MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 16/1996

Buildings

21.04 – 27.04.96

The meeting has been organized by Mark Ronan (University of Illinois at Chicago) and Peter Slodowy (Hamburg). Some of the subjects treated in the talks included:

a) Generalizations of Buildings, including Twin Buildings,

b) applications to

- i) the theory of symetric spaces
- ii) harmonic analysis
- iii) graphs and geometries

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M. RONAN:
Twin Buildings

The purpose of this talk was to give an introduction to twin buildings in preparation for the talks of several other participants. A twin building is a pair of buildings along with a codistance function (taking values in the Coxeter group) from the chambers of one building to those of the other. The automorphism group is more restricted than that of a single building. For example $GL_n(k[t,t^{-1}])$ is almost the whole automorphism group of a twin building, whereas each of the two buildings concerned admit the much larger group $GL_n(k((t)))$ and $GL_n(k((t^{-1})))$. Two main theorems were discussed. A rigidity theorem showing that a certain mild restriction on fixed point sets gives only the identity as an automorphism; this leads to a natural concept of root groups. The other theorem was a local to global theorem showing that the local structure determines the global structure in almost all cases where tree residues are excluded.

J JOST:
Harmonic Maps into Buildings and Applications of Algebraic Geometry

Harmonic maps into Euclidian buildings or more general spaces of generalized nonpositive curvature in the sense of Alexandrov are defined by some kind of infinitesimal mean value property. A general existence theorem is presented that exploits suitable convexity properties of such spaces, but does not require them to be locally compact.

Applications of this theory include results in the direction of Margulis superrigidity, as well as factorization theorems for **p**-adic representations of Kähle groups (the latter represents joint work with Kang Zuo).

E LANDVOGT:
Functorial Properties of the Bruhat-Tits Building

Since the Bruhat-Tits building is defined not very naturally, there are simple questions which cannot be answered directly, e.g. whether the Bruhat-Tits building depends functorially on the group. So the purpose of the talk was to prove the following theorem:

Theorem: Let K/k be a Galois extension of local fields and $G \subseteq H$ a k-inclusion of connected reductive k-groups. Then there exists a map $BT(G,K) \to BT(H,K)$ which is G(K)- and Gal(K/k)-equivarient and, after renormalization of the metric on BT(G,K), an isometrical inclusion.

(Here: BT(-,K) denotes the extended Bruhat-Tits building).

Consider the following map:

$$\left\{ \begin{array}{l} \iota: BT(G,K) \to BT(H,K) \\ G(K)\text{- and } Gal(K/k)\text{-equivar} \\ \text{isometrical inclusion} \end{array} \right\} \longrightarrow BT(H,K), \quad \iota \mapsto \iota(x)$$

where $x \in BT(G,K)$ is a fixed special point. Denote by μ the image of this map and by Z the centralizer of G(K) in H(K). Then:

Theorem:

(i) There is a bounded subset $\mu_0 \subseteq \mu$ such that $Z \cdot \mu_0 = \mu$.

(ii) If G splits over an unramified extension of K, then there is a point $\tilde{x} \in \mu$ such that μ=convex hull of $Z \cdot \tilde{x}$.

G. LEHRER:
<u>Split Buildings of Reductive Groups</u>

Let G be a connected reductive group over a fixed arbitrary field **k**. For any k-split torus S, we have the sphere $\mathcal{B}(S)$ consisting of $\frac{1}{2}$-lines in $Y(S) \otimes R$ (Y being the cocharacter group). The spherical building of G is defined as a quotient of $B_1(G) = \coprod \mathcal{B}(S)$ (disjoint union over all max k-split tori of G) by the equivalence relation $b_1 \sim b_2$ if $b_2 = {}^g b_1$ for some $g \in P(b_1)(\mathbf{k})$ where P(b) is the parabolic subgroup corresponding to $b \in \mathcal{B}_1(G)$. If L(b) is the Levi subgroup defined by $b \in \mathcal{B}_1(G)$, define a weaker equivalence on $\mathcal{B}_1(G)$ by $b_1 \approx b_2$ if $b_2 = {}^g b_1$ for some $g \in L(b)(\mathbf{k})$. <u>The split building is defined as</u> $\mathcal{S}(G) := \mathcal{B}_1(G)/\approx$. Clearly one has a surjective map $\mathcal{S}(G) \to \mathcal{B}(G)$; morever an easy lemma shows that $\mathcal{S}(G) \cong (\{(b,b') \in \mathcal{B}(G) \times \mathcal{B}(G) \mid b$ is opposite $b'\}$. The split building has the following properties: (i) it has apartments B(S) (ii) $\mathcal{S}$ is a functor (group homomorphism required to be injective) (iii) If $x \in G(\mathbf{k})$ is semisimple, $S(G)^x = \mathcal{S}(C_G(x)^\circ)$ (iv) $\mathcal{S}(G)$ is the d-fold suspension of the simplicial complex S(G), defined below, where d=k-rank Z(G)°.

