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The conference on *Inverse Problems in Medical Imaging and Nondestructive Testing* was organized by Heinz Engl (Linz), Alfred Louis (Saarbrücken) and Wiliam Rundell (Texas A&M) in the Mathematisches Forschungsinstitut Oberwolfach in the Black Forest in Germany from February 04 to 10, 1996.

Forty participants from thirteen countries presented their research results which can be grouped into the categories:

- overviews on the state of art in medical imaging and nondestructive testing
- x-ray computerized tomography in two and three dimensions
- ultrasound and microwave tomography
- electrical impedance tomography
- electron microscopy
- flow tomography
- inverse scattering
- regularization methods for nonlinear problems
- wavelets and regularization

The common background of the reseach in these fields is inverse problems. It became again evident that much progress in the underlying mathematical research was directly transformed into new algorithms for solving these real world problems in two areas which, at a first glance, seem to be very different. From a mathematical point of view similar principles are applied to measure the data although the realization can be rather different as an example in x–ray computerized tomography shows. In nondestructive testing typically the measured object is rotated and x–ray source and detectors are fixed, which, of course, is impossible in the medical application.

The lively scientific atmosphere and the inspiring setting in Oberwolfach stimulated many discussions which certainly will contribute to further progress in the field. Although one is tempted to take this invariably high standard in the Mathematische Forschungsinstitut for granted we thank the staff.

Inverse Problems in Medical Imaging and Nondestructive Testing

Beth, Th.: Wavelets and Waves For Imaging: Inverse and Illposed Problems

Starting with an example from medical X-Ray Imaging and related compression problems the outstanding role of wavelets as transform kernels is discussed. Their property of forming Quadrature Filter pairs can be exploited for highly reliable compression schemes for pictures, providing bandwidth economic preview and browary schemes with simultaneous back-up for zooming and lossless documentation. Examples of compressed NMR pictures, which show no subjective deterioration at compression rate 134:1 and 28.03dBPSNR are presented and compared with results of other schemes such as JPEG.

A Family of symbolic signal processors for high-speed wavelet transforms based on algorithms designed through the algebraic equations of the Daubechies wavelets has been presented.

Using the technique of QMF pairs, a genetic algorithm for the evolution of high quality data compression schemes is developed. A comparism of our algorithm family btj.box with known algorithms such as standardized IPEG, Industrial Wavelets or the newly fractal coding methods shows a superior performance w.rt. compression rate on PSNR, and - as far as the latter methods are concerned - also highly superior algorithmic behaviour. Examples of 24Bit-Colour Pictures of 4.5 MB original size are shown at a compression rate of up to 1:521.

The second part of the talk has address the availability of optical transform techniques for any kind of linear filtering approach! A short recollection of properties of Fourier optics and Wavefront manipulation techniques describes the setup of socalled 4f-systems, in which paraxial farfield wave equations can be handled. The problem of the solution of the famous inversion of Kirchhoff's Diffraction Integral is briefly discussed while the main problem of optical transform computing in the paraxial far-field regime is reduced to the illposed inverse problem of designing unitary diffractive elements of specified behaviour: Given a modulus distribution $|f(x,y)|^2$ for a complex function $f : \mathbb{R}^2 \to C$ in $(x,y) \in D \subset \mathbb{R}^2$ find a complex phase distribution $\phi : \mathbb{R}^2 \to \mathbb{C}$ with $|\phi(v,u)| = 1$ for $(v,u) \in E \subset \mathbb{R}^2$ such that $\phi = \mathcal{F}^{-1}(f)$, i.e. f is the Fouriertransform of ϕ.

This is a highly nonlinear-problem even in the large set of all possible solutions. New methods developed at the authors institution based on homotopy no's of local phase rotations can be used to minimize the number of so called speckles. New applications in industrial pattern recognition and printing devices are presented.

Blaschke, B.: Some Newton type methods for the regularization of nonlinear ill-posed problems

This lecture is concerned with regularizing algorithms for nonlinear ill-posed problems

$$F(x) = y$$

based on Newton- and Quasi-Newton-type iterations. For the resulting methods - the iteratively regularized Gauss-Newton method, a Landweber type method, and a Newton type method based on regularization by projection - we show convergence and convergence rates as the iteration index goes to infinity, in the noise free situation. In the case of noisy data (with noise level δ) we propose a priori and a posteriori stopping rules that, under appropriate conditions, yield the convergence rate $O(\sqrt{\delta})$.

