Mathematisches Forschungsinstitut Oberwolfach

Tagungsbericht 3/1996

Kombinatorik

14. bis 20.1.1996

Die Tagung fand unter der Leitung von Herrn Deuber (Bielefeld), Herrn Jackson (Waterloo) und Herrn Jungnickel (Augsburg) statt.

One of the main aims of this meeting was to bring together people from different parts of combinatorics, such as enumeration theory, finite geometry and Ramsey theory. This was reflected by talks on a large variety of topics, which contained interesting material for both specialists and nonspecialists. A lot of interest showed up in a problem session, where everyone was invited to give a short talk on a recent research question.

Vortragsauszüge

1. Hauptvorträge

N. Alon:
Max-Cut: Algorithmic, Probabilistic and Algebraic Aspects
The (simple) max-cut problem is the problem of determining the maximum number of edges in a cut of a given input graph. This algorithmic problem

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is motivated by questions in various areas, including statistical physics and theoretical computer science, and its study combines combinatorial and geometric ideas. We discuss some recent (and less recent) developments in the study of this problem and some related extremal questions, focusing on the techniques used, and mentioning several related open problems.

F. Bergeron:
MacDonald Polynomials and Associated Modules

For a given partition μ of an integer n, we consider the determinant $\Delta_\mu :=$ det $|x_i^{a_j} y_i^{b_j}|$, where the pairs (a_j, b_j) are coordinates of the squares in the Ferrer's diagram of μ. We then consider the linear span $\mathcal{H}_\mu$ of all partial derivatives of Δ_μ. Garsia and Haiman have conjectured that the bivariate Frobenius characteristic of $\mathcal{H}_\mu$ is given by "MacDonald" polynomials. We give new results and conjectures concerning the intersection $\bigcap_{\nu \in S} \mathcal{H}_\nu$, for $S \subset$ predecessors(μ) in the Young lattice order, as well as for the associated bigraded characters.

F. Brenti:
Kazhdan-Lusztig Polynomials from a Combinatorial Point of View

We introduce the concept of the "KL-polynomial" of a weighted, locally finite digraph, and show that this coincides with the classical Kazhdan-Lusztig polynomials for certain weightings of the Bruhat graph of a coxeter group. We then discuss two long-standing conjectures on Kazhdan-Lusztig polynomials, namely the nonnegativity and the combinatorial invariance conjectures, and survey what is known about them as well as present some new conjectures.

S. Fomin:
Matrix Factorisation and Total Positivity Criteria

An elementary Jacobi matrix is a matrix of the form $I + eE_{i,i+1}$, $I + fE_{i,i-1}$ or $I + (h-1)E_{i,i}$, where I is the identity matrix and $E_{i,j}$ is the matrix with entry 1 in the position (i, j) and entries 0 in all other positions. A product of such matrices, with nonnegative parameters e, f, h, is always (weakly) totally positive (TP), i.e., all its minors are nonnegative. By an old result of A.Whitney, the converse is also true: any $n \times n$ TP matrix can be factored in this way. Efficient TP criteria can be obtained from explicit formulas for such factorizations. We give these formulas via a combinatorial Ansatz

based on (i) a representation of factorization schemes by "double pseudo-line arrangements", and (ii) a peculiar involution on the set of TP matrices.
This is a joint work is with A.Zelevinsky. See the (Oberwolfach February 95) abstract for the connections with the combinatorics of Lusztig's canonical basis and his theory of TP in reductive groups. See also the relevant paper at http://www-math.mit.edu/~fomin/.

C. Godsil:
Antipodal Covers of Complete Graphs

A distance-regular graph with diameter d is **antipodal** if "is at distance d from" is an equivalence relation on its vertices (the 3-cube is an example). An antipodal distance-regular graph with diameter d is necessarily a covering graph of a distance-regular graph with diameter $\lfloor \frac{d}{2} \rfloor$ (so the 3-cube covers K_4). I am concerned with antipodal distance-regular graphs of diameter three. These graphs are related to a wide range of combinatorial structures - Hadamard matrices, projective planes, Moore graphs and group divisible designs, for example.
My talk provides an overview of these graphs, and a description of recent work, which gave new constructions from subspaces of non-degenerate quadratic forms over $GF(q)$, q even.

