Tagungsbericht 11/96

Diophantine Approximations

17.–23.03.1996

Die Tagung fand unter Leitung von H.P. Schlickewei (Ulm), R. Tijdeman (Leiden), M. Waldschmidt (Paris) und J. Brüdern (Stuttgart) statt. Im Mittelpunkt standen klassische Themen wie diophantische Approximation und diophantische Gleichungen, Einheitengleichungen, Irrationalität und Transzendenz, Linearformen in Logarithmen, diophantische Geometrie. Erstmals war auch eine kleine Gruppe analytischer Zahlentheoretiker geladen, die vor allem neuere Entwicklungen im Bereich der Hardy-Littlewoodschen Kreismethode vorstellte.

Vortragsauszüge

F. AMOROSO: **Algebraic numbers close to 1**
Given a rational function R and a real number $p \geq 1$ define $\tilde{h}_p(R)$ as the L^p-norm of $\max(\log|R|, 0)$ on the unit circle. We studied the behaviour of $\tilde{h}_p(R)$ and gave various bounds for it. The results lead to an explicit construction of algebraic numbers close to 1 having small Mahler's measure and

small degree. This shows that a lower bound for the distance $|\alpha - 1|$ recently given by Mignotte and Waldschmidt is almost sharp.

R.C. BAKER: **Goldbach's problem in short intervals**
It has been known since the 1930's that, for any $A > 0$,

$$E(X) = \#\{2n \leq X : 2n \neq p_1 + p_2\}$$

has $O(X(\log X)^{-A})$ members, so that almost all even integers are the sum of two primes. Montgomery and Vaughan sharpened this to $O(X^{1-\delta})$ in 1975, though subsequent efforts by J.R. Chen and others to give a numerical value for δ, e.g. $\delta = 1/25$, contain serious errors. Here the problem is considered in short intervals. One asks for an estimate of the form

$$E(X + X^{\theta}) - E(X) = O(X^{\theta}(\log X)^{-A})$$

with a small value of θ. This problem has attracted some attention since Perelli and Pintz obtained $\theta = 7/36 + \epsilon$ in 1993, and Mikawa, Jia and Li Hong Ze have given better results, the latter being $\theta = 7/81 + \epsilon$. In joint work with Harman and Pintz it is now shown that $\theta = 11/60$ is admissible in this problem. The method requires the Hardy-Littlewood method, the sieve of G. Harman and some new mean value results for Dirichlet polynomials (although the latter are based on the usual principles of Montgomery and Halasz).

A. BALOG: **An additive property of stable sets**
An infinite set of positive integers $\mathcal{A}$ is called *stable* if for any $t > 1$ fixed one has

$$\#\{n \leq x : n \in \mathcal{A} \text{ but } tn \notin \mathcal{A}, \text{ or } tn \in \mathcal{A} \text{ but } n \notin \mathcal{A}\} = o(x).$$

Clearly any set of zero density is stable. Typical examples of less trivial stable sets are

$$Q_{\alpha} = \{n : P(n) > n^{\alpha}\},$$

here $0 < \alpha < 1$ and $P(n)$ is the greatest prime factor of n. We are interested in solving a binary linear equation inside a stable set. The following generalization of a result of A. Hildebrand gives an affirmative answer to a question of E. Fouvry.

Theorem 1 *Let $a > 0, b > 0$ and c be integers such that $(a,b)|c$. Let $\mathcal{A}$ be a stable set with positive upper density $\bar{d}(\mathcal{A})$. The equation $ax - by = c$ has infinitely many solutions $x \in \mathcal{A}$, $y \in \mathcal{A}$; moreover $\bar{d}(a\mathcal{A} \cap b\mathcal{A} + c) > 0$.*

J. BECK: **Uniform distribution and the class number**
Let $\{x\}$ denote the fractional part of x. A proof of the following statement was outlined:

$$\frac{1}{N}\sum_{n=1}^{N}\sum_{k=1}^{n}(\{k\alpha\} - \frac{1}{2}) = \sum_{i=1}^{l}\frac{(-1)^i a_i}{12} + O(\max_{1\leq i\leq l} a_i)$$

where $\alpha = [a_0; a_1, a_2, \ldots]$ is the continued fraction expansion, and l is the least index for which the denominator q_l of the l-th convergent p_l/q_l of α exceeds N. This result has three interesting consequences.

1) We obtain a 3-line-proof of the famous Hirzebruch-Zagier-Meyer formula: if $p \equiv 3(4)$, $p > 3$ prime and $h(p) = 1$, then $h(-p) = \frac{1}{3}\sum_{i=1}^{k} a_i$ where $\sqrt{p} = [[\sqrt{p}]; \overline{a_1, \ldots, a_k}]$.

2) We can evaluate the diophantine series

$$\sum_{n=1}^{N} \frac{1}{n \sin(\pi n \alpha)} = c_\alpha \log N + O(1)$$

where the constant c_α can be expressed in terms of the digits in the continued fraction expansion of α and $\alpha/2$.

3) A sufficient and necessary condition for the Central Limit Theorem about the series $\sum\{n\alpha\}$. Let $\alpha = [a_0; a_1, a_2, \ldots]$. Then

$$\frac{1}{N}\left|\left\{1 \leq n \leq N : \frac{\sum_{k=1}^{n}(\{k\alpha\} - \frac{1}{2}) - f(N)}{g(N)} \leq \lambda\right\}\right| \to \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\lambda} e^{-u^2/2}\, du$$

as $N \to \infty$ holds if and only if

$$\sum_{i=1}^{n} \frac{a_i^2}{a_n^2} \to \infty \quad (n \to \infty).$$

Here $f(N)$ is the mean value (Cesaro mean) and $g(N)$ is the variance which is between two constant multiples of $\sum_{i=1}^{l} a_i^2$ where l is defined by the requirement $N \leq q_l$.

M.A. BENNETT: **Simultaneous Pell equations**

If a and b are distinct nonzero integers, then the equations

$$\begin{aligned} x^2 - az^2 &= 1 \\ y^2 - bz^2 &= 1 \end{aligned} \qquad (*)$$

have at most 3 solutions in positive integers x, y, z. This result sharpens work of Masser and Rickert (1995) and is not far from the truth in that, given $a \geq 2$ there exists an infinite family of b's for which (*) has at least 2 positive solutions. The proof uses simultaneous rational approximation to algebraic numbers via Padé approximation, gap principles and a lower bound for linear forms in two logarithms due to Laurent, Mignotte and Nesterenko (1995).

V. BERNIK: **Khintchine type theorems on manifolds**

There are two versions of the classical Khintchine theorems in metrical theory of diophantine approximation (homogeneous and inhomogeneous). In 1964 Sprindzuk proved Mahler's conjecture, and W.M. Schmidt obtained the general metric theorem for curves in the plane. In 1989 the theorem of Sprindzuk (Bernik) and in 1995 the theorem of Schmidt (Dodson, Bernik) were improved to the analogous Kintchine's theorem in the case of convergence. There are two theorems in the case of divergence. Let $\psi(q)$ be a monotone function with $\sum_{q=1}^{\infty}\psi(q)=\infty$.

Theorem 1 (Dodson, Bernik). *The inequality*

$$|a_2z^m + a_1z^n + a_0| < \psi(H)^{1/2}$$

has for almost all $z \in \mathbf{C}$ an infinite number of solutions for a set of positive measure. Here $H = \max(|a_0|, |a_1|, |a_2|)$.

