MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 21/1996

Geometric Rigidity and Hyperbolic Dynamics

2.-8.6.1996

The meeting has been organized by Werner Ballmann (U Bonn) and Anatol Katok (Penn State U). Some of the subjects treated in the talks are the following:

a) Semihyperbolic and hyperbolic geodesic flows
b) Rigidity of symmetric spaces and of spaces of nonpositive curvature
c) Asymptotic geometry
d) Billiards
e) Group actions
f) Singular spaces

The meeting was very lively. Special discussions in the evening were devoted to some of the more spectacular developments.

VORTRAGSAUSZÜGE

Viviane Baladi:

Correlation spectrum of quenched and annealed equilibrium states for random expanding maps

We show that the integrated transfer operators for positively weighted independently and identically distributed smooth expanding systems give rise to annealed equilibrium states for a new variational principle. Using work of Ruelle and Fried on generalised Fredholm determinants for transfer operators, we prove that the discrete spectrum of the transfer operators coincide with the correlation spectrum of these invariant measures (yielding exponential decay of correlations) and with the poles of an annealed zeta function. A modified integrated transfer operator describes the (relativised) quenched states studied e.g. by Kifer. Conditions (including SRB) ensuring coincidence of the quenched and annealed states are given. For small random perturbations, stability results on the quenched and annealed measures and spectra are obtained by applying perturbative results of Young and the author.

Yves Benoist:

Asymptotic properties of linear groups

Let M be a simply connected symmetric Riemannian space of nonpositive curvature, S its sphere at infinity, S^{reg} its regular part. Let Γ be a subgroup of the group G of isometries of M. Let S_Γ be the limit set of Γ in S and $S_\Gamma^{\text{reg}} = S_\Gamma \cap S^{\text{reg}}$. The objective of this lecture is to describe the structure of the limit set S_Γ when Γ is Zariski dense in G.
We denote by X the set of chambers in S and Λ_Γ the set of chambers which meet S_Γ^{reg}. Recall that any two chambers in S are canonically isometric. I prove that

1. S_Γ^{reg} is dense in S_Γ.
2. The intersection $S_\Gamma \cap C$ "does not depend on Γ" when C varies in Λ_p. Call this intersection C_Γ the limit cone of Γ.
3. C_Γ is a subset of the chamber which is closed convex, of non empty interior and invariant by the opposition involution.
4. Reciprocally, any subset of the chamber which satisfies these properties is the limit cone of a Zariski dense subgroup Γ of G.

Marc Bourdon:

Hyperbolic buildings, conformal dimension and rigidity

We adapt to a family of hyperbolic buildings some geometric quasi-conformal arguments, which are classical in the case of rank one symmetric spaces of non-compact type. In particular we compute a quasi-isometry invariant of these buildings: the conformal dimension (of Pansu) of their boundary. We also prove that their lattices are Mostow-rigid in the classical sense.

Ulrich Bunke:

The divisor of the Selberg zeta function associated to a Kleinian manifold

We consider a complete hyperbolic manifold such that its fundamental group is convex-cocompact. The associated Selberg zeta function has a meromorphic continuation. A natural question is to describe its singularities. We prove a (modified) conjecture of Patterson relating the singularities of the Selberg zeta function to the cohomology of the fundamental group with coefficients in a natural subspace of principal series representations.

As a first step towards a generalization of this result to bundles and other rank-one spaces we prove the absence of singular continuous spectrum of the Laplacian on bundles over the locally symmetric space. The same methods allow to construct a meromorphic continuation of the resolvent kernel (up to some exceptional points).

Sergei Buyalo:

Catching geodesics in Hadamard spaces

For every quasi-isometric map $f : X \to Y$ of Hadamard spaces we define its asymptotic limit s_f which sends the boundary at infinity $\partial_\infty X$ to the cone $C_\infty Y$ over $\partial_\infty Y$ and establish its analytic properties. In the case when X and Y are cocompact rank one spaces with respect to the same discrete isometry group Γ and hence Γ-equivariantly quasi-isometric we give a sufficient condition for s_f to be an equivariant homeomorphism between $\partial_\infty X$ and $\partial_\infty Y$ with respect to the standard topologies and biLipschitz homeomorphism with respect to Tits metrics. Apart of this condition there is a large number of equivariantly quasi-isometric cocompact Hadamard spaces whose boundaries at infinity are not equivariantly homeomorphic. This answers a question of M.Gromov.

Christopher B.Croke (joint work with V.Sharafutdinov):

Isospectral deformation rigidity of compact manifolds of negative curvature

For a compact Riemannian manifold (M, g) let Spec(g) be the spectrum of the Laplace-Beltrami operator. We show:

Theorem 1: If (M, g) is a compact manifold of negative curvature and g_t is a smooth 1-parameter family of metrics such that Spec(g_t) =Spec(g) and $g_0 = g$, then there are diffeomorphisms ϕ_t of M such that $g = \phi_t^*(g_t)$ for all t.

M is said to have simple length spectrum if for every pair of prime closed geodesics γ_1 and γ_2 we have that $L(\gamma_1)/L(\gamma_2)$ is not rational. This is a generic condition for negatively curved manifolds. We show:

Theorem 2: Let (M, g) have negative curvature and simple length spectrum. If p_1 and p_2 are C^∞ functions such that $\Delta + p_1$ and $\Delta + p_2$ have the same spectrum then $p_1 = p_2$.

