MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Darstellungstheorie endlicher Gruppen

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The meeting was organized by J. Alperin (Chicago), B. Huppert (Mainz) and G. Michler (Essen).

It was devoted to a broad survey of the major achievements and new developments in the different areas of representation theory. The main part of the program was a series of sixteen fifty minute lectures giving detailed overviews. The emphasis in these lectures ranged from reports on recent progress to surveys of an entire area. There were a number of exciting conjectures discussed. The great number of connections between them was a strong feature of the expositions. The program also gave indications of where the most promising areas were to be found. Another feature of the program was the number of relations with other branches of mathematics including number theory, computational algebra, combinatorics, algebraic groups, topology and the representation theory of algebras.

G. Robinson reported on his work with J. G. Thompson on Brauer's celebrated $k(B)$-conjecture where $k(B)$ denotes the number of characters in a block. They have proved an important special case (except for finitely many primes) , one that has been studied on its own for decades. This was a very surprising development!

E. Dade lectured on the famous conjectures counting characters in blocks which bear his name and on his reduction program to reduce these questions to simple groups. A number of other important conjectures are consequences of the Dade conjectures including very old questions of Brauer. K. Uno gave a detailed survey of work on these conjectures in the case of simple groups, a very involved situation. Some computational "miracles" suggest strongly that there are some basic discoveries to be made in the structure of finite reductive groups to explain the results.

Broué lectured on his conjectured relation between blocks (with abelian defect groups) and the theory of derived categories. The work suggests that constructions first found by Deligne and Lusztig for finite reductive groups have generalizations to all finite groups and these in turn would give structural results to explain the Dade conjectures in the abelian case. Malle lectured on the startling new results on finite complex reflection groups and Hecke algebras which where motivated by these ideas. Rickard explained the progress with derived categories. Linckelmann followed this up with his new results generalizing Rickard's ideas and brought the basic ideas of Puig on source algebras and the theory of G-algebras into this area. Puig reported on recent basic work in the area of source algebras.

C. Bessenrodt reported on Kleshchev's dramatic progress on the representation theory of the symmetric groups leading up to the solution (in joint work with B. Ford) of the Mullineux conjecture as well as her work with J. Olsson giving a quite significant simplification. Kleshchev's work includes some impressive new combinatorial contructions. The symmetric groups also appeared in a short talk by I. Kiming who exposited the ideas in number theory, including the use of modular forms, which has led to a complete solution of the very basic question of the existence of p-blocks of defect zero in symmetric groups.

A number of other reports covered a number of areas. A survey lecture by B. Külshammer was devoted to the theory of integral representations and its connections with number theory. C. Casolo lectured on several remarkable questions about degrees of complex characters (the most basic data in representation theory), their connections with solvable groups, and unexpected connections with questions about conjugacy class sizes. T. Keller discussed the recent progress in the representation theory of solvable groups. Geck and Hiss covered the representation theory modulo l of the finite reductive groups which involves all of block theory, the theory of algebraic groups as well as extensive machinery from algebraic groups. K. Erdmann discussed the interaction between the theory of group representations and the representation theory of algebras. A new conjecture would have immediate applications to a very old conjecture about the number of characters. J. Carlson lectured on the cohomology of groups and his programs which allow some remarkable computations of projective resolutions and the cohomolgy algebras mod p. H. Gollan described a new existence proof of the sporadic simple group of Lyons. It uses a new algorithm by Cooperman-Finkelstein-York-Tselman for constructing large permutation modules by means of powerful computers.

The program was completed with shorter talks devoted to reports on recent work. The participants universally felt that the meeting was both important and timely. They look forward to future progress stimulated by the lectures and public and private discussions.

Abstracts of the talks

C. Bessenrodt: Representations of the symmetric groups and related groups

This was a survey on some recent progress in the modular representation theory of the symmetric groups and related groups. The representation theory of the symmetric groups has seen significant new development in particular in an impressive series of papers of A. Kleshchev. He has made important progress towards a modular branching theorem, i.e. a description of the restriction $D^\lambda|S_{n-1}$ for the modular irreducible representations D^λ (λ) p-regular) of S_n in characteristic $p > 0$ by the following results:

Theorem 1. $\mathrm{soc}(D^\lambda|S_{n-1}) \cong \bigoplus_{A \text{ good}} D^{\lambda\setminus A}$

Theorem 2. $D^\lambda|S_{n-1}$ is completely reducible if and only if all normal nodes of λ are good.

The combinatorial concept of good and normal nodes introduced by him have already turned out to be of importance also in other contexts. Applying Theorem 1, Kleshchev reduced the long-standing Mullineux Conjecture describing the partition λ^P with $D^\lambda \otimes \mathrm{sgn} \simeq D^{\lambda^P}$ to a purely combinatorial conjecture. This was then proved in a long technical paper jointly with Ford, and recently another short proof giving additional insights was found by Bessenrodt and Olsson. The main combinatorial ingredients and the behaviour of the Mullineux map were also described; this is currently exploited to obtain results on irreducible restrictions of the modular representations of the alternating groups. In a paper of this year, Kleshchev now determines the multiplicities of all composition factors of the form $D^{\lambda(j)}$ in $D^\lambda|S_{n-1}$ by a combinatorial formula. In particular, this provides an improved lower bound for the dimensions $\dim D^\lambda$, for which so far only very limited information is available.

R. Boltje: Virtual extensions of representations in the case of coprime action

Let G be a finite group which is acted upon by a finite group S of coprime order and let GS be their semidirect product. It is well known that each S-stable irreducible character of G can be extended to an irreducible character of GS. We present a method, using canonical induction formulae, which allows to prove virtual extendibility of S-stable G-representations of various kinds, as for example projective modules, trivial source modules, linear source modules.

M. Broué: Complex reflection groups

The following conjecture has been stated in 1988.

Let G be a finite group. Let ℓ be a prime number, and let $\mathcal{O} := \mathbb{Z}_\ell[\zeta_G]$ where $\zeta_G := \exp(2i\pi/|G|)$.

Let B be a block (*i.e.*, an indecomposable two–sided ideal) of the group algebra $\mathcal{O}G$. Assume that B has an abelian defect group D, and let B_D be the correspon-ding block of $\mathcal{O}N_G(D)$. Then the derived bounded categories $\mathcal{D}^b(B)$ and $\mathcal{D}^b(B_D)$ of, respectively, the algebras B and B_D are equivalent.

This conjecture has many arithmetic consequences about character values, which have been (and still are) extensively studied. One of its consequences is that there must be a *perfect isometry* between B and B_D. In the case of principal blocks, this is now proved, among other cases,

- for all finite groups if $\ell = 2$ or 3 (Fong–Harris),
- for all finite reductive groups over $\mathbb{F}_q$ if ℓ does not divide q (Broué–Malle–Michel),
- for all sporadic simple groups (Rouquier).

The work reported here (a joint work with Gunter Malle and Raphaël Rouquier) is part of a general program to study the above conjecture in the case where $G = \mathbf{G}(\mathbb{F}_q)$, a finite reductive group, and ℓ does not divide q. In this case, assuming for simplicity that B is the principal block, it can be shown that $N_G(D)$ is an extension of a Levi subgroup $L = \mathbf{L}(\mathbb{F}_q)$ of G by a section W_ℓ of the Weyl group W of $\mathbf{G}$ which has a natural complex faithful representation as a group generated by pseudo-reflections. In the case where $\mathbf{G}$ is split over $\mathbb{F}_q$ and where $\ell \mid (q-1)$, then $W_\ell = W$. Moreover, some of the Deligne–Lusztig varieties $X(G, L; \mathbf{U})$ associated with the pair (G, L) should play a key role in the desired derived equivalence. We conjecture that there is an action of the braid group associated to W_ℓ (see below) on the ℓ–adic cohomology of $X(G, L; \mathbf{U})$ *via* a "Hecke algebra" of W_ℓ (a suitable deformation of the group algebra of W_ℓ), and this action should provide a large part of the derived equivalence. This leads naturally to studying the complex reflection groups (groups generated by pseudo-reflections), specifically to extend to this more general context known properties of Weyl or Coxeter groups – a subprogram of the program mentioned above. This talk is the first part of a series of two talks (the second one is delivered by G. Malle).

