Tagungsbericht 23/1996

Conformal Field Theory

16.–22.06.1996

INTRODUCTION

The conference was organized by Yurii Ivanovič Manin, Don Zagier (both Max-Planck Institut für Mathematik, Bonn) and by Werner Nahm (Physikalisches Institut, Universität Bonn).

Whereas the study of topological quantum field theories has become a well-established domain of mathematical research, conformal field theory is still in its infancy. Even work on its axiomatic foundation is still in a somewhat experimental stage, though relations between modular operads, $\mathcal{D}$-modules and vertex operator algebras are partially understood. Nevertheless, conformal field theory has become an important tool for the construction of new invariants of varieties (quantum cohomology). In part, the latter can be understood as the differential geometry of moduli spaces of topological field theories. Simultaneously, conformal field theory yields automorphic forms on the moduli spaces of the classical structures.

From the physics point of view, both quantum fields on algebraic surfaces and on three-dimensional Calabi-Yau spaces seem to be important for the understanding of nature. Besides, four-dimensional Calabi-Yau spaces require investigation in string theory, since many string theories in lower dimensions arise from the reduction of a twelve-dimensional entitiy (F-Theory), though the latter does not seem to have an independent existence by itself. Up to now, these dimensional reductions are the best tool to understand duality relations between various string theories, which go beyond perturbation theory. One of these dualities describes the mirror symmetry of Calabi-Yau manifolds, for which a complete understanding seems to be in sight. The work on vertex operator, or W-algebras should become a tool for these studies, though at present it is limited to abstract nonsense and the study of elementary examples like the moonshine module.

Another direction of research concerns q-deformations on the mathematical and lattice models on the physical side. These quantum symmetries have an independent importance in the description of integrable models, but also relate to the fusion ring of conformal field theory. Fusion can be described by a tensor product on bimodules of operator algebras, but also relates to bundles over natural moduli spaces (Verlinde's formula). The latter introduces arithmetic aspects, which seem to require an explanation in the context of algebraic K-theory (polylogarithms).

COLLECTION OF ABSTRACTS:

MONDAY:

Jack Morava (Johns Hopkins University, Baltimore)

Integrality properties of the Kontsevich-Witten genus

Following work of Di Francesco, Itzykson, and Zuber, we construct a ring homomorphism $\mathrm{kw} : MU^*(\mathrm{pt}) \to \Delta^* \otimes \mathbb{Q}$ where $\Delta^* = \mathbb{Z}[q_i | i \geq 1]/(\sum_i (-1)^i q_{r-i} q_i, r > 0)$ is a Hopf algebra with coproduct $q_i \mapsto \sum_{j+k=i} q_j \otimes q_k$. The generating function $q(x) = \sum q_k x^k$ satisfies $q(x)q(-x) = 1$. So $\log(q(x))$ is an odd power series, and the genus is defined by the formal group law $\mathrm{kw}^{-1}(\mathrm{kw}(X) + \mathrm{kw}(Y)) = X +_{\mathrm{kw}} Y$ with $\mathrm{kw}(T) = T^{1/2} \log q(T^{1/2})$. This defines a candidate Δ for a quantum cohomology spectrum; we can begin to see a few things about the diagram

$$\begin{array}{ccccc} M_g & \xrightarrow{\text{monodromy}} & B\Gamma_g & \longrightarrow & BU_q \\ \downarrow & & & & \vdots \\ \overline{M}_g & \xrightarrow{\text{Tw}} & MU & \xrightarrow{\text{kw}} & \Delta \otimes \mathbb{Q}. \end{array}$$

Dimitri Lebedev (Moscow)

Central Extended Yangian Double as Dynamical Symmetry of Massive Integrable Models

An impressive success of the bootstrap approach for integrable massive models stimulated the discovery of the underlying mathematical structure. This structure is usually referred to as a dynamical symmetry of the theory. Bernard, LeClair and Smirnov strongly motivated that the Yangian double should be such a symmetry. The main obstacle to complete the understanding of a massive integrable theory from the symmetry point of view was the absence of nontrivial examples of infinite-dimensional representations of the Yangian double. This talk is based on papers [1-4] and is devoted to the investigation of the dynamical symmetry of the $SU(2)$-invariant Thirring model as a typical example. The main statement is the following: The complete dynamical symmetry of the $SU(2)$ Thirring model is the central extended Yangian Double at level $c = 1$. This is formulated in a series of theorems.

