Tagungsbericht 22/1996

Algebraische K-Theorie

9.–15.06.1996

The conference was chaired by D. Grayson (Urbana) and U. Rehmann (Bielefeld). The program consisted of 27 talks. The most important result was a proof of Milnor's Conjecture given in two talks of Voevodsky.

Vortragsauszüge

A. J. Berrick:

Row versus column operations

This joint work[1] with K. L. Boey and K. H. Leung in Singapore addresses the question: which $n \times n$-matrices A (called bireducible) have the property that each sequence of row operations on A is equivalent to a sequence of column operations, and vice versa? Most complete answers are obtained when the entries of A lie in a ring R assumed to be a commutative domain, and either $n \geq 3$ or $n = 2$ and a further condition holds (e.g. $\mathrm{SL}_2 R$ is characteristic in $\mathrm{E}_2 R$), as assumed for the results below.

The key result is:

Theorem . *A is bireducible if and only if either $A = 0$ or A has a nonzero determinant that divides every product of an entry of A with a cofactor of A.*

Corollary 1. The property that any sequence of row operations is replaceable by a sequence of column operations is equivalent to the property that any sequence of column operations is replaceable by a sequence of row operations.

Corollary 2. A is bireducible if and only if after each localization at a maximal ideal it becomes the product of a scalar and an invertible matrix.

Corollary 3. When A is bireducible, its entries generate an ideal whose class in $\mathrm{Pic}(R)$ has order dividing n. If this class is trivial, then A is the product of a scalar and an invertible matrix.

Thus, while over $R = \mathbb{Z}[\sqrt{-5}][x]$, when n is odd A is bireducible if and only if it is the product of a scalar and an invertible matrix, for $n = 2$ we exhibit

[1] to appear in J. of Algebra

an invertible matrix which is not bireducible, and for even $n \geq 4$ we exhibit a bireducible matrix which is not the product of a scalar and an invertible matrix.

H. Esnault:

Survey on recent results on characteristic flat bundles and open questions

We review Reznikov's theorem showing that higher Deligne classes of flat bundles on projective smooth complex varieties vanish (Bloch's conjecture).

We list some open questions related to this result and show how this is related — via an algebraic theory (joint with S. Bloch) — to other conjectures (e.g. of Ogus).

E. M. Friedlander:

A double cube relating K-theory, cycles and homology

We consider the following commutative diagram of spectra associated to a quasi-projective variety $X/\mathbb{C}$

$$\begin{array}{ccccccc}
\underline{K}^{\mathrm{alg}}(X) & \longrightarrow & \underline{K}^{\mathrm{semi}}(X) & \longrightarrow & \underline{K}^{\mathrm{top}}(X) & & \\
\downarrow & \searrow & \underline{G}^{\mathrm{alg}}(X) & \longrightarrow & \underline{G}^{\mathrm{semi}}(X) & \longrightarrow & \underline{G}^{\mathrm{top}}(X) \\
\underline{Z}^{\mathrm{alg}}(X) & \longrightarrow & \underline{Z}^{\mathrm{semi}}(X) & \longrightarrow & \underline{Z}^{\mathrm{top}}(X) & & \downarrow \\
 & \searrow & \underline{J}^{\mathrm{alg}}(X) & \longrightarrow & \underline{J}^{\mathrm{semi}}(X) & \longrightarrow & \underline{J}^{\mathrm{top}}(X)
\end{array}$$

The 12 terms are defined in terms of cycles: $\underline{K}^{\mathrm{alg}}(X)$ arises as the algebraic singular complex of linear cycles equidimensional over X. $(\ )^{\mathrm{alg}} \to (\ )^{\mathrm{semi}} \to (\ )^{\mathrm{top}}$ involves relaxing the condition of algebraicity: $(-)^{\mathrm{semi}}$ denotes the topological singular complex of linear cycles (topology given in terms of Chow varieties), whereas $(-)^{\mathrm{top}}$ denotes topological singular complex of analytic linear cycles. The vertical (respectively, diagonal) maps of this diagram reflect the relaxation of the linearity (resp., equidimensionality) condition.

One might speculate that the horizontal maps are mod-n equivalences (above some degree upon X), the vertical maps are rational equivalences, and the diagonal maps $\searrow$ are weak equivalences for X smooth. For certain maps, these speculations are confirmed in terms of Lawson homology/cohomology, motivic homology/cohomology, singular homology/cohomology, and various K-theories.