Theorem 2 (Beresnevich, Bernik) *The inequality $|P(x)+y| < H^{-1}\psi(H)$ has for almost all $(x,y) \in \mathbf{R}^2$ an infinite number of solutions in quadratic integral polynomials P, and $H = H(P)$ is the height.*

D. BERTRAND: **Diophantine problems on algebraic groups with real multiplications**

Let $\mathcal{O}$ be the ring of integers of a totally real number field, and let G be the universial vectorial extension of an algebraic variety A defined over a real number field k. Assume that End $A \simeq \mathcal{O}$, and that $\dim(A) = \mathrm{rk}_{\mathbf{Z}}\mathcal{O}$. Then:

Theorem 3 *A non-torsion k-rational point of G cannot lie in the maximal compact subgroup of $G(\mathbf{R})$.*

The proof consists in pushing out G to each of the extensions of A by $\mathbf{G}$ which admit multiplication by $\mathcal{O}$, and in noting that the determinant of the corresponding 1-motive can be computed in terms of periods of differential forms of the third kind on A which are eigenforms for the action of $\mathcal{O}$. Their non-vanishing then follows from Wüstholz's theorem on periods.

E. BOMBIERI: **Heights of algebraic points on subvarieties of abelian varieties and linear tori**

Let $X \subset \mathbf{A}$ be a subvariety of an abelian variety and let $\hat{h}$ be the Néron-Tate height associated to a symmetric ample divisor. It is well known that $\hat{h}(x) = 0$ if and only if x is a torsion point of $\mathbf{A}$. Bogomolov asked whether points of $\mathbf{A}(\bar{\mathbf{Q}})$ of small height had special properties, and in particular if points in $X(\bar{\mathbf{Q}})$ of small height were discrete in $\mathbf{A}(\bar{\mathbf{Q}})$ with respect to the distance $d(x,y) = \sqrt{\hat{h}(x-y)}$ on $\mathbf{A}(\bar{\mathbf{Q}})/$ tors. Of course, for this to be true one needs to remove from X all translates of abelian subvarieties.

Theorem 1 *Let $X^\circ = X \setminus \bigcup\{Y\}$ where Y runs through all translates of abelian subvarieties of dimension ≥ 1 contained in X. Suppose that $\mathbf{A}$ is a CM abelian variety. Then $X^\circ(\bar{\mathbf{Q}})$ is discrete in $\mathbf{A}(\bar{\mathbf{Q}})$ in the sense that for $y \in \mathbf{A}(\bar{\mathbf{Q}})$ the set of points $x \in X^\circ(\bar{\mathbf{Q}})$ with $d(x,y) < c$ is finite provided c is sufficiently small. The constant $c > 0$ and the number of points depend only on the degree of X and $\mathbf{A}$.*

There are several generalisations. Proofs are elementary and depend on Fermat's congruence $a^p \equiv a \bmod p$.

P. BORWEIN: **The order of vanishing of polynomials of low height**

Theorem 1 *Every polynomial*

$$p(x) = \sum_{j=0}^{n} a_j x^j \quad (a_j \in \mathbf{C}, |a_j| \le 1)$$

has at most $c\sqrt{n(1 - \log|a_n|)}$ *zeroes at* 1*; here* c *is some absolute constant.*

This is essentially sharp. Sharp estimates for the minimal norm of such polynomials are also given.

D. BROWNAWELL: **Some transcendence results in positive characteristic**

Let π be the period of the Drinfeld exponential function $e_c(z)$. Then $e_c(Tz) = Te_c(z) + (e_c(z))^q$.

Theorem 1 *a)* $z, e_c(z), e_c^{[1]}(z), \ldots, e_c^{[n]}(z), \ldots$ *are algebraically independent.*
b) $1, \pi, \pi^{[1]}, \ldots, \pi^{[q-1]}$ *are linearly independent* $/\overline{\mathbf{F}_q(T)}$*,*
where $[i]$ *denotes the divided derivative given by* $(T^n)^{[i]} = \binom{n}{i} T^{n-i}$*,* $n > 0$.

J. BRÜDERN: **Binary, quarternary and octary cubic forms**
(joint work with T.D. Wooley). Let $\Phi_1, \ldots, \Phi_4$ be binary cubic forms with integer coefficients and non-zero discrimants. Let $\mathcal{N}(P)$ be the number of solutions of the diophantine equation

$$\Phi_1(x_1, y_1) + \ldots + \Phi_4(x_4, y_4) = 0$$

subject to $|x_i| \le P$, $|y_i| \le P$ $(1 \le i \le 4)$. Then, for any $\epsilon > 0$,

$$\mathcal{N}(P) \gg P^{5-\epsilon}. \qquad (*)$$

One expects an asymptotic formula for $\mathcal{N}(P)$ with main term about P^5, and can indeed show that $\mathcal{N}(P) \ll P^{5+\epsilon}$ so that the result is close to best possible. The proof uses a p-adic iterative process within the framework of the circle method. The same method also yields that almost all natural numbers satisfying certain necessary congruence conditions can be written as $n = \Phi_1(x_1, y_1) + \Phi_2(x_2, y_2)$, with integers x_i, y_i. These results improve work of Chowla and Davenport (1961). They had to assume that Φ_1 is diagonal, and also missed the lower bound (*).

W.CHEN: **A result of Lev in irregularities of distribution**

The following surprising result of Lev was discussed: Suppose that $\mathcal{P}$ is a distribution of N points in the torus $U^k = [0,1]^k$ where $k \geq 2$. For every $\mathbf{x} \in U^k$ let $B(\mathbf{x})$ denote the aligned rectangular box with one vertex at 0 and one at $\mathbf{x}$, and write

$$D[\mathcal{P}; B(\mathbf{x})] = \#(\mathcal{P} \cap B(\mathbf{x})) - N\text{volume}(B(\mathbf{x})).$$

Also, for $\mathbf{y} \in U^k$ let $\mathcal{P} + \mathbf{y}$ be the translation of $\mathcal{P}$ by $\mathbf{y}$ modulo U^k. For $q \in [1, \infty)$ write

$$D_q(\mathcal{P}) = \Big(\int_{U^k} |D[\mathcal{P}; B(\mathbf{x})]|^q \, d\mathbf{x}\Big)^{1/q}.$$

Write also

$$D_\infty(\mathcal{P}) = \sup_{\mathbf{x} \in U^k} |D[\mathcal{P}; B(\mathbf{x})]|.$$

Theorem 1 (Lev) *There exist positive constants $A_1(k), A_2(k)$ such that for every $q \in [1, \infty)$ we have*

$$A_1(k) D_\infty(\mathcal{P}) \leq \sup_{\mathbf{y} \in U^k} D_q(\mathcal{P} + \mathbf{y}) \leq A_2(k) D_\infty(\mathcal{P}).$$

S. DAVID: **Heights on abelian verieties**
(joint work with J.B. Bost). In a recent work, Masser and Wüstholz proved an estimate on the degree of the smallest abelian subvariety containing a period of a given abelian variety in its tangent space. This theorem is well known to imply uniform isogeny estimates first establishes by Faltings. Our aim was to clarify the effectivity of the above stated result. In addition to bringing down the exponents to essentially the best possible bounds, we solve the question of effectivity by proving a totally explicit comparison estimate between the Faltings height of an abelian variety and some modular height. This proof avoids recourse to the Satake compactification.

L. DENIS: **Transcendence properties of Bessel-Carlitz functions**
Let k be a rational function field over the finite field with q elements, k_∞ its completion with respect to the infinite place and c a completion of an algebraic closure of k_∞. L. Carlitz has defined the analog of the classical Bessel function $J(z)$ and its derivative $J'(z)$ in this situation. Call these analytic functions $\tilde{J}(z)$ and $\Delta\tilde{J}(z)$ C.L. Siegel proved that if α is a non-zero algebraic number then $J(\alpha)$ and $J'(\alpha)$ are algebraically independent over $\mathbf{Q}$. It is now shown that if α is in the algebraic closure of k in c and is not zero then $\tilde{J}(\alpha)$ and $\Delta\tilde{J}(\alpha)$ are algebraically independent over k.