These Theorems generalize results of Guillemin and Kazhdan (1980), later improved by Min-Oo, which apply in 2 and in higher dimensions under stronger curvature assumptions. We consider the operator $d : S^m(T^*M) \to S^{m+1}(T^*M)$, where $S^m(T^*M)$ represents the symmetric m-tensor fields, defined by $d(T) = \sigma(\nabla T)$, where ∇ is covariant derivative and σ is the symmetrization operator. Theorems 1 and 2 as well as other interesting results are shown to follow from:

Theorem 3: Let $T \in S^m(T^*M)$, where M is a compact manifold of negative curvature. If for every closed geodesic γ we have

$$\int_\gamma T(\dot\gamma(t), \ldots, \dot\gamma(t))dt \;=\; 0$$

then $T = dS$ for some $S \in S^{m-1}(T^*M)$

Renato Feres:

Infinitesimal & local linearizations of higher rank actions

Let G be a semisimple Lie group of real rank at least 2 which acts ergodically and smoothly on a compact manifold M, such that some element of G defines a partially hyperbolic diffeomorphism of M. We ask wether one can find a (non-stationary) "Birkhoff normal form" type theorem for such actions. The following ∞-jet linearization provides some poositive evidence:

Theorem: Suppose that G preserves a smooth foliation H transverse to the orbits and that TH decomposes continuously as a direct sum of subbundles, $E_1 \oplus E_2$, corresponding to a uniform exponential splitting for some element f of G, where E_1 is 1-dimensional and is the fastest contracting subbundle for f. Assume moreover that the action is "effective to first order". Then the action can be continuously linearized up to infinite order and the linearization is associated to a linear representation of G.

Serge Ferleger:

Uniform estimate on the number of collisions in a semidispersing billiard

Let M be a C^2-smooth Riemannian manifold with bounded sectional curvature and positive injectivity radius, $\{B_i\}_{i=1}^n \subset M$ convex bodies such that ∂B_i are C^1-submanifolds of M; form the billiard $B = M | \bigcup_{i=1}^n \mathrm{Int} B_i$, $\{T^t\}_{t=-\infty}^{+\infty}$ the billiard flow.

Theorem 1: For any such billiard and $x \in B$, there is $x \in U \subset B$ a neighborhood, which the particle leaves, making only finitely many collisions.

Theorem 2: For any such billiard and $x \in B$, provided the billiard is non-degenerate (see below), there is a neighborhood $x \in U \subset B$ which the particle leaves, making no more than $\left(\frac{16}{C(x)} + 2\right)^{2(n+2)}$ collisions, where $C(x)$ is non degeneracy constant, n number of walls.

Theorem 3: If the curvature of M is non-positive, then, provided that the billiard is non-degenerate, $h_{\mathrm{top}}(T^1) < P \log n + \lim_{l\to\infty} \frac{\log H(l)}{l}$ where P is a constant, depending only on C and $H(l)$ is the homotopy growth.

Theorem 4: In a non-degenerate billiard of negative curvature, the number $P(k)$ of periodic points of period k is not greater than n^k, number of periodic trajectories of billiard flow $p(k)$ of length k is not greater than $n^{(p+1)k}$, unless $P(k)$ or $p(k)$ are equal to infinity.

Definition: Billiard is called non-degenerate at a neighborhood of a point $x \in B$, if $\forall I \subset \{1, \dots, n\}$:

$$\frac{\max_{k \in I} \operatorname{dist}(y, B_k)}{\operatorname{dist}(y, \bigcap_{k \in I} B_k)} \geq C \quad , \quad \forall g \in (U \cap B) | \bigcap_{i \in I} B_i.$$

Patrick Foulon:

Rigidity of convex sets and Finsler geometry

We prove a theorem for regular Finsler metrics of constant negative curvature.

Theorem: Let (M^n, F) be an n-dimensional closed manifold. If the regular metric F has curvature $R^F = -1$ then F is a Riemannian metric of constant negative curvature.

This is a first attempt to understand the modul spaces in Finsler geometry.
A nice corollary is that we recover the result of Benzecri for a convex bounded set of $\mathbb{R}^n$, with strictly convex smooth boundary. Namely if such a convex has a compact quotient then it is an ellipsoid.
The key point is that such a geodesic flow is an Anosov flow. By BLF-theorem we obtain a conjugacy with a Riemannian symmetric space. The rest of the proof consists in showing that this conjugacy comes from an isometry.

Boris Hasselblatt (joint work with Anne Marie Wilkinson):

Open sets of Anosov diffeomorphisms with non-Lipschitz holonomies

Theorem 1: There are open sets of $\left\{ \begin{array}{c} \text{symplectic} \\ \text{codimension 1} \end{array} \right\}$ Anosov systems whose holonomies $\left\{ \begin{array}{c} \text{---} \\ \text{along 1-dimensional leaves} \end{array} \right\}$ are non-Lipschitz almost everywhere (with respect to some fully supported ergodic invariant probability measure).