T. Geisser:

Tate's conjecture, algebraic cycles and rational K-theory in characteristic p

In this talk we connect conjectures on algebraic cycles in finite characteristic and give a description on what to expect from rational K-theory in characteristic p, assuming Tate's conjecture:

Theorem . *Assume Tate's conjecture holds for the field $\mathbb{F}_q$. Then the following statements are equivalent:*

1. $CH^i(X)_{\mathbb{Q}} = A^i_{num}(X)$ *for all smooth projective $X/\mathbb{F}_q$ and all i, i.e. rational and numerical equivalence agree.*
2. $CH^i(X)_{\mathbb{Q}}$ *is finite dimensional for all smooth projective $X/\mathbb{F}_q$ and all i.*
3. $\mathbb{Q}[\pi_X] \subseteq CH^{\dim X}(X \times X)_{\mathbb{Q}}$, *the subalgebra generated by the Frobenius endomorphism, is finite dimensional for all smooth projective $X/\mathbb{F}_q$.*
4. *There is a separated filtration on the Chow groups such that the graded pieces factor through numerical equivalence.*

Furthermore, if these conditions are satisfied, we have

$$\mathrm{K}_a(X)_{\mathbb{Q}} = 0$$

for all smooth projective varieties over $\mathbb{F}_q$ and $a > 0$ (Parshins's conjecture).

The proof uses Jannsen's semi-simplicity result, the characterization of motives via their Frobenius endomorphism and some argument on eigenvalues of Frobenius, which was first used by Soulé.

The second main theorem shows that Parshin's conjecture gives strong bounds on K-theory of fields:

Theorem . *Let k be a field of characteristic p and assume Parshins's conjecture.*

i) If $\operatorname{trdeg} k/\mathbb{F}_p = r$, *then* $\mathrm{K}_a(k) = 0$ *for* $a > r$.

ii) Let $\mathrm{K}^M_a(k)$ *be Milnor's* K*-theory, then*

$$\mathrm{K}_a(k)_{\mathbb{Q}} = \mathrm{K}_a(k)^{(a)}_{\mathbb{Q}} = \mathrm{K}^M_a(k)_{\mathbb{Q}}.$$

The proof uses the Gersten-Quillen spectral sequence and de Jong's theorem on alterations.

The theorem has some corollaries, the first one states that

$$\mathrm{K}'_a(X)_{\mathbb{Q}} = \bigoplus_{j=a}^{\min(a+d,r+d)} \mathrm{K}'_a(X)^{(j)}_{\mathbb{Q}}$$

for any variety X of dimension d over a field k of transcendence degree r. It is proved using the Gersten-Quillen spectral sequence.

The theorem also implies that the Gersten-Quillen spectral sequence degenerates at E_2, $\mathrm{K}_a(X)^{(j)}_{\mathbb{Q}} = H^{j-a}(X, \mathcal{K}_j)_{\mathbb{Q}}$.

M. Hanamura:

Motivic sheaves and the intersection Chow groups

We study the analogue of the motivic t-structure in the framework of the theory of motivic sheaves over a variety. This leads, in particular, to the motivic analogues of intersection complexes in topology, hence the theory of intersection Chow groups.

L. Hesselholt:

On the K-theory of finite algebras over perfect fields

Classically, one has for every commutative ring A the associated ring of p-typical Witt vectors $W(A)$. We extend this construction to a functor which assigns to every associative (but not necessarily commutative or unital) ring A an abelian group $W(A)$. This extended functor comes equipped with additive Frobenius and Verschiebung operators. Let $K_*(A;\mathbb{Z}_p)$ denote the p-adic K-groups of A, that is, the homotopy groups of the p-completion of the spectrum $K(A)$. We prove that if A is a finite dimensional associative algebra over a perfect field k of positive characteristic p, then

$$K_i(A;\mathbb{Z}_p) = L_{i+1}W(A)_F, \quad i \geq 0,$$

the left derived functors in the sense of Quillen of the Frobenius coinvariants of $W(A)$. We note that the l-adic groups $K_*(A;\mathbb{Z}_l)$, where l is a prime different from p, are equal to products of l-adic K-groups of division algebras over the ground field k. For k finite, these are given by Quillen's original calculation.

