

Tagungsbericht 18/1996

Kommutative Algebra und algebraische Geometrie

05.05. - 11.05.1996

Die Tagung fand unter der Leitung von Ernst Kunz (Regensburg), Josepf Lipman (West Lafayette (USA)) und Uwe Storch (Bochum) statt.

In den Vorträgen und Diskussionen wurde über neuere Ergebnisse und Probleme aus der kommutativen Algebra und algebraischen Geometrie berichtet und es wurden insbesondere Fragen angesprochen, die beiden Gebieten gemeinsam entspringen. Auch Verbindungen zu anderen Bereichen der Mathematik (Homotopietheorie, K-Theorie, Kombinatorik) wurden geknüpft. Eine Reihe von Vorträgen beschäftigte sich mit Residuen und Spuren in der Dualitätstheorie. Weitere Schwerpunktsthemen waren Fundamentalgruppen algebraischer Varietäten, birationale Geometrie, Frobenius-Funktoren, Auflösungen und Hilbertfunktionen graduierter Ringe und Moduln, insbesondere in Verbindung mit der Geometrie endlicher Punktmengen im Pⁿ und kombinatorischen Strukturen. In diesem Zusammenhang sind auch mehrere Vorträge über die Multiplizität lokaler und graduierter Ringe zu sehen.

Das große Interesse an der Tagung spiegelt sich auch in der hohen Zahl ausländischer Gäste wider. Neben den 19 deutschen Teilnehmern kamen 10 weitere aus europäischen Ländern (davon 3 aus Osteuropa), 12 aus Nordamerika und je einer aus Indien, Israel, Japan und Taiwan.

Vortragsauszüge

Abhyankar, Shreeram.S.

Local Fundamental Groups of Algebraic Varieties

For a prime number p and integer $t \ge 0$, let $Q_t(p) =$ the set of all quasi-(p,t) groups, i.e., finite groups G such that G/p(G) is generated by t generators, where p(G) is the subgroup of G generated by all of its p-Sylow subgroups; members of $Q_0(p)$ are called quasi-p groups. Extend this for p = 0 by agreeing that the identity group 1 is the only 0-Sylow subgroup. For p = a prime number or 0, let $P_t(p) =$ the set of all (p,t)-groups, i.e., the set of all finite groups G such that G/p(G) is an abelian group generated by t generators. Note that then $P_t(p) = Q_t(p)$ for $0 \le t \le 1$, but $P_t(p)$ is much smaller than $Q_t(p)$ for t > 1.

Let C_g be a nonsingular projective algebraic curve of genus g over an algebraically closed ground field k of characteristic p (where p may or may not be zero). Let $C_{g,i+1} = C_g$ minus (t+1) points. Let $\pi_A(C_{g,i+1})$ be the algebraic fundamental group of $C_{g,i+1}$ which is defined to be the set of all Galois groups of finite unramified Galois coverings of $C_{g,i+1}$. In my 1957 paper, I made the following conjecture above π_A .

Curve Conjecture. For $t \geq 0$ we have $\pi_A(C_{g,t+1} = Q_{2g+t}(p))$. In particular $\pi_A(L_{k,t}) = Q_t(p)$ where $L_{k,t}$ is the t-punctured affine line over k.

Now that Harbater and Raynaud (1994) have affirmatively settled this Curve Conjecture, it seems worthwhile to make explicit the conjecture about the local algebraic fundamental group $\pi_A^L(N_{k,t}^d)$ at a t-fold normal crossing near a simple point P of a d-dimensional algebraic variety V, with $d \geq 2$, over k, which was implicit in my 1955 paper. So let W be a (d-1)-dimensional subvariety of V. Recall that W has a t-fold normal crossing at P means, near P, W is defined by $x_1 \cdots x_t = 0$ where (x_1, \ldots, x_d) is a regular system of parameters on V at P. Assuming W to have a t-fold normal crossing at P, $\pi_A^L(N_{k,t}^d)$ is defined to be the set of inertia groups above P in finite Galois coverings of V whose branch locus at P is contained in W. The said local conjecture predicts the following:

Local Conjecture For t > 0 we have $\pi_A^L(N_{k,t}^d) = P_t(p)$.

Similarly, it seems worthwhile to make explicit the conjecture about the algebraic fundamental group $\pi_A(V-W)$ of the complement of W in V, which was implicity in my 1959 paper. Recall that $\pi_A(V-W)$ is defined to be the set of all Galois groups of unramified finite Galois coverings of V-W. The said global conjecture predicts the following

Global Conjecture If V is the d-dimensional projective space over k (with $d \ge 2$), W has only normal crossings, the number of irreducible component of W



is t+1 (with $t\geq 0$), and at least one irreducible component of W has degree 1, then $\pi_A(V-W)=P_t(p)$. In particular $\pi_A(L^d_{k,t})=P_t(p)$ where $L^d_{k,t}$ is the d-dimensional affine space over k punctured at t hyperplanes which, together with the hyperplane at infinity, have only normal crossings.