D. Hachenberger:
The Additive Group of a Finite Field

We consider the m-dimensional extension $E := GF(q^m)$ over the Galois field $F := GF(q)$. An element of E is called **free over** F provided it generates a normal basis of E over F. An element of E is called **completely free** over F provided it **simultaneously** generates a normal basis over **every** intermediate field of E over F. While the existence of free elements, i.e., the Normal Basis Theorem for finite fields, is well known (K. Hensel 1888), the existence of completely free elements in finite fields has only been settled by D. Blessenohl and K. Johnsen in 1986. They proved existence in extensions of prime power degree and affirmatively settled the general question by using a reduction argument whose analogue for the case of free elements is well known.
In our talk we have presented recent results concerning the characterizations, the structure, the enumeration, the algorithmic and explicit construction of completely free elements in finite fields.
Referring to this, we have studied certain decompositions Δ of the minimal

polynomial $x^m - 1$ of the Frobenius automorphism in E over F which correspond to particular decompositions of the additive group of E and which allow construction of arbitrary completely free elements by working independently on the direct summands of the additive group of E corresponding to the factors of Δ. Particular emphasis is focused on field extensions for which the canonical decomposition $\prod_{d|m} \Phi_d$ of $x^m - 1$ (with Φ_d being the dth cyclotomic polynomial) allows independent construction.
The results will appear in the forthcoming monograph: *Finite Fields: Normal Bases and Completely Free Elements.* Kluwer Academic Publishers, Boston, 1996.

H. Lefmann:
Multicolored Subsets in Colored Hypergraphs

We consider anti-Ramsey type problems for k-uniform hypergraphs. We study the maximum size of totally multicolored subsets $Y \subset X$ w.r.t. colorings $\Delta : [X]^k \to \mathbf{N}$, $|X| = n$, where distinct k-element sets with intersection size at least l are colored differently. It can be shown that $|Y| \geq c_k (\ln n)^{\frac{1}{2k-1}} n^{\frac{n-k}{2k-1}}$, and this is sharp up to a multiplicative constant. Extending and improving former results, various related problems on Erdös-Rado numbers, Sidon sets and geometrical selection problems are discussed.

V. Rödl:
Extremal Problems on Set Systems

For a family $\boldsymbol{F}(k) = \{\mathcal{F}_1^{(k)}, \mathcal{F}_2^{(k)}, \ldots, \mathcal{F}_t^{(k)}\}$ of k-uniform hypergraphs let $ex(n, \boldsymbol{F}(k))$ denote the maximum number of k-tuples which a k-uniform hypergraph on n vertices and not containing any member of $\boldsymbol{F}(k)$ may have. Let $r_k(n)$ denote the maximum cardinality of a set of integers $Z \subset [n]$, where Z contains no arithmetic progression of length k.
For any $k \geq 3$ we introduce families $\boldsymbol{F}(k) = \{\mathcal{F}_1^{(k)}, \mathcal{F}_2^{(k)}\}$ and prove that

$$c_k n^{k-1} \geq ex(nk^2, \boldsymbol{F}(k)) \geq (n^{k-2}) r_k(n)$$

holds. We conjecture that $ex(n, \boldsymbol{F}(k)) = o(n^{k-1})$ holds.
If true, this would imply the celebrated result of Szemerédi stating that $r_k(n) = o(n)$. By an earlier result of Ruzsa and Szemerédi, our conjecture is known to be true for $k = 3$. The main objective is to verify the conjecture for $k = 4$. We also consider some related problems.

B. Schmidt:
New Developments in the Theory of Difference Sets

The investigation of regular respectively quasiregular automorphism groups of designs, in particular finite affine and projective geometries, leads to the notion of a difference set respectively relative difference set. Difference sets have been studied intensively for more than half a century. But only recently, some really satisfactory results on the existence of difference sets have been proved. These results rely on new methods of construction, in particular the so-called K-matrix method, and new ways of combining results from algebraic number theory with combinatorial arguments. With the help of these ideas it is even possible to obtain a complete classification of certain types of difference sets. But it also will be outlined that there are a lot of important open questions concerning the classification of other types of difference sets.