M. DODSON: **Metric diophantine approximation on manifolds**
The functional relations between coordinates of points on a manifold are an obstacle to transferring classical results in Diophantine approximation to

manifolds. In joint work with Rynne and Vickers it has been shown that Khintchine's theorem on simultaneous diophantine approximation holds for smooth (C^3) manifolds M embedded in $\mathbf{R}^n$ and satisfying some curvature conditions. Similar results for the linear forms version of the theorem have also been obtained and the Hausdorff dimension of the associated exceptional sets determined.

In the case of convergence in Khintchine's theorem, the manifold is required to be 2-convex (this reduces to non-zero Gaussian curvature for surfaces in $\mathbf{R}^3$) almost everywhere. In the harder divergent case, where quasi-independence is needed, more severe restrictions are required, and certain submanifolds of M have to be 2-convex. For simultaneous approximation, a mild decay condition on the error has to be imposed; this has the effect of forcing $\dim M \geq \frac{n+1}{2}$. Although this might be a technical difficulty in the arguments it might also be connected with the positive curvature of M and the inadmissibility of the lattice $\{\mathbf{p}/q : \mathbf{p} \in \mathbf{Z}^n, q \in \mathbf{N}\}$. However, only a slightly stronger decay condition gives an asymptotic formula for the number of solutions of the system of inequalities, using a method of W.M. Schmidt.

J.H. EVERTSE: **Singular differences of powers of 2×2 matrices** (joint work with R. Tijdeman) For two matrices $A, B \in GL_2(\mathbf{C})$ let

$$\mathcal{S}_{A,B} = \{(m,n) \in \mathbf{Z}^2 : A^m - B^n \text{ is singular}\}$$

Two pairs of matrices $(A, B), (A_1, B_1)$ are said to be equivalent if there is a matrix $J \in GL_2(\mathbf{C})$ such that (JAJ^{-1}, JBJ^{-1}) is equal to one of the pairs $(A_1, B_1), (B_1, A_1), (A_1^T, B_1^T), (B_1^T, A_1^T)$ where C^T denotes the transpose.

Questions by Polligton inspired us to determine the pairs (A, B) for which $\mathcal{S}_{A,B}$ is infinite. It is easy to see that these pairs can be partitioned into equivalence classes with respect to the equivalence relation defined above. We determine all equivalence classes of (A, B) such that at least one of A, B can be diagonalised, and $\mathcal{S}_{A,B}$ is infinite. Moreover a necessary condition for a pair of non-diagonalisable matrices A, B with infinite set $\mathcal{S}_{A,B}$ is given.

The basic tool is a result of Laurent on exponential polynomial equations in several unknowns which is applied to $\det(A^m - B^n) = 0$. Laurent's result goes back to Schmidt's subspace theorem. These results were applied by Pollington in his research on normality with respect to matrices.

E. FOUVRY: **Gaussian primes**

Theorem 1 (Fouvry-Iwaniec) *Let $\lambda_l \in \mathbf{C}$ with $|\lambda_l| \leq 1$. Then*

$$\sum_{l^2+m^2 \leq x} \lambda_l \Lambda(l^2 + m^2) = \sum_{l^2+m^2 \leq x} \lambda_l \psi(l) + O(x(\log x)^{-A})$$

where Λ is von Mangoldt's function,

$$\psi(l) = \prod_{p \nmid l} \Big(1 - \frac{\chi(p)}{p-1}\Big),$$

χ is the non-trivial character mod 4, A is any positive number, and the implied constant in the error term depends only on A.

By choosing for λ_l the characteristic function of a dense set $\mathcal{L}$ (which means $\#(\mathcal{L} \cap [-x,x]) \geq x(\log x)^{-c}$ for some $c > 0$ and x tending to infinity) we deduce that the number of Gaussian primes π such that $|\pi| \leq x$ and Re $\pi \in \mathcal{L}$ has the expected order of magnitude for $x \to \infty$. The proof uses large sieve techniques and properties of the polynomial $l^2 + m^2$.

K. GYÖRY: **The number of families of solutions of decomposable form equations**
(joint work with J.-E. Evertse) Consider the decomposable form equation

$$F(\mathbf{x}) = \pm b \quad \text{in } \mathbf{x} \in \mathbf{Z}^n \qquad (*)$$

where $F \in \mathbf{Z}[x_1, \ldots, x_n]$ is a decomposable form and $b \in \mathbf{Z} \setminus \{0\}$. Using a quantitative result of Schlickewei (1990) on S-unit equations the speaker obtained in 1993 explicit upper bounds for the number of maximal families of solutions of (*), as well as for the minimal number of those families of solutions whose union contains all the solutions. Considerable improvements are now possible. An explicit upper bound for the number of solutions of (*) is obtained provided that this number is finite. When the number of solutions is infinite, an asmptotic formula of the form $c(\log N)^r + O((\log N)^{r-1})$ holds for the number of solutions of (*) with $\max|x_i| \leq N$. This generalizes a result of Győry and Pethö (1977) on norm form equations.

Our results have been established in the more general situation when the ground ring is the ring of S-integers of an arbitrary number field. The main new tools are a general result of Evertse (1995) on Galois-symmetric S-unit vectors and an improved and generalized version of some results of Schmidt (1990) and Győry (1993) on decomposable form equations.

M. HATA: **An application of Beukers' integral**
Considering the double integral $\iint_S P(x)Q(y)(1-xy)^{-1-n}dx\,dy$ where $P(x) = Q(x) = (x(1-x))^n$, $n \in \mathbf{N}$ and $S = [0,1]^2$, F. Beuker's gave an elegant proof of the irrationality of $\zeta(2) = \frac{\pi^2}{6}$. It seems natural to modify his integral in order to study arithmetical properties of some other numbers. Here we change the integration domain S to the new $S_1 = [1,2] \times [\frac{1}{2}, 1]$ and obtain a non-quadraticity measure for $\log 2$:

$$|\log 2 - \xi| \geq H^{-25.051} \quad (H \geq H_0,\ H_0 \text{ effective})$$

where ξ is any quadratic number with height H. Although the double integral over S_2 diverges, it can be justified as the limit as $z \to 1$ along a curve in the upper half plane of

$$F(z) = \iint_{S_2} \frac{P(x)Q(y)}{(1-xyz)^{n+1}}\,dx\,dy.$$

Taking real and imaginary parts of $\lim_{z\to 1, \mathrm{Im} z>0} F(z)$ one obtains a simultaneous rational approximation to $\log 2$ and $\log^2 2$. The choice

$$P(x) = (x-1)^{n+[n/10]}(x-2)^{n-[n/10]}, \quad Q(y) = (y-1)^{n-[n/10]}(2y-1)^{n-[n/5]}$$

gives the result. For the estimate a complex 2-dimensional version of the saddle method is needed.

K. KAWADA: **On the representation of numbers as the sum of four cubes**

Let $E(N)$ be the number of natural numbers not exceeding N that cannot be written as a sum of four cubes. Then, for any $\epsilon > 0$ there is a $\eta > 0$ such that

$$E(N+M) - E(N) \ll M^{1-\eta}, \quad \text{providing } M \geq N^{\frac{1585}{2169}+\epsilon}.$$

This is an improvement of a result due to Brüdern and Watt who obtained the exponent $\frac{3}{4}$ in place of $\frac{1585}{2169}$. T.D. Wooley pointed out that the above result can be improved by new results due to him which he described during this conference.