J. F. Carlson: Computing projective resolutions and cohomology rings

We describe a computer program for the computation of minimal projective resolutions of modules over the modular group algebras of finite p-groups. Other programs available compute chain maps from the cohomology elements. The compositions of the chain maps give the cohomology (cup) products. For a group of reasonable size, the programs can compute generators and relations for the cohomology ring of the group with coefficients in the field with p elements. Presently the mod-2 cohomology rings of the groups of order 64 are being computed. Some sample calculations will be exhibited. As currently set up the programs run in the MAGMA environment, though it should not be too difficult to convert them to other systems.

The system has also been used for part of the calculation of the mod-2 cohomology of the Higman-Sims group, HS. The order of the Sylow 2-subgroup of HS is 512 and because of that it was only possible to compute seven steps in the projective resolution. However certain of the relations among the generators in the first seven degrees could be derived from computer calculations of the restrictions to the centralizer of the maximal elementary abelian 2-subgroups.

C. Casolo: Class lengths and character degrees

We give a brief survey of some aspects of the research on the arithmetical structure of the lengths of conjugacy classes and of the degrees of irreducible characters in finite groups. We restrict to two topics: The σ-ρ conjectures and the associated graphs.

For a broader picture, interested people are referred to B. Huppert's survey *"Research in Representation Theory at Mainz (1984–1990)"*, Progress in Mathematics series **95**, Birkhäuser, Basel 1991.

If G is a finite group, we write $\pi(G)$ for the set of all primes dividing $|G|$. If χ is an irreducible complex character of G, we denote by $\sigma(\chi)$ the set of all prime divisors of $\chi(1)$, the degree of χ. Similarly, for $g \in G$, we denote by $\sigma_G(g)$ the set of all prime divisors of $|G : C_G(g)|$, the length of the conjugacy class g^G. Then we define

$$\sigma(G) = \max_{\chi \in \mathrm{Irr}(G)} \{|\sigma(\chi)|\} \ , \ \ \rho(G) = \bigcup_{\chi \in \mathrm{Irr}(G)} \sigma(\chi) \ ,$$

$$\sigma^*(G) = \max_{g \in G}\{|\sigma_G(g)|\} \ , \text{ and } \rho^*(G) = \bigcup_{g \in G} \sigma_G(g) \ .$$

By the Ito-Michler Theorem, $\rho(G)$ is precisely the set of all primes p in $\pi(G)$ such that G does not have a normal abelian Sylow p-subgroup. On the other hand, it is an elementary fact that $\rho^*(G) = \pi(G/Z(G))$.

The so called σ-ρ conjectures were proposed some years ago by B. Huppert. The original conjecture concerns characters and states that $|\rho(G)| \leq 2\sigma(G)$ for every soluble group G. If correct, this bound would be optimal.

Theorem 1. *$|\rho(G)| \leq 2\sigma(G)$ holds if $\sigma(G) = 1$* (O. Manz) *or if $\sigma(G) = 2$ and G is soluble* (D. Gluck).

Theorem 2. *a) $|\rho(G)| \leq 3\sigma(G) + 2$ if G is soluble* (Manz & Wolf; Gluck).

b) $|\rho(G)| \leq 5\sigma(G) + c$ for all finite groups G and a computable constant c (Dolfi and Casolo).

c) $|\rho(G)| \leq 3\sigma(G)$ if G is nonabelian and simple (Alvis & Barry; Manz, Staszewski & Willems).

For conjugacy classes, it is proved that $|\rho^*(G)| \leq 2\sigma^*(G)$ when $\sigma^*(G) = 1$ (Chillag & Herzog), $\sigma^*(G) = 2$ (Casolo; Mann; P. Ferguson), and G is soluble and $\sigma^*(G) = 3$ (Casolo). Also, the inequality $|\pi(G')| < 2\sigma^*(G)$ holds for any finite group G, whence in particular $|\rho^*(G)| < 2\sigma^*(G)$ if G is a perfect group. However, the factor 2 is not correct in general:

Lemma 3. (Dolfi & Casolo) *If G is metanilpotent then $|\rho^*(G)| \leq 3\sigma^*(G)$. Moreover, there exists a family $\{G_n\}$ of supersolvable metabelian groups such that* $\lim_{n\to\infty} \frac{|\rho^*(G_n)|}{\sigma^*(G)} = 3$.

Thus we **conjecture** that

$$|\rho^*(G)| \leq 3\sigma^*(G)$$

for all finite groups G.

Theorem 4. *a) $|\rho^*(G)| \leq 4\sigma^*(G)+1$ if G is a soluble group* (P. Ferguson; Dolfi & Casolo; Z. Yiping).

b) $|\rho^*(G)| \leq 5\sigma^*(G) + 1$ for all finite groups G (Dolfi & Casolo).

E.C. Dade: On Dade's conjectures

We're going to explain the extended version of the conjectures studied in my paper

[CCB2] Counting Characters in Blocks, II. Crelle 448 (1994), 97 – 190.

As in that paper, we fix a local principal ideal domain $\mathfrak{R}$ with unique maximal ideal $\mathfrak{P} = J(\mathfrak{R})$. We assume that the field of fractions $\mathfrak{F}$ of $\mathfrak{R}$ has characteristic zero, and that the residue class field $\overline{\mathfrak{F}} = \mathfrak{R}/\mathfrak{P}$ has prime characteristic p. The finite group G in that paper is now embedded as a normal subgroup in another finite group E. We fix an epimorphism ε of E onto a third finite group F such that G is the kernel of ε. Thus we fix an exact sequence

$$1 \longrightarrow G \xrightarrow{\trianglelefteq} E \xrightarrow{\varepsilon} F \longrightarrow 1 \tag{1}$$

of finite groups. The totally split twisted group algebra $\mathfrak{A}$ of G over $\mathfrak{F}$ in [CCB2, 7.1] now becomes a totally split twisted group algebra of E over $\mathfrak{F}$.

Let C be any p-chain of G. The normalizer $\mathrm{N}_E(C)$ of C in E intersects G in the normalizer $\mathrm{N}_G(C)$ of C in G. We call the image $\mathrm{N}_F(C) = \varepsilon\big(\mathrm{N}_E(C)\big)$ of $\mathrm{N}_E(C)$ the *normalizer* of C in F. Then the exact sequence (1) restricts to an exact sequence

$$1 \longrightarrow \mathrm{N}_G(C) \xrightarrow{\trianglelefteq} \mathrm{N}_E(C) \xrightarrow{\varepsilon} \mathrm{N}_F(C) \longrightarrow 1 \tag{2}$$

of finite groups. We can use the homomorphism $\varepsilon\colon \mathrm{N}_E(C) \to F$ to turn the restriction $\mathfrak{A}[\mathrm{N}_E(C)]$ of $\mathfrak{A}$ to $\mathrm{N}_E(C)$ into an F-graded $\mathfrak{F}$-algebra with the ρ-component

$$\mathfrak{A}[\mathrm{N}_E(C)]_\rho = \dot{\sum_{\substack{\sigma\in\mathrm{N}_E(C)\\ \varepsilon(\sigma)=\rho}}} \mathfrak{A}_\sigma$$

for any $\rho \in F$. Of course this ρ-component is non-zero if and only if ρ lies in $\mathrm{N}_F(C)$, a fact we express by saying that $\mathrm{N}_F(C)$ is the *support* of the F-graded $\mathfrak{F}$-algebra $\mathfrak{A}[\mathrm{N}_E(C)]$.