Brambila, F.: Fractal tompgraphy

From cross section image we can compute the 2-d Mandelbrot dimension (using digitalization). After that we find the 3-d Mandelbrot dimension by Brambila-Mattilla's lemmas and with this 3-d Mandelbrot. We obtain the porosity of 3 dimension objects using the relation

$$(I - P)^{d_3/3} + (P)^{2/3 d_3} = 1$$

where d_3 = 3-d Mandelbrot dimension.

Work is done in vectorial Radon transform ($\bar{u}$ vectorfield $R(u,l) = \int_l u \cdot ds$) and Fractal Radon transform.

Colton, D.L.: Using Microwaves to Diagnose and Monitor Leukemia

We consider the inverse scattering problem of determining the index of refraction of an isotropic inhomogeneous medium from a knowledge of the electric near field data. This problem occurs in the problem of diagnosing and monitoring leukemia in the upper part of the lower leg. The inversion procedure is based on the so-called 'dual space method' developed by the speaker and P. Monk. Numerical examples using synthetic data at 800 MHz and 1.6 Ghz are given.

Dobson, D.C.: Recovery of blocky images in electrical impedance tomography

Due to the inherent instability of recovering high-frequency components of conductivity images in electrical impedance tomography, sharp edges of 'blocky' images are generally blurred in reconstructions. Earlier work has demonstrated that this effect can often be greatly reduced by applying nonlinear regularization techniques based on total variation minimization. This raises the interesting question: under what circumstances can images be successfully recovered by these techniques? We will present some results along these lines, both positive and negative, for several generic image recovery problems and discuss the implications for electrical impedance tomography.

Engl, H.W.: A note on regularized phase reconstruction

In 'phaseless inverse scattering', one want to reconstruct an obstacle or a potential from the amplitude of far-field data only. Also, inverse problems in diffractive optics lead to the problem of reconstructing the phase from amplitude measurements, i.e. to reconstruct a real-valued function $g \in L^2(\mathbb{R})$ from $|\hat{g}|$. This is a nonlinear, generally ill-posed problem. We investigate the regularizing properties and convergence rates for Tikhonov regularization and iterative methods applied to this problem. Ref.: B. Blaschke, H. Engl, W. Grever, M. Klibanov, An application of Tikhonov regularization to phase retrieval, to appear in Nonlinear World.

Faridani, A.: Reconstruction of density differences in Λ-tomography

Standard, or global tomography involves the reconstruction of a density function f from line integrals. Λ-tomography produces the related function $LF = \alpha(\Lambda f + \mu\Lambda^{-1} f)$ where Λ is the square root of the positive Laplacian, $-\Delta$. Reconstruction of Lf is local in the sense that computation of $Lf(x)$ requires only integrals over lines passing arbitrary close to x. We discuss the relation of Lf and f and present a method for computing approximate density jumps from Λf when f is a linear combination of characteristic functions and a smooth background. The algorithm is demonstrated with real-world examples as well as with mathematical phantoms.

Grünbaum, A.: Variations on the theme local-vs-global

The search for differential operators that commute with 'naturally arising' integral operators has recently been tied up with arbitrary solutions on the Gauss hypergeometric equation. I display 3 new explicit examples connected with 'Krall polyn.' and also point out that Gauss' equation gives - along the lines of Gelfand et al. - solutions to the F. John compatibility conditions for the Radon transform in the case of one dimensional lines in $\mathbb{R}^3$.

Hanke, M.: An inexact Newton method in electrical impedance tomography

We consider the numerical inversion of the parameter-to-solution mapping $F : a \to \Lambda$ in electrical impedance tomography. Here, Λ is the Neumann-to-Dirichlet operator associated with the selfadjoint elliptic differential equation -div($a$grad$u$) = 0 in the bounded domain $\Omega \subset \mathbb{R}^2$, and a is the corresponding positive conductivity coefficient.

