Variationsrechnung und Optimale Steuerung – Optimal Control

21.–27.1.1996

Die Tagung wurde von R. Bulirsch, TU München und K.H. Well, Uni Stuttgart geleitet.

47 Wissenschaftler aus dem In- und Ausland haben an der Tagung teilnehmen können. Davon 6 Teilnehmer aus den Vereinigten Staaten von Amerika, und eine erhebliche Anzahl aus Lettland, Weißrußland, der Ukraine und Rußland (6). Darüber hinaus nahmen auch Gäste aus Österreich, Frankreich, Italien, Polen, Israel und Vietnam teil.

In den Vorträgen wurden Grundlagenfragen ebenso behandelt wie Probleme der Praxis. Vorträge die eher Grundlagenfragen zuzurechnen sind, befaßten sich mit abstrakten optimalen Steuerungsproblemen, Garantierte Zustandsschätzung in Dynamischen Systemen (Chernousko), Stabilisierung von Dynamischen Systemen durch Methoden der optimalen Steuerung (Kirillova), Synthese Probleme für lineare und nichtlineare Systeme (Korobov), Ein neuer Ansatz um das Beobachtbarkeitsproblem für lineare nichtstationäre dynamische Systeme zu Untersuchen (Kopeijkina), Anwendung globaler Methoden der optimalen Steuerungstheorie auf einige Flugmechanikaufgaben (Krotov), Über die Steuerbarkeit der Rotation eines flexiblen Balkens (Krabs), Entwurf einer optimalen Rückkopplungssteuerung mit logarithmischen Barrierefunktionen (Leibfritz), Hinreichende Bedingungen und Sensitivitäts-Untersuchungen für nichtlineare Steuerungsaufgaben (Maurer), Konvergenz von Approximation zu nichtlinearen optimalen Steuerungsproblemen (Malanowski), Ein punktweise gültiges Maximumprinzip für Optimalsteueraufgaben mit mehrdimensionalen Integralen als notwendige und hinreichende Optimalitätsbedingung (Pickenhain), Spezifische Aspekte bei Optimalsteuerungsproblemen für elliptische Systeme (Raitums).

Eine große Anzahl von Beiträgen waren den Anwendungen gewidmet, zum Beispiel: Nichtlineare kinematische Härtung (Brokate), Numerische Lösung von Verfolgungsspielen mit Dynamischem Programmieren (Falcone), Anziehende Effekte bei optimalen Transportflüßen (Klötzler), optimale Steuerung schwappender Flüssigkeiten (Kraft), Nicht-negative Splineinterpolation (Oberle), Zwei Methoden zur Lösung von Optimalsteuerungsaufgaben mit Zustandsbeschränkungen (Phu), Variations Ansätze für elastische Mehrkörpersysteme (Rentrop und Simeon), Über Optimalsteuertechniken zur Simulation und Optimierung menschlicher Bewegungen (Spägele).

Über effiziente numerische Verfahren wurde berichtet, so über Gitterverfeinerung bei direkten Optimierungsverfahren für Probleme der Optimalen Steuerung (Betts), Mechanische Splines und ihre numerische Behandlung im Spezialfall der stückweise Euler Elastika (Reinsch), Inexakte SQP Methoden und große Optimalsteuerprobleme (E. Sachs), Chernousko's iterative Methode für spezielle Optimalsteueraufgaben (W. Schmidt), Schätzung der adjungierten Variablen in zustandsbeschränkten Optimalsteuerproblemen (von Stryk), Numerische Lösung parabolischer Optimalsteueraufgaben durch Lagrange-Newton Methoden (Tröltzsch).

