms in general (see [1]). Also, MA implies that the universe is Σ^1_3 absolute under ccc extensions. (This answers a question in [2].) More generally, given any poset P, $FA_\kappa(P)$ implies Σ^1_3 absoluteness under P-generic extensions.
In view of these results we formulate the following general conjecture: Every forcing axiom is a generic absoluteness principle. i.e., given a class of posets Γ and a cardinal κ, one can find a (natural) class of sentences Σ and a set X such that $FA_\kappa(\Gamma)$ is equivalent to the statement that the universe is absolute under generic extensions with posets from Γ, for sentences from Σ with parameters in X. The conjecture is open for PFA and MM.

References

[1] M.Goldstern and S.Shelah: The bounded Proper Forcing Axiom. JSL. 1995.
[2] H.Judah and A.Roslanowski: Martin's axiom and the continuum.

ARTHUR APTER

Laver indestructibility and the class of compact cardinals

Using an idea developed in joint work with Shelah, we show how to redefine Laver's notion of forcing making a supercompact cardinal indestructible under κ-directed closed forcing to give a new proof of the Kimchi-Magidor Theorem in which every compact cardinal in the universe satisfies certain indestructibility properties. Specifically, we show that if $K \subseteq V$ is the class of supercompact cardinals, then it is possible to force and construct a model in which the only strongly compact cardinals are the elements of K or their measurable limit points, every $\kappa \in K$ is a supercompact cardinal indestructible under κ-directed closed forcing, and every κ a measurable limit point of K is a strongly compact cardinal indestructible under κ-directed closed forcing not changing $P(\kappa)$. We then derive as a corollary a model for the existence of a strongly compact cardinal κ which is not κ^+ supercompact but which is indestructible under κ-directed closed forcing not changing $P(\kappa)$ and remains non-κ^+ supercompact after such a forcing has been done.

ALAIN LOUVEAU

Descriptive aspects of logic actions

The talk was devoted to the descriptive set theoretic properties of the Borel actions of the Polish group S_∞ (of permutations of $\mathbb{N}$), which are the ones that occur in Model Theory when studying isomorphism between countable structures. Report was given on an on-going joint work with G. Hjorth and A. Kechris about the possible descriptive complexities of the associated orbit equivalence relations, and the relationships with more structural properties of these actions.

MATTHEW FOREMAN

Weak square principles and reflection properties

The talk discussed joint work with M. Magidor and J. Cummings on weak square properties. We prove the consistency of weak square on $\aleph_\omega$ and the simultaneous reflection of stationary sets in $\aleph_{\omega+1}$. This implies the non-existence of a very good scale on $\Pi\aleph_n/_{\text{Frechet}}$, a consequence of $\square_{\aleph_\omega,\lambda}$ for $\lambda < \aleph_\omega$.

HUGH WOODIN

Chang's Conjecture and the nonstationary ideal

Theorem 1 *Assume* $AD^{L(\mathbb{R})}+$ *there exists a countable set* $\sigma \subseteq \mathbb{R}$ *such that* $HOD^{L(\mathbb{R})}[\sigma] \models AD + DC$.
Then $L(\mathbb{R})^{\mathbb{Q}_{\max}} \models$ *Chang's Conjecture + NS is* ω_1*-dense.*

Theorem 2 *(Con(There exists an* ω_1*-dense ideal on* ω_1*+ Chang's Conjecture)* $\Longrightarrow$ *Con(ZFC + there exist* ω *Woodin cardinals + there exists an* ω_1 *dense ideal on* ω_1*).*

Thus $AD^{L[\mathbb{R}]} \not\to L[\mathbb{R}]^{\mathbb{Q}_{\max}} \models$ Chang's conjecture.

I also discussed general properties of the $\mathbb{P}_{\max}$, $\mathbb{Q}_{\max}$ extensions of $L[\mathbb{R}]$.

MARTIN GOLDSTERN

Projective measurability does not imply Baire property

A set $X \subseteq \mathbb{R}$ is projective if there is a natural number n and a Borel set $B \subseteq \mathbb{R}^{n+1}$ such that $X = \{x : \exists x_1 \, \forall x_2 \, \exists x_3 \ldots : (x, x_1, \ldots, x_n) \in B\}$. In a joint work with Saharon Shelah we show that it is consistent that all projective sets are measurable, while there is a projective set without the Baire property (i.e., not equal to a Borel set modulo a first category set).
The proof uses amalgamation of forcing notions, and coding of arbitrary sets by reals.