Several authors have applied the Levenberg-Marquardt method to approximately minimize $\Lambda - F(a)$ with respect to the Hilbert-Schmidt norm. In this talk we present and motivate an inexact Newton-Krylov implementation. For a general (ill-posed) nonlinear equation $F(a) = y$ we derive assumptions on F with which we can prove convergence and regularizing properties for the case that the Landweber method is used for the inner iteration. Finally, we discuss the validity of these assumptions for electrical impedance tomography, and present some numerical results.

Isakov, V.: On identification of the diffusion coefficient from boundary measurements

We consider recovery of the coefficient a of the parabolic equation

$$u_t - \mathrm{div}(a\nabla u) = 0 \text{ in } Q, \quad u = 0 \text{ on } \Omega\times\{0\}, \quad u = g \text{ on } \partial\Omega\times(0,T), \quad Omega \subset \mathbb{R}^n \quad (n = 1,2,3)$$

from additional data

$$a\partial_\nu u = h \text{ on } \partial\Omega \times (0, T.)$$

We have no uniqueness results with one pair (g, h) except for $n = 1$, so we prescribe the map $\Lambda : g \to h$ for all smooth g, and prove uniqueness of any $a = a(x) \in C^1(\bar{\Omega})$, of $a = a_0 + kx(\tilde{Q}), (a_0,\ k \in C^2(\bar{\Omega})$ are given, $\tilde{Q}$ changing in time) and discuss, numerical results for the linearized problem $(a = 1 + f(|x|), f$ is small).

Kress, R.: Inverse obstacle scattering with the modulus of the far field pattern as data

The inverse problem we consider is to determine the shape of a sound-soft or perfectly conducting infinity long cylindrical obstacle from a knowledge of the far field pattern for the scattering of incident time-harmonic plane waves. In particular, a quasi Newton method is discussed where the Fréchet derivative of the far field operator which maps the boundary curve onto the far field pattern of the scattered wave for the unit circle as initial curve is kept fixed throughout the iteration. Both the reconstruction from the full far field and from the modulus of the far field as data is considered, the latter modulo translations of the obstacle. The question of uniqueness is adressed and numerical examples are presented, illustrating satisfactory reconstructions.

Kristensson, G.: Transient external 3D excitation of a dispersive and anisotropic slab

Propagation of a transient electromagnetic field in a stratified, dispersive and anisotropic slab, and the related direct and inverse scattering problems are investigated. The field is generated by a transient external 3D source. The analysis relies on the wave splitting concept and a 2D Fourier transform in the transverse spatial variables. The wave propagator approach provides an exact solution of the transmission operator. From this solution it is possible to extract the first (Sommerfeld) precorsor. An inverse algorithm is outlined using reflection and transmission data corresponding to four 2D Fourier parameters.

Kunisch, K.: Image Reconstruction by Minimizing Total Bounded Variation

Lagrangian and augmented Langrangian methods for nondifferentiable optimization problems arising from total bounded variation formulation of denoising and deblurring of images are studied. Conditional convergence of the Vzame algorithm and unconditional convergence of the first order augmented Lagrangian schemes are discussed. A Newton type method based on an active set strategy defined by means of a duality framework is introduced and numerical examples are given.

Langenberg, K.J.: Applied Inversion in Nondestructive Testing

Either electromagnetic or acoustoelastic waves can be utilized for nondestructive testing purposes. The aim is two-fold, either to characterize materials or to detect, localize and image defects. To solve the latter problem it is possible to model the defect as a perfectly scattering target, and this gives rise to a linearizing approximation in terms of the physical optics argument to solve the inverse scattering problem. Therefore, the talk concentrated on diffraction tomography in 3D and its possible extensions to include polarization of electromagnetic waves and mode conversion for elastic waves. Numerous simulations and applications to experimental data illustrated the potential of linear inversion algorithms in nondestructive testing.

Lobel, P.: Gauss-Newton and Gradient Methods for Microwave Tomography

We present different iterative methods and compare their performance for solving an inverse scattering problem: the reconstruction of the complex permittivity profile of inhomogeneous dielectric objects embedded in a homogeneous medium from near or farfield scattered data in the 2D-TM case. Applications concerned here are medical imaging, non destructive testing and target identification.