Eine große Anzahl von Vorträgen befaßte sich mit Anwendungen auf die Luft- und Raumfahrt, so über Existenz optimaler Lösungen für flugnavigatorische Steuerprobleme (Bittner), Robust optimale Steuerung in der Anwesenheit von unvorhersehbaren Störungen (Breitner), Designoptimierung und Mehrfachschießen (Callies), Wichtige Ratschläge zur Quadratischen Konvergenz für die Lösung von Flugbahnoptimierungsaufgaben mit der Mehrzielmethode (Chudej), Optimales aerodynamisches Design für Flüsse mit Schocks (Cliff), Treibstoffoptimaler Aufstieg von Hyperschall Flugsystemen (Grimm), Echtzeitfähige Optimale Steuerung einschließlich paralleler Algorithmen (Kugelmann), Geometrische Überlegungen in der indirekten Lösung von Optimalen Steuerungsaufgaben (Mease), Flugbahnoptimierung und die Schubkraftmodellierung (G. Sachs und Mehlhorn), Dynamik und Steuerung von luftatmenden Hyperschallflugsystemen (D.K. Schmidt), Anwendung des zurückweichenden Horizont Steuerungskonzepts für Verfolgungsspiele (Shinar), Dynamische Steuerung von automatisierten Schleppfahrzeugen auf hochfrequentierten Flughäfen (Siguerdidjane), Treibstoffoptimale periodische Steuerung und Regelung im beschränkten Hyperschallflug (Speyer und Dewell).

Collection of submitted abstracts:

Mesh Refinement in Direct Transcription Methods for Optimal Control
J.T. Betts

An optimal control problem can be transcribed into a finite dimensional nonlinear programming problem (NLP) by discretizing the state and control variables. A fundamental concern in the "direct transcription" method is to insure that the discrete solution to the finite dimension NLP subproblem provides an accurate approximation to the original optimal control. This paper describes a technique for selecting the discretization method as well as the number and location of the discretization points, such that the solution is accurately represented, and the computational efficiency is assured.

Existence of Optimal Solutions for some Problems of Spacecraft Navigation
L. Bittner

By a convexification with only one parameter α the problems of reentry into the atmosphere, of passage to an orbit, of a Brachistochrone with tangent angle constraints o.a. are transformed into so called "weakly relaxed" control problems, so that Fillipov's existence theorem is applicable. By means of the maximum principle it is shown that the optimal solution $\alpha^*(.), u^*(.), x^*(.)$ of the relaxed problems is distinguished by $\alpha^*(t) = 1$ a.e., hence $u^*(.), x^*(.)$ is optimal for the original problem. (This is due to the special character of the quoted problems.) Then a version of Fillipov's theorem for multistage processes is formulated. At last a two-stage Brachistochrone problem demonstrates how the idea of an one parameter convexification works also in the case of certain multistage processes for proving existence of an optimal solution, which is a prerequisite for the application of indirect methods.

Robust optimal control in the presence of unpredictable disturbances
M.H. Breitner

We investigate quite general control problems with unmeasurable and unpredictable disturbances. The disturbances are also present in the state and control constraints. The constraints are transformed with the help of different tequniques to deterministic state-control constraints, for which sufficient conditions for the fulfillment have been developped. The differential game theory enables to calculate neccessary conditions, which must be fulfilled by the so-called robust optimal feedback control (Isaacs equation, a partial differential equation of first order). The robust optimal control can be calculated along extremal trajectories by the characteristics method. The characteristics are computed by the multiple shooting method as the solution of multi-point boundary-value problems. At last the feedback controls can be synthezed by neural networks or other high dimensional approximation schemes. The procedure is implemented for various realistic problems, e.g. the reentry of a space shuttle under air density fluctuations and the collision avoidance against a wrong driver on a freeway.

Nonlinear kinematic hardening
M. Brokate

We discuss the wellposedness of certain rate independent elastoplastic constitutive laws, considered as functions of time only (not as function of space). We show that some such laws of kinematic hardening type, which are used in mechanical engineering for e.g. the modeling of ratchetting, can be reduced to a differential equation including a hysteresis operator (namely, the vector play), and prove the wellposedness. We conclude with some remarks on optimal control of such systems.

Design Optimization and Multiple Shooting
R. Callies

The program of design optimization is fully embedded into the formalism of the calculus of variations. Thus even complicated restrictions and constraints can be handled in a correct manner. The numerical solution of the resulting boundary-value problems is by a new version of the multiple shooting method ("Janus No. 2"). New features are dynamic nodes, a fully decoupled approach for the calculation of the sensitivity matrix, the treatment of DAE's of index one and an embedded homotopy strategy. As a real-life application, an integrated trajectory and design optimization of a small Venus spacecraft has been calculated. Reduction in calculation time by the new method was about a factor of 16.