M. LAURENT: **Linear forms in two logarithms**

Let α_1, α_2 be two non-zero algebraic numbers, b_1, b_2 be positive integers. Lower bounds for the absolute value (archimedian and p-adic) of $\Lambda = b_1 \log \alpha_1 - b_2 \log \alpha_2$ and $\alpha_1^{b_1} - \alpha_2^{b_2}$ (which is roughly the same) are obtained. We get bounds of the shape

$$\log |\Lambda| \geq -CD^4 (\log B)^2 \log A_1 \log A_2$$

where D is an upper bound for the degree $[\mathbf{Q}(\alpha_1, \alpha_2) : \mathbf{Q}]$, B is an upper bound for b_1, b_2, and $\log A_1, \log A_2$ for the absolute logarithmic height of α_1, α_2. Here C is a constant (absolute or depending upon p) which is fairly small, around some dozens in the archimedian case.

H. MAIER: **The size of the coefficients of cyclotomic polynomials**

Let $\Phi_n(z)$ be the n-th cyclotomic polynomial, $A(n)$ the absolute value of its largest coefficient. The following result is obtained: Let $\psi(n) \to \infty$ for $n \to \infty$. Then $A(n) \leq n^{\psi(n)}$ on a sequence of asymptotic density one.

The method of proof consists in the analysis of $\Phi_n(z)$ on the unit circle in combination with techniques from probabilistic number theory.

T. MATALA-AHO: **Irrationality results for p-adic logarithmic and binomial series**

In joint works with A. Heimonen and K. Väänänen we investigated some lower bounds

$$|\alpha - \frac{m}{n}|_p > H^{-m_p(\alpha)+\epsilon}, \quad H = \max(|m|, |n|) \geq H_0(\epsilon)$$

for the values α of a class of Gaussian series ${}_2F_1$. Now irrationality measures $m_p(\alpha)$ for p-adic logarithmic and binomial series are obtained. Let $1 < r/s \in \mathbf{Q}$, $p \not| s$, $er|r|_p^2 < 1$. Then

$$m_p(\log(1 - \frac{r}{s})) \leq \frac{s \log |r|_p}{2 \log |r|_p + \log r + 1}.$$

There exists a similar bound for $r/s < 1$. For binomial series $(1+z)^{1/k}$ there are analogous results.

B. DE MATHAN: **Roth's theorem in positive characteristic**
Let K be a field of positive characteristic p. It was proved by A Lasjaumias and the speaker that Thue's theorem holds for every algebraic element $\alpha \in K((T^{-1}))$ which satisfies no(!) equation $\alpha = (A\alpha p^s + B)/(C\alpha p^s + D)$ where $s \in \mathbf{N}$, $A, B, C, D \in K[T]$, $AD - BC \neq 0$. An example of such an element was given by Bucle and Robbins. For this element, Roth's theorem holds in the form $|\alpha - P/Q| \gg |Q|^{-2-\epsilon}$ but not in the stronger form $|\alpha - P/Q| \gg |Q|^{-2}$.

E.M. MATVEEV: **Elimination of the mulpile $n!$ from estimates for linear forms in logarithms**
Let $K \subset \mathbf{C}$ be a field. Suppose that $\alpha_1, \ldots, \alpha_n \in K^*$ satisfy the Kummer condition $[K(\sqrt{\alpha_1}, \ldots, \sqrt{\alpha_n}) : K] = 2^n$ with fixed values of $\log \alpha_j$. Put $D = [K : \mathbf{Q}]$ if $K \subset \mathbf{R}$ and $D = \frac{1}{2}[K : \mathbf{Q}]$ otherwise. Now write

$$A_j = \max\{1, Dh(\alpha_j), |\log \alpha_j|\}, \quad \Omega = A_1 \ldots A_n,$$

$$\rho = \mathrm{rank}_{\mathbf{R}}\{\log \alpha_1, \ldots, \log \alpha_n\}.$$

Consider a homogeneous rational linear form in logarithms

$$\Lambda = b_1 \log \alpha_1 + \ldots + b_n \log \alpha_n$$

with $b_n \in \mathbf{Z}$. It was shown that there exists an absolut constant $C > 1$ such that

$$\log |\Lambda| \geq -C^n \rho^n D^2 \Omega \log(eB) \log(C^n \rho^n D^2 \Omega / A_n)$$

where $B = \max_j\{|b_j| A_j / A_n\}$.

M. MIGNOTTE: **On the Thue equation** $ax^n - by^n = c$
We consider a binary form $F(x, y) = ax^n - by^n$ with $n \geq 3$, $a, b \in \mathbf{N}^*$, $a \neq b$ and the equation $F(x, y) = c$, $C \in \mathbf{N}^*$. In 1937 Siegel proved

Theorem A. *The inequality* $|F(x, y)| \leq c$ *has at most one solution* $(x, y) \in \mathbf{Z}^2$ *with* $\gcd(x, y) = 1$ *provided*

$$(ab)^{\frac{n}{2}-1} \geq 4c^{2n-2}\Big(n \prod_{p|n} p^{1/(n-1)}\Big)^n.$$

Gefördert durch DFG Deutsche Forschungsgemeinschaft

Here, using a recent result on linear forms in two logs of Laurent-Mignotte-Nesterenko we obtain a general lower bound of the form

$$|F(x,y)| \geq |y|^{n-C_2(\log n)^2} \quad \text{for } n \geq C_1(a,b).$$

As a corollary of this result we obtain

Theorem 1 *In the special case* $F(x,y) = (b+1)x^n - by^n$, $b \geq 1$, *suppose that* $0 < |F(x,y)| < \min\{(2^n-2)b, \frac{2}{3}n^2b^3\}$ *with* $|x| \neq |y|, xy \neq 0$. *Then* x^n *and* y^n *are of the same sign. If* $x > 0, y > 0$, *then* $y > x > 1$, $y \geq nb(y-x)$ *and* $n < 600$.

In the special subcase $(b+1)x^n - by^n = 1$ combining Theorem A and a refined version of Theorem 1 we get: if $(x,y) \neq (1,1)$ is a solution, then $n \leq 350$ and $b \leq 370$.

H. MIKAWA: **A variant of the Eratosthenes sieve**

Sieve methods seem to produce an ambiguity between integers with an odd number of prime factors and ones with an even number. From this point of view it is of some interest to note that

$$\mu(n) = 1 \quad \text{implies} \quad \sum_{\substack{d|n \\ d<\sqrt{n}}} \mu(d) = 0.$$

The relation to the sifting process and the effect on finding primes has been discussed. In particular, the problem of primes in almost all short intervals has been considered.

Y.V. NESTERENKO: **Modular functions and transcendence problems**

Let $P(z), Q(z), R(z)$ be the functions introduced by Ramanujan in 1916, $J(z) = 1728\frac{Q(z)^3}{Q(z)^3 - R(z)^2}$, and let $j(\tau) = J(e^{2\pi i\tau})$ be the modular function.

Theorem 1 *For any complex number* q, $0 < |q| < 1$, *among* $q, P(q), Q(q), R(q)$ *there exists at least three numbers algebraically independent over* $\mathbf{Q}$.

Corollary 1 Let q be algebraic, $0 < |q| < 1$. Then the numbers in each set

$$1)\ P(q), Q(q), R(q); \quad 2)\ J(q), \theta J(q), \theta^2 J(q)$$

where $\theta = z\frac{d}{dz}$, are algebraically independent. In particular all these numbers are transcendental.

The assertion 2) was conjectured by D. Bertrand in 1977.