Conjecture: For any field **k**, and any connected reductive **k**-group G, the split building $\mathcal{S}(G)$ is homotopy equivalent to a bouquet of spheres.

Theorem: For G classical, the conjecture is true.

This follows from a discrete version of the result, proved by myself with Leanne Rylands. The simplicial version of $\mathcal{S}(G)$ is defined as the poset of pairs (P,L), P a **k**-parabolic subgroup of G and L a **k**-Levi subgroup of P (HENCE: the name split building: P=LU is a splitting of P). The ordering is reverse on inclusion. The case of type A may be identified with the poset of direct decompositions of a **k**-vector space; this was treated by R. Charney. The other classical cases may be similarly identified: if V is a **k**-vector space with a sesquilinear form $\langle\ ,\ \rangle$, define $S(V) := \{(A,B) \mid A,B$ are subspaces of V; A, $B^\perp$ totally isotropic; $V=A\oplus B\}$. The ordering is inclusion of the 1st term, reverse inclusion of the second.

The space S(V) may be identified with S(G), G the isometry group of $(V,\langle\ ,\ \rangle)$ when $\langle\ ,\ \rangle$ is non-degenerate [there is a complication in type D_ℓ, but it may be overcome].

Theorem: (Leher-Rylands) S(V) is Cohen-Macauley (CM) over any field if $\langle\ ,\ \rangle$ is 0 or non-degenerate.

The proof uses the Quillen spectral sequence of the map $f:S(V)\to T(V)$ (f= 1st projection), with $E^2_{pq}=H_p(Y,H_q(f_{\geq y}))$. The fibres $f_{\geq y}$ also need to be studied and the proof depends on an induction. The following posets are proved to be CM in the process: if $A\subset B$ are subspaces of V: $\{U<V \mid U\cap A=0,\ U+B=V\}$; $\{(U,W)\in S(V) \mid U\supseteq M,\ W\supseteq N\}$ and corresponding sets in the classical case (not type A).

Applications include (i) Identities for rational functions of q arising from different expressions for the number of spheres (or dim $H_{top}(\mathcal{S}(G))$). (ii) Representation theory of G(**k**) (**k** is finite) (iii) Quillen complex $Q_\ell(G(k))$ for **k**-finite and $\ell\neq$char **k**.

J. TEITELBAUM:
P-adic Symmetric Spaces

In this lecture I presented joint work with Peter Schneider (Münster) regarding analytic properties of Drinfeld's **p**-adic symmetric space. Let $\overline{\mathfrak{X}}$ denote the compliment in $P^d_{\mathbb{R}}$ of the set of hyperplanes defined over K, where K is a local field of characteristic zero having a non-archimedean valuation. The space $\overline{\mathfrak{X}}$ can be used to construct algebraic varieties by dividing by the action of a discrete group. Schneider and Stuhler proved that $H^d_{DR}(\overline{\mathfrak{X}},K)$ is isomorphic to Hom(St,K), where St denotes the Steinberg representation of the group $GL_{d+1}(K)$. I described the construction of two maps:

$$\text{Res}: H^d_{DR}(\overline{\mathfrak{X}},K) \rightarrow \text{Hom}(St,K)$$

$$I: \text{Hom}(St,U_K) \rightarrow H^d_{DR}(\overline{\mathfrak{X}},K)$$

which are inverses to one another (Res∘I=Id). The Residue map "Res" is constructed by realizing Hom (St,K) as a space of harmonic functions on the Bruhat-Tits building $BT(GL_{d+1},K)$. The integration map I is based on viewing $\text{Hom}(St,O_K)$ as a space of bounded p-adic measures on G/P and integrating these measures against a kernel function.