The proof is by comparison with the topological cyclic homology $\mathrm{TC}_*(A)$ introduced by Bökstedt-Hsiang-Madsen. We show that the topological cyclic homology of a free associative $\mathbb{F}_p$-algebra without unit is concentrated in degree -1, and hence if $\mathbb{F}_p$-algebra, $\mathrm{TC}_*(A) = L_{*+1}W(A)_F$. We then prove that for any associative ring

$$\mathrm{TC}_{-1}(A) = W(A)_F.$$

Since it is known that $K_i(A;\mathbb{Z}_p) = \mathrm{TC}_i(A)$, for $i \geq 0$, when A is f.d. over a perfect field k of characteristic p, the stated result follows.

R. Jardine:

Localization theories for simplicial presheaves

There is a general theory of localization for simplicial presheaves and presheaves of spectra that specializes to Bousfield homology localization theories in the stable and unstable case, to the usual closed model structure for simplicial presheaves, and to a notion of localization along a geometric morphism. There is also a localization theory arising in this way for spaces, and corresponding to an arbitrary presheaf of spectra. This theory answers a question of Soulé concerning integral homology localization of simplicial presheaves.

Rob de Jeu:

Towards regulator formulas for curves over number fields

Let C be a proper, smooth, geometrically irreducible curve over a number field k, with function field $F = k(C)$. Assuming the Beilinson–Soulé conjecture on weights, we construct cohomological complexes for weights $n+1$ ($n \geq 1$) in degrees $1, \dots, n+1$, as in the conjectures of Goncharov, together with a map from H^2 to $K_{2n}^{(n+1)}(F)$. For $n = 2$ and 3, we also compute an approximation of the boundary map

$$K_{2n}^{(n+1)}(F) \to \coprod_{x \in C^{(1)}} K_{2n-1}^{(n)}(k(x))$$

on the image of H^2. (For $n = 1$ this is classical, given by the tame symbol.) Together with work of Goncharov, this yields a complete description of the image of the regulator map of $K_{2n}^{(n+1)}(C)$ for $n = 2$ or 3. This description is in fact valid without assuming the Beilinson–Soulé conjecture.

Bruno Kahn:

The Bass conjecture, the Milnor conjecture and the Beilinson-Soulé conjecture

The (motivic) Beilinson-Soulé conjecture predicts that motivic cohomology of regular schemes vanishes in non-positive degrees. The (motivic) Bass conjecture predicts that motivic cohomology groups of a regular scheme of finite type over Z are finitely generated. Using the recent results of Suslin and Voevodsky on the positive solution of the Beilinson-Lichtenbaum conjecture for 2-primary coefficients, we show that the second conjecture implies the first up to odd torsion. We get stronger consequences for fields of positive characteristic.

The proofs are conditional to resolution of singularities in nonzero characteristic and purity for motivic cohomology in the case of a general closed immersion of regular schemes. One can expect, however, that getting rid of these two conditions will be of a much lower degree of difficulty than proving either the Bass conjecture or the Beilinson-Soulé conjecture.

F. Keune:

Multirelative K-theory

Multirelative K-groups $K_n(R, \mathfrak{a}_1, \dots, \mathfrak{a}_m)$ are introduced using simplicial free resolutions of rings for m-tuples of ideals satisfying the condition

$$\Big(\bigcap_{i\in I} \mathfrak{a}_i\Big) + \sum_{j\in J} \mathfrak{a}_j = \bigcap_{i\in I} \Big(\mathfrak{a}_i + \sum_{j\in J} \mathfrak{a}_j\Big)$$

for all $I, J \subseteq \{1, \dots, m\}$. Long exact sequences are derived including K_0. Multirelative K-theory for such m-tuples can be defined as

$$K_0\Big(\bigcap_{i=1}^{m} \mathfrak{a}_i\Big).$$

Together with the property that K-groups vanish for nonunital free associative rings, this characterizes K-theory of rings. So the properties could have been taken as a set of axioms for the theory.

K. Knudson:

The homology of special linear groups over polynomial rings

We study the homology of $SL_n(F[t, t^{-1}])$ by examining the action of the group on a suitable simplicial complex. The E^1-term of the resulting spectral sequence is computed and the differential, d^1, is calculated in some special cases to yield information about the low-dimensional homology groups of $SL_n(F[t, t^{-1}])$. In particular, we show that if F is an infinite field, then $H_2(SL_n(F[t, t^{-1}]), \mathbb{Z}) = K_2(F[t, t^{-1}])$ for $n \geq 3$. We also prove an unstable analogue of homotopy invariance in algebraic K-theory; namely, if F is an infinite field, then the natural map $SL_n(F) \to SL_n(F[t])$ induces an isomorphism on integral homology for all $n \geq 2$.

