Mathematisches Forschungsinstitut Oberwolfach

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Jordan-Algebren

11.02. - 17.02.1996

An der Tagung über Jordan-Algebren, die unter der Leitung von W. Kaup (Tübingen), K. McCrimmon (Charlottesville) und H. P. Petersson (Hagen) stattfand, nahmen 46 Mathematikerinnen und Mathematiker aus Deutschland, England, Frankreich, Irland, Israel, Kanada, Neuseeland, Österreich, Rußland, Schweden, der Schweiz, Spanien, Swaziland und den USA teil.
Die während der Tagung gehaltenen Vorträge sind den Gebieten

- Algebraische Theorie der Jordan-Strukturen
- Jordan-Strukturen und Analysis
- Graduierte Strukturen und Wurzelsysteme
- Allgemeine nichtassoziative Algebren

zuzuordnen.
Neben 25 Übersichtsvorträgen, in denen der aktuelle Stand der Forschung in Teildisziplinen zusammenhängend dargestellt wurde, gab es eine Reihe von informellen „workshops", in denen in kleinerem Kreis neue Ergebnisse präsentiert und diskutiert wurden.

VORTRAGSAUSZÜGE

B. ALLISON:

Extended affine Lie algebras

We report on some recent results on extended affine Lie algebras (EALA's) which were introduced by Hoegh-Krehr and Turresani 1990 as a generalization of the important

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class of affine Kac-Moody Lie algebras. This work is joint work with S. Azam, S. Berman, Y. Gao and A. Pionzeta. By definition, any EALA possesses a nondegenerate invariant form. We have shown that this form is positive semidefinite, as conjectured by V. Kac. Using this fact, we can attach a root system called an extended affine root system (EARS), to any EALA. We have given an abstract definition of EARS and then shown how any EARS defined in this way can be constructed using semilattices. These constructions of EARS suggest several constructions of EALA's that use Jordan algebras and other structures which are finitely generated modules over the ring of Laurent polynomials in several variables.

J. A. ANQUELA:

Primitive Jordan pairs through local algebras I

Our aim is to outline the proof of the following theorem:

> „If V is a strongly prime Jordan pair having a primitive local algebra V_b^+ ($b \in V^-$) then V is primitive at b."

This, together with D'Amour and McCrimmon's result which establishes the converse, provides a so-called local characterization of primitivity for Jordan pairs. According with Zelmanov, D'Amour and McCrimmon's results on strongly prime Jordan systems, the task can be divided into two cases: The case when V_b^+ is PI, called homotope-PI case, where finite-dimensional techniques can be used, and the non-homotope-PI or hermitian case, in which Zelmanov polynomials are needed. Hermitian polynomials, introduced by Zelmanov when studying strongly prime Jordan algebras, are one of the most powerful tools in Jordan systems. The hermitian or Zelmanov polynomials needed in this case are homotope-eaters, whose construction is based on previous constructions of Zelmanov, D'Amour, Montaner and the authors. These polynomials are special-Jordan-algebra polynomials which, when evaluated in the homotope of a Jordan pair, give rise to elements which „eat" associative pair products and put them inside the original Jordan system or even in a given inner ideal of the homotope. They are used to create primitizers in a Jordan pair from primitizers of a local algebra.

J. ARAZY:

Contractive projections in C_p (joint work with Y. Friedman)

For $1 \leq p \leq \infty$ let C_p be the von Neumann-Schatten class, i.e. the Banach space of all

compact operators x on a separable infinite dimensional complex Hilbert space H, for which $\| x \|_p := (\mathrm{trace}(x^*x)^{p/2})^{1/p}$ is finite.
In this work we establish the following result.

Main Theorem. Let X be a closed subspace of C_p, $1 < p < \infty$, $p \neq 2$. Then the following four properties are equivalent:
(1) X is the range of a contractive projection from C_p;
(2) X is the l_p-sum of subspaces, each of which is canonically isometric to the C_p ideal of a Cartan factor of one of the types I-IV;
(3) $X^{p-1} := \{v(x)|x|^{p-1}; x \in X\}$ is a closed linear subspace of C_q, $(p^{-1}+q^{-1} = 1)$ where $x = v(x)|x|$ is the polar decomposition of the operator x;
(4) $V := \overline{\mathrm{span}}^{w^*}\{v(x); x \in X\}$ is closed under the triple product

$$\{u, v, w\} = (uv^*w + wv^*u)/2,$$

and is an atomic JCW^*-subtriple of $B(H)$. Moreover, X is a module over V, namely $\{VVX\} \subseteq X$ and $\{VXV\} \subseteq X$.