The identity component in the above F-grading is the subalgebra

$$\mathfrak{A}[\mathrm{N}_E(C)]_{1_F} = \mathfrak{A}[\mathrm{N}_G(C)]$$

of $\mathfrak{A}[\mathrm{N}_E(C)]$. The centralizer of $\mathfrak{A}[\mathrm{N}_G(C)]$ in $\mathfrak{A}[\mathrm{N}_E(C)]$ is just the fixed subalgebra $\mathfrak{A}[\mathrm{N}_E(C)]^{\mathrm{N}_G(C)}$ of $\mathrm{N}_G(C)$ under conjugation in the twisted group algebra $\mathfrak{A}[\mathrm{N}_E(C)]$ of $\mathrm{N}_E(C)$ over $\mathfrak{F}$. It is an F-graded $\mathfrak{F}$-subalgebra of $\mathfrak{A}[\mathrm{N}_E(C)]$, having its identity component

$$(\mathfrak{A}[\mathrm{N}_E(C)]^{\mathrm{N}_G(C)})_{1_F} = \mathrm{C}_{\mathfrak{A}[\mathrm{N}_G(C)]}(\mathfrak{A}[\mathrm{N}_G(C)]) = \mathrm{Z}(\mathfrak{A}[\mathrm{N}_G(C)])$$

as a central subalgebra. Because $\mathfrak{A}[\mathrm{N}_G(C)]$ is a split, semi-simple algebra of finite dimension over $\mathfrak{F}$, its center is the direct sum

$$\mathrm{Z}(\mathfrak{A}[\mathrm{N}_G(C)]) = \dot{\sum_{\phi\in\mathrm{Irr}(\mathfrak{A}[\mathrm{N}_G(C)])}} \mathfrak{F}1_\phi$$

of copies of $\mathfrak{F}$. Here $\mathrm{Irr}(\mathfrak{A}[\mathrm{N}_G(C)])$ is the set of all irreducible $\mathfrak{F}$-characters ϕ of $\mathfrak{A}[\mathrm{N}_G(C)]$, and 1_ϕ is the primitive idempotent of $\mathrm{Z}(\mathfrak{A}[\mathrm{N}_G(C)])$ corresponding to any such ϕ. It follows that $\mathfrak{A}[\mathrm{N}_E(C)]^{\mathrm{N}_G(C)}$ is the direct sum

$$\mathfrak{A}[\mathrm{N}_E(C)]^{\mathrm{N}_G(C)} = \dot{\sum_{\phi \in \mathrm{Irr}(\mathfrak{A}[\mathrm{N}_G(C)])}} \mathfrak{A}(C, \phi) \tag{3a}$$

of F-graded $\mathfrak{F}$-subalgebras

$$\mathfrak{A}(C, \phi) = \mathfrak{A}[\mathrm{N}_E(C)]^{\mathrm{N}_G(C)} 1_\phi \tag{3b}$$

for $\phi \in \mathrm{Irr}(\mathfrak{A}[\mathrm{N}_G(C)])$.

The group $\mathrm{N}_E(C)$ acts on the set $\mathrm{Irr}(\mathfrak{A}[\mathrm{N}_G(C)])$ by conjugation in the twisted group algebra $\mathfrak{A}$. Its normal subgroup $\mathrm{N}_G(C)$ fixes each $\phi \in \mathrm{Irr}(\mathfrak{A}[\mathrm{N}_G(C)])$. Hence the stabilizer $\mathrm{N}_E(C, \phi)$ of ϕ in $\mathrm{N}_E(C)$ has $\mathrm{N}_G(C)$ as a normal subgroup. We denote by $\mathrm{N}_F(C, \phi)$ the image $\varepsilon\big(\mathrm{N}_E(C, \phi)\big)$ of $\mathrm{N}_E(C, \phi)$ in $\mathrm{N}_F(C)$. Then (2) restricts to an exact sequence

$$1 \longrightarrow \mathrm{N}_G(C) \xrightarrow{\trianglelefteq} \mathrm{N}_E(C, \phi) \xrightarrow{\varepsilon} \mathrm{N}_F(C, \phi) \longrightarrow 1 \tag{4}$$

for each $\phi \in \mathrm{Irr}(\mathfrak{A}[\mathrm{N}_G(C)])$. We know from [CCB2, §11] that that the summand $\mathfrak{A}(C, \phi)$ in (3a) is a totally split *twisted subgroup algebra* of F over $\mathfrak{F}$ with the "stabilizer" $\mathrm{N}_F(C, \phi)$ as its support. This means that the ρ-component $\mathfrak{A}(C, \phi)_\rho$ is zero for $\rho \in F - \mathrm{N}_F(C, \phi)$, and that the restriction of (C, ϕ) is a totally split twisted group algebra of $\mathrm{N}_F(C, \phi)$ over $\mathfrak{F}$.

Now we fix a p-block B of the restriction $\mathfrak{A}[G]$ of $\mathfrak{A}$ to a twisted group algebra of G over $\mathfrak{F}$. We also fix a non-negative integer d. We denote by $\mathrm{ChIrr}(B, d)$ the family of all ordered pairs (C, ϕ), where C is a p-chain of G and ϕ is an irreducible $\mathfrak{F}$-character of $\mathfrak{A}[\mathrm{N}_G(C)]$ such that the defect $\mathrm{d}(\phi)$ of ϕ is equal to d and the p-block $\mathrm{B}(\phi)$ of $\mathfrak{A}[\mathrm{N}_G(C)]$ containing ϕ induces the p-block B of $\mathfrak{A}[G]$. The group G then acts on the set $\mathrm{ChIrr}(B, d)$ by conjugation, with any $\tau \in G$ sending any $(C, \phi) \in \mathrm{ChIrr}(B, d)$ to the pair $(C, \phi)^\tau = (C^\tau, \phi^\tau) \in \mathrm{ChIrr}(B, d)$.

We define an equivalence relation $\approx$ on the pairs in $\mathrm{ChIrr}(B, d)$ so that two such pairs (C, ϕ) and (C', ϕ') are equivalent if and only if there is some isomorphism of $\mathfrak{A}(C, \phi)$ onto $\mathfrak{A}(C', \phi')$ as F-graded $\mathfrak{F}$-algebras. This happens if and only if $\mathrm{N}_F(C, \phi)$ and $\mathrm{N}_F(C', \phi')$ are the same subgroup I of F and the restrictions of $\mathfrak{A}(C, \phi)$ and $\mathfrak{A}(C', \phi')$ to I are isomorphic twisted group algebras of I over $\mathfrak{F}$. It is easy to see that this equivalence relation is weaker than G-conjugation, in the sense that

$$(C, \phi)^\tau \approx (C, \phi) \tag{5}$$

for any $(C, \phi) \in \mathrm{ChIrr}(B, d)$ and $\tau \in G$. If $\mathcal{I}$ is any equivalence class for $\approx$ and C is any p-chain of G, then $\mathrm{k}(C, \mathcal{I})$ will denote the number of characters

$\phi \in \mathrm{Irr}(\mathfrak{A}[\mathrm{N}_G(C)])$ such that (C, ϕ) belongs to $\mathcal{I} \subseteq \mathrm{ChIrr}(B, d)$. It follows from (5) that $\mathrm{k}(C, \mathcal{I})$ depends only on the G-conjugacy class of C. Furthermore, it is easy to see that $\mathrm{k}(C, \mathcal{I})$ depends only on the normalizer $\mathrm{N}_E(C)$ of C in E. So [CCB2, 2.10] implies that the alternating sum

$$\sum_{C \in \mathcal{F}/G} (-1)^{|C|} \mathrm{k}(C, \mathcal{I}) \qquad (6)$$

is independent of the choice of $\mathcal{F}$ among the families $\mathcal{P}(G|\mathrm{O}_p(G))$, $\mathcal{E}(G|\mathrm{O}_p(G))$ and $\mathcal{R}(G)$ of p-chains of G, as defined in [CCB2, §1]. Here $\mathcal{F}/G$ is any family of representatives for the G-conjugacy classes in $\mathcal{F}$, and $|C|$ is the length n of the p-chain C: $P_0 = \mathrm{O}_p(G) < P_1 < \cdots < P_n$. The extended projective form of the conjecture in [CCB2] can now be stated as

Conjecture 7 *If* $\mathrm{O}_p(G) = 1$ *and the block* B *has defect* $\mathrm{d}(B) > 0$, *then the alternating sum (6) vanishes for any equivalence class* $\mathcal{I}$ *in* $\mathrm{ChIrr}(B, d)$.