ARNOLD W. MILLER (joint work with Juris Steprans)

Orthogonal families of real sequences

For x and y sequences of real numbers define the inner product

$$(x, y) = \sum_{n<\omega} x_n y_n$$

which may not be finite or even converge. We say that x and y are orthogonal iff (x, y) converges and equals 0. Abian asked what are the possible cardinalities of maximal pairwise orthogonal families (MOF)? Kunen proved that there exists a MOF of cardinality $\mathfrak{c}$. We prove that it is consistent that the continuum be arbitrarily large and for every cardinal κ with $\omega \leq \kappa \leq \mathfrak{c}$ there exist MOF of cardinality κ. We also show that MA implies there are no MOF's of cardinality less than $\mathfrak{c}$ which contain only finitely many elements of l_2.

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KAI HAUSER (ongoing joint work with Hugh Woodin)

An application of core model theory to descriptive set theory

We generalize a result of Leo Harrington to the third level of the projective hierarchy (making use of the Σ^1_3 correctness of the one-Woodin K).

Theorem. (Assume there are two measurable cardinals.) *If for every real x any non-empty $\Pi^1_3(x)$ set of reals contains a $\Pi^1_3(x)$ singleton then either*

1. $\mathbf{\Delta}^1_2$ *determinacy*

or

2. *For any real z for which $\Delta^1_2(z)$ determinacy fails, all reals are contained in K_z and there is such a real z which is a Π^1_3 singleton.*

The background assumptions can be weakened to "The reals are closed under sharps" by factoring a result of Philip Welch into the statement of the theorem and by modifying the models K_z.

I briefly discussed some open questions related to the "converse" of the theorem.

LEV BUKOVSKÝ (joint work with N.N. Kholshchevnikova and M. Repický)

A-sets for Rademacher and Walsh orthogonal systems

A set $A \subseteq [0,1]$ in an A^R-set (Rademacher A-set) if there is an increasing sequence $\{u_k\}_{k=0}^{\infty}$ s.t. $\{v_{u_k}(x)\}_{k=0}^{\infty}$ converges for all $x \in A$ ($\{v_u\}$ in the Rademacher orthogonal system).

Every A^R-set is meager, negligible and σ-porous. Every perfect set contains a perfect A^R-subset. The smallest size of a non-A^R-set is the splitting number $\underset{\sim}{s}$ and the covering number of the family of all A^R-sets is the refining (=reaping) number $\underset{\sim}{r}$.

The family of A^R-permitted sets is the ideal

$$\mathrm{PRM}(A^R) = \{A \subseteq [0,1]; (\forall B\ A^R - \mathrm{set}) A \cup B \text{ is } A^R - \mathrm{set}\}.$$

There exists a perfect A^R-permitted set. We denote

$$\begin{aligned} \underset{\sim}{r}' &= \min\{|\mathcal{K}|; (\forall L \in [\omega]^\omega)(\exists \mathcal{F} \in \mathcal{K})(\mathcal{F} \text{ dense in } [\omega]^\omega, \subseteq^* \text{ and} \\ &\quad (\forall K \in \mathcal{F})(K \subset^* L \text{ or } K \subseteq^* \omega - L))\}. \end{aligned}$$

Then we have

$$\mathrm{non}(\mathrm{PRM}(A^R)) = \underset{\sim}{s}, \mathrm{cov}(\mathrm{PRM}(A^R)) = \underset{\sim}{r}',$$
$$\underset{\sim}{h} \le \mathrm{add}(\mathrm{PRM}(A^R)) \le \underset{\sim}{r}'.$$

Similar results were obtained for Walsh A-sets.

ITAY NEEMAN

Continuously coded determinacy

The talk attempted to convey the general idea of the proof that continuously coded games are determined.

Def: Given a set $A \subseteq {}^{\omega}(\mathbb{R} \times \mathbb{R}) \times \mathbb{R}$, the continuously coded game G^c_A is played as follows:

I	x_0, n_0		x_1, n_1		$\dots$	x_α, n_α		$\dots$
II		y_0		y_1	$\dots$		y_α	$\dots$

Rules: At round α, player I plays a real x_α, and an integer n_α. Player II then plays a real y_α.

Player I must ensure that $\alpha \neq \beta \Rightarrow n_\alpha \neq n_\beta$ (o.w. II wins). Given that, the game ends once $\forall n \in \mathbb{N}, \exists \alpha$ s.t. $n_\alpha = n$ (The game therefore has countable length).