The proof shows this lack of regularity for the distributions, the result then follows from the following observation, which may be of independent interest:

Theorem 2: If a C^0-foliation by C^∞-leaves has α-Hölder foliations $\left\{ \begin{array}{c} \text{everywhere} \\ \text{almost everywhere} \end{array} \right\}$ then the tangent distribution is $(\alpha - \epsilon)$-Hölder $\left\{ \begin{array}{c} \text{everywhere} \\ \text{almost everywhere} \end{array} \right\}$ for any $\epsilon > 0$

(If the leaves are only C^k, $k < \infty$, then there appears to be the possibility of a genuine loss of regularity.)

The starting point is a 1-spread (everywhere) Anosov system (e.g. a linear symplectic diffeomorphism whose largest positive Lyapunov exponents differ by more than a factor of 2 - unfortunately no geodesic flows with this property appear to be known). The open set then is a neighborhood of a suitable perturbation.

The basic ingredient of this perturbation is in my thesis (ET&DS 1994)

Sergej Ivanov:

Geometry of the limit norm of a periodic metric

Let (M, g) be a Riemannian manifold and $\Gamma \subset \mathrm{Iso}(M)$ be an abelian group acting discretely and cocompactly. We say that g is a Γ-periodic metric on M. Assume $\Gamma \cong \mathbb{Z}^n$ and $n \geq \dim M$.

Examples: 1) $M \cong \mathbb{R}^n$, $\Gamma = \mathbb{Z}^n$. Then (M, g) is the universal covering of a Riemannian n-torus.

2) Universal abelian coverings of 2-surfaces of genus ≥ 2. Γ is a group of deck-transformations.

There exists a norm $||\cdot||$ on Γ and $C > 0$ such that

$$\forall x \in M \ \forall v \in \Gamma: \quad |d(x, x+v) - ||v||| \leq C$$

(D.Burago 1992). Here d is the Riemannian distance and $x + v$ is the action of v on x. The norm $||\cdot||$ is called limit norm, or asymptotic norm, or stable norm of (M, Γ, g). We include Γ into $\Gamma \otimes \mathbb{R} \cong \mathbb{R}^n$ and extend $||\cdot||$ onto $\mathbb{R}^n$. Let B denote the unit ball of $||\cdot||$.

Theorem 1: (joint work with D.Burago and B.Kleiner). Let $g \in C^2$ and $v \in \partial B$ be an irrational vector, i.e. its coordinates are $\mathbb{Q}$-independent. Then B cannot have a sharp tangent cone at v.

Theorem 2: (joint work with D.Burago). $\forall k \in \mathbb{N}\ \exists n \in \mathbb{N}$ such that for almost every vector $v \in \mathbb{R}^n$ there exists a $\mathbb{Z}^n$-periodic C^k-smooth g on $\mathbb{R}^n$ for which ∂B is not smooth at v.

The smoothness/nonsmoothness properties of ∂B are closely related to the structure of the set of minimal geodesics in (M, g) with a given direction v at infinity.

Open questions: 1) Are there C^∞-examples in Theorem 2?

2) Is Theorem 2 true for abelian coverings of surfaces in place of $\mathbb{R}^n$?

3. Can one improve Theorem 1 by finding lower bounds for the dimension of the edge of the tangent cone of B at v? For example, is $\frac{n-1}{2}$ such a bound?

Svetlana Katok:

Arithmetic and geometric coding of geodesics on the modular surface

Closed geodesics associated to conjugacy classes of hyperbolic matrices in $SL(2,\mathbb{Z})$ can be coded in two different ways. The geometric code, with respect to the standard fundamental region, is obtained by a construction universal for all Fuchsian groups, while the arithmetic code, given by "-" continued fractions, comes from the Gauss reduction theory and is specific for $SL(2,\mathbb{Z})$. Both codes are finite sequences of non-zero integers up to a cyclic permutation; the geometric code may contain positive and negative integers while the arithmetic code contains only positive integers ≥ 2. We give a complete description of all closed geodesics for which the two codes coincide.

Theorem: For a closed geodesic with the arithmetic code $(n_1, \ldots, n_m)$ its geometric code coincides with the arithmetic code if and only if for any $i \pmod m$

$$\frac{1}{n_i} + \frac{1}{n_{i+1}} \leq \frac{1}{2}$$

i.e. if the arithmetic code does not contain the following forbidden pairs:

$$\{2,q\}\ ,\ \{p,2\}\ ,\ \{3,3\}\ ,\ \{3,4\}\ ,\ \{4,3\}\ ,\ \{3,5\}\ ,\ \{5,3\}$$

Closed geodesics for which the two codes coincide are distinguished by the following regular behavior: All segments comprising such a closed geodesic γ in the standard fundamental region for $SL(2,\mathbb{Z})$, $\mathcal{F}$, are oriented clockwise, and the entire closed geodesic in $\mathcal{F}$ consists of m "bootstraps" γ_i, where m is the length of the code: $\gamma = \gamma_1 \cup \gamma_2 \cup \ldots \cup \gamma_m$. Therefore we call such closed geodesics regular. The following propositions relate the arithmetic code with the length of a closed geodesic.