As a corollary we obtain that the category of C_p ideals of atomic JCW^*-triples is stable under contractive projections.

G. BENKART:

Root graded Lie algebras

Let $\mathcal{G} = \mathcal{H} \oplus \sum_{\alpha\in\Delta} \mathcal{G}_\alpha$ be a finite-dimensional split simple Lie algebra over a field F of characteristic 0. A Lie algebra L ist Δ-*graded* if i) $L = \bigoplus_{\gamma\in\Delta\cup\{0\}} L_\gamma$; (ii) $L \supseteq \mathcal{G}$; (iii) $L_\gamma = \{x \in L \,|\, [h, x] = \gamma(h)x \text{ for all } h \in \mathcal{H}\} \supseteq \mathcal{G}_\gamma$; and (iv) $L_0 = \sum_{\alpha\in\Delta} [L_\alpha, L_{-\alpha}]$. Examples of Δ-graded Lie algebras include the affine algebras, the toroidal Lie algebras, the Tits-Kantor-Koecher construction of a unital Jordan algebra, all finite-dimensional simple Lie algebras over F containing a nontrivial toral subalgebra, and many others.
In this talk we discuss the recent classification of Δ-graded Lie algebras, their derivations, associative forms, and central extensions, with an eye towards developing a representation theory for them.

C.-H. CHU:

Some functional analytic aspects of Jordan structure

Let Z be a JB^*-triple and let $\sigma = \sigma(Z, Z^*)$ be the weak topology on Z.

Theorem 1 (Bunce and Chu [1]) The following conditions are equivalent:
(i) The map $Q(a) : z \mapsto \{aza\}$ is σ-compact for every $a \in Z$;
(ii) The map $D(a, a) : z \mapsto \{aaz\}$ is σ-compact for every $a \in Z$;
(iii) Z is an inner ideal of Z^{**};
(iv) The spectrum S_a for any $a \in Z$ is discrete.

A map between subsets of Z is called *sequentially weakly continuous* (s.w.c) if it preserves σ-convergence of sequences.

Theorem 2 (Chu and Mellon [2]). The following conditions are equivalent:
(i) The map $z \mapsto \{zza\}$ is s.w.c. for every $a \in Z$;
(ii) Z has the Dunford-Pettis property.

Theorem 3 (Chu and Mellon [2]). A JBW^*-triple Z has the Dunford-Pettis property if and only if $Z = l_\infty - \bigoplus_\alpha L^\infty(\Omega_\alpha)\overline{\otimes} C_\alpha$ where C_α is a Cartan factor and $\sup_\alpha \dim C_\alpha < \infty$.

[1] L. J. Bunce and C.-H. Chu, Pacific J. Math. 153 (1992) 249–265.

[2] C.-H. Chu and P. Mellon, J. London Math. Soc. (to appear).

T. CORTÉS:

Primitive Jordan pairs through local algebras II

We consider here the main applications of the so-called local characterization of primitivity for Jordan pairs.

The main result is the description of primitive Jordan pairs in the spirit of the classification of strongly prime Jordan pairs given by Zelmanov, D'Amour and McCrimmon. The characterization is used to establish the transfer of primitivity between a Jordan pair and its nonzero ideals, as well as between an associative pair with involution and its ample subpairs, from the corresponding known results for algebras.

We next remark that a subquotient of a primitive Jordan pair inherits primitivity from it, as a consequence of the local characterization of primitivity, so answering affirmatively the question posed by Loos and Neher.

Primitivity of a tight envelope of a primitive Jordan pair can also be obtained with the direct use of the hermitian polynomials involved in the proof of the above mentioned characterization.

Finally, we consider the case of primitive Jordan triple systems, which may have local algebras that are not primitive. Here we manage to take the problem of their description into the language of pairs, defining what we call „tight double pairs", which can be associated to any Jordan triple system and inherit from it regularity conditions.

C.M. EDWARDS and G.T. RÜTTIMANN:

Structural projections and Peirce inner ideals in JBW*-triples

Let A be a JBW*-triple. A linear subspace J of A is called an *inner ideal* in A provided that the subspace $\{JAJ\}$ is contained in J. A subtriple B in A is said to be *complemented* if $A = B \oplus \mathrm{Ker}(B)$, where $\mathrm{Ker}(B) = \{a \in A : \{BaB\} = 0\}$. A complemented subtriple in A is a weak*-closed inner ideal. A linear projection on A is said to be *structural* if, for all elements a, b and c in A,

$$\{PabPc\} = P\{aPbc\}.$$

The range of a structural projection is a complemented subtriple and, conversely, a complemented subtriple is the range of a unique structural projection.
We analyze the structure of the weak*-closed inner ideal generated by two arbitrary tripotents in a JBW*-triple in terms of the simultaneous Peirce spaces of three suitably chosen pairwise compatible tripotents. This result is then used to show that every weak* closed inner ideal J in a JBW*-triple A is a complemented subtriple in A and therefore the range of a unique structural projection on A. As an application structural projections on W^*-algebras are considered. (Joint work with K. McCrimmon.)