R.P. Stanley:
Hyperplane Arrangements and Trees

The **braid arrangement** B_n is the set of hyperplanes $\chi_i - \chi_j = 0$, $1 \leq i < j \leq n$, in $\mathbf{R}^n$. We consider some "deformations" of B_n with interesting connections to the enumeration of trees, as well as to the enumeration of internal orders. The two main examples are the **Shi arrangement** $\mathbf{S_n}$ and the **Linial arrangement** $\mathbf{L_n}$. S_n consists of the hyperplanes $\chi_i - \chi_j = 0, 1$, for $1 \leq i < j \leq n$. It has $(n+1)^{n-1}$ regions and characteristic polynomial $q(q-n)^{n-1}$. The number of regions R which are separated from a certain base region R_0 by j hyperplanes is the number of trees with vertices $0, 1, \cdots, n$ and with $\binom{n}{2} - j$ inversions. The arrangement L_n is given by $\chi_i - \chi_j = 1$, for $1 \leq i < j \leq n$. The number of regions is the number of "alternating trees" on $n+1$ vertices, and the characteristic polynomial $\chi_n(q)$ has all its nontrivial zeros on the line $\mathrm{Re}(q) = n/2$. This work has been carried out in collaboration with C.Athanasiadis, L.Pak and A.Postnikov.

A. Steger:
On evolution and threshold phenomena

The study of the evolution of a random graph, initiated by the pioneering work of Erdös and Renyi in the early sixties, is by now quite well understood and we will survey some results. Our main aim, however, is to propose some new directions of further research: the study of the evolution of graphs satisfying some additional properties. We illustrate and support this by surprising new results on the evolution of triangle-free graphs and of partially ordered sets.

J. Stembridge:
Coxeter Groups: Heaps, Reduced Words and Algebraic Deformations

Let (W, S) be a Coxeter system. For $w \in W$ let $R(w)$ denote the set of reduced words $w = s_1 \cdots s_l : s_i \in S$. Declare two reduced words equivalent if they differ by a sequence of interchanges of commuting generators in S. This equivalence relation patitions $R(w)$ into "commutivity classes"; each such class C can be expressed as the set of linear extensions of a canonically associated poset (the "heap"). If $R(w)$ has just one commutivity class, we say that w is "fully commutative" (FC). In this talk, we discuss
(1) characterazations of FC elements (for example, in S_N, w is FC iff as a permutation of $\{1, ..., N\}$ there is no decreasing subsequence of length 3),
(2) classification of Coxeter groups W with finitely many FC elements,
(3) classification of finite Coxeter groups via heaps,
(4) quotients of the group algebra and Hecke algebra with bases indexed by the FC $w \in W$ (example: for S_N, the Temperley-Leib algebra),
(5) enumeration of FC elements via representation theory of W.
Note: Several of the results we discuss have also been independently obtained by K.Fan and J.Graham in their doctoral theses (MIT 1995 and Sydney 1995, resp.).

V. Welker:
Combinatorics of Discriminants, Ordered and Unordered Configuration Spaces

We define combinatorial stratifications of discriminantal hypersurfaces in $\mathbf{R}^n$ and $\mathbf{C}^n$. We consider substrata of the standard unordered discriminant (i.e, the space of polynomials of degree n with at least one double zero) and the standard ordered discriminant (i.e, the space of n-tuples of complex numbers $(z_1, ..., z_2) \in \mathbf{C}^n$, such that at least two coordinates are equal). These stratifications are indexed by number partitions. The analysis of topological and homological behaviour in the ordered case is reduced via formulas of Goresky & MacPherson and Ziegler & Zivaljevic to the study of certain partition lattices. The unordered case requires the analysis of certain lattices of compositions. In both cases we describe the topology of the one-point compactification of some strata via analysis of the combinatorics of the associated lattices.

Q. Xiang:
Hadamard Differece Sets and Projective Codes

Hadamard difference sets are difference sets with parameters $(4m^2, 2m^2 - m, m^2 - m)$. Because of their close connection to Hadamard matrices and perfect binary arrays, Hadamard difference sets have been studied extensively. The basic question about Hadamard difference sets is for each integer m, which groups of order $4m^2$ contain Hadamard difference sets.

In this talk, we discuss a connection between reversible Hadamard difference sets and projective 3-weight (2-weight) codes. Under the condition that certain 2-weight codes exist, we give a construction for Hadamard difference sets in certain groups of order $4p^4$, where p is an odd prime. If we choose certain special spreads in $PG(3,p)$ and let $p \equiv 3 \bmod 4$, this basically gives the construction of Xia.