[1] S. Khoroshkin, Central Extension of the Yangian Double, Preprint q-alg/9602031

[2] S. Khoroshkin, D. Lebedev, S. Pakuliak, Intertwining Operators for the Central Extension of the Yangian Double, Preprint DF-TUZ/95/28, ITEP-TH-15/95, q-alg/9602030

[3] S. Khoroshkin, D. Lebedev, S. Pakuliak, Group-theoretical interpretation of bootstrap form factor integrals, in preparation

[4] S. Khoroshkin, D. Lebedev, S. Pakuliak, Traces of Intertwining Operators for the Yangian Double, Preprint q-alg/9605039

Alexander Voronov (University of Pennsylvania, Philadelphia)

Cohomology of CFT's

We describe the cohomology of algebras over certain operads of moduli spaces, including the cohomology of conformal field theories (CFT's) and vertex operator algebras (VOA's). There are three types of cohomology theories, Hochschild-like, Harrison-like (see Kimura, A.V., The cohomology of algebras over moduli spaces, in: Moduli Space of Curves, Progress in Math., vol. 126, Birkhäuser, 1995), and Chevalley-Eilenberg-like (A.V.). These cohomology theories produce a number of invariants of CFT's and VOA's, one of which is the space of their infinitesimal deformations. The problem of complete reducibility of certain CFT's and modules over VOA's can also be solved using these cohomology theories.

Martin Schlichenmaier (Universität Mannheim)

Sugawara construction for higher genus affine algebras (Krichever-Novikov algebras)

The talk reports on joint work with Oleg K. Sheinman (partly done in the framework of the "Research in Pairs" program).

After a review of the classical Sugawara construction, it is shown how to pass from highest weight representations of Krichever-Novikov vector field algebras of affine type (for many punctures) to representations of the centrally extended Krichever-Novikov vector field algebras via the generalized Sugawara construction (for arbitrary genus of the underlying Riemann surface). Relations between the weights of the corresponding representations are given and Casimir operators are constructed. There is a recent preprint (q-alg/9512016) on this subject which considers applications of the construction, too.

Bong H. Lian (Toronto, Ontario)

Large Radius Limit

A review of mirror symmetry is presented. There should exist a fibred space with Calabi-Yau fibres such that the Hodge structure near a singular point predicts the Gromov-Witten invariants of the Calabi-Yau variety. Physically, this neighbourhood corresponds to large radius. The cases that are best understood concern toric varieties P_Σ. The integral over a top form can be described by the Gauß-Manin connection. The closure of the Kähler cone $\overline{K(P_\Sigma)} \subset H^2(P_\Sigma, \mathbb{R})$ can be identified canonically as a cone in Σ_G, where Σ_G is the fan corresponding to the Gröbner compactification of $\mathrm{Hom}(H^2(P_\Sigma, \mathbb{Z}), \mathbb{C}^*)$. We obtain the following theorem (Hosono, Lian, Yau): Let $G \to G'$ be any equivariant resolution of G. Then G' contains at least one large radius limit.

