Tagungsbericht 20/1996

Four-dimensional manifolds

19.5.-25.05.1996

Die Tagung fand unter der Leitung von Ronald J. Stern (Irvine) und Stefan A. Bauer (Bielefeld) statt. Im Mittelpunkt des Interesses standen Fragen zur Topologie vierdimensionaler symplektischer Mannigfaltigkeiten und dreidimensionaler Mannigfaltigkeiten mit Kontaktstruktur. Schwerpunkte der Tagung waren der Zusammenhang zwischen Seiberg-Witten Invarianten und Donaldson Invarianten, Verklebungsformeln für Seiberg-Witten Invarianten und die Anwendung der Seiberg-Witten Invarianten auf Probleme der niederdimensionalen Topologie, sowie Fragen zur Eindeutigkeit symplektischer Strukturen.

Vortragsauszüge:

Selmar Akbulut;
Shake slice knots and Adjunction formula

(Joint work with R. Matveyev.) We show how the adjunction inequality gives examples of isomorphic but not isotopic contact structures, fake 4-manifolds with $b_2 = 1$, and obstructions to non-shake sliceness of some knots (e.g. all Whitehead doubles of some knots).

Paul M. N. Feehan:
Gluing non-abelian monopoles and the relation between Donaldson invariants and Seiberg-Witten invariants of smooth 4-manifolds

(Joint work with Thomas G. Leness). We describe our progress towards the proof of the conjectured relationship between the Donaldson and Seiberg-Witten invariants of smooth 4-manifolds using a moduli space of non-abelian monopoles proposed by V.

Pidstrigatch and A. Tyurin. This moduli space has an Uhlenbeck compactification; its singularities include both the Seiberg-Witten moduli spaces (the 'reducibles') and the moduli space of anti-self-dual connections. By cutting the non-abelian monopole moduli space along suitably chosen links of these singularities, one essentially obtains a smooth manifold-with-boundary and, in principle, this allows a comparison of the two types of invariants for any smooth 4-manifold.

One of the main analytical difficulties inherent in the program of Pidstrigatch and Tyurin is in constructing the links of the lower stratum Seiberg-Witten moduli spaces which lie at the boundary of the Uhlenbeck compactification. We have approached this problem by constructing certain gluing and ungluing maps which can be used to analyze the topology of the moduli space ends and define the links. As in Taubes' and Donaldson's constructions of gluing maps for the moduli space of anti-self-dual connections, our gluing maps are constructed by perturbing approximate solutions to the non-abelian monopole equations obtained by splicing. However, the strategy needed to solve the non-abelian equations for the required perturbation is not a straightforward modification of previous methods for anti-self-dual connections. We show that the gluing maps (togther with accompanying obstruction maps) provide a finite atlas of coordinate charts for the moduli space of non-abelian monopoles. Our gluing theorem then allows us to give local parametrizations for the links of the lower stratum Seiberg-Witten moduli spaces.

Sergey Finashin:
Quotients of complex surfaces by complex conjugation

Given a nonsingular algebraic surface defined over $\mathbb{R}$ denote by X its complex points set and by $Y = X/conj$ the quotient by the complex conjugation, $conj : X \to X$. Y is a closed 4-manifold, which inherits from X differential structure and orientation. It turns out that quotients Y tend to be decomposable into connected sums of elementary pieces, $S^2 \times S^2$, $\mathbb{C}P^2$ and $\overline{\mathbb{C}P}^2$, when Y are simply connected (note that Y is simply connected for example if X is and the fixed point set $X_{\mathbb{R}} = Fix(conj)$ is non-empty).

I proved this decomposability property for all real types of rational and Enriques surfaces and gave a new proof for real $K3$ surfaces (the first proof for $K3$ is due to Hitchin and Donaldson). I proved it also for complete intersection of arbitrary multi-degree, which can be obtained by method of small perturbation and for double planes whose branching loci are obtained by perturbation from generic configurations of lines.

This property provides a description of the differential topology of complex surfaces in terms of knottings $X_{\mathbb{R}} \subset Y$ in "standard" 4-manifolds Y. Elementary bifurcations of these knotted surfaces under deformations of real algebraic surfaces were discussed.