In the case $p \equiv 1 \bmod 4$, we discuss the recent construction of Hadamard difference sets in $Z_2 \times Z_2 \times Z_5^4$ by van Eupen and Tonchev. In the cases $p = 13, 17$, we discuss the construction of the required 2-weight codes in the construction of Hadamard difference sets in more detail. By a composition theorem, it follows that Hadamard difference sets exist in certain groups of order $4m^2$, for $m = 2^a 3^b 5^{2c_1} 13^{2c_2} 17^{2c_3} p_1^2 p_2^2 \cdots p_t^2$, where the p_i are primes $\equiv 3 \bmod 4$.

G.M. Ziegler:
Extremal Problems on Polytopes

This talk presents a survey of extremal problems of the type

- What is the maximal number of vertices?
- What is the maximal length of an increasing path?
- What is the maximal number of vertices of a 2-dimensional projection ("shadow")?

for certain classes of polytopes:

- d-dimensional polytopes with at most n facets
- d-dimensional 0/1-polytopes.

These questions are motivated by classic problems of linear and integer programming, concerning the worst-case behavior of simplex algoritms and cutting plane approaches.

Current progress includes the concept of "deformed products" of polytopes that provides a uniform description (and proofs) for virtually all exponential

lower bound constructions for variants of the simplex algorithm, as well as a $\Theta(n^{\lfloor \frac{d}{2} \rfloor})$ bound for the number of vertices in a 2-dimensional shadow (for fixed dimension d).
(This is a joint work with N. Amenta)

P.-H. Zieschang:
Structure Theory of Association Schemes

As a concept related to many combinatorial objects (codes, designs, graphs, etc.) association schemes have gained considerable interest during the last thirty years. It is widely believed that association schemes play a central role in algebraic combinarorics. On the other hand, one cannot overlook that, as mathematical objects of their own, association schemes have not yet been considered too much so far.
Following group theory (groups can be viewed as association schemes) we subdivide the general structure theory into three chapters: **local** theory, **representation** theory, **geometric** theory. The local theory allows the Jordan-Hölder Theorem. As a consequence, we shall be looking for simple objects.
The famous Feit-Higman Theorem is part of the representation theory of association schemes. In the geometric theory, we offer characterizations of (certain classes of) Bruhat-Tits buildings and Moore geometries (2-designs with $\lambda = 1$).

2. Problem Session

C. Bessenrodt:
Combinatorics related to modular representations of the symmetric groups

A report was given on some combinatorial concepts which evolved in the investigation of p-modular representations of the symmetric groups S_n. In the recent work of Kleshchev on p-modular branching results p-good nodes of partitions were introduced, which have already been recognized as important also in other contexts. Applying these, the long-standing Mullineux conjecture (describing a combinatorial algorithm for computing the tensor product of a p-modular irreducible S_n-representation with the sign representation) was reduced to an intricate combinatorial conjecture; Ford and

Kleshchev then found a long involved proof for this. In joint work with Olsson we have now found a new shorter proof using the Mullineux symbols for p-regular partitions, in which also the position of the p-good nodes was clarified. These symbols turned out to be useful for other applications as well.
For the modular *spin* representations of the symmetric groups not even the 'right' set of partition labels (corresponding to the p-regular partitions in the linear case) is known for $p > 5$. For $p = 3$ and $p = 5$ suitable partition labels were described in joint papers with Andrews, Morris and Olsson; for $p = 5$ this required a proof of a partition identity conjectured by Andrews in 1974.