Guaranteed State Estimation for Dynamical Systems
F.L. Chernousko

The guaranteed (set-membership or minimax) approach to state estimation is related to the notion of reachable, or attainable sets. We develop an approach, which makes it possible to approximate reachable sets of dynamical systems by ellipsoids which are optimal in certain sense, for example, in the sense of volume. We consider linear and nonlinear systems subject to bounded control and / or disturbances. Two-sided optimal ellipsoidal bounds for reachable sets are obtained. These ellipsoids satisfy certain nonlinear matrix differential equations. Some properties of these equations are investigated. Asymptotic behaviour of ellipsoids for small t and for $t \to \infty$ is studied. Various applications of ellipsoidal bounds are considered including estimates in optimal control, differential games, practical stability. Special attention is given to the guaranteed state estimation in the presence of observations corrupted by bounded measurement errors. The paper presents a survey of results and also some new results obtained recently. For instance, outer estimates for reachable sets are obtained for the case when the disturbance acts on the coefficients of the linear system.

Solving Trajectory Optimization Problems by Multiple Shooting – Some Crucial Advice Concerning Quadratic Convergence

K. Chudej

Multiple shooting is an indirect solution method for optimal control problems which yields precise numerical solutions. The rate of convergence is usually quadratic due to the underlying Newton type method. Applying the general necessary conditions of optimal control to problems with a quadratic control component decelerates the method often to a linear rate of convergence. We identify the crucial boundary and/or interior point condition and propose a reformulation. This restores the quadratic rate of convergence.
Applications include optimal control problems with free final time and/or state inequality constraints; especially optimization of atmospheric ascent or descent of aeroplanes or space craft.

Optimal Design of Fluid Flows with Shocks

E.M. Cliff

We study a class of optimization problems wherein it is possible to modify certain fluid flows by choice of a parameter $g \in Q \subset I\!R^k$; some parameters may change the boundary of the flow domain. Typical of these is the "inverse design" problem wherein the objective functional indicates the closeness of the obtained flow to a given target flow (i.e. $f(u) = ||u - \hat{u}||^2$). The flow is computed from a nonlinear conservation law (e.g. Euler Eq.).
Optimization approaches include "black-box" methods wherein an existing code for approximate solution of the flow equation is used to "evaluate" the flow at fixed g and a nonlinear programming algorithm is used to minimize $I(g) \equiv J(\tilde{u}^h(g))$; $\tilde{u}^h(g)$ being the computed flow solution. In this case we recommend that the gradient ∇I_g be computed via the chain-rule with the required "sensitivity" ($\frac{\partial \tilde{u}}{\partial g}$) computed as the solution of a "Sensitivity Equation" (SE). The SE is obtained by formally differentiating the flow equation wrt g. This raises the issue of "consistent" derivatives since the approximate solutions of (SE) need not be exactly $\frac{\partial \tilde{u}^h}{\partial g}$. A theory is given and some numerical results shown.
A second approach to the problem is to derive an optimality system and employ an algorithm that simultaneously moves toward optimality and feasibility (i.e. satisfy the flow equation). In this approach we believe it is important to consider algorithms that can use existing flow codes with only "high level" modifications. We describe an approach based on Reduced Sequential Quadratic Programming that requires solution of the linearized flow equation and its adjoint. The first of these is available in many flow codes that iterate to a solution. Numerical results are shown for Euler flow in a nozzle, including a single shock. Care must be taken to formulate an appropriate setting so that the map $g \to \tilde{u}^h(g)$ is smooth. This has the spirit of a shock fitting approach but only in the construction of the linearized problem.

Numerical solution of pursuit-evasion games via Dynamic Programming
M. Falcone

The solution of a pursuit–evasion game via Dynamic Programming leads to the characterization of the value function as the unique viscosity solution of the Isaac equation. This approach has been also used to develop numerical schemes which converge to the viscosity solution (i.e. the value function) of that first order partial differential equation under very general hypotheses. The above schemes can give approximate feedback controls in the domain where the solution is computed. Moreover, they provide accurate numerical results (also in cases when the value function is discontinuous) although they are very sensitive to the "rise of dimension“. We will discuss the main features of these algorithms and the issues connected with their parallelization via a domain decomposition strategy. We present some numerical result on the computation of the value function and of the trajectories of the homicidal chauffeur game.