As supporting evidence for the above Local and Global conjectures, for t=1 and p>0, I have recently shown that for integers m>1 and all powers q of p, the groups PGL(m,q) and GL(m,q) belong to $\pi_A^L(N_{k,1}^d)$ as well as to $\pi_A(L_{k,1}^d)$.

Alonso Tarrio, Leovigildo

Why Grothendieck duality on formal schemes? (joint work with A. Jeremias and J. Lipman)

We discuss some applications of the theory developed in the talk of A. Jeremias.

Let $f: X \to Y$ be a pseudo-proper map between noetherian formal schemes. We denote $f^!: D(Y) \to D_{qct}(X)$ the twisted inverse image functor, Δ -adjoint to the usual $\mathbb{R} f_*$. We construct a new functor $f^\#: D_c(Y) \to D_c(X)$ by the following formula:

$$f^{\#}\mathcal{F} := \mathsf{R}\,\mathcal{H}om_{X}^{\cdot}(\mathsf{R}\Gamma_{X}^{\prime}(\mathcal{O}_{X}),f^{!}\mathcal{F})$$

With this definition, it is possible to state a Grothendieck duality formula for coherent sheaves on formal schemes, namely,

Theorem With f as before, $\mathcal{F} \in D_c^+(Y)$, $\mathcal{G} \in D_{qc}(X)$

$$Rf_*R\mathcal{H}om_X(\mathcal{G}, f^\#\mathcal{F}) \xrightarrow{\sim} R\mathcal{H}om_Y(Rf_*R\Gamma_X'\mathcal{G}, \mathcal{F})$$

This formula allows us to recover (in an abstract fashion) previous results by Hübl, Kunz, Lipman and Hartshorne.

Aramova, Annetta

Gotzmann theorems for exterior algebras and combinatorics (joint work with H. Herzog and T. Hibi)

We show two theorems on f-vectors of simplicial complexes which correspond to Gotzmann Persistence theorems and Regularity theorems for Hilbert functions in the polynomial ring. The classical Kruskal-Katona theorem characterizes finite integer sequences which are the f-vector of a simplicial complex. It can be interpreted as a theorem on Hilbert functions. To see this we attach to each simplicial complex Δ a finite dimensional K-algebra, called the indicator algebra.

Theorem Let Δ be a simplicial complex on the vertex set $[n] = \{1, 2, ..., n\}$ with f-vector $(f_0, f_1, ..., f_{d-1})$. Then:





- a) (Kruskal-Katona) $\partial_i(f_i) \leq f_{i-1}$ for $0 \leq i \leq d-1$.
- b) (Persistence) If Δ is pure and $\partial_i(f_i) = f_{i-1}$ for some i, then $\partial_j(f_j) = f_{j-1}$ for all j < i.
- c) (Regularity) If $s \geq 1$ is an integer such that $\partial_i(f_i) = f_{i-1}$ for all $i \leq s$, then $\tilde{H}^{j-1}(\Delta^T, K) = 0$ for all $j \leq s$ and all $T \subset [n]$

Batyrev, Victor

Residue and push-forward in equivariant cohomology

Let T be a d-dimensional algebraic torus acting on an n-dimensional smooth projective algebraic variety X over C. Assume that there exist only finitely many T-fixed joints on X ($|X^T| < \infty$). Then the ring of equivariant cohomology $R = H_T^*(X, \mathbb{C})$ is a Gorenstein C-algebra of Krull dimension d. Using Atyah-Bott localization theorem, one obtains a natural 1- to 1 correspondence between elements of the fixed point set X^T and irreducible components of Spec R. Let $A := H_T^*(pt)$. Then A is isomorphic to the polynomial ring in d variables and R becomes a free module over A of rank $|X^T|$ via the ring homomorphism $\pi^*: A \to R$ induced by the projection $X \xrightarrow{\pi} pt$. Example: (M.Brion) If X is a toric variety (n = d), then R is isomorphic to the Stanley-Reisner ring of the simplicial complex of cones in the fan defining X. The push foward map $\pi_*: R \to A$ can be computed via the Bott residue theorem. This method allows to compute generating functions for intersection numbers of tautological classes on modulispaces of stable maps of rational curves and confirm predictions of physicists for "numbers" of rational curves on complete intersections in toric varieties.

Campillo, Antonio

Toric ideals and syzygies

For a monomial (or non normal toric) variety, i.e. an affine variety parametrized by monomials, we compute the defining ideal and syzygies from combinatorics, namely by using the homology of certain simplicial complexes associated to the semigroup of the involved monomials. By using this computation as unifying element, we make the algebraic-geometric study of such varieties.

Chardin, Marc and Ulrich, Bernd

Hilbert functions and residual intersections (joint work with D. Eisenbud)

We are interested in the question of when the Hilbert function of an R-ideal



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J generated by s forms is determined by the degrees of the s forms and the intersection $J^{\leq s-1} := I$ of the primary components of height at most s-1 of J.