Kenji Fukaya:
The Arnold Conjecture

(Joint work with K. Ono.) Let (X^{2n}, ω) be a symplectic manifold, $H : X \times (0,1) \to \mathbb{R}$, $H_t(x) = H(x,t)$, and put $i_{X_{H_t}} = dH_t$. Consider the system

$$\left.\frac{d\varphi_t}{dt}\right|_{t=t_0} = X_{H_{t_0}}(\varphi_{t_0}), \quad \varphi_1 = \varphi, \quad \varphi_0 = \text{identity}.$$

Theorem— Let φ be transversal. Then

$$\sharp\mathrm{Fix}(\varphi) \geq \sum \mathrm{rk} H_k(X; \mathbb{Q}).$$

The proof is based on the solution of a 'negative multi cover problem' in symplectic geometry, which is based on a Kuranishi theory with finite symmetry and a gluing theorem.

Mikio Furuta:
Simple Type Conditions

Witten proved that every Kähler manifold with $b^+ > 1$ is of simple type. In this talk I explained an idea to extend the proof to other 4-manifolds with $b^+ > 1$ when they have open Kähler submanifolds with non vanishing holomorphic 2-forms. If a 4-manifold has an embedded surface of genus $g > 0$ with intersection number zero, then a neighbourhood of the surface has a Kähler structure because it is of the form $\Sigma_g \times S^1 \times I$. When stretching this part, there are three a priori possibilities: if one takes a subsequence, then one has (1) a translation invariant monopole (2) a finite energy non trivial monopole, or (3) an infinite energy monopole. When we use a perturbation coming from a nonzero holomorphic 1-form on the surface, then (1) does not occur. In this talk I explained (2) does not occur under some condition, and (3) does occur for some 4-manifolds with $b^+ = 1$.

Lothar Göttsche:
Theta functions and Donaldson invariants

This is a report on joint work with Don Zagier. Let X be a smooth 4-manifold with $b_+ = 1$. We study the Donaldson invariants of X. Unlike the case $b_+ > 1$ they are not independent of the metric, but by Kotschick and Morgan they depend only on the period point $\omega(g)$ of g in the positive cone $H^2(X,R)^+/R^*$ of X. We extend the definition of the Donaldson invariants to the boundary of the positive cone, and show that for two classes F, G on the boundary the difference of the Donaldson invariants fullfils the k-th order simple type condition, (i.e. is zero on the ideal generated by $(p^2-4)^k$ for $p \in H_0(X,Z)$ the class of a point), for a k which is explicitely determined by F and G. There is a set of "basic" classes, such that the difference is a sum of contributions for each basic class, each given by a universal formula in terms of modular forms. The leading terms are analogous to the corresponding formulas of Kronheimer-Mrowka in the simple type case for $b_+ > 1$. If X is a rational algebraic surface the results are valid for the Donaldson invariants at the boundary points, and not just for their differences. In general, the basic classes occuring are precisely the characteristic cohomology classes of X, for which the corresponding Seiberg-Witten invariants for metrics near F and near G differ. This leads to a precise conjectural formula for the relationship between Donaldson and Seiberg-Witten invariants in case $b_+ = 1$.

Robert E. Gompf:
<u>Stein surfaces and Kirby calculus</u>

A complex surface is *Stein* if it admits a proper holomorphic embedding in $\mathbb{C}^N$ for some N.

Theorem(Eliashberg)— A smooth 4-manifold admits a Stein structure if and only if it has an open handle decomposition such that all handles have index ≤ 2 and each 2-handle is attached to the boundary of (0-handle $\cup$ 1-handles) ($\sharp S^1 \times S^2$) along a Legendrian curve, with framing obtained from the canonical framing by adding a left twist.

Here $\sharp S^1 \times S^2$ has a canonical induced contact structure $\xi = \ker(\alpha)$, $(\alpha \in \Omega^1(\sharp S^1 \times S^2))$, a curve K is Legendrian if $\alpha|K \equiv 0$, and the canonical framing on K is determined by a normal vector field to $\xi|K$.