Corollary 2 Let $\wp(z)$ be a Weierstrass elliptic function with algebraic invariants g_2, g_3 and complex multiplication over the field k; let ω be the period of $\wp(z)$. Then for any $\tau \in k$, Im $\tau \neq 0$, the numbers $\pi, \omega, e^{2\pi i\tau}$ are algebraically independent. In particular, the numbers in each set $\{\pi, e^{\pi}, \Gamma(1/4)\}$; $\{\pi, e^{\sqrt{3}}, \Gamma(1/3)\}$, $\{\pi, e^{\sqrt{D}}\}$ where D is a positive integer, are algebraically independent.

Theorem 2 *Let $q \in \mathbf{C}$, $0 < |q| < 1$ be such that the transcendence degree of the field $K = \mathbf{Q}(q, P(q), Q(q), R(q))$ equals 3 and let $\theta = (\theta_1, \theta_2, \theta_3)$ be a transcendence basis of K. Then there exists a constant $\gamma > 0$ depending only on q and θ such that for any polynomial $A \in \mathbf{Z}[x_1, x_2, x_3]$, $A \neq 0$, one has*

$$|A(\theta)| \geq \exp(-\gamma t(a)^4 (\log t(A))^{24})$$

where $t(A) = \log H(A) + \deg A$. In particular such an estimate holds for the set of numbers $\pi, e^\pi, \Gamma(1/4)$.

Theorem 3 *Let L_1, L_2 be positive integers. Then for any polynomial $A \in \mathbf{Z}[z, x_1, x_2, x_3]$, $A \neq 0$, with $\deg_z A \leq L_1$, $\deg_{x_i} A \leq L_2$ one has*

$$ord\, A(z, P(z), Q(z), R(z)) < 2(10)^{45} L_1 L_2$$

where ord denotes the order of zero at $z = 0$.

P. PHILIPPON: **Algebraic independence and K-functions**
Two applications of the new construction-extrapolation of an auxiliary function invented by Barré-Sirieix, Diaz, Gramain and Philibert are given to solve conjectures by Mahler and Manin on the transcendency of the values of the modular j function. The first application to Eisenstein series generalizes this result and contains the proof of algebraic independence of the three numbers π, e^π and $\Gamma(1/4)$ obtained by Nesterenko. The second application gives the expected lower bound for the transcendence degrees of the fields generated by certain values of functions satisfying functional equations. It generalizes results of Mahler and improves a result of Amon. The new method applies to an interesting sub-class of the G-functions ("K-functions").

M. POE: **Distribution of solutions on S-unit equations**
Consider the equation

$$\alpha p_1^{a_1} \dots p_l^{a_l} + \beta p_{l+1}^{a_{l+1}} \dots p_r^{a_r} = \gamma p_{r+1}^{a_{r+1}} \dots p_s^{a_s}. \qquad (1)$$

Assume that α, β, γ are fixed non-zero relatively prime rational integers, $p_1, \dots, p_r$ are distinct rational primes, all not dividing $\alpha\beta\gamma$. All solutions of (1) with

$$1 \leq a_i \leq M \quad (1 \leq i \leq s) \qquad (2)$$

are considered. It is shown that with at most $2s^s$ exceptions all solutions lie in a smaller box

$$a_i - \tilde{a}_i \leq \frac{s^2 \log sM}{\log p_i} \quad (1 \leq i \leq s)$$

where $\tilde{a}_i$ denotes the smallest exponent of p_i occuring in all solutions of (1) and (2).

A. POLLINGTON: **Riesz products and normality**
(joint work with G. Brown and W. Moran) For $\theta > 1$ let $B(\theta)$ denote the set of real numbers x for which $\{\theta^n x\}$ is uniformly distributed modulo 1. Then, as shown by Schmidt, $B(r) = B(s)$ $(r, s \in \mathbf{N})$ if and only if $r^n = s^m$ for some $n, m \in \mathbf{N}$. For non-integer ϕ, ψ we have

Theorem 1 *$B(\psi) \subset B(\phi)$ if and only if $\log\psi / \log\phi \in \mathbf{Q}$ and either*
1. $\psi = a^{1/n}, \phi = a^{1/m}$, $a \in \mathbf{N}$ and $\mathbf{Q}(\phi) \subset \mathbf{Q}(\psi)$,
or
2. $\psi = \theta^m, \phi = \theta^n, \theta^k \pm \theta^{-k} \in \mathbf{Z}$ and $m|n$.

We employ the method of constructing a Riesz product measure μ on a set of non-normal numbers with respect to one base and then show that with respect to this measure μ-almost all numbers are normal with respect to the other base. This uses Davenport-Erdös-LeVeque and digit arguments on transform space.

We are able to extend this method to the case of 2×2 almost integer ergodic matrices settling a question of Schmidt in this case. The application of Davenport-Erdös-LeVeque now requires a certain matrix identity to have only finitely many solutions This result was obtained by Evertse and Tijdeman, and was also reported on at this conference.

D. ROY: **Algebraic approximation to transcendental numbers and algebraic independence**
(joint work with M. Laurent and M. Waldschmidt) The problem of proving algebraic independence using interpolation determinants has been open for some time. Two solutions for this problem are proposed. The first one is based on the existence of good algebraic approximations to families of complex numbers in a field of transcendence degree one. The second uses a generalization of Gelfond's lemma in which multiplicities are taken into account.

A. SCHINZEL: **The Mahler measure for polynomials in several variables**
Let $F \in \mathbf{C}[z_1, \ldots, z_s]$ and define the Mahler measure $M(F)$ by

$$M(F) = \exp \int_{[0,1]^s} \log |F(e^{2\pi i\theta_1}, \ldots, e^{2\pi i\theta_s})| \, d\theta_1 \ldots d\theta_s.$$

Theorem 1 *Let $F = \sum_{k=1}^{l} a_k z_1^{\alpha_{k1}} \ldots z_s^{\alpha_{ks}}$, where $a_k \in \mathbf{C}^*$ and the vectors $\alpha_k = [\alpha_{k1}, \ldots, \alpha_{ks}]$ are distinct. If there exists $1 \le i, j \le s$ and a vector $v \in \mathbf{R}^s$ such that*

$$v\alpha_i < v\alpha_k < v\alpha_j \quad \textit{for all } k \neq i, j$$

then

$$M(F)^2 + \frac{|a_i a_j|^2}{M(F)^2} \le \sum_{k=1}^{l} |a_k|^2.$$

Equality holds if $F(z_1,\ldots,z_s)\bar{F}(z_1^{-1},\ldots,z_s^{-1})$ has exactly 3 terms.

This generalizes a result of V. Gongalvez.

H.P. SCHLICKEWEI: **The equation $x+y=1$**
Let $G\subset(\mathbf{C}^*)^2$ be a finitely generated subgroup of rank r. We study the equation $x+y=1$, to be solved in elements $(x,y)\in G$. Let N be the number of solutions of this equation. Two years ago, the speaker derived the first bound for N that depends only on r, namely

$$N\leq 2^{2^{26}+36r^2}.$$

This was improved in a joint paper with W.M. Schmidt to

$$N\leq 2^{13r+63}r^r.$$

In recent joint work with F. Beukers we proved

$$N\leq 2^{8r+8}.$$

W.M. SCHMIDT: **The distribution of sublattices of $\mathbf{Z}^m$**
The similarity class of a lattice of rank n may be parametrized by the orbits of the action of $GL_n(\mathbf{Z})$ on the generalized upper half plane $\mathcal{H}_n$, consisting of matrices

$$Z=\begin{pmatrix} 1 & x_{12} & \ldots & x_{1n}\\ 0 & y_2 & \ldots & x_{2n}\\ \vdots & & & \vdots\\ 0 & 0 & \ldots & y_n\end{pmatrix}$$

with $y_i>0$ $(i=2,\ldots,n)$. Put differently , it may be paramerized by the points of a fundamental domain $\mathcal{F}$ for the action of $GL_n(\mathbf{Z})$ on $\mathcal{H}_n$.