B. Köck:

Adams operations on the K-theory of group rings

We present a new construction of Adams operations (on the Grothendieck group and on the higher K-theory of schemes) which even works for the K-theory of projective modules over group rings where, in general, no exterior power operations exist. This construction uses generalization of Atiyah's cyclic power operations and shuffle products in higher K-theory.

Furthermore, we rather explicitly describe Adams operations on $K_1(C\Gamma)$ (C an algebraically closed field of characteristic 0 and Γ a finite group). Using these results we K-theoretically explain the "Adams operations" on $K_0(\mathbb{Z}\Gamma)$ defined by Cassou-Noguès and Taylor using Frölich's Hom-description and Taylor's group logarithm techniques.

M. Levine:

Chow groups of varieties of low degree

(joint work with H. Esnault and E. Vieweg)

We consider the Chow groups (with rational coefficients) $\mathrm{CH}_s(X)_{\mathbb{Q}}$ of a subset X of $\mathbb{P}^n_k$ defined by equations $f_1, \ldots, f_r$ of degrees $d_1 \geq d_2 \geq \ldots \geq d_r \geq 2$, where k is an algebraically closed field. The main result is

Theorem . *Let $l \geq 0$ be an integer. Suppose that either*

a) $d_1 \geq 3$ *or* $r \geq l+1$, *and*

$$\sum_{i=1}^{r} \binom{d_i + 1}{l+1} \leq n$$

or

b) $d_1 = \cdots = d_r = 2$, $r \leq l$ *and* $r(l+1) \leq n - r + l - 1$.

Then X contains a linear subspace L of dimension l, and $\mathrm{CH}_s(X)_{\mathbb{Q}} = \mathbb{Q}$, with generator a dimension s linear subspace of L, for $0 \leq s \leq l$.

The proof uses classical algebraic geometry, Fulton's intersection theory and the results of Kollár, Mori and Miyaoka on the rational connectivity of Fano varieties. The result gives a partial confirmation of some consequences of conjectures of Bloch and Beilinson which relate the Chow groups to singular (or étale) cohomology.

J.-L. Loday:

Dialgebras

Periodicity in algebraic K-theory is a phenomenon which is not well understood yet. However it is well understood in its additive analogue: cyclic homology. In this setting the obstruction to periodicity is Hochschild homology. What is the analogue of Hochschild homology in algebraic K-theory? The aim of the talk is to construct new algebraic objects in order to solve this question. These objects are: Leibniz algebras and dialgebras.

A Leibniz algebra is a vector space $\mathcal{Y}$ with a bracket operation satisfying

$$[x,[y,z]] = [[x,y],z] - [[x,z],y]\,.$$

A dialgebra is a vector space D equipped with two operations $\dashv$ and $\vdash$ satisfying 5 axioms (looking like associativity conditions) such that

$$[x, y] := x \dashv y - y \vdash x$$

is a Leibniz bracket.

The homology of these algebras are discussed and shown to be strongly related to planar binary trees.

References:

J.-L. Loday, *Une version non commutative des algèbres de Lie: les algèbres de Leibniz*, Ens. Math. 39 (1993), 269–293.

J.-L. Loday, *Algèbres ayant deux opèrations associatives (dialgèbres)*, C.R.A.S. Paris 321 (1995), 141–146.

A. S. Merkurjev:

K-theory of homogeneous varieties

Let G be a reductive group acting on a variety X over a field F. We consider equivariant K-groups $K'_n(G;X)$ and the restriction homomorphism

$$K'_n(G;X) \to K'_n(X).$$

Theorem . a) *The restriction homomorphism $K'_0(G;X) \to K'_0(X)$ is surjective for any X iff $\operatorname{Pic} G_E = 0$ for any field extension E/F;*

b) *If X is smooth projective and $\operatorname{Pic} G_E = 0$ for any field extension E/F, then $K'_n(G;X) \to K'_n(X)$ is a split surjection.*

Theorem . *If G is a split group and $\pi_1(G)$ is torsion free, then there is a spectral sequence*

$$E_2^{p,q} = \operatorname{Tor}_p^{R(G)}\left(\mathbb{Z}, K'_q(G;X)\right) \Longrightarrow K'_{p+q}(X),$$

where $R(G)$ is the representation ring of G. In the case of smooth projective X this spectral sequence degenerates, i.e. $E_2^{p,q} = 0$ if $p > 0$, so that

$$K'_n(X) \simeq \mathbb{Z} \otimes_{R(G)} K'_n(G;X).$$

Corollaries . 1) Let G be an arbitrary reductive group. Then $K_0(G)^{(1)}$ is a finite group.