Paul S. Green (College Park, Maryland)

A Universal Semi-Group in Conformal Field Theory

We combine the axiomatic approach of Segal with an observation inspired by the light cone gauge. Namely, given a Riemann surface with at least two boundary components, and an integral cocycle on the boundary, there is a unique section ξ of a flat unitary bundle over the surface such that $|\xi|$ is constant on each boundary component. If, in addition, a point is given on each boundary component, then each component is uniquely parametrized. This singles out a unique glueing scheme for Riemann surfaces with one incoming and one outgoing boundary component. The resulting semigroup operates on every conformal field theory, and generates a cyclic module over the vacuum for which the inner product contains information precisely equivalent to the modular functions of the theory.

TUESDAY:

Ezra Getzler (Max-Planck-Institut für Mathematik, Bonn)

Modular Operads

(joint work with M. Kapranov)

Let $\{a_{g,n} | 2(g-1)+n>0,\ g,n \geq 0\}$ be formal variables, and consider the integral

$$\log \int_{-\infty}^{\infty} \exp\left(\frac{x\xi}{\hbar} - \frac{x^2}{2\hbar} + \sum_{g,n} \hbar^{g-1}\frac{x^n}{n!}a_{g,n}\right)\frac{dx}{\sqrt{2\pi\hbar}}.$$

By Wick's theorem, this equals

$$\frac{\xi^2}{2\hbar} + \sum_{g,n} \sum_{G\in\Gamma_{g,n}} \hbar^{g-1}\frac{\xi^n}{n!}\frac{1}{|\mathrm{Aut}(G)|}\prod_{v\in\mathrm{Vert}(G)} a_{g(v),n(v)}$$

where $\Gamma_{g,n}$ is the set of (isomorphism classes of) stable graphs, which are connected graphs with n legs, and a genus $g(v) \geq 0$ at each vertex, such that $g = h^1(G) + \sum_v g(v)$ ($h^1(G)$ is the first Betti number of the graph) and $2(g(v)-1)+n(v)>0$ ($n(v)$ is the valence of v). A stable S-module is a collection $\{a((g,n)) | 2(g-1)+n>0\}$ of S_n-modules. Motivated by this formula, we define the functor on stable S-modules

$$\mathbb{M}a((g,n)) = \bigoplus_{G\in\Gamma_{g,n}}\left(\bigotimes_{v\in\mathrm{Vert(G)}} a((g(v),n(v)))\right)_{\mathrm{Aut}(G)}$$

This is a *free* modular operad. We explained what modular operads really are, gave some examples, and showed how to generalize Wick's theorem to calculate the Euler characteristic of $\mathbb{M}a$ (using symmetric functions).

Yukihiro Kanie (Tsu)

Birman-Wenzl-Murakami Algebras in Conformal Field Theory

Earlier work on conformal field theory on $\mathbf{P}^1$ with $A_1^{(1)}$ symmetries is extended to the cases $A_n^{(1)}$, $B_n^{(1)}$, $C_n^{(1)}$ and $D_n^{(1)}$. On the solution space of the KZ-equation which is satisfied by N-point functions of vertex operators and has a basis consisting of them, we can construct representations of B_N: the braid group of N-strings. In the case of $A_n^{(1)}$, these representations are factorized into the Iwahori-Hecke algebra $H_N(q)$, the resulting representations are all irreducible and mutually inequivalent, and give a full set of irreducible ones (q generic), or of unitarizable ones (q a root of 1) obtained by H. Wenzl.

In the case of $B_n^{(1)}$, $C_n^{(1)}$, $D_n^{(1)}$, the representations of B_N are factorized into the ones of $C_n(q, \mathcal{G})$, a specialization of the Birman-Wenzl-Murakami algebra $C_N(Q, A)$ with $A = A(\mathcal{G}, q)$. We construct representations of $C_N(q, \mathcal{G})$ according to Murakami's method, and we can show their irreducibility and mutual inequivalence in the case of $B_n^{(1)}$, $C_n^{(1)}$. In the case of $D_n^{(1)}$, where we take the Dynkin diagram automorphism into consideration, we get irreducible and mutually inequivalent ones. And in all cases, these give a full set of irreducible representations of the semi-simple quotient of $C_n(q, \mathcal{G})$ due to the Markov trace. In conformal field theory, we can also define the Markov trace by Verlinde-like calculus. As a result, we get representations that are irreducible and mutually inequivalent, both for generic q and q a root of 1. Of course, we must develop conformal field theory with $D_n^{(1)}$ and Dynkin diagram automorphism.