The annihilator $J^\perp$ of a weak* closed inner ideal J in A is the weak* closed inner ideal consisting of elements A in A such that $\{JAA\}$ is equal to $\{0\}$. The JBW*-triple a can be decomposed into the direct sum of J, $\ker(J) \setminus \cap \ker(J^\perp)$ and $J^\perp$. Modulo five of the generalized peirce relations, this decomposition is a grading of a of peirce type and an example is given of a weak* closed inner ideal in a JBW*-triple A for which all five fail to hold, thereby showing that the result is the best possible. It is also shown that the condition that a weak* closed inner ideal in a JBW*-triple A leads to a grading of A which is of peirce type is equivalent to several other conditions, all of a topological, rather than algebraic, nature.

J. R. FAULKNER:

Elementary groups for Kantor pairs (with B.- N. Allison)

A pair algebra (K_+, K_-) with product $K_\sigma \times K_{-\sigma} \times K_\sigma \to K_\sigma$ satisfying
(KPI) $[V_{x,y}, V_{z,w}] = V_{\{xyz\},w} - V_{z,\{yxw\}}$
(KP2) $K_{a,b}V_{x,y} + V_{y,x}K_{a,b} = K_{K_{a,b}x,y}$
where $V_{x,y}z = \{xyz\}$ and $K_{a,b}x = \{axb\} - \{bxa\}$ is called a *Kantor pair*. A standard construction gives a graded Lie algebra

$$\mathcal{G} = \mathcal{G}_2 \oplus \mathcal{G}_1 \oplus \mathcal{G}_0 \oplus \mathcal{G}_{-1} \oplus \mathcal{G}_{-2}$$

with $\mathcal{G}_\sigma = K_\sigma$ and $\mathcal{G}_{2\sigma} = K_{K_\sigma,K_\sigma}$.

The group generated by all $\exp(\mathrm{ad}(x + A))$ with $(x, A) \in \mathcal{G}_0 \times \mathcal{G}_{2\sigma}, \sigma = \pm$, is the *projective elementary group* G.

A study of elements of G that either preserve or reverse the grading of $\mathcal{G}$ leads to notions of invertibility and quasi-invertibility in Kantor pairs, and also to generalizations of the quadratic operator and the Bergman operator of Jordan pairs.

For Kantor pairs which can be embedded in a certain way in 3×3 matrices, the calculations are considerably simplified. In particular, for the Kantor pair of a hermitian form, the Bergman operator, in a special case, reduces to an Eichler transformation.

G. HESSENBERGER:

Barnes idempotents and cocapacity in Jordan pairs

We call an element (x, y) of a nondegenerate Jordan pair V *quasi-Fredholm*, if it is quasi-invertible modulo the socle. For arbitrary (x, y) the *cocapacity* of (x, y) is defined by

$$\mathrm{coc}(x, y) := \inf\{\kappa(V_2(e)) : e \in \mathrm{Soc}\, V \text{ is an idempotent}, B(y, x)V^- \supseteq V_0(e)^-\},$$

where κ denotes the capacity [2], and $V_i(e)$ are the Peirce-spaces of e. Note that the cocapacity is a non-negative integer or ∞. If $e \in \operatorname{Soc} V$ is an idempotent such that $B(y,x)V^- \supseteq V_0(e)^-$ and $\operatorname{coc}(y,x) = \kappa(V_2(e))$, it is called a *(Jordan) Barnes idempotent* to (x,y).

We show that the following conditions are equivalent:
(i) (x,y) is quasi-Fredholm,
(ii) (x,y) has finite cocapacity,
(iii) (x,y) possesses a Barnes idempotent.
This generalizes a well-known theorem of associative Fredholm-theory, which says that for invertibility modulo the socle the existence of a Barnes idempotent is necessary and sufficient [1]. Furthermore, finiteness of the cocapacity is the Jordan-theoretic counterpart of finiteness of the *nullity* „nul" and the *defect* „def", the dimensions of the kernel resp. cokernel of the image of an element of an associative algebra under a faithful representation [1].