Via Eliashberg's theorem, any Stein manifold X with only finitely many handles can be described by a handle diagram (Kirby picture) in a certain 'standard form'. The Chern class $c_1(X)$ can be computed directly from the diagram. One can also define and compute a set of invariants of the boundary of X with its induced contact structure ξ. These invariants distinguish the homotopy class of ξ as a 2-plane field on M, and are more sensible than $c_1(\xi)$.

Ian Hambleton:

The spherical space form problem

(Joint work with R. Lee.) We prove that the finite fundamental groups of closed, oriented 3-manifolds are just the finite subgroups $\Gamma \subset \mathrm{SO}(4)$ which act freely and linearly on S^3. The method is a combination of

i) techniques from algebracic number theory and surgery, and

ii) equivariant gauge theory (Donaldson, Taubes, ...).

The first ingredient is used to construct a suitable smooth 4-manifold $(Z, \partial Z)$ with $b_2^+(Z) = 0$, $\pi_1(Z) = G$, where $G = Q(8p, q)$ is any one of the groups from Milnor's 1957 list. The boundary of Z consists of (fake) space forms, together with quotients of homology 3-spheres by proper subgroup of G. In the second part, we use the ASD moduli space (for manifolds with cylindrical ends) to derive a contradiction to the existence of the fake space form Σ/G.

Dieter Kotschick:

Irreducibility and orientation-reversing diffeomorphisms

We discussed a few results which are suggested by Donaldson theory and proved using Seiberg-Witten invariants.

Beauville asked which compact complex surfaces admit a complex structure compatible with the other orientation, not the one defined by the original structure. If one changes the smooth structre, there are lots of examples. For a fixed structure there should be only very few, because often surfaces contain spheres of negative self-intersection. We now have a complete result, except for surfaces of class VII in Kodaira's classification.

Theorem 1— Let X, Y be compact complex surfaces which are orientation-reversing diffeomorphic with respect to their complex orientation. Then X (and Y) satisfies one of the following: (a) X is geometrically ruled, or

(b) $c_1^2(X) = c_2(X) = 0$, or

(c) $X = (\mathbb{H} \times \mathbb{H})/\Gamma$.

Corollary 1— If X, Y are as above and $|\pi_1| < \infty$, then X, Y are S^2-bundles over S^2.

If we assume $Kod(X) \geq 0$, then we have a complete list of examples. All these surfaces, without π_1-assumption, of course, carry compatible Thurston geometries.

Using Taubes' theorems, we proved:

Theorem 2— Let X be a minimal symplectic four-manifold with $b_2^+(X) > 1$. If X is smoothly $X_1 \sharp X_2$, then one of the X_i is a $\mathbb{Z}$-homology sphere whose π_1 has no non-trivial finite quotient.

Corollary 2— Minimal symplectic 4-manifolds with $b_2^+ > 1$ and with residually finite π_1 are irreducible, i.e. in a smooth decomposition one summand must be a homotopy sphere.

Combining Theorem 2 with Gompf's construction of symplectic four-manifolds with prescribed π_1 gives lots of counterexamples to the smooth Kneser conjecture.

Tian-Jun Li:
Seiberg-Witten invariants and the topology of symplectic 4-manifolds

We first study Seiberg-Witten invariants on 4-manifolds with $b_2^+ = 1$. On these manifolds, associated to each spin^c-structure, there are two SW-invariants. We prove a basic result expressing the difference of the two SW-invariants. This generalizes the wall crossing formula of Kronheimer and Mrowka for 4-manifolds with $b_2^+ = 1$ and $b_1 = 0$.

We take on the problem of the moduli space of symplectic structures and prove the uniqueness of symplectic structures up to deformation and diffeomorphisms on ruled surfaces and get some partial results for general symplectic 4-manifolds.