Now let $2\leq n\leq m$. Let $\mathcal{D}$ be an open subset of $\mathcal{F}$. Then the number $N(\mathcal{D},T)$ of sublattices of $\mathbf{Z}^m$ of rank n and determinant $\leq T$ whose similarity class belongs to some $Z\in\mathcal{D}$ has

$$N(\mathcal{D},T)\sim c(m,n)\mu(\mathcal{D})T^m$$

as $T\to\infty$ where $\mu(\mathcal{D})$ is the invariant measure on $\mathcal{H}_n$.

A. SHIDLOVSKI, Y.V. NESTERENKO: **Linear independence of values of E-functions**

Theorem 1 *Let $f_1(z),\ldots,f_m(z)$ be E-functions, satisfying the system of differential equations*

$$y_k'=\sum_{i=1}^m Q_{ki}y_i,\quad Q_{ki}\in\mathbf{C}(z),\ k=1,\ldots m, \tag{1}$$

and linearly independent over $\mathbf{C}(z)$. Then there exists a finite set $\Lambda\subset\mathbf{C}$, dependent on $f_1,\ldots,f_m$ such that for any $\xi\in\mathbf{A}\setminus\Lambda$ the values of $f_1(\xi),\ldots,f_m(\xi)$ are linearly independent over $\mathbf{A}$.

Let $\mathbf{p}$ be a prime ideal, generated in $\mathbf{C}[z, x_1, \ldots, x_m]$ by all in $x_1, \ldots, x_m$ homogeneous polynomials Q with the property $Q(z, f_1(z), \ldots, f_m(z)) \equiv 0$. Denote by $\mathbf{p}_\xi$ the image of $\mathbf{p}$ in $\mathbf{C}[x_1, \ldots, x_m]$ under the spezialisation $z \to \xi \in \mathbf{C}$. There exists only finitely many ξ (points of bad reduction) such that $\mathbf{p}_\xi$ is not a prime ideal. The set Λ in the theorem contains zero, singular points of the system (1) and points of bad reduction. We conjecture that the theorem holds for any $\xi \in \mathbf{A}$ distinct from zero and singular points of (1).

C. STEWART: **On prime factors of integers of the form $ab + 1$** (joint work with K. Győry and A. Sarközy). For any integer n exceeding 1 let $P(n)$ denote the greatest prime factor of n. Denote the cardinality of a set X by $|X|$. The following result is a multiplicative analogue of a result of P. Erdös, R. Tijdeman and C. Stewart.

Theorem 1 *Let $\epsilon > 0$, and let k, l be integers with $k \geq 3$ and $2 \leq l \leq \left(\frac{\log\log k}{\log\log\log k}\right)^{1/2}$. There exists a positive number $C(\epsilon)$ which is effectively computable in terms of ϵ, such that if $k \geq C(\epsilon)$ then there are sets of positive integers $\mathcal{A}$, $\mathcal{B}$ with $|\mathcal{A}| = k$ and $|\mathcal{B}| = l$ for which*

$$P\Big(\prod_{a\in\mathcal{A}}\prod_{b\in\mathcal{B}}(ab+1)\Big) < (\log k)^{l+1+\epsilon}.$$

M. SKRIGANOV: **Ergodic theory on homogeneous spaces, diophantine approximation and lattice points counting polyhedrons**
It is shown that counting lattice points for polyhedrons can be reduced to simultaneous diophantine approximations for linear forms. These diophantine problems are then interpreted in terms of certain flows on homogeneous spaces $SL(n, \mathbf{R})/SL(n, \mathbf{Z})$. As a result, we derive from ergodic theorems on semisimple groups that for any expanding polyhedron the lattice point counting has a logarithmically small error for almost all lattices with respect to the invariant measure on $SL(n, \mathbf{R})/SL(n, \mathbf{Z})$. Applications to algebraic number fields, uniform distribution and spectral theory can also be given.

J. THUNDER: **Hermite's constant for number fields**
For $n > 1$ Hermite's constant is the smallest real number γ_n such that for any positive definite quadratic form $Q(\mathbf{x})$ in n variables there is a nonzero integer point $\mathbf{z}$ satisfying $Q(\mathbf{Z}) \leq \gamma_n D^{1/n}$ where D is the discriminant of Q. This definition can be formulated using heights, and can then be used to give a definition for Hermite's constant over a general number field. Upper and lower bounds for these constants are obtained.

C. VIOLA: **A group-theoretic approach to irrationality results for the dilogarithm**
Let $L_2(z) = \sum_{n=1}^{\infty} \frac{z^n}{n^2}$ be the dilogarithm. In joint work with G. Rhin, for $L_2(1) = \pi^2/6$ the irrationality measure 5.441243 is proved, improving recent

results of M. Hata. This is obtained by applying to double integrals of the type

$$\int_0^1 \int_0^1 \frac{x^h(1-x)^i y^k(1-y)^j}{(1-xy)^{i+j-l}} \frac{dx\,dy}{1-xy}$$

the action of the permutation group related to the Gauss hypergeometric function. The method extends to yield irrationality proofs and irrationality measures for $L_2(1/q)$, for suitable integers $q \neq 0, 1$. In particular, $L_2(1/6) \notin \mathbf{Q}$. In 1993 Hata showed $L_2(1/q) \notin \mathbf{Q}$ for any integer $q \in (-\infty, -5] \cup [7, \infty)$.

R.C.VAUGHAN: **The Montgomery-Hooley asymptotic formula and some generalizations.**

Let

$$V(x,Q) = \sum_{q \leq Q} \sum_{\substack{a=1 \\ (a,q)=1}}^{q} \left|\psi(x,q,a) - \frac{x}{\phi(q)}\right|^2$$

where

$$\psi(x,q,a) = \sum_{\substack{n \leq x \\ n \equiv a \bmod q}} \Lambda(n)$$

and Λ is von Mangoldt's function. In joint work with Goldston the following theorem, improving earlier results of Barban, Davenport-Halberstam, Gallagher, Montgomery, Hooley and Friedlander-Goldston, is obtained.

Theorem 1 *Suppose that the generalised Riemann hypothesis holds and let*

$$U(x,Q) = V(x,Q) - Qx\log Q + cxQ$$

where

$$c = \gamma + \log 2\pi + 1 + \sum_p \frac{\log p}{p(p-1)}.$$

Then (i) when $1 \leq Q \leq x$ *one has*

$$U(x,Q) \ll Q^2(x/Q)^{1/4+\epsilon} + x^{3/2}(\log 2x)^{5/2}(\log\log 3x)^2$$

and (ii) there is an absolute constant C *such that when* $x/Q \to \infty$ *with*

$$Cx^{5/7}(\log 2x)^{10/7}(\log\log 3x)^{8/7} < Q \leq x$$

one has

$$U(x,Q)Q^{-2} = \Omega_{\pm}((x/Q)^{1/4}).$$

An unconditional theorem, for more general sequences than the primes, was also described.

P. VOJTA: **Roth's theorem with moving targets**

Theorem 1 *Let k be a number field, S a finite set of places of k, $C \in \mathbf{R}$, and $\epsilon > 0$. Then any infinite set of tuples $(x, (\alpha_v)_{v \in S})$ in $k^{\#S+1}$ with $\log H_k(\alpha_k) = o(\log H_k(x))$ for all $v \in S$ must have*

$$\prod_{v \in S} \|x - \alpha_v\|_v \geq \frac{C}{H_k(x)^{2+\epsilon}}$$

for all but finitely many tuples $(x, (a_v))$.