Proposition 1: The length of any closed geodesic γ with the arithmetic code $(n_1, n_2, \ldots, n_m)$ can be explicitely computed in terms of the n_i.

Proposition 2: If a closed geodesic with the arithmetic code $(n_1, \ldots, n_m)$ is regular, then the length of each individual "bootstrap" γ_i is given by an explicit formula in the n_j.

The paper "Coding of closed geodesics after Gauss and Morse" will appear in Geometriae Dedicata, 1996, and may be retrieved from
http://www.math.psu.edu/preprints/katok_s/paper.html

D.Kleinbock (joint work with G.Margulis):

Logarithm laws for flows on homogeneous spaces

We prove the following

Theorem: Let G be a connected simple Lie group, $\Gamma \subset G$ a lattice, μ a normalized Haar measure on G/Γ, $\{g_t \subset G\}$ a nonquasiunipotent (i.e. partially hyperbolic) one-parameter subgroup of G. Further, let $\{A_n\}$ be a family of measurable subsets $g \subset G/\Gamma$ such that

$$\forall \epsilon > 0\, \exists r > 0 \text{ such that } \mu(r - \text{neighborhood of } \partial A_n) \;\leq\; \epsilon\mu(A_n) \qquad (UR)$$

Then for almost every $x \in G/\Gamma$

$$\#\{1 \leq n \leq N | g_n x \in A_n\} \begin{cases} \text{is bounded if } \sum_{n=1}^{\infty} \mu(A_n) \;<\; \infty \\ = \sum_{n=1}^{N} \mu(A_n) \;+\; o\left(\sum_{n=1}^{N} \mu(A_n)\right) \text{ if } \sum_{n=1}^{\infty} \mu(A_n) \;=\; \infty \end{cases}$$

The theorem is motivated by Khinchin's theorem on diophantine approximation and by Sullivan's "logarithm law for geodesics", and both of these results can be deduced from it. Moreover we have a considerable strengthening of Sullivan's result and its generalization to arbitrary locally symmetric spaces. E.g. consider a function $\Delta : G/\Gamma \to \mathbb{R}^+$ such that
(i) Δ is uniformly continuous
(ii) its distribution function is uniformly continuous
(iii) $\mu\{x \in G/\Gamma \,|\, \Delta(x) \geq z\} \;\sim\; e^{-dz},\, d > 0$
Then it follows from the theorem that

$$\limsup_{t \to \infty} \frac{\Delta(g_t x)}{\log t} \;=\; \frac{1}{d}$$

In particular, with Δ coming from a distance function on a locally symmetric space, this gives a logarithm law for geodesics on locally symmetric spaces of finite volume and non-compact type. One can also state a multidimensional version of the above theorem, thus obtaining logarithm laws for flats in locally symmetric spaces. The proof uses ergodic properties of $\{g_t\}$-action on G/Γ, i.e. exponential decay of correlation coefficients of smooth functions. The nature of the method is very general and applications to other dynamical systems are quite possible.

Bruce Kleiner:

A higher dimensional analog of hyperbolicity for spaces of nonpositive curvature

The main results of the lecture concern nonpositively curved spaces which are not Gromov hyperbolic.

Theorem 1: (Croke-K.) For $i = 1, 2$ let $\bar{d}_i$ be a negatively curved metric on a surface N^2 of genus two, let d_i be the corresponding product metric on $M = N^2 \times S^1$ (the S^1 factor may have any length), let $\tilde{d}_i$ be the induced metric on $\tilde{M}$, and let $\partial_\infty^{\tilde{d}_i} \tilde{M}$ be the geometric boundary of $\tilde{M}$ defined by $\tilde{d}_i$. If either

i) $\mathrm{id}_{\tilde{M}} : \tilde{M} \to \tilde{M}$ carries every d_1-geodesic to within finite Hausdorff distance of some d_2-geodesic or

ii) $\mathrm{id}_{\tilde{M}}$ induces a homeomorphism $\partial \mathrm{id} : \partial_\infty^{\tilde{d}_1} \tilde{M} \to \partial_\infty^{\tilde{d}_2} \tilde{M}$

then $\bar{d}_1$ is homothetic to $\bar{d}_2$ by a homothety homotopic to id_{N^2}.

S.Buyalo and K.Ruane have related results. Theorem 1 shows that the basic property of δ-hyperbolic Hadamard spaces - that quasi-isometries induce boundary homeomorphisms - fails badly for Hadamard manifold containing flats. In these examples, however, $\partial_\infty^{\tilde{d}_1} \tilde{M}$ and $\partial_\infty^{\tilde{d}_2} \tilde{M}$ are $\pi_1(M)$-equivariantly homeomorphic.

The next example answers a question of Gromov from "Asymptotic invariants of infinite groups".

Theorem 2: (Croke-K.) There is a graph manifold M^3 carrying two nonpositively curved metrics d_1, d_2 such that the corresponding geometric boundaries $\partial_\infty^{\tilde{d}_i} \tilde{M}$ are not $\pi_1(M^3)$-equivariantly homeomorphic.

The next results indicates that hyperbolic phenomena appear above the dimension of top dimensional flats.