B. MÜHLHERR:

2-Spherical Twin Buildings

Twin buildings are structures which generalize spherical buildings in a natural way. This motivates the question about a classification of all irreductible twin buildings of rank at least 3. It turns out that one has to require that all entries of the diagram are finite; i.e. that the buildings are 2-spherical.

Under this assumption the most important theorem used in the classification of spherical buildings was proved in joint work with M.Ronan for almost all twin buildings.

The classification reduces now to the following questions: Which Moufang foundations are foundations of twin buildings?

We have the following results:

(i) Given a desarguesian foundation F, whose diagram has a vertex of valency at least 3, then if F is the foundation of a twin building the coordinatizing division algebra is a quaternion algebra or a field.

(ii) If k is a field and $\text{Aut}(k)<\infty$ then each split Moufang foundation is the foundation of a twin building.

(iii) Each finite Moufang foundation is the foundation of a twin building.

The first result is proved by analyzing Moufang $\tilde{A}_2$-buildings; using a fixed-point lemma one gets (ii) and (iii).

P. SCHNEIDER:

Verdiér duality on buildings

In this talk we considered the semisimple building X of a connected reductive group G over a nonarchimedean locally compact field K. We want to use the building in order to investigate the category of all smooth G-representations denoted by Alg(G). In joint work with U.Stuhler we constructed "localization" functions from Alg(G) to the category of G-equivariant sheaves on X as well as to the category of a G-equivariant coefficient systems (or cosheaves) on X. Exploration of the Borel-Serre compactification of X allows us to compute the (co)homology of these localized objects. As an application one can construct a natural duality theory on the category Alg(G). In the second part of the talk the obvious question was discussed how that duality is related to "classical" Verdiér duality formalism for sheaves on X. It was shown that indeed these two duality theories correspond to each other under the functor "cohomology with compact support". In order to see this one first has to redevelop the classical Verdiér duality in the G-equivariant setting which can be done without much difficulty. The key observation then is that the coefficient systems on X naturally embed into the derived category of sheaves on X as some kind of perverse sheaves. Moreover Verdiér duality maps the abelian category of constructible coefficient systems on X into the abelian category of constructible sheaves on X by the very simple procedure of passing to the linear dual on stalks.

D.E. TAYLOR:

On Outer Automorphism Groups of Coxeter Groups

Given a Coxeter system (W,R) of finite rank such that rs has finite order for all $r,s \in R$, we show that $|\mathrm{Aut}(W)/\mathrm{Inn}(W)|$ is finite. This is proved in two steps.

If V is a reflection module for W and if Aut(W) denotes the group of automorphisms of W arising from orthogonal transformations of V then, using the fact that every finite subgroup of W is contained in a finite parabolic subgroup, we show that | Aut(W): Aut(W) | is finite.

To complete the proof we show that if W is an irreducible Coxeter group of finite rank with reflection module V, root system Φ, root basis P, and if ϕ:V→V is an orthogonal transformation permuting Φ, then $\phi\Pi=\pm w\Pi$ for some w∈W. The idea is to show that N(ϕ) or N(-ϕ) is finite where $N(\phi)=\{a\in\Phi^+ \mid \phi(a)\in \Phi^-\}$. The finiteness of N($\phi$) is equivalent to ϕ preserving the Tits cone and also equivalent to ϕ preserving the dominance order on Φ.

When W is spherical, affine or hyperbolic it can be shown directly that ϕ or -ϕ preserves the Tits cone. In particular, this proves the result in the case that W has rank at most 3. Finally, if ϕ preserves dominance on all irreductible rank 3 subsystems, then ϕ preserves dominance on the entire root system.