K. Erdmann: Methods from algebras in modular representation theory

(I) The stable Auslander-Reiten quiver $\Gamma_s(\Lambda)$ of a finite-dimensional algebra Λ is an important homological invariant. We are interested in the case when Λ is a block B of some group algebra.

Suppose the defect groups of B are cyclic or dihedral, semidihedral, quaternion (that is, B is of finite or tame type). Then classification problems are solved by the following strategy. First one determines the graph structure of $\Gamma_s(B)$. Then one classifies all basic algebras Λ with $\Gamma_s(\Lambda) \cong \Gamma_s(B)$, subject to suitable regularity conditions. One obtains a list which contains the possible basic algebras for B.

A hard unsolved problem is the classification of indecomposable modules for the quaternion group algebras over characteristic 2.

All other blocks are of wild type. It has been proved:
Theorem *B is of wild type if and only if $\Gamma_s(B)$ has only components of the form $\mathbb{Z}A_\infty$ or $\mathbb{Z}A_\infty/\langle\tau^k\rangle$, and $\mathbb{Z}A_\infty$-components occur.*

Suppose B is of wild type. It is not known how to recover properties of B from $\Gamma_s(B)$ for B of wild type. For M indecomposable, define the quasi-length $ql(M)$ of M, to be the row number of M in its component. Answers to the following questions would be interesting.

If S in B is simple, is then $ql(S) = 1$; equivalently, is the heart of the projective $P(S)$ indecomposable? There are partial results by S. Kawata (see this meeting).

Is the number of simple modules in B related to properties of $\Gamma_s(B)$? Results on hereditary algebras suggest that the quasi-length $ql(M)$ of modules M with $\underline{\text{End}}(M) = K\}$ might be bounded by $\ell(B)$.

The maximal rank of tubes, i.e. the maximal period of periodic modules of B, is not known; although it is known that there is an upper bound in terms of the cohomology ring.

(II) Quasi-hereditary algebras were introduced to study highest weight modules, as they occur in the theory of semi-simple Lie algebras and algebraic groups, in the context of finite-dimensional algebras. This enabled one to use methods based on finite global dimension; an important result is the discovery of distinguished (generalized) tilting modules which have various applications. These methods and results can be used for representations of symmetric groups, via Schur algebras. In particular there is new insight into decomposition numbers and dimensions of simple modules. Quasi-hereditary algebras also occur in the context of other families of finite groups. For example, let $M = \oplus_{i \geq 0} kG/J^i$ where J is the radical of the group algebra kG; then the endomorphism ring of M is quasi-hereditary. For the case when kG is local, some work has been done but not in general.

P. Fleischmann: Finite locally semiregular groups

Let p be a prime. A finite group will be called p'-semiregular if it has a linear representation such that each p'-element acts without any fixed points. In joint work with W. Lempken and P.H. Tiep we classified all finite p'-semiregular groups, thus generalizing a classical result of Zassenhaus on semiregular groups. I will talk on this result and its applications in the theory of finite permuation groups, where it can be used to classify all primitive groups such that any two point stabilizer $G_{a,b}$ is a p-group. In the meantime we also finished the classification of all these primitive permutation groups G. The p'-semiregular groups, which occur if G has abelian socle, also appear in investigations of the multiplicative structure of Galois extensions of fields. This has been pointed out by R. Guralnick and R. Wiegand, who also obtained the classification of p'-semiregular groups (up to a certain case which is missing in their paper).

M. Geck: Character sheaves and l-modular Brauer characters

The aim of the talk is twofold:

1) To summarize some basic implications of Lusztig's theory of character sheaves and Shoji's proof of Lusztig's conjecture on character sheaves to the ordinary character theory of finite groups of Lie type. (The point being to do this in a way as elementary as possible, by avoiding the original geometric language and

formulating the results in terms of almost characters and twisted induction).

2) To apply these results to the problem of finding basic sets of l-modular Brauer characters, especially in the case where l is a bad prime. (All of the above is joint work with G. Hiß, Heidelberg)

H. Gollan: Construction of Lyons' simple group

In this lecture we describe a method by Cooperman, Finkelstein, Tselman, and York for the construction of permutation representations from matrix representations. Their algorithm was first tested with the sporadic group of R. Lyons to produce a permutation representation of degree 9606125 from a matrix representation of dimension 111 over the field GF(5). These permutations have been used in my Habilitationsschrift to give an independent existence proof for Lyons' simple group, and to produce a new presentation for it. All the relations in the 2 generators are presented in the lecture together with an outline of the proof. This new existence proof is independent of the previous, but unpublished work of C. Sims.

D. J. Green: The spectrum of the Chern subring

(joint with I.J. Leary) The mod-p cohomology ring of a finite group G can be studied using the methods of commutative algebra. Quillen described the prime ideal spectrum of the cohomology ring as a colimit over a category of elementary abelian p-subgroups of G. We study the Chern subring, a large subring of the cohomology ring which is constructed using the representation theory of G. After giving examples where the cohomology ring and the Chern subring have different spectra, we obtain a description of the spectrum of the Chern subring as a colimit over a larger category of elementary abelians.

There is a common generalization of these colimit theorems which holds for many large subrings of the cohomology ring. This in turn gives rise to a tower of natural subrings of the cohomology ring, which seems to be related to the generalized character theory of Hopkins, Kuhn and Ravenel.

J. A. Green: Quantum shuffle algebras

G. Lusztig, in his book "Introduction to quantum groups", constructs the quantum group $U_2(g)$ corresponding to a simple Lie algebra g (over a field k of characteristic zero), by first making a $k(q)$-algebra $\underline{f}$ (q is an indeterminate) which can be regarded as a quantization of $U(n^-)$, where $g = n^- \oplus \eta \oplus n^+$ is the usual "triangular" decomposition of g.

By a trivial change of Lustzig's construction, $\underline{f}$ is presented as a subalgebra of a "quantized shuffle algebra" S; this latter specializes at $q = 1$ to R. Ree's shuffle algebra (1958). Multiplication in S can be given quite explicitely, and provides a useful way of calculating in $\underline{f}$.

M. Herzog: Products of conjugacy classes in the groups PSL(n, F)

Let G be a (non-abelian) simple group, finite or infinite. We define $\mathrm{cn}(G) = r$ if r is the least integer satisfying $C^r = G$ for all nontrivial conjugacy classes C of G. Such r exists for all finite simple groups. J. Thompson conjectured that each simple group G contains at least one conjugacy class C satisfying: $C^2 = G$. This conjecture implies Ore's conjecture asserting that each element of a simple group is a commutator.