A. Björner:
The Antiprism Fan Construction

Let P be a d-dimensional convex polytope and P^* its polar dual with respect to the origin (interior to both). The map $x \to (x, -1)$ places a copy of P in the hyperplane $x_{d+1} = -1$ in $\mathbf{R}^{d+1}$ and $x \to (x, 1)$ places a copy of P^* in the parallel hyperplane $x_{d+1} = 1$. We now identify P and P^* with these copies.
Construct a d-dimensional polyhedral complex C_P sitting in $\mathbf{R}^{d+1}$ (possibly with self-intersections) as follows. Take for facets of C_P: P, P^* and all polytopes of the form $F \vee F^*$, where F is a proper face of P. Here "$\vee$" denotes the join operation, which is legal since F and its dual F^* sit in skew subspaces with complementary dimensions. Thus $F \vee F^*$ is a d-polytope.
Theorem:
1. C_P is a shellable polyhedral sphere embedded in $\mathbf{R}^{d+1}$ without self-intersection and with the origin in its interior.
2. C_P is the boundary of a convex polytope if and only if the following condition holds for P:
(*) for each proper face F of P the perpendicular in $\mathbf{R}^d$ from the origin to the affine span of F intersects aff(F) in the relative interior of F.
3. C_P is star-convex with respect to the origin of $\mathbf{R}^{d+1}$, i.e. any ray emanating from 0 intersects C_P in exactly one point.
4. The face lattice of C_P is $\cong$ to the poset of all intervals of L_P (the face lattice of P) ordered by reverse inclusion (include the empty interval also).
5. By 3 we have that C_P spans a complete fan F_P centered at $0 \in \mathbf{R}^{d+1}$. If P is rational, so is F_P. The toric h-polynomial of F_P is related to the

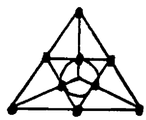
g-polynomials of P, P^* and all their faces F, F^* via:

$$h_{F_P}(x) = \sum_{F \in L_P} x^{\dim F+1} g_F(\frac{1}{x}) g_{F^*}(x).$$

A.A. Bruen:

Cotangency Sets and $(10,3)$-configurations

In a plane Π, S (a subset of the points) is a **cotangency set** if there exists an inflection f : points of S $\rightarrow$ lines of S satisfying the following properties:
(a) for P in S the line $p = f(P)$ does not contain P,
(b) for P, Q in S we have that P, Q, $f(p)$, $f(q)$ are collinear.
Our main result was that S cannot contain a quadrangle.
G. Pickert has established a connection between this result and the following $(10,3)$-configuration $\longrightarrow$



In particular, it can be imbedded in $PG(2, F)$, F a division ring, if and only if F is not a field. There is a connection between this $(10,3)$- and the Desargues configuration. We conjecture that the $(10,3)$-configuration is embedded in a finite plane Π if and only if Π is non-Pappian. Some evidence is provided. The method also provides evidence in support of a celebrated conjecture of H. Neumann on the existence of Fano configurations in non-Desarguesian planes.

W. Deuber:

Independent Subsets in Triangle Free Graphs

Recently Hajnal asked whether every triangle free graph on the positive integers contains a Hindman set. Erdős asked the finite version of it. Recently Rödl was able to settle Erdős question in the positive sense. From his approach it is clear to specialists that the same holds for artithmetic progressions, for partition regular systems of equations and for many other structures which are embeddable into parameter sets.
As every graph naturally is a metric space the problem can be formulated for metric spaces where it is widely open.
Addendum of 30th of January: Hindman found a counterexample for Hajnal's question.

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D.M. Jackson:
Some Questions Relating to Two-Cell Embeddings of Graphs and Jack Symmetric Functions

A **map** is a two-cell embedding of a graph in a locally orientable surface. The **genus series** is the generating series for such maps, having been rooted, with respect to the number of vertices, faces and edges. The underlying theory, both analytic and algebraic, appears in connexion with metric models, and there is a variety of topological questions about mappings between surfaces and monochromy, where there are constructions that reduce such questions to a combinatorial problem.

It has been shown that the genus series for hypermaps (face 2-colourable maps) can be represented in the form

$$(1+b)t\frac{\partial}{\partial t}\log\left(\sum_{\theta}\frac{t^{|\theta|}}{< J_\theta, J_\theta >_{1+b}}J_\theta(\underline{x};1+b)J_\theta(\underline{y};1+b)J_\theta(\underline{z};1+b)\right)\Bigg|_{t=1},$$

where $J_\theta(\underline{x};1+b)$ is the Jack symmetric function in $\underline{x}=(x_1,x_2,\cdots)$ indexed by a partition θ, with parameter $1+b$, the sum is over all partitions, and $< \cdot,\cdot >_{1+b}$ is the usual inner product. Restriction to orientable and locally orientable surfaces is by $b=0,1$ respectively. Goulden and I have conjectured that the expansion of this series has coefficients that are polynomials in b with nonnegative integer coefficients, and that b marks a combinatorial statistic for rooted maps.