When $\#S = 1$ and $\alpha_v \in K$ for some fixed $K \supset k$, this follows from a result of van der Poorten and Bombieri (1988).

This statement was suggested by a conjecture of R. Nevanlinna proved by C. Osgood. The proof follows Steinmetz's simplification of Osgood's proof.

J.F. VOLOCH: **Linear forms in p-adic roots of unity**
(joint work with J. Tate) Let $\mathbf{C}_p$ be the completion of the algebraic closure of the p-adics. If $a_1, \ldots, a_n \in \mathbf{C}_p$ there exists a constant $c > 0$ such that for any roots of unity $\zeta_1, \ldots, \zeta_n \in C_p$ either $\sum a_i\zeta_i = 0$ or $|\sum a_i\zeta_i| \geq c$.

We also speculate when $a_1, \ldots a_n$ are in a number field, how the constant c varies as we vary the place in the number field.

J. WOLFART: **Jacobians of CM type**
Let X be a nonsingular projective algebraic curve, defined over $\mathbf{C}$, and of genus > 1 (for simplicity). If Jac X is of CM type it is defined over a number field, and X can also be defined over $\bar{\mathbf{Q}}$.

Theorem 1 *If X is defined over $\bar{\mathbf{Q}}$ it has a covering curve Y with "large automorphism group" Aut Y, i.e. any proper deformation of Y decreases $\#\mathrm{Aut}\,Y$.* Jac *X is of CM type if and only if* Jac *Y is of CM type.*

(The proof involves the existence of a Belyi function on X and Fuchsian groups). Y is also defined over $\bar{\mathbf{Q}}$.

Theorem 2 *If X has a large automorphism group it has a Jacobian of CM type if e.g.*
- *the genus of X is 2 or 3 or*
- Aut *X is abelian.*

Examples: Klein's quartic, Fermat curves
Counterexamples: Bring's curve (g=4), Macbeath's curve (g=7).
Ideas of proof: Consider periods of first kind on X and their behaviour under Aut X, apply linear independence results for these periods (Shiga-Wolfart) coming from the analytic subgroup theorem, or transcendence results for Siegel modular functions (P.Cohen-Shiga-Wolfart).

T.D. WOOLEY: **Progress on exponential sums over smooth numbers**

The exponential sums

$$f(\alpha; P, R) = \sum_{x \in \mathcal{A}(P,R)} e(alx^k),$$

where $e(\alpha) = \exp 2\pi i\alpha$ and

$$\mathcal{A}(P, R) = \{n \in [1, P] \cap \mathbf{Z} : p|n \to p \leq R\}$$

have been fundamental in recent approaches to a number of additive problems, including Waring's problem. Hitherto, mean values of such sums,

$$U_s(P, R) = \int_0^1 |f(\alpha; P, R)|^s \, d\alpha$$

have been estimated, for s which are not even integers, by Hölder's inequality. Thus, if t is the integer with $2t \leq s < 2t + 2$, then

$$U_s(P, R) \leq U_{2t}(P, R)^{t+1-\frac{1}{2}s} U_{2t+2}(P, R)^{\frac{1}{2}s-t}.$$

In new work, we provide a method, extending Vaughan's "new" iterative method, which provides non-trivial estimates for all moments. Amongst the corollaries one has:

(I)

$$\int_0^1 \Big| \sum_{x \in \mathcal{A}(P,R)} e(\alpha x^3) \Big|^6 d\alpha \ll P^{\lambda_3 + \epsilon}$$

where $R = P^\eta$, $\eta = \eta(\epsilon) > 0$ and $\lambda_3 = 3,24959$.

(II) The number $\mathcal{N}$ of integers $n \leq x$ which are the sum of three k-th powers satisfies $\mathcal{N} \gg x^{\frac{3}{k} - e^{-k/17}}$, coming close to the expected lower bound $\mathcal{N} \gg x^{3/k}$.

(III) Also the new estimates proved important in the proof (with R.C. Baker and J. Brüdern) of:

When $\lambda_1, \ldots, \lambda_7, \mu$ are real numbers with λ_1/λ_2 irrational, $\lambda_i \neq 0$, there are infinitely many integral solutions of

$$|\lambda_1 x_1^3 + \ldots + \lambda_7 x_7^3 + \mu| < (\max |x_i|)^{-10^{-4}}.$$

Some problems posed at the problem session

D. BERTRAND: For any integer $N \geq 1$ denote by $\Phi_N(X, Y) = 0$ the modular equation of level N, and by $J(q)$ the usual modular invariant. The following would extend the theorem of Barré, Diaz, Gramain and Philibert:

Conjecture: Let q_1, q_2 be non-zero algebraic numbers of absolute value < 1 such that $J(q_1)$ and $J(q_2)$ are algebraically dependent over $\mathbf{Q}$. Then q_1 and q_2 are multiplicatively dependent; in particular, there exists an integer $N \geq 1$

such that $\Phi_N(J(q_1), J(q_2) = 0$.

In relation with the 4-exponential problem we mention the following consequence of the conjecture:

"Corollary": Let α_1, α_2 be positive real algebraic numbers distinct from 1. Then $(\log\alpha_1)(\log\alpha_2)$ and $\zeta(2)$ are linearly independent over $\mathbf{Q}$.

P. BORWEIN, D. BOYD: Does there exist a polynomial $p(z) = 1 + a_1z + \ldots + a_nz^n$ with $a_i \in \{0, 1, -1\}$ which has a repeat root of order 5 at some point α with $|\alpha| \neq 1$? In general can such polynomials have non-cyclotomic factors of arbitrary high order?

J. BRÜDERN: Let s_n be the strictly increasing sequence of those natural numbers which are the sum of two cubes of natural numbers, and define the gap set as $\mathcal{G} = \{s_n - s_{n-1} : n \geq 2\}$. I conjecture that $\mathcal{G}$ has positive density. This would follow from the following paucity type estimate. Let $T(P)$ denote the number of solutions of the diophantine inequality

$$0 < x_1^3 + x_2^3 - y_1^3 - y_2^3 = x_3^3 + x_4^3 - y_3^3 - y_4^3 \leq 8P$$

with $1 \leq x_i, y_i \leq P$. Here I expect $T(P) \ll P^3$ which would imply the conjecture.

A. SCHINZEL: Let $F \in \mathbf{C}[z_1, \ldots, z_s]$ be a polynomial, let $L(F)$ denote its length and $M(F)$ the Mahler measure. Let F_n be polynomials over $\mathbf{C}$. Does $\lim_{n\to\infty} = L(F_n - F) = 0$ imply $\lim M(F_n) = M(F)$? (D.W. Boyd has shown the answer is yes under the additional assumption that $F_n \in \mathbf{C}[\mathbf{z_1}, \ldots, \mathbf{z_s}]$ and $\deg F_n$ is bounded).

R. TIJDEMAN: A Beatty sequence is a sequence of the form $\{\lfloor n\alpha + \beta\rfloor\}_{n\in\mathbf{Z}}$ or $\{\lceil n\alpha + \beta\rceil\}_{n\in\mathbf{Z}}$ where $\alpha \geq 1$ and β are reals. The complement of a Beatty sequence is a Beatty sequence.
Is it true that for $m > 2$ there are only finitely many ways to split $\mathbf{Z}$ into m Beatty sequences with distinct α's?