Thus it is no surprise that in the case where $V = (A, A)$ is the Jordan pair of an associative algebra the cocapacity can be calculated with the help of nullity and defect: For a quasi-Fredhoolm element (x,y) we have

$$\operatorname{coc}(x,y) = \max\{\operatorname{nul}(1_A - xy), \operatorname{def}(1_A - yx)\}.$$

[1] Barnes et al., „Riesz and Fredholm theory in Banach algebras", Pitman (Boston, 1982)

[2] Loos, „Finiteness conditions in Jordan pairs", Math. Z. 206 (1991).

N. C. HOPKINS:

Some nonassociative algebras associated to differential equations

We construct a class of $\mathbb{Z}_2$-graded commutative algebras in the following fashion: Let $\mathcal{B}$ be a commutative nonassociative algebra over $\mathbb{R}$, V a vector space over $\mathbb{R}$, C a symmetric bilinear form on V, $f : \mathcal{B} \to \mathbb{R}$ linear, $Q \in \mathcal{B}$ and $M \in \operatorname{End}(V)$. We let $\mathcal{M} = \mathcal{B} \oplus V$ as a vector space and define the product on $\mathcal{M}$ by $\begin{pmatrix} x \\ y \end{pmatrix}^2 = \begin{pmatrix} X^2 + C(Y,Y)Q \\ f(X)M(Y) \end{pmatrix}$. Then $\mathcal{A}$ is a $\mathbb{Z}_2$-graded algebra with $\mathcal{A}_0 = \mathcal{B}$ and $\mathcal{A}_1 = V$. This class includes all $\mathbb{Z}_2$-graded algebras over $\mathbb{R}$ for which $\dim \mathcal{A}_0 = 1$ or $\dim \mathcal{A}_1 = 1$ and hence all $\mathbb{Z}_2$-graded algebras of dimension 2 or 3. As we are interested in the solutions to the differential equation $\frac{dz}{dt} = z^2$ for $z \in \mathcal{A}$, we consider two algebraic properties known to be important to the study of such differential equations, namely nilpotence and simplicity. For both we give necessary conditions and sufficient conditions for $\mathcal{A}$ to have the property. We close by outlining some open problems. This is joint work with Michael Kinyon.

A. ILT'YAKOV:

On polynomial and rational invariants of simple exceptional linear algebraic groups

It is known that minimal linear representations of simple exceptional algebraic groups are related to certain simple nonassociative algebras. Using this connection, we describe generators of the algebra (field) of polynomial (rational) invariants in several vector variables. Also rationality of the field of rational invariants is proved in the case of groups of type F_4 and E_6.

I. KANTOR:

A generalization of Jordan approach to symmetric Riemmanian spaces

By a bisymmetric space we understand a homogeneous fibrebundle with symmetric fiber and symmetric base.

Theorem 1. Given a Jordan triple system of second order φ the subalgebra $L(\varphi) = U_{-2} + U_{-1} + U_0 + U_1 + U_2$ contains a subalgebra $S(\varphi) = H_0 + E_1 + E_2$, which has the structure of the Lie algebra of a bisymmetric space. Moreover an orbit of $S(\varphi)$ on an R-space (corresponding to a Lie algebra $L(\varphi)$) is a domain on M which has the structure of a bisymmetric space.

We will say that a Lie algebra $S^-(\varphi) = H_0 + iE_1 + E_2$ which has the same space and the same commutators but where the commutators $[E_1, E_2]$ have opposite signs is a *Lie algebra of a dual bisymmetric space.*

Theorem 2. Among all Jordan triple systems of second order with the same $L(\varphi)$ there is one for which the domain coincides with the whole M. The orbit of the Lie algebra $S^-(\varphi)$ of a dual bisymmetric space is a projectively bisymmetric domain.

G. LETAC:

Recent results on Wishart distributions on Euclidean Jordan algebras

Wishart distributions are here defined as probabilities on the associated symmetric cone $\overline{\Omega}$ by

$$C^{te}\exp(-\text{Trace }\Theta x)\mu_p(dx);$$

where Θ is in Ω and μ_p is the measure on $\overline{\Omega}$ with Laplace transform $(\det\Theta)^{-p}$, and p belongs to the Wallach set $\{\frac{d}{2},\dots,\frac{d(r-1)}{2}\}\cup(\frac{d(r-1)}{2},+\infty)$ (r = rank, d = Peirce constant). We survey some results about these distributions:
1) Continued fractions $(x_1+(x_2+(x_3+\dots)^{-1})^{-1})^{-1}$, where x_i are Wishart (Bernadac).
2) Homogeneous quadratic variances characterize Wishart (Casalis).
3) Peirce decompositions and expectations (Massam, Neher).
4) Quadratic conditional expectations characterize Wishart (Casalis, Letac, Massam).