In recent work (with I.P. Goulden and J. Harer) on the orbifold characteristic of the moduli spaces of real curves, we have shown that this characteristic can be obtained by a simple expression involving a parameter α that can be specialized to obtain the complex case (treated by Harer and Zagier). There are some technical reasons to believe that α is related to the Jack parameter mentioned above, and that knowledge of the combinatorial interpretation of one can be transported to the other context.

D. Jungnickel:
Signed Steiner Forests

We pose the following combinatorial optimization problem, which comes from the study of decoding algorithms for ternary graphical codes.

Problem ("Signed Steiner Forest"):

(*) Let $G=(V,E)$ be a connected digraph with girth g and consider a set of $2w$ vertices half of which have sign $+$, while the others have sign $-$. Find a forest S of smallest cardinality such that the discrepancy between the number of plus-vertices and the number of minus-vertices in each component tree is a multiple of 3.

For our purposes, it would be sufficient to find an efficient algorithm for this problem under the additional assumptions $g \geq 2w+1$ and that a forest with the required properties and cardinality at most $(g-1)/2$ exists. Under these assumptions we have an $O(|E|^2)$-algorithm, but we hope that there is a simpler optimization approach solving Problem (*) in general or at least under the assumption $g \geq 2w+1$.

D.M. Kulkarni:
Combinatorics of the Ladder Determinantal Ideal

Let X be a matrix of indeterminates. A subset S of X is said to be saturated if the following holds: if the principal diagonal of any minor lies in S, so does the hole minor. The ideal generated by the $p \times p$ minors of S in the corresponding polynomial ring is shown to be prime (Abhyankan, Narsimhan), Hilbertian (Abhyankar, Kulkarni) and Cohen-Macaulay (Herzog, Trung). We discuss the method of constructing its Hilbert polynomial by counting nonintersecting p-tuples of paths lying within S and having a fixed number of north-west turns. A nice expression for even a special case of Ferrer's diagram shape in general ($p > 2$) is not known. The question of log-concavity of its h-vector is still open. Of course, it is a special case of Stanley's conjecture about h-vectors of a standard graded Cohen-Macaulay domain.

K.Leeb:
Tarski's "logical notions" applied

I have, in a very compressed form, reported one positive and one contrasting negative result I had obtained last summer: Dedekind-Lindenbaum-Tarski's "Kettentheorie" supports the distributive law $K \cdot B - K \cdot a = K \cdot (B - a)$ for non-rigid K only up to $K = 2$. For non-rigid $K \geq 3$ this law fails.

For details I have refered to my "Kombinatorik der Komposition..." (and supplements pp. 121-183 and pp. 185-196), where the following facts are presented:

page	for		comments
187, 193	pointed $\underset{\cdot}{3}$	$\underset{\cdot}{3} \times 2 \leq \underset{\cdot}{3} \times 3 \not\Rightarrow 2 \leq 3$	$\leq$ means injections
187	pointed $\underset{\cdot}{3}$	$\underset{\cdot}{3} \times 3 \leq \underset{\cdot}{3} \times 4 \not\Rightarrow 3 \leq 4$	$\leq$ means surjections
188/9, 195	non-rigid 2	$2 \mid B \wedge 2 \mid a \Rightarrow 2 \mid B - a$	classification of all complementations
194	non-rigid 2	$3 \mid B \wedge 3 \mid a \not\Rightarrow 3 \mid B - a$	
190	signed sets	$\sqrt[n]{0} = 0$	
	n rigid		
191	K variably pointed by $A \to K$, $B \to K$	$K \times A = K \times B \Rightarrow A = B$	the ultimate in canceling $K \times \cdot$
133	n rigid	${}^nX = {}^nY \Rightarrow X = Y$	

Related material can be found on pp.134,135,168-178.

I. Schiermayer:
3-Colourability and Combinatorics

We count the maximum number of maximal independent sets for paths and cycles to provide an improved (excact) algorithm for the 3-colourability problem. Motivated by Mycielski's graph, we construct classes of graphs with chromatic number 4. We apply a ramsey argument for a 3-colouring of an odd cycle to show that these graphs are not 3-colourable.