2) If $H \subset G$ is a connected subgroup, then $K_0(G/H)$ is finitely generated.

3) For any G-variety X, the group $G(F)$ acts trivially on $K'_0(X)$.

S. Mueller-Stach:

Higher Chow groups via transcendental methods

By a theorem of Spencer Bloch, Quillen's K-groups for quasiprojective varieties can be computed at least rationally by higher Chow groups. In this talk we show how representation theory on cohomology groups may be used to study the image of certain regulators from graded pieces of K-groups to cohomology. First we recall 5 different proof of the Noether-Lefschetz theorem on surfaces in projective 3-space. Then — quasi as a generalization of that theorem — we study the group $\mathrm{CH}^2(X,1) = H^1(X,\mathcal{K}_2)$ on a smooth complex variety X. It consists of divisors carrying rational functions such that their divisors sum up to zero on X. This group has a natural subgroup generated by the image of $\mathrm{Pic}(X)\otimes\mathbf{C}^*$. We call the quotient group the indecomposable part. Its image in Deligne cohomology modulo $\mathrm{NS}(X)\otimes\mathbf{C}^*$ is countable by Beilinson's rigidity argument. Many interesting algebraic surfaces have been studied which have non-trivial indecomposable part even modulo torsion: Nori and Collino (abelian varieties), Voisin/Oliva and myself (K3 surfaces) and Shioda sextics of general type (by myself). Examples over number fields were given much earlier by Beilinson, Ramakrishnan, Flach and Mildenhall. Over the complex numbers the groups of indecomposables is even not finitely generated modulo torsion (Collino).

At the end we explain some existing conjectures and further problems. Details can be found in my paper in the duke eprint server (Nov 1995). See also Pedrini's abstract for a sheaf theoretic approach to this problem.

A. Nenashev:

Algebraic description for K_1 of an exact category

Definition . A *double short exact sequence* in an exact category $\mathcal{A}$ is a pair of short exact sequences

$$0 \to A \xrightarrow{f_1} B \xrightarrow{g_1} C \to 0, \qquad 0 \to A \xrightarrow{f_2} B \xrightarrow{g_2} C \to 0$$

of the same objects. We write such data in the form

$$l = \left(A \underset{f_2}{\overset{f_1}{\rightrightarrows}} B \underset{g_2}{\overset{g_1}{\rightrightarrows}} C\right).$$

Given a double short exact sequence l, we assign to it a loop $\mu(l)$ in $G.\mathcal{A}$ (the G-construction of Gillet and Grayson):

$$l \mapsto \begin{array}{ccc} (A,A) & \xrightarrow{e(l)} & (B,B) \\ {\scriptstyle e(A)} \nwarrow & & \nearrow {\scriptstyle e(B)} \\ & (0,0) & \end{array}$$

and let $m(l)$ be the class of $\mu(l)$ in $\pi_1(G.\mathcal{A}) = K_1(\mathcal{A})$.

Theorem . (i) *$\forall x \in K_1(\mathcal{A})\ \exists l$ such that $x = m(l)$.*
(ii) *The elements $m(l)$ are subjects to the relations*
(a) $f_1 = f_2,\ g_1 = g_2 \quad \Rightarrow \quad m(l) = 0$;
(b) *if we are given a diagram of the form*

$$\begin{array}{ccccc} A' & \rightrightarrows & A & \rightrightarrows & A'' \\ \downdownarrows & & \downdownarrows & & \downdownarrows \\ B' & \rightrightarrows & B & \rightrightarrows & B'' \\ \downdownarrows & & \downdownarrows & & \downdownarrows \\ C' & \rightrightarrows & C & \rightrightarrows & C'' \end{array}$$

in which the 1-st arrows commute with the 1-st arrows and the 2-nd arrows commute with the 2-nd arrows, then

$$m(l_A) - m(l_B) + m(l_C) = m(l') - m(l) + m(l'') .$$

(iii) *Any other relation is a consequence of* (a) and (b).

Ivan Panin:

On the Gersten type conjecture for Azumaya algebras

A variant of Gersten's Conjecture was proved for the K-theory of Azumaya algebras over a smooth local ring of geometric type. A corollary of this is the positive solution of the conjecture of Grothedieck on principal homogeneous spaces for the special linear group $SL_{1,D}$.