We shall consider $G = \mathrm{PSL}(n, F), F$ any field. A matrix $T \in \mathrm{GL}(n, F)$ is called cyclic if the Jordan form of T over the algebraic closure of F has a unique block corresponding to each eigenvalue of T. A conjugacy class of PSL(n, F) is cyclic if it contains an image of a cyclic matrix in SL(n, F).

Theorem 1 Let $G = \mathrm{PSL}(n, F)$, $n \geq 3$, F any field and C_1, C_2, C_3 cyclic conjugacy classes of G. Then: $C_1C_2C_3 \geq G - \{1\}$.

Theorem 2 Let $G = \mathrm{PSL}(n, F)$, $n \geq 2$, F is algebraically closed, C_1, C_2 are any conjugacy classes of G. Then $C_1C_2 = G \Leftrightarrow C_2 = C_1^{-1}$ and C_1, C_2 are cyclic.

Theorem 3 Let $G = \mathrm{PSL}(2, F), F$ is algebraically closed and C any nontrivial conjugacy class of G. Then: $C^2 = G$. In particular $\mathrm{cn}(G) = 2$.

Theorem 4 All simple PSL$(n, F), F$ any field, satisfy the Thompson conjecture.

Theorem 5 Let $G = \mathrm{PSL}(n, F)$, $n \geq 4$ and F any field, satisfy $|F| \geq 4$. Then: $\mathrm{cn}(G) = n$. These results were obtained by my Ph.D. student Arie Lev.

G. Hiß: Decomposition numbers and blocks of finite groups of Lie type

In this talk I give a survey on some new results on decomposition numbers of classical groups. Furthermore, I shall report on the theory of blocks of finite groups of Lie type in non-defining characteristics.

The talk centers around Harish-Chandra philosophy. First of all I shall present a theorem of Geck, Malle, and myself on the classification of the irreducible representations of a finite group of Lie type in non-defining characteristic. This is a theorem of ℓ-Harish-Chandra theory.

Next the d-Harish-Chandra theory of Broué, Malle, and Michel is sketched, and their main theorem on the distribution of the ordinary characters into blocks is described.

Then I shall talk about the new results of Gruber and myself on the computation of decomposition matrices of classical groups in the case of so-called linear primes. Finally, I shall sketch the definition of the new q-Schur algebras of type B and D, which played a crucial role in the determination of these decomposition matrices.

Finally, if time allows, I shall report on the algorithm of Lascoux, Leclerc, and Thibon for the determination of decomposition numbers of q-Schur algebras at roots of unity and the connection of this theory to groups of Lie type.

S. Kawata: On the Auslander-Reiten components and simple modules for finite group algebras

Let G be a finite group, k a field of characteristic $p > 0$ and B a block of the group algebra kG. Erdmann showed that if B is a wild block, then all AR-components of the stable Auslander-Reiten quiver of B have tree class A_∞. Here we ask where simple modules lie in the AR-component with tree class A_∞ and we consider what happens when some simple module does not lie at the end.

1. Let Λ be a symmetric algebra and Θ an AR-component containing a simple module. Suppose that the tree class of Θ is A_∞ and some simple module does not lie at the end of Θ. Then for some simple Λ-modules $S, T_1, T_2, \cdots, T_n$, the projective covers P_i of T_i are uniserial and their composition factors, from the top, are given: $T_i, T_{i-1}, \cdots, T_1, S, T_n, T_{n-1}, \cdots, T_i$. In particular the Cartan matrix for Λ is as follows:

$$\begin{pmatrix}
2 & 1 & 1 & \cdots & 1 & 0 & \cdots & 0 \\
1 & 2 & 1 & & \vdots & \vdots & & \vdots \\
1 & 1 & \ddots & \ddots & 1 & 0 & & \vdots \\
\vdots & & \ddots & 2 & 1 & 0 & \cdots & 0 \\
1 & 1 & \cdots & 1 & & & & \\
0 & 0 & \cdots & 0 & & & & \\
\vdots & & & \vdots & & & * & \\
0 & \cdots & \cdots & 0 & & & &
\end{pmatrix}$$

2. For a wild block B of kG, under the following condition (2.1) or (2.2), all simple modules in B lie at the end.

(2.1) G is p-solvable and k is algebraically closed.

(2.2) G has a non-trivial normal p-subgroup and k is algebraically closed.

T. M. Keller: The derived length and the number of irreducible character degrees in solvable groups

Let G be a finite solvable group, $\mathrm{dl}(G)$ the derived length of G and $\mathrm{cd}(G)$ the set of all irreducible complex character degrees of G. We are interested in bounding $\mathrm{dl}(G)$ in terms of $|\mathrm{cd}(G)|$. The first result on this problem was obtained by K. Taketa in 1930, who proved

$$\mathrm{dl}(G) \leq |\mathrm{cd}(G)|$$

for monomial groups. G. Seitz conjectured that this bound also holds for arbitrary solvable groups. T.R. Berger established this conjecture for groups of odd order. D. Gluck proved that $\mathrm{dl}(G) \leq 2|\mathrm{cd}(G)|$ for all solvable groups. We discuss these bounds for small values of $\mathrm{dl}(G) = |\mathrm{cd}(G)|$. Such groups G are only known for $\mathrm{dl}(G) \leq 5$. Furthermore we ask whether a linear bound is asymptotically best possible. For p groups, a logarithmic bound seems more probable, as recent results of B. Huppert, I.M. Isaacs and A. Previtali on Sylow subgroups of linear groups indicate. However, one is far away from being able to improve Taketa's result for p-groups in general. So to attack the linear bound, it makes sense to regard classes of groups where the p-group problems do not occur. If G is a solvable group such that all its Sylow subgroups are abelian, and its $\mathrm{dl}(G) \geq 16$, then

$$\mathrm{dl}(G) \leq 6\frac{\log|\mathrm{cd}(G)|}{\log\log|\mathrm{cd}(G)|} + 5.$$

I. Kiming: Arithmetic of some partition problems

Let p be an odd prime and let n be a natural number. Let S_n be the symmetric group of degree n and denote by $\hat{S}_n$ a double cover of S_n.

We give elementary proofs of the following two theorems.

Theorem 1: (Granville-Ono). If $p \geq 5$, then for all $n \in \mathbb{N}$, S_n has a faithfull, irreducible character of p-defect 0.

Theorem 2: (Erdmann-Michler for $p = 7$, Kiming for $p \geq 11$). If $p \geq 7$, then for all $n \in \mathbb{N}$, $\hat{S}_n$ has a faithfull, irreducible character of p-defect 0.

Denote by $t_p(n)$ and $s_p(n)$ respectively the number of "p-core partitions" of n and the number of "$\bar{p}$-core partitions" (in the sense of J. B. Olsson) of n respectively.

Then theorem 1 is equivalent to the statement that $t_p(n) > 0$ for $p \geq 5$, $n \in \mathbb{N}$. Similarly theorem 2 is equivalent to the statement that $s_p(n) > 0$ for $p \geq 7$, $n \in \mathbb{N}$. By work of Garvan, Kim and Stanton, one knows that $t_p(n)$ equals the number of integral solutions to:

$$n = \sum_{i=0}^{p-1} \left(\frac{p}{2} \cdot x_i^2 + i x_i\right) \qquad \text{and} \qquad \sum_{i=0}^{p-1} x_i = 0 \, .$$

By Olsson's theory, one has that $s_p(n)$ equals the number of integral solutions to:

$$n = \sum_{i=1}^{\frac{p-1}{2}} \left(p \cdot \frac{1}{2} y_i (y_i - 1) + i y_i\right) .$$

Our proofs of the above theorems for $p \geq 11$ consist in proving the existence of solutions (for each $n \in \mathbb{N}$) to the above equations. At the heart of the proof stands in both cases an application of Gauss' theorem on the representation of integers as sums of 3 squares.