Fuel-Optimal Ascent of a Hypersonic Vehicle
W. Grimm

Fuel-optimal ascent means to maximize the final mass on attaining a specified energy. Three approachs are covered:
1. Solution based on a reduced dynamical model: Using the energy-state model the optimal control can be computed pointwise as a feedback control. The optimal trajectory is the solution of an initial value problem.
2. Guidance based on the reduced solution: An altitude controller tracks the altitude history of the reduced solution. The propulsion control can be extended to a neighbouring optimal control which accounts for small mass variations.
3. The computation of the optimal control with the point mass model fails for long trajectories. The breakthrough is achieved using internal path controllers in the dynamics. The most successful one is an internal path controller.

Stabilization of Dynamic Systems by Optimal Control Methods
F.M. Kirillova

The stabilization problem is one of the central problems of the control theory. The most existing results on the problem deal with given a priori structures. Besides it is seldom that restrictions on control functions are taken into consideration. In the talk the approach which is based on optimal control methods, is suggested. The scheme of constructing the stabilizes includes 1) formation of the auxiliary optimal control problem, 2) method of realization of optimal feedback and 3) proof of the stabilizability of feedback under consideration. We construct the stabilizing feedback with the help of minimum intensity problem, minimum energy-resticted control problem and consider the stabilization of dynamic systems under uncertainty. Nonlinear dynamic systems are investigated using linear approximations and traditional conditions on the right part of the system. The stabilization problem of pendulum in the upper unstable state of equilibrium is solved. Inverted pendulum on moving trolley is considered.

Attraction Effects in Optimal Transportation Flows
R. Klötzler

Transportation flow problems are mixtures of classical transportation problems and problems of optimal control. They allow too the inclusion of combined transports of several objects under some markdowns. This generates with the intention of cheapest transport an interest for collective transports and an attraction of transportation projects. The paper gives a mathematical approach to these phenomena via transportation flow technique, examples, and discussions on the solvability of such problems.

A New Approach to Researching of the Observability Problem for Linear Nonstationary Dynamic Systems
T. B. Kopeijkina

A new universal method is proposed to investigate the observability problem for linear nonstationary dynamic systems (LNDS) (ordinary, time delay systems, neutral type systems) of ordinary differential equations and for linear nonstationary singularly perturbed dynamic systems (LNSPDS) (ordinary, with constant time delay, neutral type systems). This method uses a uniform approach for construction of determining equations. It takes into account the specific character of systems being investigated (their nonstationarity, the presence of time delay, singularity) and does not require the analysis of the conjugate control systems. In terms of components of solution of new determining equations we formulate some results (both new ones and well known previously) of complete observability, relative x-, y-, {x,y}-observability for the above mentioned systems. The rules for the construction of new type determining equations are very simple ones. They reflect the type of the systems being investigated by the natural way and for the stationary systems coincide with the rules well known previously.

Problem of Admissible and Optimal Synthesis for Certain Classes of Systems
V.I. Korobov

Consider the system of differential equations $\dot{x} = f(x, u), x \in I\!\!R_n, u \in \Omega \subset I\!\!R_r$. By positional synthesis of a continuous control we mean the problem of finding a function $u = u(x) \in \Omega$ which is continuous for $x \neq 0$ and is such that the trajectory of the system $\dot{x} = f(x, u(x)), x(0) = x_0 \in Q$ (a neighborhood of zero), terminates at the point $x_1 = 0$ at a finite time $T(x_0)$, i.e. $\lim_{t \to T(x_0)} x(t) = 0$. The set Ω is assumed to be bounded and to contain the point 0 in its interior. We give an approach to the solution of this problem. We construct, for controllable linear system, a set of positional controls, which solves the synthesis problem for a domain Q. (For example, if the system is stable, the problem is solved for any arbitrary bounded neighborhood of zero.) On the basis of this set of controls, we give an admissible maximum principle for the solution of the synthesis problem. For linear system we give a solution of the optimal control problem.