J. TITS:
<u>Twin Trees</u>

In this lecture, trees are always thick, i.e. all vertices have valency ≥3. Let T_+, T_- be two such trees and let $V(T_+)$, $V(T_-)$ be their sets of vertices. A <u>codistance</u>, or a <u>twinning</u>, between T_+ and T_- is an integer-valued function $d^*: V(T_+) \times V(T_-)\to \mathbf{N}$ such that, for $x\in T_+$ and $y\in T_-$,

(CD1) if $d^*(x, y)=0$, then $d^*(x_1, y)= d^*(x, y_1)=1$ for all neighbours x_1 of x in T_+ and all neighbours y_1 of y in T_-, and

(CD2) if $d^*(x,y)= n\geq 1$, then, with the same notation as in (CD1), $d^*(x_1 ,y)= d^*(x,y_1)= n-1$ for all x_1, y_1, except for a single x_1 and a single y_1, for which $d^*(x_1, y)= d^*(x,y_1)= n+1$. An easy consequence of these conditions is that if two vertices of $T_+\cup T_-$ at even distance or codistance have equal valencies; thus T_+ and T_- are isomorphic semi-homogenous trees.

Twin trees (i.e. pairs of twinned trees) are the "simplest" examples of twin buildings, though some techniques used in higher ranks are not applicable here. Motivation for their study can be found in the references given below.

There is a standard example of twin trees attached to the groups $SL_2(\mathbf{k}[t,t^{-1}])$ (k any field), where T_+ and T_- are the trees attached in the well-known fashion to $SL_2(\mathbf{k}((t)))$ and $SL_2(\mathbf{k}((t^{-1})))$. One can show, using the Moufang property, that the full automorphism group of the twin tree in question is generated by $SL_2(\mathbf{k}[t,t^{-1}])/\{\pm 1\}$, Aut $\mathbf{k}$ and two groups isomorphic with $\mathbf{k}^\times$: the group of conjugations by $\begin{pmatrix} c & 0 \\ 0 & 1 \end{pmatrix}$ ($c \in k^\times$), and the group of parameter changes, $t \mapsto ct$ ($c \in k^\times$).

The problem of constructing twin trees is discussed from various angles in [T1], [F], [RTII]. All twin trees which "naturally" turned up so far satisfy the already mentioned Moufang condition which is a strong homogeneity condition implying, for instance, that the automorphism group of a Moufang twin tree is transitive on the set of pairs of opposite edges (an edge of T_+ and and edge of T_- are said to be opposite if each vertex of the first is at codistance 0 of a vertex of the second). Formulas given in [T1] describe "in principle" all Moufang twin trees; they yield in particular the existence of uncountably many nonisomorphic twinnings of homogeneous trees of valency 3. The content of [F] was the subject of the D.G. Fon-Der-Flaass talk at this meeting. The present lecture was mainly focused on [RTII].

The method developed in the latter (as yet unpublished) paper consists in associating to any semi-homgeneous tree T a certain graph T°, the universal twin of T, in which any tree twinned with T is canonically embedded. All twinnings involving T can be obtained by constructing the twin T_- inside T°. The group Aut T operates naturally on T° and two twinnings (T,T_-) and (T,T'_-) are isomorphic if and only if T_-, T'_- are conjugate (in T°) by an element of Aut T. An analysis of the structure of T° leads to the following result, among others:

Theorem: If α is the cardinality of T, there exist 2^α non isomorphic twinnings (T,T_-). (A twinning is said to be rigid if its automorphism group is reduced to the identity).

References

[F] D.G.Fon-Der-Flaass, A combinatorial construction for twin trees, European Journal of Combinatorics, 17 (1966), 177-189.

[RT I] M. Ronan and J Tits, Twin Trees I, Invent. Math 116 (1994), 463-479

[RT II], [RT III], ..., Twin Trees II, III, in preparation.

[T1] J. Tits, Resumé de Cours, Annuaire de Collège, 89e année (1988-89), 81-95

[T2] J. Tits, Twin Buildings and groups of Kac-Moody type, in Groups, Combinatorics and Geometry, Lond. Math. Soc. Lecture Notes 165, Cambridge U.P., (1992), 249-286.

V. USTIMENKO:

BN-pairs and Extremal Graph Theory

Buildings of rank 2 turn out to be extremely useful in extremal graph theory and its applications.

An example: By ex(v,n) we denote the greatest number of edges (size) in a graph on v vertices with girth > n.