The nature of this problem is strongly non linear and ill-posed when quantitative imaging is requested. During the past 15 years, intensive studies have concerned reconstruction algorithms able to give an efficient solution to quantitative imaging. Among them, Newton-Kantorovich or Levenberg-Marquardt algorithms, Gradient and Modified Gradient method have been applied to this problem. Starting from an exact integral representation of the EM field, the moment method is utilized to generate matrix relations. Different iterative optimization procedures for the inverse problem have been used. For example, one based on the minimization of a cost functional decomposed into two residual errors or with a functional using a non linear matrix relation between the scattered field and the contrast profile. From this non linear formulation, gradient method is used or a Newton type algorithm using Tikhonov regularization with identity operator or gradient operator in order to deal with ill-conditioned matrices which have to be inverted. The different iterative procedures have been constructed in order to incorporate multiincidence or multifrequency configurations.

Investigations on the initial of the complex permittivity profile have been made with a back-propagation scheme using the adjoint operator which allows to provide an estimate of the induced current inside the inhomogeneous object.

Different configurations of interest have been studied for different applications of microwave imaging i.e. medical applications, non destructive testing in civil engineering and target identification from bistatic RCS measurements. Influence of the initial guess (without a priori information and using the backpropagation scheme) are also investigated.

Louis, A.K.: Filter design techniques for X-ray CT and ultrasound

The solution of $Af = g$, where A is a linear operator between the Hilbert spaces X and Y, and especially $X = L_2(\mathbb{C}^3)$, $Y = \mathbb{C}^N$ for finitely many data, is achieved by applying the approximate inverse.

For an arbitrary mollifier or wavelet $e_\gamma(x,y)$ the minimum norm solution of $A^*\Psi_\gamma(x) = e_\gamma(x,\cdot)$ is computed. If Y is a finite dimensional space, this is always possible, otherwise smoothness conditions on e_γ are required depending on A^*. Then the approximate inverse is

$$S_\gamma g(x) = \langle g, \Psi_\gamma(x) \rangle = \langle Af, \Psi_\gamma(x) \rangle = \langle f_M, e_\gamma(x,\cdot) \rangle$$

where f_M is the minimum-norm solution of $Af = g$. The advantage of this approach is that the reconstruction kernel Ψ_γ can be precomputed independently of the data.

Invariance properties of A reduce dramatically the storage need for the reconstruction kernel and the computer time for evaluating $S_\gamma g$.

This technique is applied for developping reconstruction kernels for X-ray CT and ultrasound. In the case of X-ray CT for finitely many data the inversion is of filtered backprojection type. Reconstructions from real data in 2D (fan beam) and 3D (cone beam) show the applicability of this technique.

Maaß, P.: Accelerated regularization methods

Tikhonov regularization and iteration methods are amongst the most frequently used regularization methods for solving linear and non-linear inverse problems. The numerical efficiency of these methods depends on fast algorithm for the evaluation of the involved operators (Ax or $F(x)$).

If the operator is replaced by a family of sparse approximating operators, then the speed of the algorithm can be improved considerably. However convergence and convergence rates can only be proved if the approximation levels are updated during the iteration. Optimality results are proved for linear problems (accelerated Tikhonov - and Landweber - method) and non-linear problems (accelerated Landweber iteration).

Natterer, F.: A numerical method for ultrasound tomography

Ultrasound tomography is modelled by the inverse problem of a 2D Helmholtz equation at fixed frequency with plane-wave irradiation. It is assumed that the field is measured outside the support of the unknown potential f for finitely many incident waves. Starting out from an initial guess f^o for f we propagate the measured field through the object f^o to yield a computed field whose difference to the measurements is in turn backpropagated. The backpropagated field is used to update f^o. The propagation as well as the backpropagation are done by a finite difference marching scheme. The whole process is carried out in a single-step fashion, i.e. the updating is done immediately after backpropagating a single wave. It is very similar to the well known ART method in x-ray tomography, with the projection and backprojection step replaced by propagation and backpropagation.

Pidcock, M.: Electrical Impedance Tomography

The advantages and disadvantages of EIT are well known. In particular EIT systems are modest in size, cost and complexity but image quality remains modest. In order to make EIT systems useful it is important to make improvements which retain the advantages. In the move to 3D imaging the modelling and computational costs are high and it is vital that both are performed in the best way possible, particularly, when using iterative reconstruction algorithms. In this talk I will discuss two such issues: First the problem of electrode modelling which the violent behaviour of the current density distribution at the edge of the electrodes causes significant computational problems and second: The choice of optimal current injection patterns and voltage measurement patterns.