On the Controllability of the Rotation of a Flexible Arm
W. Krabs

We consider the rotation of a flexible arm in a horizontal plane around an axis through the arm's fixed end driven by a motor whose torque is controlled. The model was derived and investigated computationally by Sakawa and co-authors for the case that the arm is described as a homogeneous Euler beam. The resulting equation of motion is a partial differential equation of the type of a wave equation which is linear with respect to the state, if the control is fixed, and non-linear with respect to the control.
Considered is the problem of steering the beam, within a given time interval, from the position of rest for the angle zero into the position of rest under a certain given angle. At first we show that, for every L^2-control which is suitably bounded, there is exactly one (weak) solution of the initial boundary value problem which describes the system without the end condition.
Then we present an iterative method for solving the problem of controllability and discuss its convergence. Finally we demonstrate the method by numerical results.

On Optimal Control of Sloshing Liquids
D. Kraft

In large industrial storage systems containers have to be moved from position A to position B in minimum time. The controlling forces $f(t)$ accelerate the container and the including liquid. The forces are to be applied such that the liquid does not slosh beyond the boundary of the container. The resulting mathematical model consists of a set of ODEs (container movement) and a set of PDEs (sloshing liquid). We consider laminar, viscous flow (Navier Stokes equations). The coupling of the ODEs and the PDEs performs via the accelerated container. The optimal control problem is transformed into an NLP-problem by discretization of the controlling forces f and by solving initial- and initial-boundary-value problems for the ODEs and PDEs, respectively.

Application of global methods of optimal control theory to some problems of flight dynamics

V.F. Krotov

Some mathematical problems of optimal control are considered. Solutions provided absolute optimum are constructed by global methods of optimal control theory. These solutions are used for problems related to control of spacecraft trajectories. The following maneuvers are considered: 1. The flight with constant mass, controlled by angle of attack; 2. The flight of spacecraft with variable mass, controlled by angle of attack and engine thrust. The minimum of consumption for maneuvers is proved and the computer algorithm is constructed.

Real-time Computation of Feedback Controls Including Parallel Approaches

B. Kugelmann

In this paper a feedback method is presented which allows the computation of control corrections in real-time. The scheme is based on the indirect method for optimal control problems and can be applied to a wide class of different and realistic problems. It also has the property of being applicable to different requirements with respect to real-time issues. These features are discussed along with two applied problems: the re-entry problem of an orbiter under several state and control constraints and the minimum energy guidance of three aircraft within dense traffic near some airport.

Logarithmic Barrier Methods For Solving the Optimal Output Feedback Problem

F. Leibfritz

We consider the problem of designing a feedback control law when a complete set of state variables is not available. The resulting nonlinear and nonconvex optimization problem for determining the optimal feedback gain will be solved by a logarithmic barrier approach. The method is tailored to the particular structure of the constant output feedback problem. We consider two algorithms, an Anderson-Moore type and an inexact Newton method, which are embedded in the logarithmic barrier framework. The local and global convergence properties of the algorithms are discussed in detail. Using test examples from optimal output feedback design we can also verify these results numerically.

Convergence of Approximations to Nonlinear Optimal Control Problems

K. Malanowski

An approach of studying convergence of approximations to cone constrained optimization problems in Banach spaces is proposed. In this approach stability results for parametric optimization problems, based on Robinson's implicit function theorem for generalized equations, are exploited. An estimate of the rate of convergence is obtained, provided that the approximations are uniformly strongly regular.
The abstract results are applied to estimate the rate of convergence of Euler's approximations to nonlinear optimal control problems subject to mixed control-state constraints. It is shown that if some constraint qualifications and coercivity conditions are satisfied, the approximations are locally convergent with the same rate as that of space discretization.

Sufficient Conditions and Sensitivity Analysis for Nonlinear Control Problems

H. Maurer

We review recent second order sufficient conditions for optimal control problems with control-state inequality constraints. Second order theory is an important tool for conducting a stability and sensitivity analysis of perturbed control systems. We derive conditions that ensure solution differentiability of the perturbed optimal solutions with respect to parameters in the system. This property is crucial for designing real-time approximations of perturbed solutions. The underlying theoretical concepts are illustrated by the Rayleigh problem from electrical engineering. Second order conditions can be checked via Riccati equations. Sensitivity derivatives are computed explicitly and a convergence analysis of direct optimization methods is performed. A second order theory is developed for the Rayleigh problem with free final time.