From Erdös' Even Cycle Theorem follows, that

$$ex(v,2k) \leq cv^{1+1/k}$$

It is known that the bound above is sharp for k=2,3,5. We can just consider the incidence graph I_k for the geometry of groups $A_2(q)$, $B_2(q)$, $G_2(q)$ (generalized m-gons, m=3,4,6). Size of I_k is an upper bound.

Other problem: Let $\{G_i\}_{i\geq 1}$, be a family of graphs such that $\{G_i\}$ is anti-regular of increasing order v_i and girth g_i. Following Biggs we say that $\{G_i\}$ is a family of graphs with large girth if $g_i \geq \gamma^{\log_{r-1}(v_i)}$. Known explicit results belong to Margulis and Lubotzky, Phillips and Sarnak. Actually $\gamma<2$, bigger γ corresponds to a bigger size. The best known case is $\gamma=\frac{4}{3}$.

Construction: Let G be locally finite Tits group of Moufang type. P_1 and P_2 are standard parabolics containing B^-. The orbit $\mathcal{D}S\ P_1$ ($\mathcal{D}S\ P_2$) of U^+ on the set $(G:P_1)$ ((G: P_2) respectively) which has the largest order (i.e. largest dimension), we shall refer to as a dual Schubert cell. The restriction of the incidence relation for the geometry of G on $(\mathcal{D}S\ P_1) \cup (\mathcal{D}S\ P_2)$ we shall refer to as dual Schubert structure $\mathcal{D}S(G)$.

Theorem: There is an infinite family of quotients of $\mathcal{D}S(G)$ which is a family of graphs with large girth.

Certain deformations of $\mathcal{D}S(G)$ gives us a family with $\gamma=\frac{4}{3}$ (Lazebnik, Ustimenko, Woldar, 1985).

The quotients of $\mathcal{D}S(G)$ turn up as solutions of the following problems.

Problem 1 (Lubotzky): Prove that for every k≥3, there are infinitely many k-regular Ramanujan graphs.

Problem 2: Prove that for every k≥3, there are infinitely many k-regular Cayley expanders.

The solutions of both problems are explicit.

H. VAN MALDEGHEM:

Some Results on Automorphism Groups of Affine Buildings of Rank 3

The Moufang condition for spherical buildings contains essentially two ingredients: one requirement is a transitivity condition (implying a regular action); a second requirement is that the automorphisms under consideration fix all chambers which have a panel in a certain root (but not on the boundary); or, equivalently, some commutation relation holds. There are several ways to generalize this to the general case. Actually, Tits already introduced this and it is our aim to find, in the special case of rank 3 affine buildings, alternative, but weaker conditions. This is obtained by considering (i) a transitivity condition on chambers "at the boundary" of a root, and, a commutation relation on "root groups", (ii) a transitivity condition on apartments through a root, and, requiring that all automorphisms under consideration fix chambers as above. For all rank 3 affine buildings, (i) implies Moufang at infinity. For all $\tilde{A}_2$-buildings, (ii) implies Moufang at infinity. We have also: a locally finite $\tilde{A}_2$-building with a group transitive on pairs of chambers at fixed Weyl distance, is classical (e.g. all strongly transitive $\tilde{A}_2$-buildings). Finally, we introduce a p-adic Moufang condition. Most hyperbolic rank 3 buildings cannot satisfy such a condition. Everything is joint work with Kristel Van Steen.

M. RAPOPORT:

Problems on Bruhat-Tits Buildings Arising in the Theory of Shimura Varieties

The problems referred to in the title arise in describing the Dieudonné modules of abelian varieties over $\bar{\mathbf{F}}_p$. Two types of general results were discussed:

1). Let (V,ϕ) be an isocrystal of height n over $\bar{\mathbf{F}}_p$, with Newton vector $\nu(\phi)\in\mathbf{Q}^n\cap\bar{C}$. For a lattice $M\subset V$, let $\mu(M,\phi)\in\mathbf{Z}^n\cap\bar{C}$ be the relative position of M and $\phi(M)$. Then

$$\nu(\phi)\leq \mu(M,\phi)$$

in the usual partial order on the positive Weyl chamber.