This question has an affirmative answer if R is Cohen-Macaulay and $s \le ht(J) + 1$. In general, however, one needs to impose conditions on I:

Definition. I is s-residually S_2 if, for every $i \leq s$ and every i-residual intersection K, the ring R/K satisfies Serre's condition S_2 .

Theorem. Let R be a homogeneous Cohen-Macaulay ring over an infinite field and I a homogeneous s-generated ideal satisfying G_{s-1} . Set $I = J^{\leq s-1}$. If I is (s-2)-residually S_2 , then the Hilbert function of R/J is determined by I and the degrees of the s generators of J.

Here, G_s means that $\mu(I_{\mathfrak{p}}) \leq \operatorname{ht}(\mathfrak{p})$, $\forall \mathfrak{p} \in \operatorname{Supp} R/I$ $\operatorname{ht}(\mathfrak{p}) \leq s-1$. A criterion for the residually S_2 condition is given by the following:

Theorem. Let R be Gorenstein and I an ideal of height g satisfying G_s . If $\operatorname{Ext}_R^{g+j}(R/I^j,R)=0$ for all $1\leq j\leq s-g+1$, then I is a s-residually S_2 .

Cutkosky, Dale

Local factorization of Birational maps

The local factorization theorem of Zariski and Abhyankar shows that an inclusion of regular local rings of dimension two with a common quotient field can be factored by a unique finite product of quadratic transforms. In higher dimensions this statement is false, as shown by examples of Hironaka, Sally and Shannon showing that regular local rings $R \subset S$ of dimension 3 cannot be a product of monoidal transforms.

We prove the following theorem on local factorization:

Theorem Suppose that R, S are excellent regular local rings, of dimension 3, containing a field k of characteristic zero, with a common quotient field K, such that S dominates R. Let V be a valuation ring of K which dominates S. Then there exists a regular local ring T, with quotient field K, such that T dominates S, V dominates T, and the inclusions T and T are factored by sequences of monoidal transforms.

An affirmative answer to a question of Abhyankar follows from this theorem. As a corollary, we give a geometric interpretation of this theorem for complete morphisms. Complete is the analogue of proper for a not necessarily separated morphism.

Foxby, Hans-Bjorn

Gorenstein dimension and local homomorphisms





Consider local rings (R,m) and (S,n), and a ring homomorphism $\varphi:R\to S$ which is local (i.e. $\varphi(m)\subseteq n$). Of central interest is the ability of φ to transfer a given property from the source R to the target S - and vice versa. Under the additional assumption that S is of finite flat dimension over R this has been scrutinized by Avramov and me for a variety of properties (including "Gorenstein" and "'Cohen-Macaulay"). The talk describes a wider class of homomorphisms with excellent transfer abilities for the "Gorenstein" property: the "flat dimension" can be replaced by "Gorenstein dimension" which is a finer invariant. It was introduced by Auslander for finite moduls - and extended in the talk to general ones.

Eisenbud, David

Resolution of the ideal of a general set of points

The Minimal Free Resolution Conjecture asserts that for a general set Γ of γ points in P^r_k , the free resolution of the ideal $I_\Gamma \subset k[X_0,\ldots,X_r] =: S$ of the points satisfies dim $\mathrm{Tor}_1^s(I_\Gamma,k)_j = 0$ or dim $\mathrm{Tor}_{i+1}^s(I_\Gamma,k)_j = 0$ for every i and j. The MRC is known to be true for $r \leq 4$ (Gaeta, Ballico-Geramita, Walter) and for $\gamma >> r$ (Hirschowitz-Simpson). But Schreyer made computation suggesting strongly that it fails in P^6 for 11 points, P^7 for 12 points, and P^8 for 13 points. (No counterexample occurs in P^9 for ≤ 50 points (Kreuzer)). With Sorin Popescu I proved:

Theorem for $r \ge 6$, $r \ne 9$ there is a number $\gamma(r)$ such that the MRC fails in P^r for $\gamma(r)$ points. Explicitly, if we write

$$r = \binom{s+1}{2} + k \quad \text{with} \quad 0 \le k \le s$$

then we may take

$$\gamma = r + s + 2 = \binom{s+2}{2} + k + 1$$

Huang, I-Chiau

Residue Theorem via an Explicit Construction of Traces

Let X be a locally Noetherian scheme and let $\phi: Y \to X$ be a morphism of finite type whose fibers have bounded dimensions. Assume that X admits a residual complex K_X which is **explicitly** given, that is, an injective hull $M(\mathfrak{p})$ for each point $\mathfrak{p} \in X$ is chosen and how K_X is built up from the $M(\mathfrak{p})$'s is specified. I have constructed canonically an injective hull $M(\mathfrak{P})$ for each point $\mathfrak{P} \in Y$ and described a map $M(\mathfrak{P}) \to M(\mathfrak{Q})$ for each pair of points $\mathfrak{P}, \mathfrak{Q} \in Y$ so that these





information give rise to a residual complex K_Y on Y. In the talk, I will give a short review of these constructions and give some concrete examples. I will then describe explicitly a map

$$\operatorname{tr}_{\mathfrak{P},p}:M(\mathfrak{P})\to M(\mathfrak{p})$$

for each pair of points $\mathfrak{P}\in Y$ and $\mathfrak{p}\in X$ and compare it with the Tate's trace. The maps $\mathrm{tr}_{\mathfrak{P},\mathfrak{p}}$ induce a map

$$\operatorname{tr}_{Y/X} = \phi_* K_Y^{\cdot} \to K_X^{\cdot}$$

of \mathcal{O}_X -moduls. Residue theorem asserts that $\operatorname{tr}_{Y/X}$ is a map of complexes if ϕ is proper.