We also consider for a fixed p the asymptotics of the numbers $t_p(n)$ and $s_p(n)$. Using modular forms, it is only an exercise to find an asymptotic formula for $t_p(n)$. The case of $s_p(n)$ is somewhat more complicated. Again using modular forms (and in particular the Ramanujan-Petersson conjecture, proved by Deligne) we have obtained asymptotic formulae for $s_p(n)$ in the cases: ($p \equiv 1$ (4) and $p \geq 13$).

Suppose for example that $p \equiv 5$ (8), $p \geq 13$. Put $k := \frac{p-1}{4}$, and let χ denote the Dirichlet character belonging to $\mathbb{Q}(\sqrt{-1})$, so that $\chi(x) = (-1)^{\frac{x-1}{2}}$ for odd $x \in \mathbb{Z}$. Then if $n \in \mathbb{N}$ and we write $N := 4n + \frac{(p-1)(p-2)}{12} = p^s m$, where $p \nmid m$, we have for all $\epsilon > 0$:

$$s_p(n) = (-1)^{\frac{k+1}{2}} \cdot \frac{2k}{B_{k,\chi}} \cdot \frac{2}{p^k - 1} \cdot N^{k-1} \cdot \sum_{d|m} \chi(d) d^{1-k} + O(n^{\frac{k-1}{2}+\epsilon}) \, .$$

Here $B_{k,\chi}$ is the k'th Bernoulli number belonging to the character χ.

B. Külshammer: Some recent results in integral representation theory

This survey talk will be mainly concerned with the following topics:

I. Realizing finite group representations over rings of algebraic integers.
II. Galois-stability of lattices.

I concentrate on results by G. Cliff, G.-M. Cram, O. Neiße, J. Ritter and A. Weiss. The following questions will be addressed, for a finite group G and an irreducible character χ of G.

1. Suppose that χ is afforded by a representation $G \to \mathrm{GL}(n, K)$, for an algebraic number field K. Is χ also afforded by a matrix representation $G \to \mathrm{GL}(n, \mathcal{O}_K)$ where $\mathcal{O}_K$ is the ring of integers in K.

2. Is χ always afforded by a representation $G \to \mathrm{GL}(n, \mathbb{Z}[\zeta_t])$ where $t := \exp(G)$ and $\zeta_t = e^{2\pi i/t}$.

3. Over which cyclotomic fields and rings can χ be realized?

4. Let V be an absolutely irreducible FG-module affording χ where F is a Galois extension of $K := \mathbb{Q}(\chi)$. Then $\Gamma := \mathrm{Gal}(F|K)$ acts on the set of isomorphism classes of $\mathcal{O}_F G$ lattices on V. Is there a Γ-stable isomorphism class?

In some cases answers are known for solvable groups only.

M. Linckelmann: Splendid equivalences for non principal blocks

Jeremy Rickard developped the notion of a splendid derived equivalence for which he then proved that at least for principal p-blocks of finite groups with same p-local structure such an equivalence induces derived equivalences at all local levels of the considered blocks and shows in particular, that the blocks are isotypic. We slightly modify Rickard's definition of a splendid derived equivalence in order to prove the analogous results for arbitrary blocks with a common defect group and same p-local structure.

G. Malle: Complex reflection groups and cyclotomic Hecke algebras

Let $W \leq \mathrm{GL}(V)$, $V = \mathbb{C}^n$, be a finite complex reflection group. In this lecture we presented recent results on the structure of the associated generic cyclotomic Hecke algebra $\mathcal{H}(W, g)$. Let $M = V - \bigcup_{H \in \mathcal{A}} H$ be the complement of the set $\mathcal{A}$ of reflecting hyperplanes of W and $B(W) = R_1(M/W, x_0)$ be the fundamental group of the space of regular orbits, the braid group associated to W. For $H \in \mathcal{A}/W$ let $U_{H,1}, \ldots, U_{H,e_H}$ be indeterminants, where $e_H = |\mathcal{C}_w(H)|$, and $u = (U_{H,i} | H \in \mathcal{A}/W, i)$. Let $f_H(X) := \prod_{i=1}^{e_h}(X - U_{H,i})$.

Then the cyclotomic Hecke algebra $\mathcal{H}(W, u)$ is the quotient of $\mathbb{Z}\{u, u^{-1}\}B(W)$ modulo the ideal generated by the $f_{H,}(s)$ for all generators of the monodromy around the $H \in \mathcal{A}$.

By a result of Broué, Rouquier and the author, this gives the same object as previous definitions starting from generalised Coxeter diagrams, up to finitely many possible exceptions. It is known that $\mathcal{H}(W, u)$ is a free $\mathbb{Z}\{u, u^{-1}\}$-module of rank (W), hence isomorphic to the group algebra of W over $\overline{U(u)}$ by Tits' deformation theorem, in almost all cases.

By a result of Brenche and the author $\mathcal{H}(W,u)$ carries a canonical symmetrizing form, up to finitely many possible exceptions. In the talk we stated a conjecture as to what the relative degrees associated to this form should be. This is known to be true for small cases. The relative degrees, as conjectured have the correct specialization to character degrees in blocks of finite groups of Lie type, as predicted by the conjectures of Broué and the author.

Finally it was stated that a certain standard specialization of the relative degrees leads to a set of polynomials which can be extended to a set $\mathcal{E}(W)$ of so-called unipotent degrees for W (in the case that W is imprimitive and generated by $n = \dim V$ reflections). These degrees share many combinatorial properties with the sets of degrees of unipotent characters of finite groups of Lie type.

G. Pazderski: On groups all of whose characters are quasi-primitive

An irreducible character χ of a finite group G is said to be quasi-primitive if its restriction χ_N to any normal subgroup N of G decomposes homogeniously, i.e. if χ_N is a multiple of a certain irreducible character of N. In case that every irreducible character of G is quasi-primitive we will call G quasi-primitive. Obviously each abelian group is quasi-primitive, and it is known (see Isaacs, I.M.: Character theory of finite groups, p.96) that a quasi-primitive solvable group necessarily is abelian. In this talk a complete characterization of all quasi-primitive groups is presented. It says that quasi-primitive groups are exactly the direct products with amalgamated centers of quasi-simple groups. This result uses the classification of finite simple groups.

Furtheron two generalizations of quasi-primitivity are considered. The first one relates to groups for which the restriction of irreducible characters merely to characteristic subgroups decomposes homogeniously, the second one relates to groups which have on the conjugacy classes and on the irreducible characters of any normal subgroup similar permutation representations. This is joint work with René Bartsch.

L. Puig: Source algebras of blocks from the source algebras of their splitting extensions

It is well-known that, when studying a block b of a finite group G over a perfect field k of characteristic p, the inertial quotient $I = N_G(P,e)/PC_G(P)$ of a maximal Brauer pair (P,e) associated with b is *not* necessarily a p'-group. Precisely, if we assume that b is ***absolutely primitive*** in $Z(kG)$ (i.e. $Z(kGb)/J(Z(kGb)) = k$) and set $\hat{k} = Z(kC_G(P)e)/J(Z(kC_G(P)e))$, the Sylow p-subgroups of $\mathrm{Gal}(\hat{k}/k)$ and I are isomorphic. In my talk I will show that a source algebra of b is a *cros-*

sed product of a source algebra of a block $\hat{b}$ in $\hat{k}Gb$ and a suitable $\hat{k}^*$-extension of this inertial quotient I, over the corresponding central $\hat{k}^*$-extension of the inertial quotient of a maximal Brauer pair $(p, \hat{e})$ associated with $\hat{b}$. I will apply it to the case where $\hat{b}$ is a nilpotent block.