A. F. LÓPEZ:

Orders in Jordan pairs with DCC on principal inner ideals

In a joint work with Eulalia Garcia Rus, we have given a notion of order in Jordan pairs which extends the definition of order in Jordan algebras given by Zel'manov, in the following sense: If a Jordan algebra J is an order in a unital Jordan algebra Q then the Jordan pair (J,J) is an order in the Jordan pair (Q,Q). Our notion of order in Jordan pairs is "local" and it is inspired by our recent works on Jordan algebras satisfying local Goldie conditions. We prove that if a Jordan pair V is an order in a nondegenerate Jordan pair W with dcc on principal inner ideals, then
(i) V is nondegenerate and satisfies acc on the annihilators of its elements. Moreover, V is strongly prime iff W is simple.
(ii) V is an essential subdirect product of Jordan pairs V_i each of which is an order in a simple Jordan pair W_i with dcc on principal inner ideals.
(iii) If W is locally artinian then any element $x\in V^\sigma,(\sigma=+,-)$ has finite Goldie dimension, i.e., the principal inner ideal $[x]$ does not contain infinite direct sums of inner ideals of V.

C. MARTINEZ:

GK-dimension in Jordan algebras

If A is a finitely generated algebra (not necessarily associative) over a field K, V is a finite-dimensional vector space that generates A and V^n denotes the linear span of all products of elements of V of length $\leq n$, then $GK\text{-dim}(A) = \sup \lim_{n\to\infty} \frac{\ln(\dim V^n)}{\ln n}$ (*Gelfand Kirillov dimension*).
It is known that the definition does not depend on V and that finite- dimensional algebras are those having GK-dimension 0. If A is not finite-dimensional, then GK-dim $(A) \geq 1$. Bergman proved that if A is associative and $1 \leq GK\text{-dim}(A) < 2$ then GK-dim$(A) = 1$ and also the following result by Small-Stafford and Warfield is known in associative algebras:
Theorem: Let A be a finitely generated associative algebra. Then: (a) $N(A)$ is nilpotent, (b) $A/N(A)$is a finite module over a finitely generated subalgebra of the center.
Here it is proved that if J is a finitely generated Jordan algebra, then:
(a) If J is special and A is an associative enveloping algebra, $GK\text{-dim}(A) = GK\text{-dim}(J)$.
(b) $GK\text{-dim}\, M(J) \leq 2\, GK\text{-dim}(J)$ where $M(J)$ is a universal mult. enveloping algebra.
(c) If J is special, then $GK\text{-dim}\, M(J) = 2\, GK\text{-dim}(J)$
Also the following result (with E. Zel'manov) is given:
Theorem: Let J be a finitely generated Jordan algebra and $1 \leq GK\text{-dim}(J) < 2$. Then (a) $GK\text{-dim}(J) = 1$.
(b) $N(J)$ is nilpotent, where $N(J)$ denotes the McCrimmon radical of J.
(c) $J/N(J)$ is a finite module over a finitely generated subalgebra of the associative center.

J. MARTINEZ:

Inner derivations on ultraprime Banach algebras

We show that, for every ultraprime real Banach algebra A, there exists a positive number γ satisfying $\gamma \parallel a + Z(A) \parallel \leq \parallel D_a \parallel$ for all a in A, where $Z(A)$ denotes the centre of A and D_a denotes the inner derivation on A induced by a. Moreover, the numer γ depends only on the „constant of ultraprimeness“ of A.

F. MONTANER:

On Goldie theory for Jordan algebras

A Jordan version of Goldie's theory was given by Zelmanov in the late 80's. In this talk two related problems are considered. First, extending Zelmanov's results to quadratic Jordan Algebras, and second, the extension to Jordan theory of some of the ideas involved in Goldie's work, such as nonsingularity and uniform dimension.

The results presented here have been obtained in joint work with A. Fernandez Lopez and E. Garcia Rus. Our main result states that a Jordan Algebra is a (classical) order in a simple (semisimple) artinian Jordan Algebra if, and only if, it satisfies the analogous conditions to the ones given by Zelmanov in the linear case, if, and only if, it is strongly prime (resp. nondegenerate), nonsingular, and has finite uniform dimension, with an adequate definition of these concepts.

The proof makes use, among other ideas, of a GPI-type theorem, which is based on recent work by D'Amour and McCrimmon, and some Jordan polynomials defined by Anquela and Cortes. These ideas also allow a deeper study of the uniform dimension of a Jordan algebra.