Geometric Considerations in the Indirect Solution of Optimal Control Problems

K.D. Mease

Indirect methods for the solution of optimal control problems require the solution of a Hamiltonian boundary value problem (BVP). The fact that the corresponding Hamiltonian vector field preserves volume in the phase (state-costate) space explains the sensitivity to initial conditions that often complicates the solution of the BVP. This sensitivity is paricularly problematic when, in the region of the solution of interest, the flow of the Hamiltonian vector field evolves on two (or more) time scales and exhibits "boundary-layer" behaviour. At the same time, this behaviour implies a geometric structure for the phase flow. The theory of Fenichel establishes, for sufficiently large time-scale separation, the existence of an isolated slow invariant manifold with associated fast stable and unstable manifolds. This structure offers the opportunity for constructing a solution algorithm for this class of problems, in which the sensitivity is actively suppressed. A conceptual overview of an approach being developed is given. The basic strategy is to determine a phase-dependent basis that splits the Hamiltonian vector field into slow, fast/stable and fast/unstable components. The invariance properties of the slow and fast manifolds, along with some guidance from the phase-dependent eigenvalues and eigenvectors of the linear part of the Hamiltonian vector field, are used to determine the desired basis. Forward integration suppressing the fast/unstable motion and backward integration suppressing the fast / stable motion are used to construct an approximate solution to a given BVP by adjusting the remaining degrees of freedom in the unknown initial and final conditions. This is a computational analog of the analytic method of matched asymptotic expansions, with the important difference that no a priori identification of slow and fast variables and small parameters is required.

Non-negative Spline Interpolation
H.J. Oberle

We investigate splines from a variational point of view, which have the following properties: (a) they interpolate given data, (b) they stay non-negative, when the data are positive, (c) for a given integer $k > 0$ they minimize the functional $f(x) = \int_a^b x^{(k)}(t)^2 dt$ for all non-negative, interpolating $x \in W_2^k[a,b]$. Known results for $k = 2$ are extended to larger k, in particular to $k = 3$ and general necessary conditions for solutions of this restricted minimization problem are developed. These conditions imply that solutions are splines in an augmented grid. In addition, we find that the solutions are in $C^{2k-2}[a,b]$ and that they consist of piecewise polynomials in Π_{2k-1} with respect to the augmented grid. It is proved that for general, odd $k \geq 3$ there will be no boundary arcs which means (nontrivial) subintervals in which the spline is identically zero. It is shown also that the occurrence of a boundary arc in an interval between two neighboring knots prohibits the existence of any further knot in that interval. For $k = 3$ it is proved that between given neighboring interpolation knots, the augmented grid has at most two additional grid points. In the case of two interpolation knots (the local problem) polynomial equations are developed for the additional grid points which can be used directly for numerical computation. For the general (global) problem we propose an algorithm which is based on a Newton iteration for the additional grid points and which uses the local spline data as an inital guess. There are extensions to other types of constraints such as two sided restrictions, also ones which vary from interval to interval. Several numerical examples including graphs of splines computed by MATLAB- and FORTRAN-programs are given.

Two Methods for Solving Some Optimal Control Problems with State Constraints

H.X. Phu

To overcome difficulties caused by state constraints, we developed two methods for solving optimal control problems. The first one is called the method of region analysis. Using some function h defined on the state region G, we analyse G to get subdomains, where h is positive, or negative, or equals zero. By this structure of G, we can obtain some information on switching points of the optimal control and on contact intervals of the optimal trajectory with the boundary of the state region G. In some cases, this method can deliver the complete form of the optimal solution. Another possibility for solving optimal control problems with state restrictions is based on the method of orienting curves. Here, optimal trajectory can be constructed part by part, by means of the so-called orienting curves and bottle-neck points. These methods have been used for solving some practical problems, for instance, optimal control of hydroelectric power plants, optimal inventory problems, manipulator trajectory planning ...

A Pointwise Maximum Principle in Optimal Control with Multiple Integrals – as a Necessary and Sufficient Optimality Condition

S. Pickenhain

In the last years a weak version of a maximum principle for control problems with multiple integrals was developed. This maximum principle holds without restricting assumptions, however a numerical use of this principle is quite difficult since in one of its two formulations the adjoint multipliers y are measures, $y \in [L_\infty^{nm}(\Omega)]^*$, and in the other formulation the maximum condition holds in the so-called "integral form" only. In this paper we are able to verify a pointwise maximum condition with multipliers $y \in L_q^{nm}(\Omega), 1 < q < \infty$, for a subclass of problems. If the Hamiltonian is assumed to be convex with respect to the state variable the developed maximum principle is at the same time a sufficient optimality condition.