Terry Gannon (Max-Planck-Institut für Mathematik, Bonn)

Galois, Kac-Moody algebras, and the Classification of RCFT

I review the problem of classifying modular invariant partition functions, which are sesquilinear combinations of affine Kac-Moody characters at fixed level. I pay particular attention to a Galois symmetry inherent in the problem, which has played an important role in the modern attacks on the problem.

Boris A. Dubrovin (SISSA, Trieste / Steklov Institute, Moscow)

Painlevé transcendents and 2D topological field theory

Equations of associativity of Witten, Dijkgraaf, E. Verlinde, and H. Verlinde (WDVV) proved to be the defining relations of two-dimensional topological field theory, at least at tree-level approximation. Assuming semisimplicity of the primary chiral algebra at generic point of the space of parameters, we give an expression for the general solution of the WDVV equations via certain Painlevé-type transcendents (i.e., solutions of equations of isomonodromy deformations of certain linear differential operators with rational coefficients). We also discuss the problem of specifying solutions of WDVV with good analytic properties. We provide some "experimental" evidence that for these "good" solutions the monodromy group of the associated linear differential operation with rational coefficients is discrete. An important example is the quantum cohomology of the complex projective plane.

Aleksander Beilinson (Cambridge)

Geometry of Chiral Algebras

The talk, based on a joint work with V. Drinfeld, dealt with the following two subjects: (a) The $\mathcal{D}$-module approach to chiral algebras, and (b) the construction of a new family of chiral algebras — Hecke chiral algebras — associated to an arbitrary reductive group and a negative integral level. If our group is a torus, the Hecke chiral algebra coincides with the usual chiral Heisenberg algebra associated to the lattice.

Edward Frenkel (Harvard University, Cambridge)

Quantum Virasoro algebra

The quantum Virasoro algebra is a one-parameter family of deformations of the universal enveloping algebra of the Virasoro algebra. It is an associative algebra depending on two parameters, p and q, such that if we set $p = q^{1-\beta}$ with fixed β and take q to 1, we obtain the ordinary Virasoro algebra with central charge $c = 1 - 6(1-\beta)^2/\beta$. Another interesting limit is $p \to q$, in which the algebra becomes a commutative Poisson algebra, isomorphic to the center of $U_q(\widehat{sl_2})$ at the critical level. Both the Poisson structure and the product structure of the deformed Virasoro algebra are elliptic. They are closely connected with various models of statistical mechanics and quantum field theory associated to elliptic solutions of the Yang-Baxter and star-triangle equations.

WEDNESDAY:

András Szenes (MIT, Cambridge)

Intersection numbers on the moduli space of vector bundles

There are several ways to regularize the formulas of Witten for these numbers: Θ-function regularization, Bernoulli polynomials, residues. These approaches are shown to be equivalent.

Andreas Wißkirchen (Universität Bonn)

A class of (0,2) string vacua

This talk is based on work with Ralph Blumenhagen and Rolf Schimmrigk. After reviewing some facts about string theory, especially the heterotic case, Gepner's construction for exactly solvable (2,2) string vacua is shown in detail. An example of an identification with a point in the moduli space of a Calabi-Yau manifold is given by the $3^{\otimes 5}$ model. States and couplings are considered. This construction is extended to the (0,2) case where the gauge group is given by E_{9-r} $(r \geq 4)$. The main difficulty is to find a modular invariant partition function for a $(c, \bar{c}) = (6+r, 9)$ nondiagonal conformal field theory. An outlook to other subjects is given. Furthermore, it is shown that by an orbifold construction the symmetry algebra can be extended — as in the example of the space of $c = 1$ theories. In Gepner's construction such a phenomenon also occurs: $SO(10) \times U(1) \to E_6$.