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Hübl, Reinhold

Geometric Applications of Residues (joint work with E. Kunz)

A classical result of Humbert states that for a plane curve C and a line H intersecting C in $\deg(C)$ many points, the sum $\sum_{P \in C \cap H} \cot(\alpha_P)$ is an invariant of the points of C and H at infinity. Here α_P denotes the angle between H and C at P. A formula of Reiss states that $\sum_{P \in C \cap H} \frac{\kappa_P(C)}{\sin^3(\alpha_P)} = 0$, where $\kappa_P(C)$ denotes the curvature of the curve C at P. The residue theorem on curves allows to generalize these results to the intersection of arbitrary curves $C \subseteq A_k^n$ with hypersurfaces $H = \{f = 0\} \subseteq A_k^n$.

Jeremias López, Ana

Grothendieck duality and flat base change for formal schemes (this is part of a joint work with I. Alonso and J. Lipman)

Given $\mathcal J$ an ideal of definition of a locally noetherian formal scheme X, consider the functor

$$\Gamma_X'M:=\varinjlim_{n>0}\mathcal{H}om_{\mathcal{O}_X}(\mathcal{O}_X/\mathcal{J}^n,\mathcal{M})\quad (\mathcal{M}\in\mathcal{A}(X))$$

Let $\mathcal{A}_{qct}(X)$ be the full subcategory of the category of quasi-coherent sheaves \mathcal{F} verifying $\Gamma'_X(\mathcal{F}) = \mathcal{F}$. Let $D_{qct}(X)$ be the subcategory of D(X) of complexes whose homology is in $\mathcal{A}_{qct}(X)$.

Let $f: X \to Y$ be a map between locally noetherian formal schemes, let $\mathcal{J} \subset \mathcal{O}_Y$, $J \subset \mathcal{O}_X$ be ideals of definition of Y and X such that $\mathcal{J}\mathcal{O}_X \subset J$.





Set $X_{[0]}$ (resp. $Y_{[0]}$) for the map of ordinary schemes induced by f. The map $f: X \to Y$ is separated iff $f_{[0]}: X_{[0]} \to Y_{[0]}$ is a separated map of schemes. We say $f: X \to Y$ of pseudo-finite type (resp. pseudo-proper) if $f_{[0]}: X_{[0]} \to Y_{[0]}$ is a finite type (resp. proper) map of schemes.

Theorem 1. (Grothendieck duality on formal schemes) Let $f: X \to Y$ be a separated pseudo-finite type map of noetherian formal schemes. Then the triangular functor $Rf_{\bullet}: D_{act}(X) \to D(Y)$ has a triangular right adjoint.

For a pseudo-proper map of noetherian formal schemes $f: X \to Y$ we will write $f^!: D(Y) \to D_{qct}(X)$ for the right adjoint to $Rf_{\bullet}: D_{qct}(X) \to D(Y)$, as usual for proper maps of schemes.

Theorem 2. (Flat base change isomorphism) Let $f: X \to Y$ be a suitable pseudo-proper map and $u: U \to Y$ an adic flat map of noetherian formal schemes. Complete the diagram (f, u) to a fiber square with projection maps $v: V \to X$ and $g: V \to U$. Then there is a "base change" isomorphism

$$v^*f^!\mathcal{F} \xrightarrow{\sim} g^!u^*\mathcal{F} \qquad (\mathcal{F} \in \mathcal{D}^+_{\stackrel{\sim}{\sim}}(Y))$$

(when we write f' for f^X , and the index \vec{c} means the "homology is a direct limit of coherent moduls" over Y)

As consequence we get: Sheafified version of Grothendieck Duality on formal schemes: For any pseudo-proper maps $f: X \to Y$ of noetherian formal schemes, every $\epsilon \in \mathsf{D}_{\mathsf{qct}}(X)$ and $\mathcal{F} \in \mathsf{D}^+_{\mathsf{qct}}(X)$, there is a bifunctorial isomorphism

$$Rf_*R \mathcal{H}om_{\mathcal{O}_X}(\epsilon, f^!\mathcal{F}) \xrightarrow{\sim} R \mathcal{H}om_Y(Rf_*\epsilon, \mathcal{F})$$

Jouanolou, Jean-Pierre

Inertia and acyclicity

We present an "inertia lemma" that is an effective version of the acyclicity lemma in commutative algebra. More precisely, the use of resultant ideals permits to have a lot of information on the cohomology modules of complex of finite free modules. Applications are given: effective criterion for a sequence of polynoms to be a regular sequence using a description of the regular locus of the resultant variety, torsion of quotient rings, more precise version of the Falting's lemma,...