Berichterstatter: Jörg Brüdern, Stuttgart

Tagungsteilnehmer

Prof.Dr. Francesco Amoroso
Dipartimento di Matematica
Universita di Pisa
Via Buonarroti, 2

I-56127 Pisa

Prof.Dr. Vasilii Bernik
Institute of Mathematics
Academy of Sciences of Belarus
ul. Surganova 11

Minsk 220072
BELORUSSIA

Prof.Dr. Roger C. Baker
Department of Mathematics
Brigham Young University

Provo , UT 84602
USA

Prof.Dr. Daniel Bertrand
Institut de Mathematiques Pures et Appliquees, UER 47
Universite de Paris VI
4, Place Jussieu

F-75252 Paris Cedex 05

Prof.Dr. Antal Balog
Mathematical Institute of the
Hungarian Academy of Sciences
P.O. Box 127
Realtanoda u. 13-15

H-1364 Budapest

Prof.Dr. Enrico Bombieri
School of Mathematics
Institute for Advanced Study
Olden Lane

Princeton , NJ 08540
USA

Prof.Dr. Jozsef Beck
Dept. of Mathematics
Rutgers University
Busch Campus, Hill Center

New Brunswick , NJ 08903
USA

Prof.Dr. Peter Borwein
Dept. of Mathematics
Simon Fraser University

Burnaby , B.C. V5A 1S6
CANADA

Prof.Dr. Michael A. Bennett
Department of Mathematics
The University of Michigan
3220 Angell Hall

Ann Arbor , MI 48109-1003
USA

Prof.Dr. W.Dale Brownawell
Department of Mathematics
Pennsylvania State University
218 McAllister Building

University Park , PA 16802
USA

Prof.Dr. Jörg Brüdern
Mathematisches Institut A
Universität Stuttgart
Pfaffenwaldring 57

70569 Stuttgart

Dr. Laurent Denis
Institut de Mathematiques Pures et Appliquees, UFR 920
Universite de Paris VI
4, Place Jussieu

F-75252 Paris Cedex 05

Prof.Dr. Peter Bundschuh
Mathematisches Institut
Universität zu Köln
Weyertal 86-90

50931 Köln

Dr. Maurice M. Dodson
Dept. of Mathematics
York University

GB-Heslington, Yorkshire YO1 5DD

Prof.Dr. William W.L. Chen
School of MPCE
Macquarie University

Sydney NSW 2109
AUSTRALIA

Dr. Jan-Hendrik Evertse
Department of Mathematics and Computer Science
Rijksuniversiteit Leiden
Postbus 9512

NL-2300 RA Leiden

Prof.Dr. Sinnou David
Institut de Mathematiques Pures et Appliquees, UFR 920
Universite de Paris VI
4, Place Jussieu

F-75252 Paris Cedex 05

Dr. Roberto Ferretti
ETH Zürich
Mathematik
HG G 28.1

CH-8092 Zürich

Prof.Dr. Bernard De Mathan
Centre de Recherche en Mathematique de Bordeaux, CNRS
Universite de Bordeaux 1
351 Cours de la Liberation

F-33405 Talence Cedex

Prof.Dr. Etienne Fouvry
Mathematiques
Universite de Paris Sud (Paris XI)
Centre d'Orsay, Batiment 425

F-91405 Orsay Cedex

Prof.Dr. Kalman Győry
Institute of Mathematics
Lajos Kossuth University
Pf. 12

H-4010 Debrecen

Prof.Dr. Helmut Maier
Abteilungen für Mathematik
Universität Ulm

89069 Ulm

Prof.Dr. Masayoshi Hata
Division of Mathematics
Faculty of Integrated Human Studies
Kyoto University

Kyoto 606 01
JAPAN

Prof.Dr. Tapani Matala-aho
Department of Mathematical Sciences
University of Oulu
PL 333
Linnanmaa

SF-90571 Oulu

Dr. Martin Helm
Mathematisches Institut A
Universität Stuttgart

70550 Stuttgart

Prof.Dr. Eugene M. Matveev
Bld. 38, Apt. 24,
Oktyabrskaya Street

127018 Moscow
RUSSIA

Prof.Dr. Koichi Kawada
Dept. of Mathematics
Faculty of Education
Iwate University
Ueda

Morioka 020
JAPAN

Prof.Dr. Maurice Mignotte
Institut de Mathematiques
Universite Louis Pasteur
7, rue Rene Descartes

F-67084 Strasbourg Cedex

Prof.Dr. Michel Laurent
Laboratoire de Mathematiques
Discretes
Case 930
163 Avenue de Luminy

F-13288 Marseille Cedex 9

Prof.Dr. Hiroshi Mikawa
Institute of Mathematics
University of Tsukuba
Tsukuba-shi

Ibaraki 305
JAPAN

Prof.Dr. Yuri V. Nesterenko
Dept. of Mathematics
Moscow State University

119899 Moscow
RUSSIA

Prof.Dr. Damien Roy
Department of Mathematics
University of Ottawa
585 King Edward

Ottawa , Ont. K1N 6N5
CANADA

Prof.Dr. Patrice Philippon
Problemes Diophantiens-UMR 9994
Universite P. et M. Curie
Mathematiques, T. 46-56, 5eme et.
4 Place Jussieu

F-75252 Paris Cedex 05

Prof.Dr. Andrzej Schinzel
Institute of Mathematics of the
Polish Academy of Sciences
P.O. Box 137
ul. Sniadeckich 8

00-950 Warszawa
POLAND

Prof.Dr. Milja Poe
Dept. of Mathematics
Harvard University
1 Oxford Street

Cambridge , MA 02138
USA

Prof.Dr. Hans Peter Schlickewei
Abteilung für Mathematik II
Universität Ulm

89069 Ulm

Prof.Dr. Andrew Pollington
Dept. of Mathematics
Brigham Young University

Provo , UT 84602
USA

Prof.Dr. Wolfgang M. Schmidt
Dept. of Mathematics
University of Colorado
Campus Box 395

Boulder , CO 80309-0395
USA

Prof.Dr. Alfred J. van der Poorten
School of MPCE
Macquarie University

Sydney NSW 2109
AUSTRALIA

Dr. Maxim Skriganov
St. Petersburg Branch of Steklov
Mathematical Institute - POMI
Russian Academy of Science
Fontanka 27

191011 St. Petersburg
RUSSIA

Prof.Dr. Cameron L. Stewart
Dept. of Mathematics
University of Waterloo

Waterloo, Ontario N2L 3G1
CANADA

Prof.Dr. Paul A. Vojta
Department of Mathematics
University of California
at Berkeley
721 Evans Hall

Berkeley , CA 94720
USA

Prof.Dr. Jeff L. Thunder
Dept. of Mathematics
Northern Illinois University

DeKalb , IL 60115
USA

Prof.Dr. Jose Felipe Voloch
Dept. of Mathematics
University of Texas at Austin
RLM 8.100

Austin , TX 78712-1082
USA

Prof.Dr. Robert Tijdeman
Department of Mathematics and
Computer Science
Rijksuniversiteit Leiden
Postbus 9512

NL-2300 RA Leiden

Prof.Dr. Michel Waldschmidt
Institut de Mathematiques Pures et
Appliquees, UFR 920
Universite de Paris VI
4, Place Jussieu

F-75252 Paris Cedex 05

Prof.Dr. Robert Vaughan
Dept. of Mathematics
Imperial College of Science
and Technology
180 Queen's Gate, Huxley Bldg

GB-London , SW7 2BZ

Prof.Dr. Jürgen Wolfart
Fachbereich Mathematik
Universität Frankfurt
Postfach 111932

60054 Frankfurt

Prof.Dr. Carlo Viola
Dipartimento di Matematica
Universita di Pisa
Via Buonarroti, 2

I-56127 Pisa

Trevor D. Wooley
Department of Mathematics
The University of Michigan
3220 Angell Hall

Ann Arbor , MI 48109-1003
USA

Prof.Dr. Kunrui Yu
Department of Mathematics
University of Science a. Technology
Clear Water Bay
Kowloon

Hong Kong
HONG KONG