J. Rickard: Derived categories and applications to group representation theory

Let $\mathcal{O}$ be a complete discrete valuation ring with field of fractions K of characteristic zero and residue field k of characteristic $p > 0$. For a block algebra $\mathcal{O}A$ of a finite group, we denote by kA the corresponding block algebra over k. For $R = \mathcal{O}$ or k, we denote by $D^b(RA)$ the derived category of bounded complexes of finitely generated RA-modules.

Some time ago, we proved that for block algebras RA and RB finite groups G and H, the derived categories $D^b(RA)$ and $D^b(RB)$ are equivalent (as triangulated categories) if and only if there is a bounded complex X of finitely generated RA-RB-bimodules, projective over RA and over RB, such that $X \otimes_{RB} X^* \cong RA$ and $X^* \otimes_{RA} X \cong RB$ in the derived categories of RA-bimodules and RB-bimodules. X is then called a "tilting complex". Such equivalences are conjectured by Broué to be very common. For example:

Conjecture (Broué): If G is a finite group with abelian Sylow p-subgroup P, then the principal blocks of $\mathcal{O}G$ and $\mathcal{O}N_G(P)$ have equivalent categories.

Many phenomena observed in the evidence for this conjecture remained unexplained by simply an equivalence of derived categories. This led us to make the following definitions which applies in the case where G, H have a common Sylow p-subgroup P:

Definition: A tilting complex X for block algebras RA (of RG) and RB (of RH) is called *splendid* if $X \otimes_{RB} X^* \cong RA$ and $X^* \otimes_{RA} X \cong RB$ is the appropriate chain homotopy categories, of complexes of bimodules, and the terms of X are (when regarded as $R[G \times H]$-modules) direct summands of permutation modules that are induced from subgroups of $\Delta P = \{(\pi, \pi) \in G \times H : \pi \in P\}$. We call the equivalence of derived categories induced by such a complex a *splendid equivalence*.

Evidence suggests that it is reasonable to hope that the equivalences predicted by Broué's conjecture should be splendid. The definition also has good consequences, as described in the following theorems:

Theorem: In the context of Broué's conjecture, if there is a splendid equivalence between the principal blocks of kG and $kN_G(P)$, then for every $Q \leq P$ there is a splendid equivalence between the principal blocks of $kC_G(Q)$ and $k[C_G(Q) \cap N_G(P)]$.

Theorem: If there is a splendid equivalence between blocks kA and kB, then there is a splendid equivalence between $\mathcal{O}A$ and $\mathcal{O}B$.

Together these theorems give a structural explanation of a phenomenon at the level of characters, called an "isotopy" by Broué, that has been observed in numerous examples.

G. R. Robinson: On Brauer's $k(B)$-problem for p-solvable groups

Brauer asked whether it is the case that when B is a block with defect group D of a finite group, then $k(B) \leq |D|$ ($k(B)$ denotes the number of ordinary irreducible characters in B). In the general case, Brauer and Feit (1958) proved that $k(B) \leq \frac{1}{4}|D|^2 + 1$, a bound which has resisted significant improvement.

In 1962, H. Nagao (using results of P. Fong) established that to prove the $k(B)$ conjecture for p-blocks of p-solvable groups, it suffices to prove the "$k(GV)$-conjecture": if G is a finite p'-group, and V is a faithful irreducible $\mathrm{GF}(p)G$-module, then $k(G) \leq |V|$. This problem has been extensively studied. Between 1980 and 1984, R. Knörr introduced powerful new ideas, which were sufficient, for example, to establish the truth of the $k(GV)$-conjecture when G is supersolvable, or $|G|$ is odd (the latter case was done independently by D. Gluck, making use of Knörr's methods). In 1990, R. Knörr showed that the conjecture is correct if there is some $v \in V$ such that $\mathrm{Res}^G_{C_G(V)}(V)$ is a permutation module (for any given GV). In 1993, R. Gow showed that the conjectured inequality holds if $V \cong V^*$ as $\mathrm{GF}(p)G$-module.

In 1995, J.G. Thompson and I proved the following theorem, which establish the $k(GV)$-conjecture for p sufficiently large:

Theorem 1: Let G, V be as above. Suppose that there is a vector $v \in V$ such that $\mathrm{Res}^G_{C_G(V)}(V)$ has a faithful self-dual submodule. Then $k(GV) \leq |V|$.

Theorem 2: Let G, V be as above, and suppose that $p > 5^{30}$. Then there is a vector $v \in V$ such that $\mathrm{Res}^G_{C_G(V)}(V)$ has a faithful permutation module as a summand. In particular, $k(GV) \leq |V|$.

The proof of Theorem 2 relies on a result of M. Liebeck, which asserts that if G, V are as above, and $E(G)$ is quasi-simple with $F(G) = Z(G)$, then if $p > 5^{30}$, G has a regular orbit on V, unless $E(G) \cong A_m$ where $\dim(V) = m - 1$ and $m < p$. This improves an earlier result of Hall, Liebeck and Seitz.

As to improving the bounds, a student of Liebeck (Dominic Goodwin) has made significant progress in reducing the 5^{30} bound to 5^6. Another case which needs to be considered (after Clifford theoretic reductions) is when $O_q(G)$ is of symplectic type for some prime q. It can then be shown that a vector with the necessary properties exists if $p > 6875$. These techniques also yield:

Theorem 3: Suppose that G is a finite solvable group, $p > 751$ is a prime, and B is a p-block of G. Then $k(B) \leq |D|$, where D is a defect group for B.

B. Srinivasan: Green polynomials of symplectic groups

This talk is based on joint work with T. Shoji. Let $\mathbf{G}$ be a connected reductive group defined over $\mathbb{F}_q$, $F : \mathbf{G} \to \mathbf{G}$ a Frobenius morphism and $G = \mathbf{G}$ a finite group of Lie type. Let $\mathbf{T}_1 \subset \mathbf{B}_1$ be an F-stable maximal torus and Borel subgroup respectively, and $W = N_{\mathbf{G}}(\mathbf{T}_1)/\mathbf{T}_1$ the Weyl group. Then

representations for the maximal tori of G can be written as $\{T_W |\ w$ a representative of an F-conjugacy class of $W\}$. For each T_W we have a Green function $Q^G_{T_W}$ on the unipotent elements of G. The values $Q^G_{T_W}(u)$ form a part of the character table of $G\,(u \in G)$.

Let $u \in G$. By a theorem of Springer we have a representation of W on $H^*_c(\mathcal{B}_u, \overline{\mathbb{Q}}_l)$ where $\mathcal{B}_u$ is the variety of all Borel subgroups of $\mathbf{G}$ containing u. Then we can write

$$Q^G_{T_W}(u) = \sum_i (-1)^i \mathrm{Tr}(wF, H^i_c(\mathcal{B}_u)), \text{at least for good}\, p = \mathrm{char}\, \mathbb{F}_q.$$

Also $\widehat{A(U)} = C_{\mathbf{G}}(u)/C^0_{\mathbf{G}}(u)$ acts on $\mathcal{B}_u$, so we can talk of $(H^*_c(\mathcal{B}_u))\phi$ where $\phi \in \widehat{A(u)}$. Now let $G = \mathrm{Sp}(2n, \mathbb{F}_q)$, $G = \mathrm{Sp}(2n, \mathbb{F}_q)$. Then $W = W'D$, where $D \lhd W$, $W' \cong S_n$. Let $\pi : W \to W'$ be the natural map. We define a map f : $\{\text{Unipotent classes of}\, \mathrm{Sp}(2n, \overline{\mathbb{F}}_2\} \to \{\text{Unipotent classes of}\, \mathrm{GL}(n, \mathbb{F}_q)\}$. Then fix $W_1 \in W'$, and consider $\pi^{-1}(W_1)$. We would like to compare

$$\frac{1}{|W|} \sum_{W \in \pi^{-1}(W_1)} (\widetilde{Q^{\mathrm{Sp}}_{T_W}(u)})(q)$$

where $\tilde{}$ means arranging over G-conjugacy classes contained in the G-conjugacy class of u, and

$$\frac{1}{|W'|}(Q^{\mathrm{GL}}_{T_{W_1}}(f(u)))(q^2)$$

This means that we compare

$$H^{2i}(\mathcal{B}^{\mathrm{GL}}_{f(u)}) \,\text{with}\, H^{4i}(\mathcal{B}^{\mathrm{Sp}}_u)^D_1 \,\text{as}\, W' = S_n\text{-modules.}$$

(i.e. the part of $H^{4i}(\mathcal{B}^{\mathrm{Sp}}_u)$ which is fixed by D and where $A(u)$ acts by 1, the trivial character).