Piero, A.R. De: Parameters Choice using Generalized Cross-Validation in Emission Tomographie

We analyze theoretical and practical problems arising from the application of Generalized Cross-Validation (GCV) as a stopping criterion for iterative methods for solving large scale ill-posed problems. We present some results for linear stationary methods and for the conjugate gradient method that allow us to derive efficient algorithms to compute the GCV functional. Finally we apply the results to PET (positron emission tomography) data.

Pike, E.R.: Three-dimensional Superresolving Confocal Laser Scanning Fluorescence Microscopy.

The problem of inversion of the image data in high-numerical-aperture confocal scanning laser microscopy of three-dimensional fluorescent specimens is defined and a solution based on the use of an optical mask placed in the image plane, calculated by a singular value decomposition of the Ignatowsky Richards and Wolf imaging kernel is discussed. The numerical techniques used will be described and examples of inversions of both simulated and real data will be given.

Plato, R.: On the regularization of Abel's integral equation

Several applications like the spectroscopy of cylindrical gas discharges or seismic imaging of the sub-surface lead to Abel integral equations of the first kind

$$(Au)(t) := \frac{1}{\Gamma(1/2)} \int_0^t (t-s)^{-1/2} u(s)\, ds = f_*(t), \quad t \in [0,a], \tag{1}$$

(with $0 < a < \infty$, and Γ denotes Euler's gamma function) in $X = L_2[0,a]$. For the approximate solution of (1) with noisy data, i.e.,

$$f_* \in \mathcal{R}(A), \quad f^\delta \in X, \quad \|f_* - f^\delta\|_2 \leq \delta, \tag{2}$$

we consider (for fixed integer $m \geq 2$) Lavrentiev's m-times iterated method which for real $\gamma > 0$ generates an $u_\gamma^\delta \in X$ by

$$\begin{aligned} (\gamma I + A)v_n &= \gamma v_{n-1} + f^\delta, \quad n = 1,2,\ldots,m, \\ u_\gamma^\delta &:= v_m, \end{aligned} \tag{3}$$

with $v_0 = 0$. The regularizing properties of the discrepancy principle as accompanying parameter choice are investigated.

Ramm, A.G. :1) Inversion of fixed-energy noisy data: 3D inverse scattering. 2) Finding discontinuities of functions from tomographic data.

1) An algorithm for inversion of noisy fixed-energy 3D scattering data is described. Stability

estimates for the inversion method are given.
2) New methods for finding discontinuities of functions from tomographic data are described.

Rieder, A.: A multilevel iteration for cone-beam reconstruction: first ideas

In an abstract setting we introduce a multilevel method which is designed for the solution of the normal equation which arises by applying the method of least squares combined with a Tikhonov regularization to an operator equation of the first kind. We give a convergence result and demonstrate the performance of the method by numerical simulations in the context of 1D-integral equations.

Next we discuss the potential of the iteration for the solution of the cone-beam reconstruction problem in 3D-tomography. We consider an a-priori estimate of the convergence speed and we look carefully at the algebraic structure of the normal equation. Our investigations show that sparse matrix techniques can be used to store the normal equation and to speed up the method. Our preliminary results vindicate a further exploration of the proposed method.

Sabatier, P.C.: Patchwork approach to boundary measurements

This is an attempt to appraise the information contained in boundary measurements on a local topic vs a global one. The reconstruction of a three-dimensional domain from boundary measurements is studied by means of local methods, applying the subdomains, and coupled together, being understood that a priori information encloses the unknown parameters in these subdomains and that they are reasonally far from each other. It is also understood that for each local subdomain, the direct and the inverse problem could be given solutions if the other subdomains did not exist. The general study is therefore that of a patchwork whose texture would be known and features are under study.

The first part of the lecture deals with a linear problem, where the measurements produce values of the Neuman-Dirichlet operator on a part of the surface of the domain. Thanks to the formal overdetermination (the operators depends on 4 variables, the parameter v in the differential equation - the Schrödinger one, depends on 3 ones), one can hope to obtain the situation described above. In many cases, this is true and an algorithm yields the result. Some variations on nonlinear cases are then given. Finally, a one dimensional example is exactly calculated.