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Gerald Höhn (Santa Cruz)

Analogies between codes, lattices and conformal field theories

Since many years the analogies between codes and lattices are studied. The construction of the moonshine module by Frenkel, Lepowsky and Meurmann established Vertex Operator Algebras (VOAs) as a next step in this analogy. In this talk I want to explain how many — if not all — of the well known combinatorial properties of self-dual codes and lattices can be extended to VOA's and better understood in this context. There is a fourth step before binary codes, namely codes over the Kleinian fourgroup $\mathbb{Z}_2 \times \mathbb{Z}_2$. Self-dual "Kleinian" codes are extending the definition of the so called Type IV codes over $\mathbb{F}_4$, giving a combinatorial, more natural theory.

As in the code and lattice case one has for VOA's general results for arbitrary ranks and special results for smaller ranks (≤ 26). This special results are related to the existence of the Hexacode, the Golay Code, the Leech lattice and the moonshine module. Other results described are: a "no input" definition of the moonshine module as a "lexicographic VOA", characterisation of VOAs with some extremal properties, the description of the shorter moonshine module $VB^{\#}$ and an example of a sub-VOA inside $V^{\#}$ belonging to the N=1 supersymmetric minimal series. The analogy between codes, lattices and VOAs should be understood as the orthogonal way to VOAs.

Wolfgang Eholzer (DAMTP, Cambridge)

Does Rationality imply Unitarity for $N = 2$ superconformal theories?

We show that all rational models of the $N = 2$ super Virasoro algebra are unitary. Our derivation relies on the coset realisation of the algebra in terms of $\widehat{su(2)}_k$ and two free fermions. Most of our arguments generalise to the Kazama-Suzuki models indicating that all rational $N = 2$ supersymmetric models might be unitary.

As an independent non-trivial check we calculate Zhu's algebra $A(\mathcal{H}_0)$ in some examples. We also analyse the modular properties of the vacuum characters. For more details, see [EG] W. Eholzer, M.R. Gaberdiel, *Unitarity of rational $N = 2$ superconformal theories*, preprint DAMTP-96-06, hep-th/9601163.

THURSDAY:

Akihiro Tsuchiya (Nagoya)

Spectral Decomposition of Path Spaces in Solvable Lattice Models

We give the spectral decomposition of the path space of the $U_q(\widehat{sl_2})$ vertex model with respect to the local energy function. By using this spectral decomposition, we get a new character formula of the integrable module $L(l, k)$ of the affine Lie algebra $\widehat{sl_2}$. The results suggest a hidden Yangian module structure on the $\widehat{sl_2}$ level l integrable module $L(l, k)$.

Alexei Morozov (Moscow)

Liouville models from the group theory point of view

Some results about Liouville and Toda models can be deduced directly from group theory. Namely, the Hamiltonians of these models are certain reductions of Laplace (quadratic Casimir) operators for the simple groups G. The wave functions are certain matrix elements of G and naturally possess amusing integral representations. Their asymptotics — the Harish-Chandra functions — are given by products of inverse Γ-functions over all the positive roots of G. In this approach the 1 + 1-dimensional Liouville model is associated to $G = \widehat{SL}(2)$, and the Harish-Chandra function exhibits peculiar "elliptic" properties, which were earlier discovered in the study of 3-point functions of Liouville Conformal Field Theory.