Köck, Bernhard

The Equivariant Grothendieck-Riemann-Roch Theorem





Let G be a group or a group scheme. We established formulas for the equivariant Euler characteristic of locally free G-modules on a projective G-scheme X: We proved an Adams-Riemann-Roch theorem and, under a certain continuity assumption for the push-forward map, a Grothendieck-Riemann-Roch theorem in (higher) equivariant algebraic K-theory. Furthermore, we presented the following application: In case of a flag variety G/B, the above continuity assumption has be verified, and the Grothendieck-Riemann-Roch theorem for this situation yields a new proof of the Weyl character formula.

Korb, Thomas

On Rees algebras with a Gorenstein Veronese-subring (joint work with Manfred Herrmann and Eevo Hyry)

The talk should illustrate that the assumption which is mentioned in the title provides a very good setting for considering properties and the structure of blow up algebras. The following question was i.p. considered: Suppose the blow up algebras of an ideal I in a local ring A have some "good properties". What can then be said about upper bounds for the reduction exponent r(I) of I?

We recalled some known answers to this question (e.g. Goto/Shimoda as generalized by Johnston/Katz; Aberbach/Huneke/Trung for the Gorenstein-case etc.) and mentioned then that all these results follow immediately from the following a-invariant formula:

$$a(G_A(I)) = \max(r(I) - \ell(I), \delta(G(I)))$$

which holds in case $G_A(I)$ is Cohen-Macaulay. Using this formula we can even generalize some of the above mentioned results to:

$$(\operatorname{ht}(I)>1, G_A(I)CM)R_A(I^s)\operatorname{Gor.}\Rightarrow r(I)\leq \ell(I)-s-1$$

The question is now whether we can obtain the same upper bound for r(I) in other cases (i.e. when $G_A(I)$ is not CM). We presented a theorem in this direction which is bases on a new duality result (coming from the Sancho de Salas sequence and Lipman's local-global duality), the theory of multi-Rees algebras and the fact that we have in our setting: $\omega_{G_A(I)} \cong G_A^*(I)(-s-1)$, where $G_A^*(I)$ denotes the form ring of the so called Ratliff-Rush filtration coming from I.

Kreuzer, Martin
Kähler Differentials for Points in Pⁿ



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For a 0-dimensional reduced subscheme $X\subset P^n$ with homogeneous coordinate ring $R=P/I=K[X_0,\ldots,X_n]/I$, we study the Hilbert function of its first module of Kähler differentials $\Omega^1_{R/K}$. We showed that this Hilbert function H_Ω contains more information about the geometry of X than H_X or the minimal P-resolution of R. The function $H_\Omega(i)$ was described for small i, for i>>0 (by giving a sharp upper bound for the regularity of $\Omega^1_{R/K}$), and for an intermediate range. We also presented a number of results about the torsion submodul $\tau_{R/K}$ of $\Omega^1_{R/K}$, including an explicit system of generators and – strengthening the positive answer to Berger's conjecture—its nonvanishing in all suitable degrees. As a consequence, we also obtain a lower regularity bound for $\Omega^1_{R/K}$ which is sharp. Finally, the relation between H_Ω and the Hilbert function of the symbolic square of I was exhibited and used – together with the Alexander-Hirschowitz theorem on systems of double points – to calculate H_Ω in the case of generic sets of points X.

Kwiecinski, Michal

Tensor powers with applications to symmetric algebras

For a morphism of complex algebraic varities $f:X\to Y$ we define a topological invariant $\varphi(f)$ equal to the maximal number of points in any give fibre, which lie in a limit of one sequence of general fibres. When X,Y are affine this invariant can be expressed algebraically: $\varphi(f):=\sup\{i:\bigotimes_{\mathbb{C}[Y]}^i\mathbb{C}[X]/\sqrt{0} \text{ is torsion free over }\mathbb{C}[Y]/\sqrt{0}\}$.

Several results about this invariant (also over general fields) describe some aspects of how general fibres degenerate to special fibres. Two applications of these results are:

- 1) If $f: X \to Y$ is a generically finite map, with an isolated special fibre of dimension w > 0, then there are at least $\left[\frac{\dim Y 1}{w}\right]$ points in the generic fibre.
- 2) Let K be an algebraically closed field, R a finitely generated K-algebra which is a regular ring and M a finitely generated R-module. If Spec SymM is equidimensional and P is a minimal associated prime of $\operatorname{Fitt}_{r-1}M$ and not of Fitt_rM then $\operatorname{codim} P \leq r(r rkM)$. (Results of Simis and Vasconcelos give lower bounds for such a codimension).

Except for the Symmetric Algebras part, this is joint work with Piotr Tworzewski.

Neeman, Amnon

Homotopy theoretic methods in algebraic geometry





Let $f:X\to Y$ be a morphism of schemes. Let $f_\bullet: D(X)\to D(Y)$ be the induced map of derived categories of chain complexes of quasicoherent modules. Grothendieck proved that f_\bullet has a right adjoint, denoted $f^!$. The original proof was based on trace map. One explicitly gave the counit of adjunction $f_\bullet f^!(x)\to x$, often called the residue map, and then computed that it gives an adjuntion.