E. NEHER:

Structures graded by root systems

The basic concept introduced in the lectures was that of a root system grading of a Jordan pair $V = (V^+, V^-)$: a decomposition $V = \oplus_{\alpha \in R_1} V_\alpha, V_\alpha = (V_\alpha^+, V_\alpha^-)$, where R_1 is the 1-part of a 3-graded root system $R = R_1 \dot\cup R_0 \dot\cup R_{-1}$, such that $Q(V_\alpha^\sigma)V_\beta^{-\sigma} \subset V_{2\alpha-\beta}^\sigma, \{V_\alpha^\sigma V_\beta^{-\sigma} V_\gamma^\sigma\} \subset V_{\alpha-\beta+\gamma}^\sigma$ and $D(V_\alpha^\sigma, V_\beta^{-\sigma}) = 0$ for $\alpha \perp \beta \quad (\sigma = \pm, \alpha, \beta, \gamma \in R_1)$.

If such a grading is induced by idempotents (Example: V is covered by a grid) there is hope to obtain a classification, and in the talk the present stage of the classification was discussed.

Lie algebras graded by root systems are related to Jordan pairs with a root system grading via the following

Theorem: A Lie algebra L has a root system grading if and only if L is a central extension of the Tits-Kantor-Koecher algebra of a Jordan pair covered by a grid.

The second talk reported on joint work with O. Loos on Steinberg groups for Jordan pairs. Let $V = \oplus_{\alpha \in R_1} V_\alpha$ be a Jordan pair with a root system grading $\mathcal{R} = (R, R_1)$. The Steinberg group $\mathrm{St}(V, \mathcal{R})$ is given by a presentation: generators are $x_\sigma(v), v \in V^\sigma$, and the relations are some of the basic relations valid in the projective elementary group $\mathrm{PE}(V)$. Thus, one has a canonical epimorphism $\pi : \mathrm{St}(V, \mathcal{R}) \to \mathrm{PE}(V)$. Examples for $\mathrm{St}(V, R)$ are the classical Steinberg groups $\mathrm{St}_n(A)$, A a ring [here V is a rectangular matrix pair] and the unitary Steinberg groups [here V is a hermitian matrix pair]. Among the theorems presented was the following:

Theorem: If the root system grading is induced by a family of idempotents and if R is irreducible of rank ≥ 5, then $\mathrm{St}(V, \mathcal{R})$ covers central extensions: Whenever $G \xrightarrow{\Psi} \mathrm{PE}(V)$ is a central extension there exists a unique homomorphism $\varphi : \mathrm{St}(V, \mathcal{R}) \to \mathrm{PE}(V)$ such that $\pi = \psi \circ \varphi$.

Hence $\mathrm{St}(V, \mathcal{R})$ is the universal central extension whenever $\pi : \mathrm{St}(V, R) \to \mathrm{PE}(V)$ is a central extension. For example this is so if R has infinite rank (and is irreducible) - a result proven earlier by Milnor for R of type A and by Bak for R of type C. This is also so if V is simple, nondegenerate and Artinian (earlier work of Steinberg and Deodhar).

J. M. OSBORN:

Lie algebras of class W*

Let F be a field of characteristic 0, and let B be the F-span of the monomials $x_1^{\alpha_1}, x_2^{\alpha_2} \ldots, x_n^{\alpha_n} = x^\alpha$ where $\alpha_i \in \Delta_i \subset F$. Each Δ_i is either the nonnegative integers, or is an additive subgroup of F containing 1. B is a commutative associative algebra using the product $x^\alpha x^\beta = x^{\alpha+\beta}$. The set $A = \{\sum_{i=1}^{n} f_i \partial_i \mid f_i \in B\}$ where ∂_i is the partial derivative with respect to x_i is a Lie algebra using the product $[f_i \partial_i, g_j \partial_j] = f \partial_i(g_j)\partial_i - g_j \partial_j(f_i)\partial_i$. The class of algebras defined in this way is called W^*.

For a fixed monomial $h \in B$, define $L(h)$ to be the linear span of the set $\{D_{ij}^h(f) = h\partial_j(f)\partial_i - h\partial_i(f)\partial_j\}$. The derived algebra $L'(h) = [L(h), L(h)]$ is simple, and the algebras defined in this way form the class S^*.

In this talk we discussed the derivation and automorphisms of these algebras, and also the question of when two of these algebras can be isomorphic to each other.