Specific Aspects of Optimal Control Problems for Elliptic Systems

U. Raitums

The talk is devoted to properties of optimal control problems for elliptic systems $-\mathrm{div}A(x,t,u,\nabla u) + a(x,t,u,\nabla u) = 0, u = (u_1,\dots,u_m), x \in \Omega \subset I\!R^n$. The emphasis is on the case where the set of admissible controls is not convex, for instance, the optimal layout of two materials. The first item of the talk is to describe great difference in properties (such as relaxability, the type of the necessary optimality conditions, etc.) for $m \leq n-1$ and for $m \geq n$. The second item is to discuss the possibilities of the extension of problems for $m \geq n$, especially for the case with a non weakly cost functional.

Mechanical Splines and their Numerical Treatment in the Special Case of Piecewise Euler Elastica

K.D. Reinsch

Mechanical splines are defined as plane curves which represents the central line of thin rods (with circular round cross section) passing through (in a plane) given frictionslessly rotating slides and loaded with given complanate forces (acting at fixed points of the rod). In equilibrium the functional of potential energy of the rod assumes a stationary value. The curve can be piecewisely expressed in terms of elliptic functions.
If stretching is vanishing, the spline is consisting of pieces of Euler's elastica. In this case it will be presented a procedure in numerical calculation of the mechanical spline and specialized in the two applications:
1) The rod only passes given frictionslessly rotating slides.
2) The rod is buckled only by complanate forces (acting at fixed points of the rod).

Variational Approach for Elastic Multibody Systems

P. Rentrop and B. Simeon

We present a variational approach for multibody systems composed of rigid and elastic bodies. The resulting model consists of differential-algebraic equations (DAEs) and partial differential equations (PDEs). As an application a slide crank mechanism is studied. Separation of time and space and a Ritz ansatz in space yields an index-3 DAE (in time). Theoretical and numerical properties are discussed and simulations are presented.

Inexact SQP Interior Point Methods and Large Scale Optimal Control Problems

E. Sachs

Optimal control problems with partial differential equations lead to large scale nonlinear optimization problems with constraints. An efficient solver which takes into account this structure and also the size of the problem is an inexact SQP method, where the quadratic problems are solved iteratively. Based on a reformulation as a mixed nonlinear complementary problem we give a measure when to terminate the iterative QP solver. For the latter we use an interior point algorithm. Under standard assumptions the local linear and superlinear convergence can be proved. The numerical application is an optimal control problem from the nonlinear heat conduction.

Flight Path Optimization Problems Related to Vehicle Fixed Thrust Direction

G. Sachs and R. Mehlhorn

An unresolved problem in airplane performance optimization concerns the modelling of the thrust force direction. There are fundamental optimality difficulties when the thrust force is considered fixed on the vehicle. An approach is presented which is capable of producing a solution for the above problem. A relaxed optimization problem is constructed as an equivalent of the original system and a solution is developed on this basis. A range cruise problem of hypersonic flight is used as a numerical example.

Chernousko's Iterative Method for Special Integral Processes

W.H. Schmidt

For several kinds of optimal control processes described by Volterra or Fredholm integral equations the research group in Greifswald proved necessary optimality conditions in view of pointwise maximum principle. The necessary conditions are used here to construct iterative methods for optimal control processes without boundary or phase constraint. Additional constraints are to add to the objective functional as penalty terms. The idea is to start with a dispatcher control, to solve the corresponding state equation and then the adjoint equation. We put the calculated state and costate in the Pontryagin function and find a new control by the maximum principle. A result of Popov is generalized which guarantees the convergence of the sequences of states, costates and controls in special cases, but under very strong assumptions. The limits fulfill the maximum principle. Then quasilinear integral processes with polyhedral control domain are considered. Some modifications of the Chernousko method are discussed. At every iteration step a linear programming problem is to solve. The maximum principle is fulfilled "in the limit". Some simple examples illustrate the methods.