Theorem 3: Let X be a locally compact Hadamard space so that $\text{Isom}(X)$ acts cocompactly on X. Then the following are equivalent:

1. There is an n-flat in X.
2. There is an n-quasiflat in X.
3. There is an isometric embedding of a standard $(n-1)$-sphere in $\partial_{\text{Tits}}X$.
4. $H_k(\partial_{\text{Tits}}X) \neq 0$ for some $k \geq n-1$
5. There is a compact set $K \subseteq \partial_{\text{Tits}}X$ with topological dimension $\geq n-1$
6. Some asymptotic cone of X contains an n-flat.
7. For some asymptotic cone X_ω of X, we have a point $x \in X_\omega$ so that the local homology group $H_k(X_\omega, X_\omega \setminus x)$ is nontrivial for some $k \geq n$
8. There is an asymptotic cone of X which contains a compact set with topological dimension $\geq n$

G.Knieper:

Asymptotic geometry of manifolds with non-positive curvature

In this talk we gave the precise asymptotics of the volume growth of geodesic spheres on Hadamard spaces admitting a compact quotient. More precisely, we obtained the following result:

Theorem A: Let X be a non flat Hadamard manifold admitting a compact quotient. Then if $p \in X$ and $\text{vol}S_r(p)$ denotes the volume of the geodesic sphere of radius r about p

$$\text{vol}S_r(p) \approx r^{\frac{\text{rank}X-1}{2}} e^{hr}$$

where $h > 0$ and $\approx$ means that the ratio of both functions is uniformly bounded away from 0 and infinity.

Furthermore we considered the growth rate of closed geodesics. Let

$$P(t) = \#\{\text{primitive closed geodesics of period } \leq t, \text{ modulo free homotopy}\}$$

We obtain:

Theorem B: If M is a compact rank one manifold then

$$\frac{1}{at}e^{ht} \leq P_{\text{hyp}}(t) \leq P(t) \leq ae^{ht}$$

where $a > 1$ and $P_{\text{hyp}}(t)$ counts only rank one (hyperbolic) closed geodesics.

The rank one part of Theorem A we obtain studying Busemann densities (conformal densities) on the sphere at infinity. In particular we showed that they are uniquely determined and realized by a Hausdorff measure.

F.Labourie:

An example of lamination: the space of convex surfaces

The purpose of the talk was to describe a lamination of a compact space by surfaces, associated to every 3-dimensional compact manifold M with $K \leq -1$, having the following properties:

(i) The reunion of compact leaves is dense.
(ii) A generic leave is dense.
(iii) Stability in the following sense: if M_g is the space associated to M with metric g; if g' is close to g; then M_g and $M_{g'}$ are homeomorphic by an homeomorphism that preserves the leaves.

The space contains natural subfoliations: for instance, let M_1 be the bundle over UM, whose fiber at n consists of the circle of normal vectors to n, then $M_1 \subset M$ and furthermore

(iv) Let C_g be the reunion of compact leaves of genus g then $\overline{C_g} \supset M_1$.
(v) Let $A_{g_0} = \cup_{g \geq g_0} C_g$ then $A_{g_0} = M$.

Another interesting subfoliation is in the case of $M = \mathbb{H}^3/\Gamma$, M_2 the Grassmanian of two-planes foliated by totally geodesic planes then

(vi) $M_2 \subset M$.

To define the space, let's define a k-surface to be a surface in M satisfying

(a) $\det B = \frac{1}{2}$,
(b) $\text{tr}(B)(B\cdot,\cdot)$ is a complete metric where B is the shape operator. Let's now define

$$\mathcal{M}_0 = \{(x, \Sigma), \Sigma \text{ is a } k\text{-surface }, \ x \in \Sigma\}.$$

k-surfaces (as other Monge Ampere problems) have a nice compactness property. It turns out that with the topology of convergence on every compact set, $\mathcal{M}_0$ can be compactified by adding $\mathcal{M}_1$, i.e. k-surfaces degenerate into geodesics. Then

$$\mathcal{M} = \mathcal{M}_0 \cup \mathcal{M}_1$$

is the definition of our space. To prove the properties asserted, one has to solve the "Plateau problem" with different variants, according to k-surfaces.

Urs Lang (joint work with V.Schroeder):

Quasiflats in Hadamard spaces

Let X be a Hadamard space in the sense of A.D.Alexandrov. We consider k-flats F in X for which the projection $\pi : X \to F$ satisfies the following uniform

Contraction property: There exist $s > 0$, $\lambda > 1$, such that the following holds: If Q is a k-dimensional cube of edge length $3s$ in F, $Q_1, \ldots, Q_{2^k}$ are the cubes of edge length s located in the corners of Q, and $x_1, \ldots, x_{2^k}$ are points in X such that $d(x_i, F) = s$ and $\pi(x_i) \in Q_i$, then $d(x_i, x_j) \geq \lambda d(\pi(x_i), \pi(x_j))$ for some $1 \leq i < j \leq 2^k$.