S. PUMPLÜN:

Composition algebras over a ring of fractions

Composition algebras over the Ring $R = \{\frac{g(t)}{f(t)^j} \in k(t) \mid j \geq 0,\ g \in k[t] \text{ with } \deg g \leq 2j\}$, with k a field of characteristic not two, and $f(t) \in k[t]$ a monic irreducible polynomial of degree two, are enumerated and partly classified. Crucial steps include
1. a proof that every composition algebra C over R without zero divisors or rank > 2 contains a composition subalgebra of half rank, and thus is a generalized Cayley-Dickson doubling of the latter,
2. a proof that for any composition algebra without zero divisors of rank ≤ 4 which is defined over k there is up to isomorphism at most one non-classical Cayley-Dickson doubling which results in a composition algebra without zero divisors not defined over k.

After classifying the composition algebras of rank 2 (the *tori*) over R it turns out that every composition algebra of rank > 2 is the classical Cayley-Dickson doubling of a suitable torus.

Classification is facilitated by the fact that the subalgebra defined over k mentioned in 1. is uniquely determined up to isomorphism if C is not defined over k.
It should be emphasized that results of this kind are not to be expected for arbitrary rings. The most important feature of R used here is its close relation to the projective line, i.e. $\operatorname{Spec} R = \mathbb{P}^1_k - \{(f(t))\}$.

B. RUSSO:

On the Iwasawa decomposition of the automorphism group of a bounded symmetric domain

Let G be the connected component of the holomorphic automorphism group of a bounded symmetric domain D in C^n with Lie algebra $\mathcal{G}$. Let $E, \{\cdot,\cdot,\cdot\}$ be the JB^*-triple for which D is the open unit ball, and let $v_1, \ldots, v_r$ be a frame for E with $v = v_1 + \ldots + v_r$. Denote the joint Peirce decomposition with respect to this frame by $E = \sum E_{kl}$, and for $b \in E$, write $b = \sum b_{kl}$. This decomposition could be refined by replacing the Peirce projections by $P^{\varepsilon}_{kl} = P_{kl}(I + \varepsilon Q(v))/2$ ($\varepsilon = \pm$). Then $b^{\varepsilon}_{kl} = P^{\varepsilon}_{kl} b$.
Denote by $G = PK, \mathcal{G} = \mathcal{K} + \mathcal{P}$ the Cartan decompositions, so that $\mathcal{P} = \{\xi_b : b \in E\}$, where $\xi_b(z) = b - \{zbz\}$, and let $G = KAN$, $\mathcal{G} = \mathcal{K} + \mathcal{A} + \mathcal{N}$ be the Iwasawa de-

compositions constructed by use of a regular element ξ_{b_0}, where $b_0 = \sum_j s_j v_j$ with $s_1 > \ldots > s_r > 0$. Let $L = AN$ and $\mathcal{L} = \mathcal{A} + \mathcal{N}$ denote the corresponding solvable components.

Theorem $\mathcal{L} = \{R_0(b_0, \mu(b_0)b) + \xi_b : b \in E\}$, where $R_0(a,b)$ is the curvature tensor at the origin of the Bergman metric: $R_0(a,b)z = 2\{abz\} - 2\{baz\}$, and $\mu(b_0)$ is a "refined' ' Schur multiplier with respect to the frame $v_1, \ldots, v_r$:

$$\mu(b_0)b = \sum_{1 \leq k \leq l \leq n} \left(\frac{1}{s_1 k + s_l}\right) b_{kl}^- \sum_{1 \leq k < l \leq n} \left(\frac{1}{s_k - s_l}\right) b_{kl}^+ + \sum_{l=1}^{n} \frac{1}{s_l} b_{0l}.$$

This theorem describes, in algebraic and geometric terms, the element of $\mathcal{K}$, namely $R_0(b_0, \mu(b_0)b))$, which needs to be added to a given element ξ_b of $\mathcal{P}$ in order to obtain an element of $\mathcal{L}$. The "correction term" $R_0(b_0, \mu(b_0)b)$ can be shown to be independent of the choice of b_0. Moreover, this term is a new "alternating curvature" with respect to the frame $\{v_j\}$, acting on $b \in E_{kl}^{\varepsilon}$ as $R_0(v_k - \varepsilon v_l, b)$.
There is a version of the Theorem at the Lie group level: if $\sigma \in L$ has Cartan decomposition $\sigma = \varphi_\alpha \circ V_\alpha$ with $\varphi_\alpha \in P$ and $V_\alpha \in K$, then V_α can be expressed in terms of $\alpha = \sigma(0)$ by means of the fundamental operators of a JB^*-triple. For example, if σ belongs to the nilpotent group N and the domain D is of tube type, then $V_\alpha = B(\alpha, \alpha)^{-\frac{1}{2}} B(\alpha, v)$, where $B(x,y)$ is the Bergman operator.
(This is joint work with Yaakov Friedman.)