Application of Receding Horizon Control Strategy to Persuit-Evasion Problems

J. Shinar

Time-optimal interception of an arbitrary maneuvering evader is investigated using a simplified kinematic model of planar constant speed motion. Against such an uncertain evader behaviour a new and robust pursuer strategy, inspired by the "receding horizon" control concept, is developed. Numerical results of an extensive simulation study show that for all cases that have been tested the optimal horizon length is approximately 60% of the minimum time required to capture a straight flying evader. Moreover, the performance loss compared with the optimal control and game solutions is less than 1%. The proposed guidance algorithm, based on the receding horizon control concept and using a singular perturbation approximation, is very simple to implement and leads almost always to a near-optimal outcome.

Dynamic Control of Steering Vehicles at Busy Airports

H. Siguerdidjane

This work deals with the automatic control of tractor systems in order to perform a cooperative rendezvous between these systems and the aircrafts. We are interested in this work on taxing phase of the aircrafts. Since the traffic is becoming so important than a fully automatic ground motion should be considered in the future. Feedback nonlinear controls are derived in such a way that the tractor system may track given prescribed trajectories. These trajectories are generated by the computer according to the airport configuration and in an optimal way. The tractor system consists of two vehicles which are described in the presentation. A successful cooperative rendezvous is performed.

An Optimal Control Technique for the Simulation and Optimization of Human Movements

T. Spägele

Motivated by the need for better understanding the coordination of human movements by the central nervous system, optimal control theory is used as a framework to study the intermuscular control of human multi-joint movements. Obviously muscle forces are determined by the central nervous system according to some optimization schemes. Therefore, dynamic optimization techniques can be used to calculate muscle forces necessary for measured human movements as well as for the optimization of movements. The appropriate biomechanical model of the human musculoskeletal system consists of differential equations describing the kinematics and dynamics of the skeleton and of the human muscles. The results for an optimal limb movement and the simulation of a human vertical jump are presented.

Fuel-Optimal Periodic Control and Regulation in Constrained Hypersonic Flight

J.L. Speyer and L.D. Dewell

The concepts of optimal periodic control theory prove to be particularly beneficial in its application to hypersonic cruise. In this study, it is demonstrated that optimal periodic solutions may yield fuel savings over the best static cruise of over 10 %. Moreover, such a periodic flight path possesses a more favorable aerothermodynamic environment than static cruise. These results were obtained in spite of a stringent constraint on the vehicle acceleration. These optimal periodic trajectories can be mechanized via a regulating guidance law in which the second variation of the cost is minimized while also guaranteeing asymptotic convergence to the extremal orbit. Finally, the regulator is extended to include a slowly-varying vehicle mass by application of the method of multiple time scales.

Estimates of Adjoint Variables in State Constrained Optimal Control
O. von Stryk

Nowadays direct transcription methods have become efficient tools for solving a great variety of optimal control problems. In contrast to indirect methods knowledge of optimal control theory is not necessarily needed in order to apply them. Also they are remarkably robust with respect to bad initial estimates of the solution.
A further great benefit of some direct methods is that they can provide with reliable estimates of adjoint variables (even in the presence of active state constraints) although adjoint variables do not explicitly appear in the formulation of the direct methods.
These estimates make it possible to use the information related to adjoint variables without having to deal with adjoint differential equations.
Therefore the subsequent use of an indirect method can be facilitated enormously. Or consistency checks of the computed discretized control by an (estimated) switching function as well as a local, numerical synthesis of the feedback control is made possible.
In the talk it is described how these estimates of adjoint variables can be obtained by a direct transcription method based on collocation. Numerical results for problems of various complexity are presented. Results for a real industrial robot are shown in a 16 mm film.

Numerical Solution of Parabolic Control Problems by Lagrange-Newton Methods
F. Tröltzsch

We consider the application of Lagrange-Newton-SQP methods to the numerical solution of optimal control problems governed by the heat equation with nonlinear boundary condition. On transforming the optimality system of the problem into a generalized equation we are able to invoke known results on the convergence of the corresponding Newton method. In this way the quadratic convergence of a SQP method can be shown for the control problem. Any numerical implementation of this technique has to deal with the high dimension of the generated discretized problems. We report on first numerical experience with a multigrid strategy w.r. to the control function applied to the (constrained) linear-quadratic subproblems. Some high-precision examples are presented.

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