For instance, flats of maximal dimension in a locally compact and cocompact Hadamard space X possess this property, with s, λ depending only on X. We prove:

Theorem: For all $l \geq 1$, $c \geq 0$, $k \in \mathbb{N}$, $s > 0$, $\lambda > 1$ there exists a (computable) constant D such that the following holds: If X is a Hadamard space containing a k-flat F with the (s, λ)-contraction property, and if $f : \mathbb{R}^k \to X$ is an (L, C)-quasiisometric map satisfying

$$\limsup_{r \to \infty} \frac{\sup\{d(f(x), F) : |x| \leq r\}}{r} < \frac{1}{L},$$

then the Hausdorff distance between $f(\mathbb{R}^k)$ and F is not larger than D.

Avery similar result has recently been proved by different methods by B.Kleiner.

F.Ledrappier (joint work with M.Babillot):

Geodesic paths and horocyclic flow

Consider a regular $\mathbb{Z}^a$ cover $\bar{S} \to S$, where S is a closed hyperbolic surface.

Theorem: The horocyclic flow is ergodic on $\bar{S}$ for the Liouville measure.

In fact, there is a whole family $\{\mu_n, n \in \mathbb{R}^a\}$ of invariant measures for the horocyclic flow, with μ_0 being the Liouville measure.

Theorem: For any $n \in \mathbb{R}^a$ μ_n is ergodic for the horocyclic flow.

The proof yields a precise estimate of the number of geodesic paths with length between $T - \delta$ and T and with prescribed Frobenius element.

Bernhard Leeb (joint work with M.Kapovich):

Large-scale geometry of 3-manifold groups

We study quasiisometry invariants for fundamental groups of 3-manifolds. Denote by $\mathcal{M}$ the class of all closed irreducible 3-manifolds with infinite fundamental group which are geometrisable but not geometric, i.e. have a nontrivial canonical decomposition in the sense of Jaco-Shalen and Johannson. There is a strong link between these manifolds and the geometry of nonpositive curvature (npc):

1) A manifold $M \in \mathcal{M}$ generically admits a npc metric. This is, for instance, the case if an atoroidal piece occurs in the JSJ decomposition.
2) For all $M \in \mathcal{M}$, $\pi_1 M$ is large-scale nonpositively curved in the sense that there is a npc $M_1 \in \mathcal{M}$ such that $\pi_1 M$ and $\pi_1 M_1$ are quasiisometric.

This has various direct implications for geometric properties of $\pi_1 M$, e.g.

a) geometric components and $\mathbb{Z}^2$-subgroups are quasiisometrically embedded.
b) quadratic isoperimetric inequality.

We use the presence of npc to find more quasiisometry invariants:

1) We classify all 2-dimensional quasi-flats in $\pi_1 M$. In the npc case the equivalence classes of Hausdorff close quasi-flats correspond to simple closed curves in the Tits boundary.
2) We show that the canonical decomposition is a quasiisometry invariant.
3) We conclude, using work of Tukia and R.Schwartz that finitely generated groups quasi-isometric to a $\pi_1 M$, $M \in \mathcal{M}$, are finite extensions of groups commensurable to $\pi_1 N$, $N \in \mathcal{M}$. It is an open question which groups are quasi-isometric to Sol.

Jochen Lohkamp:

Dispersive metrics

A metric is considered "dispersive" if the associated geodesic flow on the sphere bundle is (in some weak sense) hyperbolic. The classical example of such metrics is provided by metric whose geodesic flow is Anosov. In turn the Anosov property has the (central) implication of ergodicity of the flow. Even more, the flow has the Bernoulli property and positive entropy. The question we discussed in this talk was whether there is a difference of impact of Anosov versus Bernoulli property. The results are that the Anosov type manifolds have a topological rigidity property, while there is no implication from the Bernoulli property, which is a "local" property.

Grigorii Margulis (Yale University):

Decay of matrix coefficients and existence of compact quotients of homogeneous spaces

We give some new examples of homogeneous spaces which do not admit compact quotients by discrete subgroups. The idea is to use the asymptotic properties of matrix coefficients of unitary representations.

Definition: Let G be a locally compact group, H a closed subgroup of G, and K a compact subgroup of G. Let Θ denote (left invariant) Haar measure on H. We say that H is (G, K)-tempered if there exists a function $q \in L^1(H, \theta)$ such that

$$\langle \rho(h) w_1, w_2 \rangle \leq q(h) ||w_1|| \, ||w_2||$$

for any $h \in H$, any $\rho(K)$-invariant vectors w_1 and w_2 and any unitary representation ρ of G without non-trivial $\rho(G)$-invariant vectors.

Theorem: Let G be a unimodular locally compact group, H a closed subgroup of G, and Γ a discrete subgroup of G. Suppose that H is (G, K)-tempered and Γ is not a lattice. Then $\Gamma L \neq G/H$ for any compact subset L of G/H. In particular if H is not compact then, for any closed subgroup $F \subset H$, G/F has no compact quotients by discrete subgroups (we consider quotients with respect to left actions on G/F).

The proof of this theorem is rather simple. Using estimates for spherical functions we get the following corollaries.

Corollary 1: Let π_n denote the n-dimensional irreducible representation of $\mathrm{SL}(2, \mathbb{R})$. Suppose that $n \geq 4$. Then $\mathrm{SL}(n, \mathbb{R})/\pi_n(\mathrm{SL}(2, \mathbb{R}))$ has no compact quotient by discrete subgroups.