2) Let (V,ϕ) be an isocrystal over $\bar{\mathbf{F}}_p$. For all c>0 there is C>0 with the following property: For any lattice $M\subset V$ with

$$p^{c}\cdot\phi(M)\subset M\subset p^{-c}\,\phi(M)$$

there exists a lattice M_o with

$$p^c \cdot M_o \subset M \subset p^{-c} \cdot M_o$$

such that M_o is decomposable w.r.t. the slope decomposition of (V,ϕ) and rational w.r.t the rational structure over $\mathbf{Q}_p$ on V defined by it.

We explained how these results may be viewed as statements on the Bruhat-Tits building of GL_n and how to extend them to an arbitrary reductive algebraic group over $\mathbf{Q}_p$. We also related these results to the following result of G.Rousseau.

Theorem (Rousseau): Let B be a building of Euclidian type and let $\sigma \in Aut(B)$ with $B^\sigma \neq \emptyset$. For $x \in B-B^\sigma$ let x_o be the unique point in B^σ closest to x. There exists $\theta>0$ such that

$$\measuredangle_{x_0}([x,x_o],[\sigma(x),x_o]) \geq \theta$$

and hence $d(x, \sigma(x)) \geq 2d(x,x_o) \sin \frac{\theta}{2}$.

H. BEHR:

Finiteness Properties of S-arithmetic Groups

Survey of Results: On finiteness properties (generation, presentation, type FP_n) of S-arithmetic subgroups of simple algebraic groups over function fields.

Methods: (a) Action on (euclidean or twin) buildings, (b) Filtration of buildings (gallery distance, distance to infinity), (c) Reduction to local topological properties of subcomplexes of finite spherical buildings, (d) In the case of finite presentation: Reduction to amalgamations of finite stabilizers.

New Approach: (Program details not checked!) (a) Action of $\Gamma=SL_n(F_q[t])$ on subcomplex Y of the affine building X of codimension 1 ("boundary of the non-stable region"), (b) Description of Y as a subcomplex Y'_∞ of the "split building" $Y_\infty=\{(P,P') \mid P \in X_\infty(F_q(t)),\ P' \text{ opposite } P\}$.

If (i) Y'_∞ is homeomorphic to Y

(ii) Y_∞ is (n-1)-spherical

(iii) Y'_∞ is a retract of Y

Then Γ is of type F_{n-2} .

P. ABRAMENKO:

Twin Buildings and Groups Acting on Them

This is a brief survey on topological properties of certain subcomplexes of twin buildings and some of their group theoretic applications.

Let $\Delta=(\Delta_+,\Delta_-,d^*)$ be a twin building and $c_+ \in D_+$ a fixed chamber. Consider the subcomplex $\Delta^o(c_+)$ of Δ_- consisting of all chambers opposite c_+ and all of their faces. Then one obtains:

Theorem 1: If Δ is a spherical building of type A_n, C_n or D_n such that every panel of Δ is contained on at least $2^{2n+1}+1$ chambers (respectively $2^{n-1}+1$ chambers for type A_n), then $|\Delta^o(C_+)| \simeq \vee S^{n-1}$.

Remark: There exist counter-examples to the conclusion of Theorem 1 for "small" spherical buildings.

Theorem 2: Let Δ be an n-dimensional Moufang twin building of irreducible affine or compact hyperbolic type satisfying

(S) $|\Lambda(x)| \simeq \vee S^{\ell}$ for all ℓ-dimensional ($\ell < n$) links $\Lambda \subset \Delta$ and all chambers x of Λ

Then $|\Delta^o(C_+)|$ is (n-2) - but not (n-1) - connected.

Using a criterion of Ken Brown, Theorems 1 and 2 can be combined to get the following applications:

Corollary 1: Let $\mathcal{G}$ be a classical $\mathbf{F}_q$-group, $q \neq 2^{2n-1}$, with $n := \mathrm{rk}_{\mathbf{F}_q} \mathcal{G} > 0$. Then $\mathcal{G}(\mathbf{F}_q[t,t^{-1}])$ and $\mathcal{G}(\mathbf{F}_q[t])$ are of type F_{n-1}, and $\mathcal{G}(\mathbf{F}_q[t])$ is not of type F_n.