By generalizing Kronheimer and Mrowka's ideas, we prove for symplectic 4-manifolds a generalized adjunction inequality. Suppose M is a symplectic four-manifold with $b_2^+ = 1$ and ω is a symplectic form. Let C be a smooth connected embedded surface with nonnegative self-intersection. If $[C].\omega > 0$, then $2g(C) - 2 \geq K.[C] + [C]^2$.

We prove that rational or ruled surfaces can be characterized among all symplectic 4-manifolds by the existence of smoothly embedded essential spheres with nonnegative squares, or the existence of 'exotic' embedded $--1$ spheres.

Gordana Matic:
Tight contact structures and Seiberg-Witten invariants

(Joint work with Paolo Lisca.) A Stein structure on a 4-manifold with just 0- and 2-handles can be described by a framed link in S^3 with Legendrian components. Eliashberg shows that the necessary and sufficient conditions for such a link to describe a Stein structure is that is satisfies the condition $fr = tb - 1$. We construct examples of different Stein structures on the same smooth 4-manifold such that they have different c_1. We then show induced tight contact structures on the boundary are non-isotopic hence proving:

Theorem 1— There exist homology 3-spheres with arbitrarily large numbers of homotopic, non-isomorphic tight contact structures.

To study contact structures on boundaries of convex domains in Stein surfaces we prove an symplectic embedding theorem for compact domains:

Theorem— Let X be a Stein manifold, and $\phi : X \to \mathbb{R}$ a smooth strictly plurisubharmonic function. Let $r \in \mathbb{R}$ be a regular value of ϕ, and $X_r = \{\phi < r\} \subset X$. Then, there exists a holomorphic embedding of X_r as a domain inside a smooth projective variety with ample canonical bundle S having a Kähler form whose pull-back to X equals $\omega_\phi = dJ^*(d\phi)$. Moreover, when X has complex dimension two S may be chosen so that $b_2^+(S) > 1$.

We than use the above theorem and Seiberg-Witten invariants to show the following theorem (also proved by Kronheimer-Mröwka):

Theorem— Let X be a smooth 4-manifold with boundary. Suppose J_1, J_2 are two Stein structures with boundary on X with associated Spin^c-structures Θ_1 and Θ_2. If the induced contact structures ξ_1 and ξ_2 on ∂X are isotopic, then Θ_1 and Θ_2 are isomorphic (and in particular have the same c_1).

This together with the construction, which produces Stein structures on the same smooth manifold with different c_1, proves theorem 1.

Dusa McDuff:

From symplectic deformation to isotopy

Let (X, ω) be a symplectic 4-manifold which is not of SW-simple type in the sense that there are non-trivial Gromov invariants $Gr(A)$ occuring in classes $A \in H_2$ with $k(A) = c_1(A) + A^2 > 0$, where $c_1 = c_1(X, J) = -K$ and K is the canonical divisor. By the results of Li-Liu and Liu these are precisely the symplectic manifolds s.t. $b^+ = 1$ and, in case $b_1 \neq 0$, have non-trivial cup product map $H^1 \times H^1 \to H^2$. Using the inflation procedure, I showed that if ω_t is a family of symplectic forms on such X with $[\omega_0] = [\omega_1] \in H^2(X; \mathbb{R})$, then $\{\omega_t\}$ is homotopic to an isotopy ω_t' relative endpoints. In particular, this means there is a fmaily g_t of diffeomorphisms of X with $g_0 = \mathrm{id}$ and $g_1^* \omega_1 = \omega_0$.

As applications, one gets uniqueness of structure results for ruled surfaces, certain products $\Sigma_1 \times \Sigma_2$ (Σ_i Riemann surfaces) and results on the symplectic blow-up and existence of packings. In particular, all cohomologous blow-ups of $\mathbb{CP}^2$ are equivalent, and, as Biran showed, there is a full filling of $\mathbb{CP}^2$ by k equal balls for any $k \geq 9$. (This means one can symplectically embed k disjoint balls into $\mathbb{CP}^2$ to fill as much of the volume as one wishes.)

G. Mikhalkin:
Adjunction inequality for a pair of real algebraic curves

I describe a construction which yields applications of the adjunction inequality for the Seiberg-Witten classes in real algebraic geometry. It produces new theorems on the arrangement of the ovals of plane curves (starting from degree 7), and mutual arrangement of the ovals of a pair of curves.