Corollary 2: Let H be a noncompact simple Lie group. Then there exist only a finite number of finite dimensional irreducible representations π of H such that $\mathrm{SL}(\dim \pi, \mathbb{R})/\pi(H)$ has compact quotients by discrete subgroups.

Shahar Mozes (joint work with Marc Burger and Bob Zimmer):

Lattices in the automorphism group of a product of trees

We study uniform lattices $\Gamma < \mathrm{Aut}(T_1) \times \mathrm{Aut}(T_2)$ where each T_i is a (bi)-regular tree whose vertex degrees are ≥ 3. Given such a lattice let $H_i = \overline{\mathrm{pro}_i(\Gamma)}$ be the closure of the projection into $\mathrm{Aut}(T_i)$.

Definition: A subgroup $H < \mathrm{Aut}(T)$ will be called primitive if H acts transitively on the edges of T and for every vertex v of T, $\mathrm{Stab}_H(v)$ acts primitively (in the sense of permutation groups) on the set of edges adjacent to v.

Theorem: Let $\Gamma < \mathrm{Aut}(T_1) \times \mathrm{Aut}(T_2)$ be a uniform lattice such that each $H_i = \overline{\mathrm{pro}_i(\Gamma)}$ is primitive. Then one of the following holds:

1. Γ is reducible.
2. For every homomorphism $\rho : \Gamma \to \mathrm{GL}_n(\mathbb{C})$ the image $\rho(\Gamma)$ is finite.
3. $\exists$ primes p_1, p_2 and semisimple p_i-adic groupes L_i, $i = 1, 2$, and surjective continuous homomorphisms $\phi_i : H_i \to L_i$, $i = 1, 2$, such that $\ker\phi_i$ is discrete torsion free, $(\phi_1 \times \phi_2)(\Gamma) < L_1 \times L_2$ is an arithmetic lattice. The kernels of ϕ_1 and ϕ_2 are trivial if and only if L_1 and L_2 are of rank one.

Theorem 2: Let $\Gamma < \mathrm{Aut}(T_1) \times \mathrm{Aut}(T_2)$ and $\Gamma' < \mathrm{Aut}(T_1') \times \mathrm{Aut}(T_2')$ be uniform lattices. Assume that $H_i = \overline{\mathrm{pro}_i(\Gamma)}$ are primitive, $i = 1, 2$, and that there is an isomorphism $\pi : \Gamma \to \Gamma'$. Then π is induced by an isometry $\tau : T_1 \times T_2 \to T_1' \times T_2'$.

Carlos Olmos (joint work with Bernardo Molina):

Manifolds all of whose flats are closed

Let M^n be a compact Riemannian manifold such that any geodesic is contained in a flat ($k \geq 2$). It is not in general true that the manifold, if locally irreducible, must be symmetric. There is a large *non* symmetric family with the above property, constructed by Spatzier-Strake (the homogeneous examples were already known to Ernst Heintze). Of course, if we want to obtain a locally symmetric space of compact type, the k-flats must be (immersed) compact. This condition turns out to be sufficient for the local symmetry as it is shown in the following

Theorem: Let M be a compact Riemannian manifold such that every geodesic with initial condition in an open dense subset of TM is contained in a compact flat of dimension at least 2. Assume furthermore that M is locally irreducible at any point of an open nonempty subset of M. Then M is a locally symmetric space of the compact type.

Viktor Schroeder (joint work with Urs Lang):

Kirszbraun's theorem for Alexandrov spaces

The classical Kirszbraun theorem states:

Theorem: Let $S \subset \mathbb{R}^n$ be arbitrary and $f : S \to \mathbb{R}^m$ be an L-Lipschitz map. Then there exists a Lipschitz extension $\bar{f} : \mathbb{R}^n \to \mathbb{R}^m$ with the same Lipschitz constant L.

We generalize this result in the context of Alexandrov spaces. Let M_k^2 be the modul space of constant curvature k and let $D_k := \mathrm{diam}(M_k^2)$. For triangles Δ in a geodesic space X with Perimeter(Δ) $< 2D_k$ there exists a unique comparison triangle Δ_k in M_k^2. Δ is called k-thick (k-thin) if the distance between to points on Δ is $\geq$ ($\leq$) the distance between the corresponding points in Δ_k.

Theorem: Let X (resp.Y) be geodesic length spaces such that all triangles Δ in X (resp.Y) with perimeter $< 2D_k$ are k-thick (k-thin). Assume in addition that Y is complete. Let $S \subset X$ be arbitrary and $f : S \to Y$ be a 1-Lipschitz map with $\mathrm{diam} f(S) \leq \frac{1}{2} D_k$. Then there exists a 1-Lipschitz extension $\bar{f} : X \to Y$.

We remark that the condition on $\mathrm{diam} f(S)$ is sharp. The main technical tool in the proof is the definition of a "scalar" product on the tangent cones of an Alexandrov space. This scalar product has some semiadditivity properties in the case that the space has an upper or lower bound.