Corollary 2: Let $\mathcal{G}$ be a (minimal, split) Kac-Moody group of rank n+1 and of irreducible affine or compact hyperbolic type. Set $G := \mathcal{G}(\mathbf{F}_q)$ and assume that the associated twin building Δ satisfies (S). Then G and all its parabolic subgroups are of type F_{n-1}, whereas no proper parabolic subgroup is of type F_n.

Assumption (S) in Corollary 2 cannot be dropped:

Example: Is $\mathcal{G}$ is of type  (n+1=3, all Coxeter numbers =4) and Γ a proper parabolic subgroup of $\mathcal{G}(\mathbf{F}_2)$, then Γ is not of type F_1, i.e. not finitely generated. There is work in progress concerning generalizations of Theorem 2 (Corollary 2) to other types of twin buildings (Kac-Moody groups).

D. FON-DER-FLAASS:
A Combinatorial Construction for Twin Trees

The definition of a twin tree $J=(T_+, T_-, cod)$ can be found in J.Tits' abstract.

I begin my construction with defining a pairing $\pi: T_+ \leftrightarrow T_-$ as a graph isomorphism such that $cod(x,T(x))=0$ for all vertices x. Every twin tree admits such pairings; if J is of valency 3 then every pair of opposite edges v_+w_+, v_-w_- uniquely determines a pairing for which $\pi(v_+)=v_-$, $\pi(w_+)=w_-$.

Thus given a twin tree J with a pairing π, we can define a halved twin tree $(T, c: T\times T \to Z^{\geq 0})$ where the codistance function c is defined by $c(x,y)=cod(x_+, y_-)$.

All halved twin trees can be constructed inductively. Let c_n be a partial codistance; $c_n(x,y)$ is determined for those $x,y \in T$ for which $d(x,y) \leq n$ (d is the graph distance in T). It can be shown (ref[F] in J Tits' abstract) that any c_n can be extended to c_{n+1} and that any partial codistance c_n has uncountably many different codistance functions extending it.

For valency 3, the function c_2 is unique up to isomorphism, and we have $G_0 \rightleftharpoons Aut_0(T,c_2) \simeq Z_3 * Z_2$ acting simply transitively on the edges of T [Aut_0 is the group of type-preserving automorphisms] and $Aut(T,c_2) \simeq Z_3 * Z_2$ is simply transitive on flags (directed edges).

Theorem 1° A twin tree J of valency 3 is Moufang if and only if for some pairing π, $Aut_0(J, \pi)=G_0$ and $Stab(v, \pi(v))=S_3$.

2° There are uncountably many halved twin trees (T,c) such that $Aut_0(T,c)=G_0$.

3° For any Moufang twin tree J of valency 3, $Aut_0T=\langle x,y,z\rangle$, $|x|=|y|=3$, $|z|=2$, $\langle x,y\rangle=Z_3 * Z_3$, $\langle y,z\rangle=S_3$.

The bad news is that there is apparently no easy combinatorial way to ensure that a halved twin tree (T,c) admitting G_0 admits also the extra automorphism z.

L.KRAMER:
Algebraic Polygons

A generalized n-gon (P,L,F) is called K-algebraic if the point set P and the line set L are K-varieties, and if the map $(x,y) \to proj_y x$ is a K-morphism on the set

$\{(x,y)\in(P\cup L)^2 \mid d(x,y)=n-1\}$ of almost opposite vertices. Here, K is an algebraically closed field of characteristic 0. We prove

Theorem: There are precisely 3 K-algebraic polygons, namely the projective plane, the symplectic quadrangle, and the split Cayley hexagon over K.

The proof uses Knarr's Theorem on compact n-gons, some complex algebraic geometry, a recent result of Schroth-Van Maldeghem, and some model theory. As an application we obtain:

Corollary: Let (G,B,N,S) be an irreducible effective, spherical Tits systems of rank ≥2. If G is a K-algebraic group and if $B\subseteq G$ is Zariski-closed, then G is simple and B is a Borel subgroup.

This is joint work with Katrin Tent.