I. SHESTAKOV:

Exceptional Jordan superalgebras

New constructions for simple exceptional finite dimensional Jordan superalgebras are given. The simple Jordan superalgebras J_{12} and J_{21} of characteristic 3 and of dimensions 12 and 21, respectively, are represented in the form $H_3(B, {}^*)$ for a suitable simple alternative superalgebra B with a superinvolution.
The Kac superalgebras K_{10} and K_9 are represented in the form $J + U + V$, where J is a simple Jordan superalgebra of dimension 3 or 4, U and V are two isomorphic copies of a certain irreducible 3-dimensional superbimodule over J, $U^2 = V^2 = 0$, $UV \subseteq J$.

S: SKRYABIN:

Representation theory for a class of Lie algebras of derivations

Representations of the Lie algebras of vector fields in the modules of sections of vector bundles serve as a motivating example for a purely algebraic study of representations in quite general settings. Let R be a commutative associative and unital algebra, W a Lie algebra of its derivations which is a projective R-module of rank n. I introduce a certain category of W-modules which carry additonal structures of an R-module and a module over a certain Lie R-algebra g which is just the general linear Lie algebra $gl(n, R)$ in the simplest case of height one representations. The general principle is that every W-submodule of an object of this category is also an R-submodule and a g-submodule and that every W-module homomorphism between two objects respects the R-module and g-module structures als well. There is, however, a certain class of exceptional representations realized in the modules of differential forms whose theory is more complicated.

A. SLINKO:

Linearly compact algebras and coalgebras

It is proved that a coalgebra is locally finite if and only if its dual algebra is profinite-dimensional, i.e., isomorphic to the inverse limit of finite-dimensional algebras.
We specify a large class of varieties for which all linearly compact algebras are profinite-dimensional.
This class includes all Jordan and alternative algebras. Necessary and sufficient condition for a linearly compact Lie algebra to be profinite-dimensional is obtained. In particular, linearly compact algebraic Lie algebras are profinite-dimensional. Moreover they must be algebraic of bounded degree.

H. UPMEIER:

Semisimple Jordan algebras and quantization of non-convex tube domains

The classical theory of Toeplitz operators on the Hardy space $H^2(\Omega)$ over the positive cone Ω in a formally-real Jordan algebra X can be generalized as follows: For any semi-simple Jordan algebra $\tilde{X}$, a connected component $\tilde{\Omega}$ of its regular set is a non-convex pseudo-symmetric space. The associated tube domain $T(\tilde{\Omega})$ is homogeneous but not symmetric. It has a symmetric envelope $\hat{T}(\tilde{\Omega})$ recently contructed by Faraut and Gindikin. The associated Hardy space $H^2_q(\tilde{\Omega})$ consists of $\bar{\partial}$-closed L^2-cohomology classes of $(0, q)$-forms on $\hat{T}(\tilde{\Omega})$, where q is the concavity index of $\tilde{\Omega}$. In joint work with U.

Hagenbach we have shown that the associated Toeplitz C^*-algebra has all its irreducible representations realized on the faces (boundary components) of $\hat{T}(\tilde{\Omega})$.

E. ZELMANOV:

On graded Jordan superalgebras

All algebras are considered over an algebraically closed field of characteristic 0.
A *superconformal algebra* is a simple $\mathbb{Z}$-graded Lie superalgebra $L = \sum_{i \in \mathbb{Z}} L_i$ such that all dimensions $\dim L_i$'s are uniformly bounded and the even part of L contains the Virasaro algebra Vir. If L_0 is not solvable then the even part of L_0 contains a subalgebra $sl_2 = < e, f, h >$. The action $\mathrm{ad}_L(h)$ has finitely many eigenvalues. In the particular (but important!) case when the eigenvalues are $-2, 0, 2$, L is isomorphic to a Tits-Kantor-Koecher construction $K(J)$ of a simple graded Jordan superalgebra J.
Theorem (V. Kac - C. Martinez - E. Z.) Let $J = \sum J_i$ be a simple graded Jordan superalgebra such that the dimensions of J_i's are bounded from above and $K(J)_{\bar{0}} \supseteq$ Vir. Then there exists a $\mathbb{Z}/n\mathbb{Z}$-graded vector space $V = V_0 + \cdots + V_{n-1}$ and a generalized Poisson superbracket $[\,,]$ on the twisted algebra A of functions on odd variables from V and one Laurent variable t such that J is the Kantor double of $(A, [\,,])$.
Then $K(J)$ is a contact Lie superalgebra.

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