Tom Mrowka:
Contact Structures and Monopole Invariants

(Joint work with Peter Kronheimer.) This talk described some techniques which give rise to invariants of a pair (X,ξ) where X^4 is an oriented connected four-manifold with non-empty boundary and $\xi \to \partial X$ is a contact structure compatible with the boundary orientation. The contact structure determines a Spin^c-structure, s_0, on ∂X. The domain of the invariant is the set of isomorphism classes of pairs (s,i) where s is a Spin^c-structure on X and $i : s|_{\partial X} \to s_0$ is as isomorphism. Call this set $\mathcal{I}(X,\xi)$. The invariants generalize the Seiberg-Witten invariants and take the form of a function $\mu : \mathcal{I}(X,\xi) \to \mathbb{Z}$. These invariants enjoy the following property:

If there is symplectic form ω on X compatible with the orientation of X so that $\omega|_\xi > 0$ then $\mu(s_\omega = \pm 1$, where s_ω denotes the Spin^c-structure determined by ω.

Recall that a contact structure (Y,ξ) is called fillable in case there is a pair (X,ω) as above with $\partial X = Y$ and $\omega|_\xi > 0$. One of the consequences of the existence of these invariants is

Theorem— Only finitely many components of the space of two-plane fields can contain a fillable contact structure.

Via a recent result of Thurston and Eliashberg we also get

Corollary— Only finitely many components of the space of two-plane fields can contain a taut foliation.

We can also recover the recent result of Lisca and Matic regarding Stein surfaces.

Theorem— Suppose X is a smooth four-manifold carrying a pair of exact symplectic forms $\omega_1 = d\eta_1$, $\omega_2 = d\eta_2$ and contact structures ξ_1, ξ_2 on ∂X so that $\omega_i|_{\xi_i} > 0$, $i = 1,2$. Then if ξ_1 is isotopic to ξ_2, then the Spin^c-structures s_{ω_1} and s_{ω_2} are isomorphic.

Vicente Munoz:

Basic classes for connected sums along Riemann surfaces

Let $\bar{X}_1$ and $\bar{X}_2$ be two compact oriented connected four-manifolds with $b_1 = 0$, $b^+ > 1$ and of simple type, such that there are embedded Riemman surfaces $\Sigma_i \subset \bar{X}_i$ of the same genus g and self-intersection zero and representing odd homology classes. Consider a connected sum $X = \bar{X}_1 \# \bar{X}_2$ of $\bar{X}_1$ and $\bar{X}_2$ along the Riemann surfaces (there might be many). We want to use Floer homology to compute the Donaldson invariants of X in terms of the invariants of $\bar{X}_1$ and $\bar{X}_2$. When the genus is $g = 1$, we get explicitly the invariants of general connected sums along tori. As an application, we get the invariants of the elliptic surfaces with $b_1 = 0$, of a generalised logarithmic transform and of the manifold $S = \mathbb{CP}^1 \times \mathbb{T}^2$ (with respect to suitable metrics).

When the genus is $g = 2$, we use a suitable version of the Atiyah-Floer conjecture to get that X is of simple type and that for every pair L_1, L_2, with $L_i \in H^2(X_i; \mathbb{Z})$, the sum of the coefficients of all the basic classes $K \in H^2(X; \mathbb{Z})$ such that $K|_{X_i} = L_i$, $i = 1, 2$, is zero unless L_i are the restriction of basic classes $\bar{L}_i$ for $\bar{X}_i$, $i = 1, 2$, such that $\bar{L}_1 \cdot \Sigma_1 = \bar{L}_2 \cdot \Sigma_2 = \pm 2$, in which case is ± 32 times the product of the coefficients of $\bar{L}_1$ and $\bar{L}_2$. We generalise to manifolds of (potentially) non-simple type and to the case of $b^+ = 1$, notably proving that any manifold X with an embedded Riemman surface of genus 2 of self-intersection zero and representing an odd homology class is of finite type.