Somersalo, E.: Impedance Tomography Algorithms

The problem under consideration is to estimate an unknown resistivity distribution in a body based on current-voltage measurements on the surface of the body. In the talk, two numerical algorithms were presented. The first one is a variant of gradient based optimization algorithms. Here, the novel features are the use of a realistic electrode model by introducing contact impedances, and the use of the total variation of the resistivity distribution as a penalty term. The second algorithm is based on the use of a Riccati equation that propagates the Neumann-to-Dirichlet operator in the radial direction of a circular body. Both methods have been tested by using synthetic and real data.

Sparr, G.: Flow tomography

We consider the problem of recovery of a vectorfield on a compact region in the plane from information collected along lines. Motivated by the modeling of ultrasound Doppler measurements, the Doppler Spectral Transform (DST) is introduced, by associating to the vectorfield $\bar{u}$ and the directed line $\bar{l}$ the distribution of projections onto $\bar{l}$ of the field vectors $\bar{u}$ along $\bar{l}$. It is conjectured that, under some regularity conditions, knowledge of $DST(\bar{u}, \bar{l})$ for all $\bar{l}$ determines $\bar{u}$ uniquely up to a radially symmetric field. This has been verified for an analogous discrete problem. Using parts of the information, only the first moment of the spectrum, earlier results (Juhlin, Tech. report, Lund 1992, Sparr et al, Inverse Problems 1995) have been implemented and used on experimental data (Jansson et al, Technical report, Lund 1995).

Sylvester, J.: Layer Stripping via the Hilbert Transform

The technique of layer-stripping has been proposed as a method for solving a variety of inverse problems. There have been a number of numerical experiments applied to one dimensional, as well as multidimensional problems. However, these methods have not been amenable to a rigorous mathematical analysis of convergence or stability.

In this talk, we will summarize a mathematically rigorous and numerically stable layer-stripping algorithm to solve the inverse scattering problem for the one dimensional Helmholtz equation, summerizing the approach in [S-W-G] as well as subsequent developments.

The new features of the theory include a nonlinear plancherel equality, nonlinear Paley-Wiener and sampling theorems, and a nonlinear Hilbert transform, which enforces causality and eliminates the need for trace formulas which have been a part of previous implementations.

[S-W-G] Sylvester, Winebrenner, Gylys-Colwell, Layer Stripping for the Helmholtz Equation, SIAM Journal of Applied Mathematics, June 1996.

Vainikko, G.: Stability estimates of some fast solvers of periodic integral equations

A class of periodic integral equations of the first kind is considered. The corresponding integral operators build isomorphisms between Sobolev spaces H^λ and $H^{\lambda-\alpha}$ of periodic functions for any $\lambda \in I\!R$ ($\alpha \in I\!R$ is the order of equation). Fast solvers are constructed on the basis of trigonometric Galerkin method, two-grid iterations and asymptotical expansions of integral operators (treated as periodic pseudodifferential operators). The stability properties of the method with respect to errors in the data (free term and the kernel) are established. Choosing the discretization parameter in correspondence to the error level of data, the (self) regularization of the approximate solution to the (ill-posed) problem is achieved.

Vessella, S.: Stability estimates in an inverse problem for 3D heat equation

We consider the problem of determining an insulating body, D, contained in a conducting one, Ω. If $u(x,t)$ is the temperature of $\Omega\backslash D$ and the following data are assigned $u = \varphi$ on $\partial\Omega \times [0,T_1]$, such that $\frac{\partial\varphi}{\partial t} > 0$, and $u(x,0) = 0$; if the flux $\frac{\partial u}{\partial n}$ is measured on $\Gamma \times [T_0,T_1]$ $(\Gamma \subset \partial\Omega)$, then D is determined.

A logarithimic stability estimate is found if some a priori assumptions are given on D.

Yagola, A.G.: Inverse Problems in Electronic Minosepty

Considered 3 inverse problems in data processing of electronic microscopy: 1) image processing at electronic microscopy autoradiography of surfaces. 2) reconstruction of 2D magnetic fields; 3) induced currents methods for semiconductors nondestructive testing. Regularized procedures for data processing were proposed and applied.

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