Reference:

I. A. Panin, A. A. Suslin, *On a Grothendieck Conjecture for Azumaya Algebras.* Preprint POMI, 1996, submitted to St.-Petersburg Math. Jour.

C. Pedrini:

Bloch's conjecture and decomposability of higher Chow groups

Let X be a smooth projective variety over the complex field $\mathbb{C}$ and let

$$CH^p(X,n)$$

be the *higher Chow groups* as defined by S. Bloch.

Definition . We say that $CH^p(X,n)$ is *decomposable* if there exist positive integers r and s with $0<r<p$ and $0<s<p$ such that the map induced by the product structure for higher Chow groups :

$$(CH^r(X,l)\otimes CH^s(X,m))\otimes\mathbb{Q}\to CH^p(X,n)\otimes\mathbb{Q}$$

(where $l+m=n$) is surjective.

Because of the isomorphisms

$$CH^2(X,1)\simeq H^1(X,\mathcal{K}_2)\ \text{ and }\ CH^2(X,2)\simeq H^0(X,\mathcal{K}_2)$$

decomposability of $CH^2(X,1)$ and $CH^2(X,2)$ are equivalent respectively to the cokernels of the maps

$$\operatorname{Pic}X\otimes\mathbb{C}^*\to H^1(X,\mathcal{K}_2)\ \text{ and }\ K_2(\mathbb{C})\to H^0(X,\mathcal{K}_2)$$

being torsion.

Let $A_0(X)$ be the group of zero cycles of degree 0 and $T(X)$ the kernel of the *Albanese map* $A_0(X)\to \mathbf{Alb}X$; then Bloch's Conjecture asserts that, when X is a surface, then $p_g=\dim H^2(X,\mathcal{O}_X)=0$ implies $T(X)=0$.

This conjecture has been proven for all surfaces but those of general type (in which case also $q=\dim H^1(X,\mathcal{O}_X)=0$). Bloch and Srinivas showed that if $p_g=0$ and Bloch's conjecture holds then the group $CH^2(X,1)$ is decomposable and that the same result holds for $CH^2(X,2)$ if also $q=0$.

Several authors have given examples of surfaces with $p_g\neq 0$ such that $CH^2(X,1)$ is not decomposable.

We prove the following :

Theorem . *Let X be a smooth projective surface over $\mathbb{C}$; assume*

$$H^1(X,\mathcal{O}_X)=H^2(X,\mathcal{O}_X)=0.$$

Then the groups $CH^2(X,n)$ are decomposable for $n\leq 2$ iff Bloch's Conjecture holds for X.

The proof is based upon a description of the cokernels of the maps

$$\operatorname{Pic}X\otimes\mathbb{C}^*\to H^1(X,\mathcal{K}_2)\ \text{ and }\ K_2(\mathbb{C})\to H^0(X,\mathcal{K}_2)$$

in terms of the sheaf maps

$$\mathcal{K}_1\otimes\mathbb{C}^*\to\mathcal{K}_2$$

and of the Chern class map $\mathcal{K}_2\to\mathcal{H}^2_{\mathcal{D}}(\mathbb{Z}(2))$, where $\mathcal{H}^2_{\mathcal{D}}(\mathbb{Z}(2))$ is the Zariski sheaf associated to the Deligne-Beilinson cohomoly groups.

D. Quillen:

Morita invariance of K-theory for h-unital rings

We extend the category mod(R) of unitary modules over a unital ring R to nonunital rings A, which are idempotent: $A = A^2$, as follows. We call an A-module M firm when $A \otimes_A M \xrightarrow{\sim} M$, $a \otimes m \mapsto am$, and we say that A is a firm ring when $A \otimes_A A \xrightarrow{\sim} A$. Let $\mathfrak{M}(A)$ denotes the full subcategory of firm modules in all A-modules. Then $\mathfrak{M}(A)$ is abelian, because it is equivalent to the quotient abelian category of all A-modules by the Serre subcategory of modules killed by A.

Two idempotent rings A and B are said to be Morita equivalent when $\mathfrak{M}(A)$ and $\mathfrak{M}(B)$ are equivalent. A natural question is to what extent results about Morita invariance for unital rings extend with this more general setting.