R.-H. Schulz:
Error Correcting Check Digit Systems

Given a finite abelian group $(A, +)$ as alphabet and automorphisms $\beta_1, ...,$ $\beta_{n+1}, \gamma_n, \gamma_{n+2}$, furthermore $\beta_{n+2}, \gamma_{n+1} \in \mathrm{Aut}\ A \cup \{0\}$, one can consider the check digit system $A^n \to A^{n+2}$ determining to every word $a_1 \cdots a_n$ two check digits a_{n+1}, a_{n+2}, such that the equations $\sum_{i=1}^{n+2} \beta_i(a_i) = 0 = \sum_{i=1}^{n+2} \gamma_i(a_i)$ are fulfilled. Under certain conditions on the β_i and γ_i, one is able to detect double errors and to correct single errors and neighbour transpositions $(a_1 \cdots a_i a_{i+1} \cdots a_{n+2} \to a_1 \cdots a_{i+1} a_i \cdots a_{n+2})$. This is a generalization of the work by Sethi, Rajaraman and Kenjale published in Inform.Proc.Letters 7/2, p.72-77 in 1978.
By the Hamming bound, $n + 2 \leq |A|$. It is not known to us whether the upper bound can be improved.

T.I. Visentin:
Counting Maps in Orientable Surfaces of Arbitrary Genus

A map is a 2-cell embedding of a graph in a surface. The genus series for a class of maps is the formal generating series for the number of such maps with respect to the genus of the underlying surface. By appealing to the embedding theorem for orientable surfaces, one can encode a map on $2n$ edges by a permutation ν on 2n symbols whose cycle lengths are the degrees of the vertices. When ν is multiplied by a canonical representative of the conjugacy class of fixed point free involutions, one obtains a permutation whose cycle lengths give the degrees of the faces of the map. Using the Euler-Poincaré formula, the genus of the underlying surface is now easily obtained. The combinatorial problem of counting maps in surfaces now becomes a problem in the group algebra of the symmetric group and the genus series can be expressed in terms of sums of characters of irreducible representations.

In this talk we describe this combinatorial encoding of maps and discuss the results which one can obtain using these methods. For example, if $M(u, x, y, z)$ is the genus series for all rooted maps and $Q(u, x, y, z)$ is the genus series for rooted quadrangulations with the indeterminate u marking genus and x, y, z marking numbers of vertices, faces, edges, respectively, then one can show that

$$2Q(u^2, x, y, z) = M(4u^2, x + u, x, z^2 y) + M(4u^2, x - u, x, z^2 y).$$

When the coefficient of u^0 is extracted on both sides, one obtains the well-known fact that the number of quadrangulations of the sphere is equal to the number of all maps on the sphere. This can be shown to be true, by a simple combinatorial construction, but no such construction has been found which explains this result for higher genera. We have recently found a generalization which describes a relationship between Eulerian maps and all hypermaps and we hope that the extra information given by this new result will help us find the combinatorial construction which we seek.

The result relating quadrangulations and all maps is of some importance in theoretical physics as it implies a relationship between two matrix models for 2-dimensional quantum gravity, namely the ϕ^4 model and the Penner model.

M. Wachs:
Whitney Homology of Semipure Shellable Posets

It is demonstrated in recent work of Sundaram that Whitney homology of a (pure) Cohen-Macaulay poset P can be used as a powerful tool for computing the character of an automorphism group G acting on the top homology of P. Here we generalize this technique to semipure sequentially Cohen-Macaulay posets by introducing a refinement of Whitney homology for semipure posets. We apply the technique to computing the representation of the homology (in each dimension) for a general class of semipure posets of partitions of $[n]$. Namely, for fixed positive integers m, d, the poset of partitions whose block sizes are of the form $m + id$, $i \geq 0$, is shown to be semipure and shellable ($\Rightarrow$ sequentially Cohen-Macaulay), and a plethystic expression for the Frobenius characteristic of its homology representation is given. This generalizes formulas of Calderbank-Hanlon-Robinson and Sundaram.

D.G. Wagner:
Chow Homology for Partially Ordered Sets

We apply the presentation for Chow homology of a complete toric variety, due to Fulton and Sturmfels, to the case arising from the order polytope P of a finite partial order (with $\hat{0}$ and $\hat{1}$). In this case, the homology is free; the codimension one cohomology is the cycle space of the Hasse graph of P with a new edge joining $\hat{0}$ and $\hat{1}$; and the dimension one homology is the free abelian group generated by the connected components of P with $\hat{0}$ and $\hat{1}$ removed. We give an interpretation for all Betti numbers when P belongs to a certain small class of posets. The general interpretation of these Betti numbers is open.

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