Once the game ended, we let $f : \omega \to \mathbb{R} \times \mathbb{R}$ be the function given by

$$f_{(n)} = \langle x_\alpha, y_\alpha \rangle \text{ for the unique } \alpha \text{ s.t. } n_\alpha = n.$$

We let $w \in \mathbb{R}$ be the w.o. of ω given by $n_\alpha \leq n_\beta$ iff $\alpha \leq \beta$.
Then I wins iff $\langle f, w \rangle \in A$.

Thm: *Assume that there exist $\kappa < \tau < \lambda$ s.t.*

– *κ is $\tau + 1$-strong,*

– *τ is a Woodin cardinal, and*

– *λ is measurable.*

Let $A \subseteq {}^{\omega}(\mathbb{R} \times \mathbb{R}) \times \mathbb{R}$ be $\underset{\sim}{\Pi}^1_1$. Then G^c_A is determined.

ALAN DOW

Two questions of set theory from topology

Motivated by the question of whether $\omega \cup \{p\}$ for an ultrafilter p embeds in a nice space we pose the problem of the existence of a stationary subset of $[\omega_2]^\omega$ with the properties that the sup function is 1-1 and such that it reflects stationarily often.

Secondly we prove that it is consistent to have $\mathfrak{p} = \omega_1 < \mathfrak{c}$ and the following principle: Given ideals $\mathcal{A}, \mathcal{B} \subset [\omega]^\omega$ such that $|\mathcal{A}| + |\mathcal{B}| < \mathfrak{c}$ and $\mathcal{A} \cap \mathcal{B} = [\omega]^{<\omega}$, then there is a $C \subset \omega$ such that $|C \cap A| = \omega$ for all $A \in \mathcal{A}$ and $|C \cap B| < \omega$ for all $B \in \mathcal{B}$. In fact there is a σ-centered poset which adds a set C as required and which does not "fill" any tower. This model was constructed so as to produce a model in which every compact, separable radial space is Fréchet. (A space is radial if $x \in \bar{A} \Rightarrow$ there is a well-ordered sequence from A converging to x – and Fréchet if the sequence can always be countable.)

JÖRG BRENDLE

Cardinal invariants related to ultrafilters on ω

Given a free ultrafilter $\mathcal{U}$ on ω, let us define the following four cardinal invariants:

$$p(U) = \min\{|\mathcal{F}|;\ \mathcal{F} \subseteq U, \neg\exists A \in U\,\forall B \in \mathcal{F}(A \subseteq^* B)\}$$

p measures the P-pointness of U; in particular U is a P-pt iff $p(U) \geq \omega_1$

$$\begin{aligned}\pi p(U) &= \min\{|\mathcal{F}|; \mathcal{F} \subseteq U, \neg\exists A \in [\omega]^\omega \forall B \in \mathcal{F}(A \subseteq^* B)\}\\ \pi\chi(U) &= \min\{|\mathcal{F}|; \mathcal{F} \subseteq [\omega]^\omega, \forall A \in U \exists B \in \mathcal{F}(B \subseteq^* A)\}\end{aligned}$$

the π-character of U.

$$\chi(U) = \min\{|\mathcal{F}|;\ \mathcal{F} \subseteq U, \forall A \in U \exists B \in \mathcal{F}(B \subseteq^* A)\}$$

the character of U. We discuss the relation between these cardinals and some of the classical cardinal invariants of the continuum, like the unbounding number, the dominating number, the splitting number and the reaping number, we also state several consistency results showing that the cardinal coefficients may be different for different ultrafilters, and sketch the proof of the following result.

THM.
(a) CON (*there are Ramsey ultrafilters U and ϑ s.t. $p(U) = \omega_2 = c$ and $\pi p(\vartheta) = \omega_1$*).
(b) CON (*there are Ramsey ultrafilters U and ϑ s.t. $\chi(U) = \omega_1$, and $\pi\chi(\vartheta) = \omega_2 = c$*).

TOMEK BARTOSZYNSKI (results of joint work with Saharon Shelah)

Strongly meager sets

Def. A set $X \subseteq 2^\omega$ is strongly meager if $X + H \neq 2^\omega$ for every measure zero set $H \subseteq 2^\omega$.

Let SM = the collection of all strongly meager sets.

Theorem 1. *Suppose that $\kappa > \aleph_0$ is a regular cardinal. It is consistent that*

$$MA_{<k} + SM = [\mathbb{R}]^{<\kappa}.$$

Theorem 2. *Suppose that $\kappa > \aleph_0$. It is consistent that $SM = [\mathbb{R}]^{<\kappa}$. In particular, if $cf(k) = \aleph_0$ then SM is not a σ-ideal.*

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