We also relate these results with the ones of Morgan-Szabó-Taubes about the behaviour of Seiberg-Witten invariants for connected sums along Riemann surfaces.

Victor Pidstrigatch:

Donaldson Invariants and Seiberg-Witten invariants

(Joint work with A. Tyurin.) We define a moduli space $M_B(p_1(E), c_1(E) + c)$ of solutions of the system

$$D_a\phi = 0 \qquad (F_A^+)_0 = -(\phi \otimes \bar{\phi})_{00},$$

where $a \in \mathcal{A}_\omega = \{b \in \mathcal{A}_E | trF_b = \omega\}$ is a connection on a rank 2 hermitian vector bundle E on a 4-manifold X, $W^\pm$ are Spin^c-bundles on X, $\phi \in \Gamma(W^+ \otimes E)$, $c_1(W^\pm) - c$. Reducible solutions of this system are either points of type $(a, 0)$ subject to the condition $F_a^+ = 0$, or of type $(\lambda_1 \oplus \lambda_2, 0 \oplus \phi_2)$ subject to $F^+_{2\lambda_2+\nabla} = -(\phi_2 \otimes \bar{\phi}_2)_{00}$. Reducibles of the first type correspond to antiselfdual connections and those of second type to solutions of abelian Seiberg-Witten equations. We proof a transversality theorem and construct a compactification for M_B. M_B is a bordism between modifications of the moduli space of asd connections and links of moduli spaces of reducibles of the second type in M_B. These latter ones are described in terms of moduli spaces of Seiberg-Witten M_{SW} and 'universal' gluing parameter spaces $GL_r(\alpha)$ described by Kotschick and Morgan. To get a relation between Donaldson and Seiberg-Witten invariants we assume that the Conjecture of Kotschick and Morgan holds true: Then

$$\int_{L_r(\alpha)} \mu_\Sigma^i \nu^j u^{4r-1-d} = \sum \mathrm{km}_{ij\ell}^{rd} |\Sigma|^{2\ell} \alpha(\Sigma)^{i-2\ell}$$

with coefficients km that depend only on $H_2(X, \mathbb{Z})$ and the intersection form Q.

The bordism M_B provides a formula for the Donaldson polynomial

$$\gamma_{w_2}^d(\Sigma) = \sum_\beta \int_{Link(M_{SW}(\beta))} \mu_\Sigma^d \lambda^{ind-1},$$

where $w_2 = c_1(E) \mod 2$, $d = -p_1(E) - \frac{3}{2}(b_2^+ 1 + 1)$. This gives

Theorem— Let X be a four-manifold of simple type with Donaldson series given by

$$\gamma(\Sigma) = \exp(|\Sigma|^2/2) \cdot \left(\sum_{\alpha \in Bs_{KM}} m_\alpha \exp(\alpha(\Sigma)) \right)$$

and SW-invariants $\{\beta, n_\beta\}_{\beta \in Bs_{SW}}$. Then $Bs_{KM} = Bs_{SW}$ and there is a constant C depending on $H_S(X, \mathbb{Z})$ and Q only such that $n_\beta = C.m_\beta$.

Zoltan Szabó:

Gluing Formulae for Seiberg-Witten Invariants

(Joint work with J. Morgan.) We investigate SW-invariants of smooth closed 4-manifolds, split along the three-torus $X = X_1 \cup_{T^3} X_2$. We prove the existence of relative SW-invariants $SW : H^2(X_i, T^3; \mathbb{Z}) \to \mathbb{Z}$, compute the SW-invariants of $X_i(\Phi) = X_i \cup_\Phi D^2 \times T^2$, where $\Phi : \partial X_i \to \partial(D^2 \times T^2)$ in terms of SW_{X_i}. We also compute the SW-invariants of X in terms of SW_{X_i}.

Note that the character variety of T^3 contains a unique singular point, where the Dirac operator has a nontrivial kernel. In order to define the relative SW-invariants, we have to investigate the structure of the cylindrical end moduli space around the singular boundary value. This is done by computing the stable set of the singular point in the centre manifold.