Theorem . *K_1 is Morita invariant for firm rings. In other words, a Morita equivalence $\mathfrak{M}(A) \simeq \mathfrak{M}(B)$ with A, B firm gives rise to a canonical isomorphism $K_1(A) \simeq K_1(B)$.*

Theorem . *Cyclic type homology: HH_*, HC_*, etc. is Morita invariant for h-unital algebras flat over a commutative unital ground ring.*

Conjecture . K_* is Morita invariant for h-unital rings. (These are defined by $\mathrm{Tor}_n^{\tilde{A}}(\mathbb{Z}, A) = 0 \ \forall n$.)

W. Raskind:

On the Chow group of 0-cycles of the self-product of a CM elliptic curve over $\mathbb{Q}$ (joint with A. Langer)

Let E be an elliptic curve over $\mathbb{Q}$ with CM by the ring of integers of an imaginary quadratic field K. Let $X = E \times E$, and denote by $\mathrm{CH}_0(X)$ the group of 0-cycles modulo rational equivalence. Let N be the conductor of E, and assume for simplicity that $(N, 6) = 1$. We show that for p a prime where E has good reduction, the p-primary component of $\mathrm{CH}_0(X)$ is finite ($p \neq 2, 3$).

J. Rognes:

Topological cyclic homology of the integers and the stable pseudoisotopy space of a point

The topological cyclic homology $TC(A)$ of a ring A is assembled from the fixed points $THH(A)^C$ of its topological cyclic homology, for the various finite subgroups $C \subset S^1$, by means of structure maps $R, F : THH(A)^C \to THH(A)^B$ for $B \subset C \subset S^1$, called the restriction and Frobenius maps. We compute the mod 2 homotopy of the fixed points $THH(\mathbb{Z})^{C_{2^n}}$ for all n, where $\mathbb{Z}$ denotes the integers, together with the homomorphisms induced by the R- and F-maps. As a result we can compute the mod 2 homotopy of $TC(\mathbb{Z})$.

By theorems of McCarthy and Hesselholt–Madsen, this amounts to a calculation of the mod 2 algebraic K-groups of the 2-adic integers, $K_*(\mathbb{Z}_2; \mathbb{Z}/2)$. By a further topological analysis, we obtain the 2-adic homotopy type of $K(\mathbb{Z}_2)$, which may be described by the following two fiber sequences of implicitly 2-completed infinite loop spaces and maps:

$$BK(\mathbb{F}_3) \longrightarrow K^{red}(\mathbb{Z}_2) \longrightarrow BBU$$

$$K^{red}(\mathbb{Z}_2) \longrightarrow K(\mathbb{Z}_2) \xrightarrow{\text{red}} K(\mathbb{F}_3)$$

The connecting maps for these fiber sequences may also be described.

As a corollary, we see that the natural map $K(\mathbb{Z}) \to K(\mathbb{Z}_2)$ induces an isomorphism of 2-completed groups modulo torsion, in all degrees $4k+1$ for $k \geq 1$.

As a different application, we use a theorem of Dundas identifying the homotopy fiber of the linearization map $A(*) \to K(\mathbb{Z})$ with that of the linearization map $TC(*) \to TC(\mathbb{Z})$. Here $A(*)$ is Waldhausen's algebraic K-theory of the one-point space $*$, though of as the K-theory of the sphere spectrum. Likewise $TC(*)$ is the topological cyclic homology of a point, as defined and computed by Bökstedt, Hsiang and Madsen, thought of as the TC-theory of the sphere spectrum.

By the calculations above, $TC(*)$ and $TC(\mathbb{Z})$ are known 2-adically in a range, allowing us to express $A(*)$ in terms of $K(\mathbb{Z})$ in a similar range. Using Waldhausen's theorem $A(*) = Q(S^0) \times B^2\mathcal{P}(*)$ where $\mathcal{P}(*)$ is the stable pseudoisotopy space of a point, and the calculation of the 2-torsion in $K_*(\mathbb{Z})$ done during the conference, we find

$$\pi_*\mathcal{P}(*) = (0, \mathbb{Z}/2, 0, \mathbb{Z}, 0, \mathbb{Z}/2, 0, \dots)$$

modulo odd groups in degrees 3 and above. This has an interpretation in terms of the space of diffeomorphisms of high-dimensional discs.