We show the existence of the commutative diagram

$$\begin{array}{ccc} H^i(B^{\mathrm{GL}}) & \xrightarrow{\sigma_0} & H^i(B^{\mathrm{Sp}})^D \\ \bar{\phi}\downarrow & & \downarrow\phi \\ H^{2i}(B^{\mathrm{GL}}_{f(u)} & \xrightarrow{\sigma} & H^{4i}(B^{\mathrm{Sp}}_u)^D_1 \end{array}$$

and show σ is injective.

An upshot is that $(1) = (2) +$ error term. The error term is 0 for certain classes (u), e.g. where U is parametrized by a partition of $2n$ with all parts even.

J. Thévenaz: The Dade group of a p group

Let k be an algebraically closed field of characteristic p and let P be a finite p-group. The Dade group $D(P)$ of P is an equivariant version of the Brauer group of a field. It is an abelian group made of equivalence classes of simple P-algebras over k having a P-invariant basis. Many important invariants in modular representation theory lie in $D(P)$ (e.g. sources of simple modules, block invariants, etc.). The structure of $D(P)$ is only known if P is abelian (Dade, 1978).

It was proved 15 years ago by Puig that $D(P)$ is finitely generated but, until recently, no significant progress was made about its structure. There are now some results about the torsion-free rank of $D(P)$.

A suitable subgroup $T(P)$ of $D(P)$ plays a crucial role. In particular $\mathbb{Q} \otimes D(P)$ embeds in the product of the groups $\mathbb{Q} \otimes T(N_p(\mathbb{Q}/Q)$, where Q runs over all subgroups of P up to conjugation.

Theorem 1 (Alperin, 1995): Let X be the poset of elementary abelian subgroups of P of rank ≥ 2. Then $\dim(\mathbb{Q} \otimes T(P))$ is the number of conjugacy classes of connected components of X.

Note that the connected components of X can be described explicitly. Alperin's proof uses relative syzygies and I have a proof using tensor induction.

Theorem 2: $\dim(\mathbb{Q} \otimes D(P)) \leq \sum_Q \dim(\mathbb{Q} \otimes T(N_p(Q)/Q))$ where Q runs over all subgroups of P up to conjugation.

Equality is expected to hold, at least in most cases. Another open problem is the description of the torsion subgroup of $D(P)$. It is expected to be a 2-torsion group.

A. Turull: Character quotients for coprime acting groups

Let A be a finite group acting on the finite group G with $(|A|, |G|) = 1$. Let P be the semidirect product of A and G. Let χ be an irreducbile character of Γ whose restriction to G is irreducible. Let $\mathcal{C}(A)$ be the set of all restrictions of χ to A where χ runs over all those that arise with G a direct product of extraspecial groups. In a paper of the author and B. Hartley we proved that $\mathrm{Res}_A^\Gamma(\chi) \in \mathcal{C}(A)$ as soon as the simple sections of G satisfy some condition of the Green functions of groups of Lie type. Here we prove that the function $Q(\chi) : S \to \mathbb{C}\, Q(\chi)(s) = \frac{|C_G(s)|}{\chi(s)}$ is also an element of $\mathcal{C}(A)$ under the same hypotheses.

K. Uno: List of the cases where some form of Dade's conjecture has already been verified

1. SIMPLE GROUPS

M_{11}	final	Dade [D1]
M_{12}(covering groups, outerauto.incl.)	final	Dade
M_{22}(covering groups, outerauto. incl.)	final	Huang
M_{23}, M_{24}	final	Schwartz, An, Conder
J_1	final	Dade [D1]
J_2(covering groups, outerauto.incl.)	final	Dade
J_3(covering groups, outerauto. incl.)	final	Kotlica
McL	invariant ordinary $p \neq 2$	Murray
Ru	ordinary	Dade
He	final	An, preprint
Co_3	final	An, in preparation
$L_2(q)$	final	Dade
$L_3(q)$	final $p\|q$	Dade
$Sz(2^{2n+1})$	final	Dade
$G_2(q)$	ordinary, $p \nmid q$	An [A1]
${}^2G_2(3^{2n+1})$	final, $p \neq 3$	An [A2]
${}^2F_4(2^{2n+1})$	ordinary, $p \neq 2$	An [A3]
${}^2F_4(2)'$(Tits group, outerauto. incl.)	final	An [A4]

2. OTHER GROUPS

$\mathrm{GL}(n,q)$	ordinary, $p\|q$	Olsson, Uno [OU1]
S_n	ordinary, $p \neq 2$	Olsson, Uno [OU2]
S_n	ordinary, $p = 2$	An, preprint

3. GENERAL RESULTS

cyclic defect group case	final	Dade [D4] + α
tame block case	invariant ordinary	Uno [U]
abelian defect unipotent block	ordinary	Broué Malle, [BM]
abelian defect principal block $p = 2$	ordinary	Fong, Harris [FH]
abelian defect small inertia index	ordinary	Usami, preprints

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W. Wheeler: Stratifying the rank variety of a module

Suppose that E is an elementary abelian p-group, k is an algebraically closed field of characteristic p, and M is a finitely generated kE-module. By considering the direct sum decomposition of M upon restriction to cyclic shifted subgroups $< u_\alpha >$, it is possible to decompose the rank variety $V^r(M)$ into a disjoint union of locally closed subspaces. Specifically, let $X(M; n_1, \ldots, n_p)$ denote the set of all $\alpha \in V^r(k)$ such that the i-dimensional indecomposable module has multiplicity n_i as a summand of the restriction $M_{<u_\alpha>}$ for $1 \le i \le p$. Then $X(M; n_1, \ldots, n_p)$ is locally closed in $V^r(k)$. Moreover, the closure of the subspace $X(M; n_1, \ldots, n_p)$ can be described in terms of deformations of modules over a group of order p.

A. E. Zalesskii: Eigenvalues of matrices in representations of quasi-simple groups

The talk discusses the problem of determining the degrees of minimal polynomials of p-elements in representations of quasi-simple finite groups. The main result describes the pairs (G, n) where G is a quasi-simple group and n is the degree of a non-trivial irreducible representation of G over an algebraically closed field of characteristic 0 or p, provided G has a cylic Sylow p-subgroup. For $n < p$ the problem was solved earlier by Blau-Zhang. For $n \ge p$ the result can be stated as follows.

Theorem. Let G be a quasi-simple finite group, and ρ be an irreducible representation of G of degree $n \ge p$. Suppose G has a cyclic Sylow p-subgroup and there is a p-element $g \in G$ such that the degree of the minimal polynomial of $\varphi(g)$ is less than $|g|$. Then $n = 2(p-1)$, and one of the following holds:

i) $p = 5$, $G/Z(G) \in \{A_8, A_9, \mathrm{Sp}_6(2)\}$,
ii) $p = 7$, $G/Z(G) \in \{G_2(4), \mathrm{Suz}\}$,
iii) $p = 13$, $G/Z(G) = \mathrm{Co}_1$.

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