Using a theorem of Brown, originally conceived in homotopy theory, it is easy to give a short, direct proof of the existence of Grothendieck's adjoint.

Peeva, Irena

Homology of codimension 2 lattice ideals (joint work with Bernd Sturmfels)

Let $S = K[X_1, ..., X_n]$ be a polynomial ring over a field K. We study homoloy of lattice ideals; a case of particular interest are prime lattice ideals, i.e. toric ideals.

For a lattice ideal I we prove that the projective dimension of S/I is $\leq 2^{\operatorname{codim}(I)} - 1$. When $\operatorname{codim}(I) = 2$ we construct the minimal free resolution of S/I and prove that $\operatorname{reg}(\hat{I}) \leq \operatorname{deg}(I)$. The latter inequality is strict for prime ideals - this shows that Eisenbud-Goto's conjecture is true for codimension 2 toric varieties.

Sastry, Pramathanath

Global Torelli Theorems for Moduli Spaces of Vector Bundles

Recall that a polarized abelian variety is a pair (M, [w]), where M is an Abelian variety and $[\omega] \in H^2(M, \mathbb{Z})$ is a polarizing class on M. A map between polarized Abelian varieties is a map of the underlying Abelian varieties which is compatible with the polarizing classes.

A compact Riemann surface of genus $g \geq 1$ gives a (principally) polarized Abelian variety $(J(X), [\omega_X])$ where J(X) is the "Jacobian" and $[\omega_X]$ is ω_X is given by $\omega_X(\alpha \wedge \beta) = \#\alpha \cdot \beta$ for $\alpha, \beta \in H_1(X, \mathbb{Z})$. We then have the classical Torelli's theorem:

Theorem (Torelli): If X and X' are compact Riemann surfaces (of genus ≥ 1) such that

$$(J(X),[\omega_X]) \xrightarrow{\sim} (J(X'),[\omega_{X'}])$$

as polarized Abelian varieties, then

$$X \xrightarrow{\sim} X'$$

The above theorem was generalized by Mumford-Newstead (in a special case) and Narasimhan-Ramanan in the following way: For a Riemann surface X, let



 $M_X^2(n)$ denote the moduli space of rank n bundles V, with determinant det V equal to L.

Theorem (Narasimhan-Ramanan/Mumford-Newstead): Let $n, d \in \mathbb{Z}$, with (n, d) = 1. Let $X_1, X_2 \in \mathfrak{M}_g$ (the moduli space of genus g curves), $g \geq 3$, and let $L_i \in \text{Pic } X_i$, $d = \deg L_i$. Suppose

$$M_{X_1}^{L_1}(n) \xrightarrow{\sim} M_{X_2}^{L_2}(n)$$

Then

$$X_1 \xrightarrow{\sim} X_2$$

We generalize this to include the case when n and d are not necessarily coprime. The work is joint work with D. Arapura. The principal difficulties in the "non co-prime" case are

- a) $M_X^2(n)$ is not smooth.
- b) It is not a "fine" moduli space (i.e. it does not represent the moduli functor).

 Our result is:

Theorem (D. Arapura & P.S.) Fix n. Let $M_X = M_X^{O_X}(n)$. Then

$$M_X^s \xrightarrow{\sim} M_{X'}^s \Rightarrow X \xrightarrow{\sim} X'$$

Here M_X^{\bullet} is the stable locus of M_X . V.Balaji has a related result when n=2.

Schenzel, Peter

Applications of Koszul homology to syzygies

 $R=\bigoplus_{n\geq 0}R_n$ denote a graded K-algebra of finite type with $R_0=K$ a field and $R=R_0[R_1]$. For a finitely generated graded R-module M and a "sufficiently" general system of linear forms $\underline{\ell}=\ell_1,\ldots,\ell_s$, $s\geq \dim M$, the following cohomology modules $H_i(\underline{\ell};H^j_{R_+}(M))$, $i,j\in \mathbb{Z}$ are studied. This mixture of Koszul homology of local cohomology is a local analogue to the $K^i_{p,q}$ -invariants introduced by M. Green in 1984.

Results: 1) Finiteness conditions for $H_i(\underline{\ell}; H^j_{R_+}(M))$, 2) Bounds on the minimal numbers of generators for ideals, in particular in small codimension, 3) A new description of the Castelnuovo-Mumford regularity, and 4) A generalization of M. Green's duality theorem. The usefulness of this generalization is described in studying the minimal free resolution of non-singular rational curves in P^n_K in terms of their Hartshorne-Rao module.