Samson Shatashvili (Yale University, New Haven)

Chiral Lagrangians, Anomalies, Supersymmetry and Holomorphy

We investigate higher-dimensional analogues of the bc systems of 2D RCFT. When coupled to gauge fields and Beltrami differentials defining integrable holomorphic structures the bc partition functions can be explicitly evaluated using anomaly and holomorphy. The resulting induced actions generalize the chiral algebras of 2D RCFT to $2n$ dimensions. Moreover, bc systems in four and six dimensions are closely related to supersymmetryc matter. In particular, we show that $d = 4$, $N = 2$ hypermultiplets induce a theory of self-dual Yang-Mills fields coupled to self-dual gravity. In this way the bc systems fermionize both the algebraic sector of $WZNW_4$ theory and the classical open $N_{WS} = 2$ string.

Rolf Schimmrigk (Universität Bonn)

F-Theory and String Theory Dualities in Four Dimensions

The nonperturbative structure of string theory allows the construction of compactifications on manifolds which are not of Calabi-Yau type. The resulting theories are most succinctly interpreted as ground states of a twelve-dimensional theory compactified on elliptically fibered manifolds — so-called F-theory. The focus so far has been on the understanding of F-theory on K3 surfaces and Calabi-Yau threefolds defining theories in $D = 8$ and $D = 6$ respectively. In this talk, we initiate a systematic investigation of F-theory in the physical dimension $D = 4$ by considering elliptically fibered Calabi-Yau fourfolds which are also fibered with generic fiber a Calabi-Yau threefold. For such manifolds we conjecture the duality relation $F_{12}(CY_4) \longleftrightarrow \mathrm{Het}(CY_3)$. By generalizing the twist map of hep-th/9512138 to Calabi-Yau hypersurfaces of arbitrary dimensions, predictions are obtained for the Hodge numbers of the fourfold via the above conjectured duality relation. These predictions are confirmed in a number of examples by performing the first computations of the complete Hodge diamond of elliptic fourfolds.

Richard E. Borcherds (Berkeley)

CFT and automorphic forms with singularities on Grassmannians

There seems to be a mysterious correspondence associating automorphic forms on Grassmannians to CFT's. We will give a few examples of this, and also describe a method due to Harvey and Moore of constructing these automorphic forms directly.

Albert Schwarz (Davis)

Grassmannians and CFT

We give a construction of a modular operad consisting of the Sato Grassmannian. For a large family of infinite-dimensional subgroups of gl(∞) we construct suboperads of this operad. We define formally "string amplitudes" related to the Grassmannian operad. For that purpose, we use a general definition of string amplitudes for equivariant modular operads.

FRIDAY:

Reinhold W. Gebert (Universität Hamburg)

The Sugawara operators at arbitrary level

The talk is based on joint work with K. Koepsell and H. Nicolai.

We present an explicit formula for the affine Sugawara operators for arbitrary level in terms of a Heisenberg algebra, which generalizes the well-known expression for level 1. This is achieved by employing a physical string vertex operator realization of the affine Lie algebra at arbitrary level. The underlying model describes a toroidally compactified subcritical ($2 < d < 26$) bosonic string with a nondegenerate even Lorentzian lattice as momentum lattice. An essential new feature of our construction is the appearance, beyond level 1, of new types of poles in the operator product expansion in addition to the ones at coincident points, which entail (controllable) non-localities in our formulas. We also present a new formula for the affine step operators in terms of the DDF oscillators and the Lorentz generators of the string model. The corresponding Lorentz boosts are nothing but affine Weyl translations.

Andreas Recknagel (ETH Zürich)

Generalization of the Knizhnik-Zamolodchikov equation to quasi-rational CFTs

(joint work in progress with A. Alekseev and V. Schomerus)

It has been shown by W. Nahm that there exist special finite-dimensional subspaces of the highest weight representations of quasi-rational CFTs which behave submultiplicatively under fusion. This fact can be used to set up first order differential equations on the correlators of fields corresponding to these subspaces. As an application, these generalized Knizhnik-Zamolodchikov equations provide a canonical construction of a quantum symmetry algebra $\mathcal{G}$ of a quasi-rational CFT; moreover, $\mathcal{G}$ is generated by (suitable restrictions of) the generators of the observable algebra itself.