Mina Teicher:

New invariants for compelx surfaces

The moduli space $M_{c_1^2,c_2}$ of all algebraic complex surfaces with given c_1^2, c_2 is not irreducible. The fundamental group of a complement of the branch curve of a generic projection does not change when one moves in a connected component. Thus these groups can distinguish between surfaces lying in different components. An algorithm to compute such groups was introduced and a few examples were presented. Unlike former expectations these groups turn out to be 'almost solvable', i.e. they contain a solvable subgroup of finite index.

In the process of computing such groups, we found new algebraic surfaces: Surfaces of general type which are simply connected, spin, and have nonnegative signature. These surfaces were introduced: They are Galois covers of Hirzebruch surfaces.

Andrei Teleman:

The coupling procedure and G-monopoles

(Joint work with C. Okonek and A. Schmitt.) We explain first the 'Coupling Procedure' in Geometric Invariant Theory. This means to study 'correlation functions' on a GIT $\mathrm{GL}(m)$-moduli space by coupling the given group operation with a new one (which leads to a simpler moduli space). We get in this way a group-operation of $\mathrm{GL}(m)$ in the projective space $\mathbb{P}(A \oplus B)$ of a direct sum of vector spaces. Suppose $\mathbb{C}^*$ acts by homotheties, but with different weights on A and B. Our strategy consists in studying the 'master space' $Q := \mathbb{P}(A \oplus B)^{ss}/\!\!/\mathrm{SL}(m)$ as a $\mathbb{C}^*$-space. Factorising by $S^1 \subset \mathbb{C}^*$, we get a homological equivalence which allows us to express correlation functions on the given moduli space $Q_{source} = \mathbb{P}(A)^{ss}/\!\!/\mathrm{SL}(m)$ in terms of correlation functions on $Q_{sink} = \mathbb{P}(B)^{ss}/\!\!/\mathrm{SL}(m)$ and on the space of 'reductions' (fixed points of the $\mathbb{C}^*$-action which do not lie on $Q_{source} \cup Q_{sink}$.)

We construct a Gieseker moduli space of semistable 'oriented pairs' (triples $(E, \varphi, \varepsilon)$ with E torsion free, $\varphi : E \to E_0$, $\varepsilon : \det(E) \to L$, $E_o \in \mathrm{Coh}_X$ torsion free, $L \in \mathrm{Pic}(X)$ fixed) and we describe this moduli space as a 'master space', by identifying the source and the sink. We show that previous results using 'Geometric approximation" can be recovered easier using the 'coupling procedure' in our sense.

We introduce the concept of G-monopole equations (associated to a $\mathrm{Spin}^G(4)$-structure on a 4-manifold) and we discuss possible applications: Construction of new invariants for 4-manifolds or of moduli spaces relating different versions of Donaldson theories to different versions of Seiberg-Witten theories. The $Spin^G(4)$-monopoles arise naturally by applying a differential geometric coupling procedure to the ASD equations.

Shuguang Wang:
Four-manifolds as branched covers

We show the following result:

Theorem— For a Kaehler surface X with $H_1(X,\mathbf{Z}) = \{1\}$, let $\Sigma \subset X$ be the fixed point set of an anti-holomorphic involution. Suppose that Σ consists of orientable surfaces only and $[\Sigma] = o \in H_2(X,\mathbf{Z}_2)$. When Σ is connected, assume in addition that $[\Sigma] \neq 0 \in H_2(X,\mathbf{Z})$. Then the double cover of X branched along Σ admits a symplectic structure.

This theorem is interesting in that the other construction, the quotient of X under the anti-holomorphic involution, is often without symplectic structure. Furthermore the double cover in the theorem may well be non-Kaehler, thus giving another possible construction of non-Kaehler symplectic 4-manifolds.

For a gerneral smooth branched cover, we also discuss some relation of Seiberg-Witten invariants on the base and cover manifolds, and interpret the relation in terms of deforming the holonomy of singular Seiberg-Witten solutions. This part of research is in progress, joint with Yongbin Ruan.

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