Seibert, Gerhard

Frobenius Functors and Invariant Factors

We characterize the noetherian local rings (R, \mathfrak{m}) of prime characteristic with the following property: If M is a finite generated R-module then there exists $n, \tau \in \mathbb{N}$ and elements $a_1, \ldots, a_\tau \in \mathfrak{m}$ such that $F_R^n(M) \cong \bigoplus_{i=1}^\tau R/a_iR$ where $F_R^n(-)$ denotes the n-th power of the Frobenius functor with respect to R. These are exactly the noetherian local rings (R, \mathfrak{m}) of prime characteristic and dimension less than or equal to 1 which are geometrically unibranch in the sense of Grothendieck, i.e. R_{red} is an integral domain, the integral closure $\overline{R_{red}}$ of R_{red} in its field of fractions is local and the residual field extension of $\overline{R_{red}}/R_{red}$ is purely inseparable; for example noetherian local complete domains of prime characteristic and dimension less than or equal to 1 and algebraically closed residue field are such rings.

Srinavasan, Hema

Bounds for Multiplicities

Let R be a polynomial ring over a field and I be a graded ideal of R. Let S = R/I. The minimal graded resolution of S over R is

$$0 \to \sum_{i=1}^{b_s} R(-d_{sj}) \to \cdots \to \sum_{i=1}^{b_t} R(-d_{tj}) \to \cdots \to \sum_{i=1}^{b_t} R(-d_{1j}) \to R \to S \to 0$$

So, $P\dim S=s$. S is said to have a pure resolution if $d_{tj}=d_t$ $1\leq j\leq b_t$ $1\leq t\leq s$. In 1986 Huneke and Miller have proved that if S is Cohen-Macaulay with a pure resolution

$$0 \to R^{b_a}(-d_a) \to \cdots \to R^{b_t}(-d_t) \to \cdots \to R^{b_1}(-d_1) \to R \to S \to 0$$

then the multiplicity of S, $e(s) = \frac{d_1 \cdots d_s}{s!}$. Let us denote by $m_t = \min_j d_{tj}$ and $M_t = \max_j d_{tj}$, the minimal and maximal shifts in the t^{th} place. Then we conjecture that:

If S is Cohen-Macaulay, then

$$\frac{\prod_{t=1}^{s} M_t}{s!} \ge e(S) \ge \frac{\prod_{t=1}^{s} m_t}{s!}$$

This is clearly true when S is a graded complete intersection. If S ist not Cohen-Macaulay the lower bound fails in general. In joint work with Jürgen Herzog we prove the following:



Theorem 1. If S is Cohen-Macaulay with the minimal graded resolution satisfying $m_t \ge M_{t-1}$ for all $1 < t \le s$, then the conjecture is true.

Theorem 2. If I is a stable monomial ideal or a square free strongly stable monomial ideal then the conjectured upper bound holds for S = R/I and the lower bound holds for the multiplicity of S if S is Cohen-Macaulay. We also prove the conjecture for height 2 Cohen-Macaulay ideals I. The conjectured upper bounds are verified in some other cases including the grade 3 Gorenstein ideals and S = R/I with a p-linear resolution.

Stückrad, Jürgen

Composition series and a new invariant of local rings (joint work with W. Vogel)

Let A be a local ring or a graded K-algebra (K a field). For any (graded) f. g. A-module M we consider the following two numbers:

 $n_A(M) := \sup\{ \tfrac{\ell(M/QM)\}}{\epsilon(Q;M)} | Q \text{ (homogeneous) } \mathfrak{m}_A - \text{primary ideal} \} \in \mathsf{R}^+ \cup \{\infty\}$

 $\tilde{n}_A(M) := \sup\{\frac{\ell(M/QM)}{\epsilon(Q;M)}|Q \text{ (homogeneous) parameter ideal of } M\} \in \mathbb{R}^+ \cup \{\infty\}$ The problem is to characterize those M for which $n_A(M)$ or $\tilde{n}_A(M)$ is finite. Here are the main results (M) is always f.g.):

- 1.) If $n_A(M) < \infty$ then M is quasiunmixed (in the sense of Nagata).
- 2.) If A is local and $\tilde{n}_A(M) < \infty$ then M is quasiunmixed.
- 3.) If A is graded then $\tilde{n}_A(M) < \infty$ for all f. g. graded A-modules M.
- 4.) If for all complete regular local rings R (or all polynomial rings R) $n_R(R/P) < \infty$ for all (homogeneous) prime ideals P of R, then $n_A(M)$ is always finite, provided M is quasiummixed.
- 5.) $n_R(R/I) < \infty$ if R is a polynomial ring and I is a quasiunmixed monomial ideal of R. In this situation

$$n_R(R/I) \le r - \deg R/I + 1$$

if there is a composition series $0 = M_0 \subsetneq M_1 \subsetneq \cdots \subsetneq M_r = R/I$ of length r such that M_0, M_1, \ldots, M_r are graded R-modules with $M_i/M_{i-1} \cong R/P_i$ (up to shifts of degree), where P_i are monomial prime ideals of R, $i = 1, \ldots, r$. Moreover, $\tilde{n}_R(R/I) \leq \frac{r}{\deg R/I}$ for arbitrary monomial ideals of R, where r is defined as above.