V.Sharafutdinov:

Inverse problem of determining the source in the stationary transport equation on a Riemannian manifold

In physical terms the problem is posed as follows: One has to determine a source distribution of particles (or radiation) in a bounded domain M from the known flow emitting through the boundary. Particles are assumed to move along geodesics of a Riemannian metric. The domain contains a medium that can absorb and scatter the particles. We assume that only direction of a particle vary in scattering while velocity is preserved. In physical terms this means that collisions of the particles with medium atoms are elastic, the medium atoms are unmoving and much more heavy than the particles.
Mathematical posing of the problem: Let (M,g) be a compact Riemannian manifold with boundary,

$$TM = \{(x,\xi) | x \in M, \xi \in T_x M\}$$

be its tangent space, and

$$\Omega M = \{(x,\xi) \in TM \,|\, |\xi|^2 = \langle \xi, \xi \rangle = g_{ij}(x)\xi^i\xi^j = 1\}$$

be the unit sphere bundle. By $H : C^\infty(\Omega M) \to C^\infty(\Omega M)$ we denote the geodesic flow vector field. The (stationary, isotropic, unit velocity) transport equation is the following equation on the manifold ΩM:

$$(H + a(x))u(x,\xi) = \frac{1}{\omega} \int_{\Omega_x M} s(x, \langle \xi, \xi' \rangle) u(x,\xi') d\omega_x(\xi') + f(x) \qquad (1)$$

Here the coefficient $a \in C^\infty(M)$ is called absorption (or attenuation), the coefficient $s \in C^\infty(M \times [-1,1])$ is called the scattering diagram, and the term f is called the source.

Integrating in (1) is performed over the unit sphere $\Omega_x M = T_x M \cap \Omega M$ at the point x, and $d\omega_x$ is the natural measure on $\Omega_x M$. Equation (1) is supplemented by the homogeneous boundary condition (absence of incoming flow): $u|_{\partial_-\Omega M} = 0$ where

$$\partial_-\Omega M = \{(x,\xi) \in \Omega M | x \in \partial M, \ \langle \xi, \nu(x) \rangle \leq 0\}$$

is the manifold of inner boundary vectors ($\nu(x)$ being the outer normal to the boundary). We discuss a tzheorem solving the inverse problem of determining the source $f(x)$ from the known outgoing flow $u|_{\partial_+\Omega M} = u_{\text{out}}$.

References:
1. Siberian Math.J., 1995, V.36, No.3, 664-700.
2. Siberian Math.J., 1996, V.37, No.1, 211-235.

Jacek Świątkowski:

Homogeneous polygonal complexes of nonpositive curvature and their automorphism groups

Polygonal complex is a polyhedral cell 2-complex. We say that it is homogeneous, if
(1) $\exists k \geq 3$: each cell is a k-gon;
(2) $\exists$ univalent graph L: each link is isomorphic to L.
Simply connected polygonal complex satisfying (1) and (2) is called a (k, L)-complex.

The following inequality expresses combinatorically the nonpositive curvature condition:

$$g(L) \geq \frac{2k}{k-2}, \qquad (NPC)$$

where $g(L)$ is the number of edges in the shortest nontrivial circuit in L. It is known (see [BB]) that for (k, L) satisfying (NPC) (k, L)-complexes exist. The following result answers the question of uniqueness:

Theorem 1: Let (k, L) satisfy (NPC).
(a) If all embeddings of a star of edge in L into L extend to automorphisms of L and $k \geq 4$, then (k, L)-complex is unique.
(b) If $k = 3$ and all embeddings of star of star of vertex in L into L extend to automorphisms of L, then (k, L)-complex is unique.
(c) In both cases (a) and (b) the automorphism group of resulting complex is uncountable and acts transitively on flags of the complex.
(d) For all other cases, there are uncountably many non-isomorphic (k, L)-complexes.

Next theorems deal with existence and classification of symmetric polygonal complexes in some of the nonuniqueness cases.

Theorem 2: Assume L is regular (i.e. Aut$L$ acts transitively on oriented edges of L) and 3-valent, (k, L) satisfy (NPC).

(a) There exist exactly two (k, L)-complexes admitting flag-transitive groups of automorphisms, except in the cases (a) and (b) of Theorem 1.

(b) If the pointwise stabilizer of a star of vertex in graph L is nontrivial, then the groups of automorphisms of complexes from (a) are uncountable.

(c) If Aut$L$ contains a subgroup acting simply transitively on oriented edges of L then the group of automorphisms of a (k, L)-complex from (a) contains a subgroup acting simply transitively on flags.

Theorem 3: Let L be isomorphic to a Cayley graph $C(H, S)$ of a finite group H with presentation $H = \langle S \mid R \rangle$. Assume that $S \cap S^{-1} = \emptyset$, and for each $s \in S$ choose $k_s \geq 3$ satisfying (NPC) condition $g(L) \geq \frac{2k_s}{k_s - 2}$. Let Γ be given by the presentation

$$\Gamma = \langle S \cup \{\tau\} | R \cup \{\tau^2\} \cup \{(\tau s)^{k_s}\} : s \in S \rangle.$$

Then Γ acts simply transitively on oriented edges of a (NPC) polygonal complex X, all links of which are isomorphic to L.

Proves of theorem 1-3 go by performing the inductive construction of [BB] carefully.

[BB] W.Ballmann, M.Brin, "Polygonal Complexes and Combinatorial Group Theory", Geometriae Dedicata 50, 1994.

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