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M. Rost:

On splitting varieties for 3-symbols mod 3

Consider a symbol $\{a_1, a_2, a_3\} \in K_3^M F/3$. Let $A = (a_1, a_2)$ be the degree 3 algebra given by a_1, a_2. Then a generic splitting variety for $\{a_1, a_2, a_3\}$ is given by

$$Z = \{\, x \in A \mid \mathrm{Nrd}(x) = a_3 \,\}.$$

Let $\bar{Z}$ be a smooth compactification of Z. Using the theory of exceptional Jordan algebras one can show that the direct image maps ($d = \dim Z = 8$)

$$H^d(\bar{Z}; \mathcal{K}_{d+i}) \to K_i F$$

are injective for $i = 0, 1$.

S. Saito:

Filtration on Chow groups
and generalized normal functions

Let $X \xrightarrow{f} S$ be a projective smooth morphism of non-singular algebraic varieties. We define a map

$$\rho^{r,\nu}_{X/S} \colon F^{\nu}_S \,\mathrm{CH}^r(X) \to \Gamma(S, DJ^{r,\nu}_{X/S}).$$

Here $\mathrm{CH}^r(X)$ is the Chow group of cycles of codimension r in X, $F^{\nu}_S\,\mathrm{CH}^r(X)$ ($\nu \geq 0$) is a filtration which is a relative version of Bloch-Beilinson filtration, $DJ^{r,\nu}_{X/S}$ is a sheaf on S_{an} and $\Gamma(S, DJ^{r,\nu}_{X/S})$ is the space of the sections which are called generalized normal functions. For $\nu = 1$, $F^1_S\,\mathrm{CH}^r(X)$ is the subgroup of cycles homologically equivalent to zero fiberwise and $\rho^{r,1}_{X/S}$ is the classical map associating normal functions due to Griffits. We investigate $\rho^{r,\nu}_{X/S}$ in the case that F is the universal family of complete intersections and show a modest generalization of Abel's theorem.

V. Voevodsky:

Milnor's conjecture

Abstract not provided by the speaker.

C. Weibel:

The 2-torsion in $K_*\mathbb{Z}$

For i even, define $\omega_i = \omega_i(\mathbb{Q})$ to be the largest power of 2 dividing $4i$; this equals the order of $H^0(\mathbb{Q}, \mu_{2^\nu}^{\otimes i})$ for $\nu \gg 0$, and also equals the even part of the denominator of $\zeta(1-2i)$. It also equals the order of the 2-Sylow subgroup of $Im(J)_{2i-1}$. Then:

$K_0(\mathbb{Z}) = \mathbb{Z}$	$K_{8n}(\mathbb{Z}) = (\text{odd}) \quad \text{for } n \geq 1$
$K_1(\mathbb{Z}) = \mathbb{Z}/2$	$K_{8n+1}(\mathbb{Z}) = \mathbb{Z} \oplus \mathbb{Z}/2 \oplus (\text{odd})$
$K_2(\mathbb{Z}) = \mathbb{Z}/2$	$K_{8n+2}(\mathbb{Z}) = \mathbb{Z}/2 \oplus (\text{odd})$
$K_3(\mathbb{Z}) = \mathbb{Z}/48$	$K_{8n+3}(\mathbb{Z}) = \mathbb{Z}/16 \oplus (\text{odd})$
$K_4(\mathbb{Z}) = 0$	$K_{8n+4}(\mathbb{Z}) = (\text{odd})$
$K_5(\mathbb{Z}) = \mathbb{Z} \oplus (\text{3-torsion group})$	$K_{8n+5}(\mathbb{Z}) = \mathbb{Z} \oplus (\text{odd})$
$K_6(\mathbb{Z}) = (\text{odd})$	$K_{8n+6}(\mathbb{Z}) = (\text{odd})$
$K_7(\mathbb{Z}) = \mathbb{Z}/240 \oplus (\text{odd})$	$K_{8n+7}(\mathbb{Z}) = (\mathbb{Z}/\omega_i) \oplus (\text{odd}), \quad i = 4(n+1).$

Here the symbol "(odd)" denotes a finite group of odd order. In particular, note that up to odd torsion we have $\frac{1}{2}$ times Lichtenbaum's conjectured formula:

$$\frac{|K_{4i-2}(\mathbb{Z})|}{|K_{4i-1}(\mathbb{Z})|} = \frac{1}{2}\zeta(1-2i).$$

The method is to use Voevodsky's version

$$E_2^{pq} = H_{et}^{p-q}(\mathbb{Q}; \mathbb{Z}/2) \Rightarrow K_{p-q}(\mathbb{Q}; \mathbb{Z}/2), \quad (p \leq 0,\ p \geq q)$$

of the Bloch-Lichtenbaum spectral sequence.

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