Examples show that even for square free monomial ideals $\tilde{n}_R(R/I) \leq n_R(R/I) \ (< \infty)$.

Conjectures: 1.) $n_A(M) < \infty$ for all f. g. quasiunmixed (graded) A-modules M.

2.) $n_A(M)$, $\tilde{n}_A(M) \in \mathbf{Q}$ if these numbers are finite.

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Thoma, Apostolos

Set-theoretic complete intersections

A monomial curve in the n-dimensional affine or projective space is set theoretic complete intersection if it is defined as a set by n-1 equations. We presented a technique which from a given projective monomial curve which is set-theoretic complete intersection provides infinitely many monomial curves which are set-theoretic complete intersections in the next dimension affine or projective space.

Vasconcelos, Wolmer

Homological Degree of Modules

This is a proposed extension of the notion of multiplicity to allow for a priori estimates usually requiring CM conditions. For a graded module M over a standard graded Gorenstein algebra S, one looks for numerical functions $Deg(\cdot)$ satisfying:

- (0) Deg(M) = deg(M) = multiplicity of M if it is CM
- (1) If $L = \Gamma_{\mathfrak{M}}(M)$, $\operatorname{Deg}(M) = \ell(L) + \operatorname{Deg}(M/L)$
- (2) If $h \in S_1$ is a regular hyperplane section

$$\operatorname{Deg}(M) \ge \operatorname{Deg}(M/hM)$$

(2) may look counter-intuitive but it has the derived goal.

The proposed "big" degree is given as

$$Deg(M) = deg(M) + \sum_{i>0} c_i \cdot Deg(Ext_S^i(M, S))$$

where c_i' 's are certain binomial coefficients. For example, if $d=\dim M=\dim S$, $c_i=\binom{d-1}{i-1}$. It can be computed through Gröbner basis techniques.

One proves that (2) hold, for generic, regular hyperplane sections. A similar definition can be given in the local case.

One of the properties of any such $\operatorname{Deg}(\cdot)$ is $\operatorname{Deg}(M) \geq \nu_{\mathcal{S}}(M) = \min$ number of generators of M.

Watanabe, Keiichi

Characterizing terminal singularities via Frobenius map

Our aim is to characterize terminal (resp. canonical, log-terminal, log-canonical) singularities using Frobenius after taking mod p reduction. For that purpose we define F-terminal ring in char p > 0.



Definition Let A be a ring of char = p > 0 s.th. $F: A \to A$, $a \to a^{1/p}$ is a finite morphism and let I be the defining ideal of $\mathrm{Sing}(A)$. Then we say that A is F-terminal if for every $q = p^e >> 0$ there exists $c \in I^q$ such that the map $A \to A^{1/q}$ sending 1 to $c^{1/g}$ splits as A-module homomorphism. We show that if A (char = 0) has F-terminal reduction for p >> 0, then A is a terminal singularity and the converse holds if $\dim A \leq 3$. We can also show many interesting properties of F-terminal rings very easily. (Hopefully getting ringtheoretic properties of terminal singularities.)

Winiarski, Tadeusz

Some new applications of Gröbner bases (joint work with J. Apel, J. Stückrad (Leipzig), P. Twonewski (Krakau)).

We introduce a notion of Gröbner reduction of everywhere convergent power series over the real or complex numbers with respect to ideals generated by polynomials and an admissible term ordering.

Our main theorem states the existence of a formula for the division of everywhere convergent power series over the real or complex numbers by a finite set of polynomials. If the set of polynomials is a Gröbner basis then the reminder of their division depends only on the equivalence class of the power series modulo the ideal generated by the polynomials. When the power series which shall be derived is a polynomial the division formula leads to a usual Gröbner representation well-known for the polynomial rings.

Finally the results are applied to prove the closedness of ideals generated by polynomials in the ring of everywhere convergent power series, to give a very simple proof of the affine version of Serre's graph theorem, to extend the effective Hilbert Nullstellensatz to the ring of entire functions and to prove that the ring of entire functions is flat over the ring of polynomials. In the second part we present a method for determining the reduced Gröbner basis with respect to a given admissible term order of string type of the intersection ideal of an infinite sequence of polynomial ideals. As an application we discuss the Lagrange type interpretation on algebraic sets and the "approximation" of the ideal I of algebraic sets by 0-dimensional ideals, whose affine Hilbert functions converge towards the affine Hilbert function of I.

Yekutieli, Amnon

On the Intersection Cohomology of a Curve (an Application of Beilinson Completion Algebras)



Suppose Y is a smooth variety over R and X is an irreducible subvariety. Denote by D_Y the sheaf of differential operators on Y. There is a simple D_Y -module $\mathcal{L}(X,Y)$, with the property that its DeRham complex $DR\mathcal{L}(X,Y)$ is the intersection cohomology sheaf of X with middle perversity. (Brylinski-Kashiwara). We give an explicit algebraic characterization of $\mathcal{L}(X,Y)$, using Beilinson Completion Algebras and algebraic residues, when X is a curve. In fact our description is valid over any field of characteristic 0.

Berichterstatter: R. Waldi

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