Michael Rösgen (Universität Bonn)

Path Algebras and K-Theory in Conformal Field Theory

(joint work with J. Kellendonk, A. Recknagel, R. Varnhagen)

Path space representations of conformal field theories are derived from character identities of the generalized Rogers-Ramanujan type. A quasiparticle decomposition of the path spaces allows for the implementation of an explicit local $su(1,1)$ operation on paths, which should extend to lattice realizations of the Virasoro algebra. K-theory of the corresponding path algebras (AF-algebras) is used to relate the path description to the fusion rules. Ordinary as well as supersymmetric minimal models provide examples for this construction.

Antony J. Wassermann (PMMS, Cambridge)

Operator Algebras and Conformal Field Theory

We explain how to make level l positive energy representations of $LSU(N)$ into a braided category using Connes' notion of fusion for bimodules over a von Neumann algebra. This ties up the braiding operators of Doplicher-Haag-Roberts with the monodromy representations of the braid group on products of primary fields.

Tristan Hübsch (Howard University Physics Department, Washington DC 20059)

Mirror Symmetry as a Prediction of Conformal Field Theory

Historically, the still conjectural mirror map has been suggested as a consequence of certain relatively well defined transformations in a class of Conformal Field Theories. A prototype of this map is however present in all (2,2)-supersymmetric 2-dimensional field theories, and the conformal symmetry merely focuses attention to target spaces of trivial canonical class and clarifies (partly) the geometrical content. This map and a few related methods of constructions of mirror pairs of manifolds are described, together with a conjecture about a general construction of pairs of mirror models.

On the other hand, the existence of the mirror map (and a few other developments) would also characterize the precise category of mirror models and the relevant cohomology to be used; some resulting 'derivative' conjectures are also presented.

Mikhail Kapranov (Northwestern University, Evanston)

The elliptic curve in the S-duality theory and geometric Eisenstein series for Kac-Moody groups

Let S be an algebraic surface and G a semisimple algebraic group. The S-duality conjecture is a statement about the collection of the $Bun_S(G,n)$, the moduli spaces of (semistable) G-bundles on S with $c_2 = n$. Namely, the series

$$\sum_n \mu(Bun_S(G,n))\, q^n, \quad |q| < 1 \tag{1}$$

(where μ is an Euler characteristic type invariant) is expected to have modular properties with respect to identifications of the elliptic curves $\mathbb{C}^*/q^{\mathbb{Z}}$ for different q.
We consider a more general generating function

$$E(q, z_1, ..., z_r) = \sum_{n,d_1,...,d_r} \mu(Bun_{S,X}(G,n)_{d_1...d_r}) q^n z_1^{d_1} \cdots z_r^{d_r} \tag{2}$$

where $Bun_{S,X}(G,n)_{d_1...d_r}$ is the moduli space of G-bundles with $c_2 = n$, equipped with a parabolic structure along a curve $X \subset S$ so that the d_i are the degrees of the quotients on X. The series can be regarded as a kind of Eisenstein series but for the Kac-Moody group $\hat{G}$. The formalism of Eisenstein series for reductive groups can be pushed to the case of $\hat{G}$ and gives that $E(q, z_1, ..., z_r)$ satisfies a functional equation with respect to $\hat{W}$, the affine Weyl group of $\hat{G}$. This implies that for any q, $E(q, z_1, ..., z_r)$ is a rational section of a natural theta-bundle on the symmetric power of the curve $\mathbb{C}^*/q^{\mathbb{Z}}$. The natural "numerator" of E, denote it $N(q, z_1, ..., z_r)$, is a symmetric theta-function (without poles). This provides one possible explanation of why the formal variable q in (1) should be thought of as related to elliptic curves at all and suggests a generalization of the S-duality conjecture to the effect that $N(q, z_1, ..., z_r)$ should be a Jacobi form (with respect to a congruence subgroup in $SL_2(\mathbb{Z})$).

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