D. CARTWRIGHT:

A Family of Groups Acting Simply-Transitively on the Vertices of a Building of Type $\tilde{A}_n$

Let q be any prime power, and let n≥2 be any integer. Let Δ be the thick building of type $\tilde{A}_n$ associated with the local field $\mathbf{F}_q((x))$. In joint work with T.Steger it is shown that there is a subgroup Γ of $PGL(n+1,\mathbf{F}_q((x)))$ which acts simply transitively on the vertices of Δ. This group is realized as a finite index subgroup of $A(\mathbf{F}_q[\frac{1}{x}])$, where A is the automorphism group of a suitable cyclic simple algebra defined over $\mathbf{F}_q(x)$.

G. ROBERTSON:

A Haagerup Inequality for $\tilde{A}_2$ Buildings

In 1979 U. Haagerup proved an inequality relating the operator norm and the ℓ^2 norm for functions on a free group. This inequality and its generalizations were used in proving Banach space approximation properties for function spaces and for proving the Novikov conjecture for word hypebolic groups.

Attempts to obtain higher rank analogues of Haagerup's inequality were unsuccessful; until now. Ramagge, Robertson, and Steger have proved such an inequality for $\tilde{A}_2$ groups.

C. BENNETT:

Affine Λ-Buildings

In this lecture I discussed the question of generalizing the work on Λ-trees of Morgan, Shalen, R.Alperin, H.Bass, and others to general affine buildings of higher rank. In this case we consider Λ to be a general totally ordered abelian group, the two main examples being $\Lambda = \mathbf{Z} \overset{lex}{\underset{x}{\rightarrow}} \mathbf{Z}$ and Λ=**R**. We compared and contrasted the definitions given by myself in 1994/1990 and the definition of Parshin (1994/1996).

My definition of an affine building uses the language of J.Tits' 1984 Como conference paper working on embeddings of apartments. Difficulties encountered in this definition include the definition of an apartment structure that a group can act upon, and of defining a W-invariant Λ-metric structure. These obstacles are overcome first by using the spherical root system to define the apartment structure and then using a modified Minkowski metric for a W-invariant metric. This technique works for arbitrary Λ, but loses the simplicial structure useful in other cases.

Parshin's work on the other hand provides a simplicial structure, but as a result only works for the case L= **Z** × **Z** × ... × **Z**. This is more useful in providing a G-equivariant space.

In both cases, a fundamental example is given in the case K=k(x,y). v:K→ **Z** × **Z** a 2-dimensional valuation and we work through this example thoroughly.

G. ROUSSEAU:

Twin Buildings Associated to Forms of Kac-Moody Algebras

To a Kac-Moody algebra is associated a group acting transitively on pairs of opposite chambers of some twinned buildings B^+ and B^-. A form of a Kac-Moody algebra over a field K of characteristic 0 is a Lie algebra $\mathfrak{g}_K$ such that $\mathfrak{g}_K \otimes \bar{K}$ (if $\bar{K}$ is the algebraic closure of K) is isomorphic to a Kac-Moody algebra $\mathfrak{g}$. Fixing such an isomorphism one gets an action of the Galois group $\Gamma = Gal(\bar{K}/k)$ of $\mathfrak{g}$, G and $B^+ \cup B^-$. The form is called almost split if G stabilizes B^+ and B^-. Then I prove that $B^{+\Gamma}$ and $B^{-\Gamma}$ are twinned buildings and that G_K acts transitively on the pairs of opposite chambers. This gives interesting examples of twin buildings and allows one to develop a Borel Tits theory for almost split forms of Kac-Moody algebras (see J of Algebra 171(1995), 43-96)

P.-H. ZIESCHANG:

An Algebraic Treatment of Buildings via Association Schemes

In a first step, the class of buildings will be embedded into the class of association schemes. Here, in the class of association schemes, the buildings play "exactly" the role which is played by the Coxeter groups in group theory.

The above-mentioned embedding provides us with two possibilities to characterize buildings:

1: Let A be an association scheme generated by two "fundamental involutions". Suppose these two involutions generate the Hecke algebra of A.

Then A "is" a finite building of rank 2 or a Moore geometry.

2: The concept of apartments will be generalized to association schemes. Then we can prove:

Buildings and certain quotients of buildings, viewed as association schemes, are characterized by their apartments.

References:

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